Intellectual Property Protection, non-homothetic Preferences, and Income Distribution

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Abstract

This paper analyzes how changing the (expected) duration of intellectual property (IP) protection affects distribution and growth in a product variety model with non-homothetic preferences. In equilibrium, poor households consume a smaller variety of goods than richer ones and a uniform increase in the length of IP protection increases the rate of growth and reduces current consumption of poor households more than that of richer ones. Consequently, poorer households prefer shorter IP protection. When there is no inequality among households who can afford to purchase IP protected goods, changing the length of IP protection does not affect growth. If IP protection is unexpectedly prolonged for future innovations but not for previously issued IPRs, the rate of growth can (permanently) fall if the wealth distribution is sufficiently unequal. Independently of the strength of IP protection, an increase in impatience can increases the rate of growth ("paradox of thrift") if wealth is unequally distributed.

1 Introduction

Intellectual property rights (IPRs) are commonly used in order to stimulate innovation and growth. There is a large literature analyzing which design and strength of IPRs serves this purpose and optimally trades off (possible) dynamic gains resulting from increased innovation incentives and static efficiency losses resulting from monopoly distortions. Recently, mounting evidence of increasing income and wealth inequality in many developed countries (see for example Piketty, 2014) has brought distributional concerns back to policy debates. For policymakers it is therefore not only important to know what aggregate effects IP policies might have, but also to be able to evaluate their distributional consequences.

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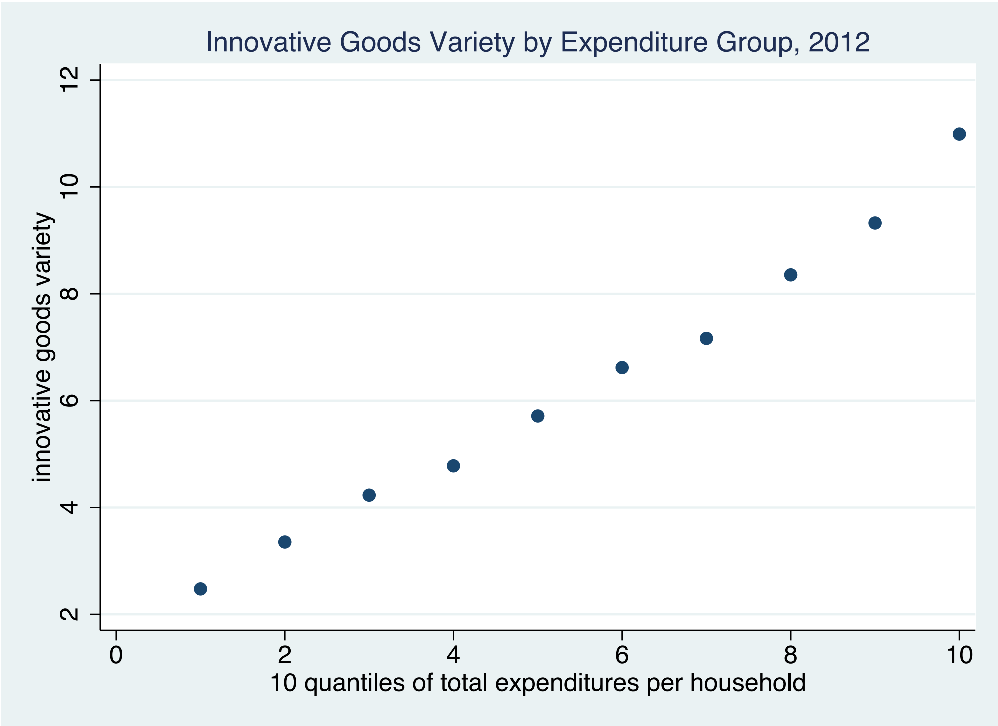
While strengthening IP protection can have direct redistributive effects by raising the relative wages of workers performing R&D intensive tasks, the focus of this paper lies on a different and less explored mechanism: when households with different incomes differ with respect to the consumption of IP protected goods, they might have conflicting views about the optimal strength of IP protection. The following graph shows how US households in different expenditure quantiles differed with respect to the variety of some selected "innovative" goods that they consumed in 2012[1]:



The graph shows that here is a strong positive correlation between the variety of "innovative" goods consumed by a household and the household expenditures, implying that a considerable number of such goods are (on average) only purchased by rich households but not by poorer ones. The differences in consumption pattern might be even larger when consumers in rich and poor countries are compared. Given that the supply and the prices of these "innovative" goods are affected by the strength of intellectual property protection, rich and poor households might therefore be affected in a different way by different (global) IP policies.

This paper analyzes the consequences that varying the (expected[2]) length of IP protection has on distribution and growth in a product variety model of endoge-

[1] Using the US consumer expenditure survey (CEX), the measure "innovative goods variety" is defined as follows: it counts the average number of "innovative" goods listed in the consumer expenditure survey of which households in a certain income quantile have bought at least one unit in 2012. 70 out of over 600 goods were classified as "innovative", among them goods like digital audio players, apps/games/ringtones for devices, photographic equipment, digital book readers, portable memory, computer software and new cars. I thank Liliya Khabibulina for providing this graph.

[2] IP protection is assumed to expire with a constant hazard rate

nous growth with non-homothetic preferences. The analysis is based on Föllmi and Zweimüller (2006) who analyze the effects of inequality on growth in the case of full (infinite) IP protection. Unlike in models with homothetic preferences that are commonly used in the growth literature, rich households consume a larger variety of goods than poorer households in this setting. It is assumed that goods are symmetric[3] and that households consume either zero or one unit of any invented good. Firms with IP protection engage in monopoly pricing while goods are sold at marginal cost once IP protection has expired. As households first consume the cheapest goods on which IP protection has expired before they spend the remaining income on IP protected goods, rich households purchase a larger variety of IP protected goods than poorer ones.

If the (expected) duration of IP protection is increased for both new and previously issued IPRs, this increases the incentives to innovate and growth. At the same time, it reduces the variety of goods consumed by poor households more than that consumed by richer households for the following reason: the increase in IP protection increases the share of monopolized industries so that relatively more goods are sold at a markup. This harms all households in a similar way as they all consume the same amount of goods on which IP protection has expired. However, markups decrease in the share of monopolized industries and rich households benefit more from this effect than poorer ones as they consume more IP protected goods. While all households (by construction) benefit in a similar way from an increase in the rate of growth which increases wages, poorer ones loose more in terms of current consumption than richer ones if the duration of IP protection increases. Because of that, poorer households prefer a shorter duration of IP protection.

There is no efficient level of protection on which all income groups agree if transfer payments are permitted. Households that are so poor that they do not consume any IP protected goods at all benefit from an increase in IP protection that increases wage growth. The analysis therefore shows that agreeing on a uniform (international) level of IP protection might be difficult and that poor households or countries might need to be compensated in order to support strong IP protection.

The paper furthermore shows that the effects that strengthening IP protection has on growth can depend on the distribution of income: If all consumers that are rich enough to purchase IP protected goods have the same level of income, increasing the (expected) duration of IP protection for both new and previously issued IPRs does not affect the incentives to innovate and growth. The reason for this is that the increased duration of monopoly power is exactly offset by a fall in equilibrium markups, leaving the value of an innovation unchanged. If IP protection is unexpectedly prolonged for future innovations but not for previously issued IPRs, the value of the latter falls due to falling markups. If wealth is unequally distributed, this leads to a reduction in inequality which again implies a further reduction of markups and, for a given strength of IP protection, discourages innovation in this way (this is the main result of Föllmi and Zweimüller (2006) who show that inequality increases growth in this setting). It is shown that in the case where the distribution of wealth is sufficiently unequal, the latter effect can be so strong that

[3] This is different to Föllmi and Zweimüller (2006) who analyze the case of more general hierarchical preferences.

an increase in IP protection can actually reduce the rate of growth.

A further result of the paper is that an increase in the discount rate can lead to a faster rate of growth, independently of the strength of IP protection. The reason for this is that an increase in the discount rate goes along with an increased propensity to consume out of interest income and therefore increases the inequality in consumption expenditures. As the latter implies larger markups and profits, the value of an innovation can actually increase even if a higher discount rate also leads to more discounting due to an increase in the interest rate. Such a "paradox of thrift" does not arise in standard growth models and has not been derived by Föllmi and Zweimüller (2006)[4].

The paper is structured as follows: In Section 2, the related literature is discussed. Section 3 describes the model setup and Section 4 analyzes the equilibrium. While the main part of the analysis focuses on the case of two income groups, Section 4.6 analyzes the case in which there are many income groups. Section 4.6.1 discusses how the model can be applied to an international context. Proofs and graphs are collected in the Appendix (Sections 6 and 7).

2 Related literature

The literature about the optimal design of intellectual property rights is extensive and I only mention a few papers here: Gilbert and Shapiro (1990) argue that it is optimal to have an infinite patent life but a narrow patent breadth (scope) to minimize the deadweight losses associated with a given strength of R&D incentives. However, if patent breadth determines the ease of entry into the protected market, Gallini (1992) argues that patents should be broad in scope and short. This result depends on the assumption that the innovator cannot license its technology to potential entrants. Maurer and Scotchmer (2002) allow for such licensing and again obtain the result that patents should be long and narrow.

Cysne and Turchick (2012) study the optimal IP expiration rate in a lab-equipment product variety model with homothetic preferences and show that less than full protection can be optimal in specific cases. In this setup, increasing the (expected) length of IP protection always increases growth. Föllmi and Zweimüller (2002b, shorter version 2008) introduce hierarchical preferences in an endogenous growth model with expanding product variety and find that a finite patent length is optimal if markups cannot be restricted. Horowitz and Lai (1996) study a quality-ladder model and find that a finite length of patent protection maximizes the (average) rate of innovation if R&D is carried out by monopoly innovators. Their result is driven by the Arrow replacement effect that implies that incremental profits fall in a firm's lead if innovation is cumulative. This effect, however, does not arise in product variety models and I am not aware of any other paper that, like the present one,

[4]Their analysis is less general than the one undertaken in this paper as they only consider the special case in which initial wealth is distributed in the same way as labor endowments and in which an exogenous change in the labor endowment distribution is always accompanied by a corresponding exogenous change in the distribution of wealth. Contrary to that the present paper solves for the transition dynamics in the wealth distribution if there is an exogenous change in the distribution of labor endowments.

finds that increasing protection against imitation can reduce growth in a product variety model.

The main difference between the present paper and those mentioned above is, however, that they do not look at distributional implications of the patent system. In standard growth models like the ones of Romer (1990) or Grossman and Helpman (1992), preferences (or the function that aggregates intermediate goods into a final good) are assumed to be of the homothetic CES type, implying that an increase in the number of goods increases all utilities proportionately and that rich and poor consumers face the same trade-off between growth and current consumption. Because of this, there is no disagreement about the optimal strength of patent protection (at least as long as wealth is distributed proportionally to incomes, eliminating valuation effects). While the assumption of homothetic preferences brings great advantages in terms of tractability, its implication that the shares of income spent on any given good are the same for rich and poor agents (so that the rich consume the same goods as the poor but proportionately more of each good) is at odd with empirical evidence that shows that the variety of goods purchased increases in household income (see Jackson, 1984 and Falkinger and Zweimüller, 1996).

Chu (2010) studies the effects of patent strength on inequality and growth within the context of a quality ladder model in which assets are unequally distributed. He finds that an increase in patent protection increases growth and the rate of return on assets, so that agents that initially hold more assets benefit more. An increase in patent protection therefore increases income inequality, while the effect on consumption inequality is ambiguous and depends on the elasticity of intertemporal substitution. In this setup, markups are constant and inequality does not affect the incentives to innovate due to the assumption of homothetic preferences. Because of that, it is, unlike in the present paper, not possible that an increase in patent protection can reduce growth by affecting the level of inequality.

Saint- Paul (2004) analyzes a model in which high skilled (rich) workers work in the R&D sector and low skilled (poor) in the production sector so that the high skilled benefit more from an increase in the protection of intellectual property rights then the low skilled do. He finds that under some conditions the low skilled can benefit from a reduction in intellectual property right protection while the high skilled loose but that it is always preferable to have a maximal protection of IP and to compensate the low skilled via transfer payments, even if those are distortionary (as they affect the intersectoral allocation of labor). This approach differs from the one used in this paper as the channel through which a conflict about the optimal degree of IP protection arises is that incomes of low and high skilled (that means of poor and rich) agents are affected differently by the degree of IP protection while in the present paper differences in labor productivities are exogenously given and disagreement about the optimal IP policy arises due to differences in the composition of demand and due to valuation effects.

A closely related paper is Saint- Paul (2002) in which preferences are also non-homothetic. It is assumed that the utility derived from the consumption of any single good is bounded from above, so that there are "limited needs". In this setup, richer consumers are closer to the satiation point than poor consumers as they consume more of each good and because of that they value an increase in product variety (through innovation) more than the poor do. That means that poorer agents compared to richer ones prefer that less resources are used for R&D and more for the production of already invented goods. Saint- Paul (2002) shows that a social planner that puts equal weight on each consumer prefers to allocate less resources to R&D than are actually allocated in equilibrium if patents are infinitely lived. However, the paper does not analyze whether lower patent protection would actually benefit poorer consumers by increasing their current consumption and by decreasing growth. While this seems to be the likely result, analyzing finite patent length and doing a welfare analysis might be analytically difficult in such a model. Contrary to that, in the present model the simplifying assumption is made that agents either consume one or zero units of each variety, which facilitates the welfare analysis considerably. Moreover, this specification makes it easier to study the interaction of intellectual property protection, inequality and exclusion, where the latter means that some ("exclusive") goods are sold at such high prices that only the rich buy them while others are sold at lower prices to both rich and poor consumers. Contrary to that, Saint- Paul (2002) restricts attention to a set of parameters for which monopolists always want to sell to the whole population so that there is no exclusion.

Another closely related paper is Hatipoglu (2012) who studies the effects of inequality on growth in a model with hierarchical preferences, income distribution and finite patent length. He assumes that markups are exogenously given so that there are only market size but no price effects (unlike in Föllmi and Zweimüller (2006) and the present article where markups are endogenous). In his model, innovations reduce the costs of producing differentiated goods, but in order to generate equal prices for patented and non patented goods he assumes that once a patent expires also the cost reduction disappears so that there is no benefit of reducing patent length whatsoever (as it does not lower costs for consumers and increases costs for producers and reduces innovation incentives). This makes his model unsuitable to study welfare implications of different patent policies.

A somewhat related literature analyzes the incentives of developed and less developed countries (or more generally of heterogeneous countries within a global economy) to enforce intellectual property rights (see for example Grossman and Lai (2004), Diwan and Rodrik (1991), Helpman (1993) and Saint- Paul (2007)). However, these models usually do not address the question whether there might be disagreement about the optimal uniform global strength of IP protection as for example specified in the TRIPS agreement (which can be addressed with the model here) and instead they analyze whether it is in the interest of a single country to deviate (that means to change its own IP policies unilaterally) given the policies of the other countries.

Furthermore, the paper relates to an extensive literature about the relationship between inequality and growth and specifically to a few papers in which inequality affects growth through the channel of demand: Building on Föllmi and Zweimüller (2006), Föllmi, Zweimüller and Würgler (2014) introduce the possibility to undertake cost-saving innovations (that go along with quality reductions) and Würgler (2010) the possibility to invest in the improvement of the quality of goods and both papers analyze how inequality affects both the incentives to introduce new varieties and the incentives to cut costs or to increase the quality of existing varieties. Matsuyama

(2002) studies a model with hierarchical preferences where productivity increases due to learning by doing.

3 The model setup

3.1 Preferences

There is a continuum of potentially producible differentiated goods indexed by $j \in [0, \infty)$. In a given period, only one or zero units of any of these goods can be consumed: $c(j,t) \in \{0,1\}$.

Households are infinitely lived and intertemporal utility is given by:

$$U(\tau) = \int_{t=\tau}^{\infty} ln \left(\int_{j=0}^{\infty} c(j,t)\, dj \right) e^{-\rho(t-\tau)} dt \tag{1}$$

where $\rho > 0$ denotes the rate of time preference. The strong assumption of indivisibilities in the consumption of goods ("$0-1$ consumption") is made in order to introduce non-homothetic preferences in a simple and tractable way[5].

3.2 Technology

Suppose that at point in time t, the measure $N(t)$ of goods has been invented. The factors of production are homogenous labor and the "stock of knowledge" $N(t)$. Producing one unit of an invented good requires $\frac{b}{N(t)} \geq 0$ units of labor input. Attaining an innovation in sector j is associated with setup costs (R&D costs) equal to $\frac{F}{N(t)}$ units of labor[6], so that there are increasing returns to scale. It is therefore assumed that labor productivity increases in the stock of knowledge $N(t)$ in all sectors. This assumption is made in order to allow for exponential growth[7].

3.3 Intellectual property protection and prices

The labor market is assumed to be competitive and there is free entry into R&D. An inventor who has invented good j obtains intellectual property (IP) protection on it which allows her to exclude others from producing this good. The intellectual property right, however, does not allow to appropriate any of the spillovers which

[5]Intertemporal utility is assumed to be logarithmic in order to simplify the welfare analysis. The results, however, do not change in a qualitative way if a more general CES intertemporal utility function is assumed.

[6]The possibility of duplicative R&D is ruled out

[7]In standard models of endogenous growth it is assumed that only R&D productivity increases in N while the productivity of the production sector stays constant. This is, however, not possible in the model here if $b > 0$ as the assumption of $0-1$ consumption rules out the possibility to produce and consume less of each differentiated good when the number of goods that are consumed increases. Therefore, the assumption that the productivity of the production sector increases in $N(t)$ is needed to obtain endogenous growth (only in the special case of a "digital economy" where $b = 0$, these spillovers are not required). If there are no spillovers in the R&D sector, there is linear growth but the qualitative results are the same.

accrue in other sectors due to the innovation (the market for knowledge is therefore incomplete[8]). IP protection is assumed to expire with hazard rate γ (that means with probability γdt in time interval dt), so that γ is an inverse measure of the strength of IP protection (the expected length of protection is equal to $\frac{1}{\gamma}$, so that IPRs are infinitely lived if $\gamma = 0$ and not protected at all if $\gamma \to \infty$).

After the IPR of a good j has expired, anyone can freely produce this good and it is supplied at marginal cost due to perfect competition [9]. The market clearing wage is denoted by $w(t)$. In order to obtain constant prices for the competitively supplied goods, the wage of a productivity-adjusted unit of labor is normalized to one, meaning that the wage for one unit of labor is normalized to $w(t) = N(t)$. Due to this normalization, the marginal production costs of a good and therefore the price of goods on which IP protection has expired is given by $p(j,t) = b$. Due to the chosen normalization and the assumption of competitive R&D, the innovation costs are constant over time and given by F. It is assumed that a firm that has IP protection in sector j cannot observe the income of a consumer and therefore cannot price discriminate between consumers with different willingness´to pay.

Agents are not financially constrained and can borrow and lend at the interest rate $r(t)$.

3.4 Distribution

The size of the population and the total labor endowment of the economy are normalized to 1. While all households have the same utility function, it is assumed that there are poor (P) and rich (R) households with population shares β and $1-\beta$ ($0 < \beta < 1$). The labor endowment of a poor household is given by $l_P = \vartheta$ ($0 < \vartheta \leq 1$) and that of a rich household therefore by $l_R = \frac{1-\beta\vartheta}{1-\beta} \geq 1$ (because $\beta l_P + (1-\beta) l_R = 1$ must hold). As labor is homogenous, labor endowments represent (per capita) labor incomes in each group as a share of average labor income in the whole economy.

Inequality is said to increase if β increases or if ϑ decreases[10]. Note that an increase in the population share β of the poor increases the labor endowment of a rich household if that of a poor (ϑ) is not changed.

[8]As R&D productivity increases in the stock of knowledge $N(t)$, future inventors benefit from the R&D undertaken by previous inventors. IP protection could therefore be broadened by granting inventors some blocking power over future inventions which would enable them to extract licensing fees from future inventors. But as each inventor uses knowledge of previous inventors as inputs for its own R&D activity and also contributes to the stock of knowledge on which future inventors build, each inventor would then have to pay licensing fees upon entry and only get delayed licensing income in the future. As total licensing fees can maximally grow at the rate of growth of the economy and as this rate must be lower than the interest rate in equilibrium, the effect of licensing on the present discounted profits of inventors is negative, so that broadening intellectual property protection in this way would reduce innovation incentives and growth. This is formally shown in Appendix A8.

[9]The analysis focuses on intellectual property rights as the only factor granting a monopoly position. If such a position can be obtained through other factors like trade secrecy, the same analysis applies to these factors as long as there are the same spillovers and as long as the monopoly position can be lost due to imitation with hazard rate γ.

[10]This also results if one compares Gini coefficients

At the initial date $t = \tau$ the economy is endowed with wealth in the form of previously granted non-expired IPRs that are worth the discounted sum of future profit incomes that accrue to their owners. The initial wealth of a rich household is denoted by $V_R(\tau)$ and that of a poor household by $V_P(\tau)$ and it is assumed that $V_R(t) \geq V_P(t) \geq 0$ holds[11].

3.5 Consumption choices

The intertemporal budget constraint of a household of type i ($i \in \{R, P\}$) is given by:

$$\int_{t=\tau}^{\infty} N(t) l_i e^{-R(t,\tau)} dt + V_i(\tau) \geq \int_{t=\tau}^{\infty} \left(\int_{j=0}^{\infty} p(j,t) c_i(j,t) \, dj \right) e^{-R(t,\tau)} dt \qquad (2)$$

where $R(t,\tau) = \int_{s=\tau}^{t} r(s) ds$ is the cumulative discount rate between dates t and τ. The left hand side represents the the discounted sum of wage income (note that $w(t) = N(t)$) plus the value of initial wealth and the right hand side the discounted sum of consumption expenditures.

A household maximizes intertemporal utility (1) subject to this budget constraint (2). Setting up the Lagrangian and deriving with respect to $c(j,t)$ gives the following first order conditions[12]:

$$c_i(j,t) = \begin{cases} 1 \;\; if & p(j,t) < \frac{e^{R(t,\tau) - \rho(t-\tau)}}{\mu_i C_i(t)} \equiv z_i(t) \\ 1 \; or \; 0 \; if & p(j,t) = z_i(t) \\ 0 \;\; if & p(j,t) > z_i(t) \end{cases} \qquad (3)$$

where $C_i(t) = \int_{j=0}^{\infty} c_i(j,t) \, dj$ indicates the measure of goods consumed by household i in period t and $z_i(t)$ denotes household i s willingness to pay for a good. μ_i is the Lagrange multiplier and represents the marginal utility of wealth at the initial date τ. As goods enter symmetrically into the utility function, households consume all goods the prices of which lie below their willingness to pay and a nonnegative measure of goods the prices of which are equal to their willingness to pay. Given that rich households spend more on consumption in a given period than poor households, they also consume a larger variety (measure) of goods. In equilibrium, the intertemporal budget constraints are satisfied with equality and the willingness to pay of the rich exceeds that of the poor, so that $z_R(t) > z_P(t)$ (and $\mu_R < \mu_P$).

[11] The analysis can be extended to cases where $V_R(t) < V_P(t)$, where $V_i(t) < 0$ for one of the groups (debt), or where $\vartheta > 1$. As long as the distribution of labor endowments and initial wealth is such that a rich household is overall richer than a poor household and spends more in every period, this does not change the (qualitative) results.

[12] The second order conditions are satisfied so that we get a maximum

3.6 Monopoly pricing

A firm that has IP protection on good j sets the price $p(j,t)$ in order to maximize profits. If $z_R(t) > z_P(t)$, market demand for any good j in period t is given by a step function: for a price higher than the willingness to pay of the rich ($p(j,t) > z_R(t)$), there is no demand for the good, for a price equal to or below the willingness to pay of the rich but above that of the poor ($p(j,t) \in (z_P(t), z_R(t)]$), demand is given by the population size of the rich, $1-\beta$, and for a price below or equal to the willingness to pay of the poor ($p(j,t) \leq z_P(t)$) demand is equal to one (the size of the whole population)[13].

Figure 1:

In order to maximize profits, an IPR holding firm then either sets $p(j,t) = z_R(t)$ and sells only to the rich (point A in Figure 1) or charges $p(j,t) = z_P(t) < z_R(t)$ and sells to both rich and poor households (point B). In the first case, profits are given by $\pi_R(t) = (1-\beta)(z_R(t) - b)$ and in the second case by $\pi_P(t) = z_P(t) - b$ (note that marginal production costs are equal to b). The firm therefore charges low prices and sells to the whole population if $\pi_P(t) > \pi_R(t)$ and charges high prices and sells only to the rich if $\pi_R(t) > \pi_P(t)$, and is indifferent between the two strategies if $\pi_P(t) = \pi_R(t)$.

3.7 Equilibrium price structure in period t

The subset of sectors in which IP protection has expired is denoted by $M(t) < N(t)$ and the definition $m(t) \equiv \frac{M(t)}{N(t)}$ is used. While prices are equal to b in these sectors, the prices in the sectors in which IPRs are protected (of which there is a measure $N(t) - M(t)$) depend on the distribution of income and other parameters of the model.

Given that rich and poor households spend the amounts $Y_R(t)$ and $Y_P(t)$ ($< Y_R(t)$) in period t, two different regimes can potentially arise in equilibrium: $C_P(t) < C_R(t) < N(t)$ and $C_P(t) < C_R(t) = N(t)$. The case where $C_P(t) = C_R(t) = N(t)$ cannot be an equilibrium for the following reason: If rich households purchased the same measure of goods as poor households (at the same prices), they would not exhaust their budgets, implying that their willingness to pay for an additional good, $z_R(t)$, would be infinitely large. Then, some firms would have an incentive to increase their price and to sell exclusively to rich households, implying that poor households would not purchase one of each of the invented goods anymore. The case where $C_P(t) < C_R(t) < N(t)$ can arise if even rich households do not spend enough to be able to purchase all invented goods, even if they are sold at marginal cost (e.g. if $Y_R(t) < bN(t)$). As goods are perfect substitutes, competition between firms implies that in this regime no good is sold at a price that exceeds the marginal cost of b in equilibrium. Therefore, profits are zero even for IP holding firms. Given that the economy stays in this regime, there are no incentives to undertake costly R&D and there is no growth. For this reason, the following analysis focuses on the

[13] here, the cases are considered in which consumers actually buy the good if their willingness to pay is equal to its price.

regime in which $C_P(t) < C_R(t) = N(t)$ and in which rich households consume one of each of the invented goods while poor households just the fraction $c_P(t) \equiv \frac{C_P(t)}{N(t)}$ of those goods. Two cases have to be distinguished:

In **Regime A**, $C_P(t) > M(t)$ (Condition A) so that the poor not only consume goods the IPRs of which have expired but also some more expensive IP protected goods[14]. Part of the IP protected goods are then exclusively sold to the rich at the price $p_R(t) = z_R(t)$ while others are sold to both rich and poor households at the price $p_P(t) = z_P(t)$. As IP holding firms supply symmetric goods, they must be indifferent between both strategies so that $\pi_R(t) = (1-\beta)(p_R(t) - b) = \pi_P(t) = (p_P(t) - b)$ has to hold. This implies that

$$p_P(t) = z_P(t) = \beta b + (1 - \beta)p_R(t) \tag{4}$$

Firms that sell to both groups therefore charge a price that is lower than the price $p_R(t) = z_R(t)$ charged by firms that only sell to rich households.

In **Regime B** the spending of the poor households is so low relative to $M(t)$ that they only consume goods the IPRs of which have expired (this happens if $Y_P(t) < bM(t)$, in which case Condition A is violated). In this case, all IP holding firms charge the same price $p_R(t) = z_R(t)$ (where $b < z_R(t)$) and exclusively sell to rich households.

In the following sections the equilibrium of the dynamic model is derived and the endogenous variables $N(t)$, $M(t)$, $Y_i(t)$, $C_i(t)$, $p_j(t)$, $\pi_j(t)$, $r(t)$ and $z_i(t)$ are derived as functions of the exogenous parameters.

4 The general equilibrium

This section studies the general equilibrium of the model. The analysis focuses on regime A where even the poor purchase some IP protected goods in equilibrium, so that $C_P(t) > M(t)$ (Condition A) holds. Regime B will be briefly analyzed in Subsection 4.4.

4.1 The allocation of resources across sectors

In regimes A and B the demand for production labor L_D in period t is given by $L_D(t) = \int_{j=0}^{\infty} \left(\frac{b}{N(t)}\right) [\beta c_P(j,t) + (1-\beta)c_R(j,t)]\, dj = b\beta c_P(t) + b(1-\beta)$ as $\frac{b}{N(t)}$ units of labor are needed in order to produce one unit of the good and as the population size of the poor (rich) is given by β $(1-\beta)$. The simplification arises as $C_R(t) = \int_{j=0}^{\infty} c_R(j,t)\, dj = N(t)$ and as the poor only consume a subset $c_P(t) \equiv \frac{C_P(t)}{N(t)}$ of the existing goods. The demand for R&D workers L_R depends on how much research

[14] Only the case is considered in which $Y_R(t) > bN(t)$, so that the equilibrium prices of IP protected goods lie above the marginal production costs b. The case in which $Y_R(t) = bN(t)$ is not interesting as all profits are then equal to zero.

is undertaken, that means on $\overset{\bullet}{N}(t) = \frac{\partial N(t)}{\partial t}$. As the invention of a new product requires $F/N(t)$ units of labor, the demand for R&D workers is given by: $L_R(t) = F\frac{\overset{\bullet}{N}(t)}{N(t)} = Fg(t)$ where $g(t)$ denotes the rate of growth of the stock of knowledge $N(t)$. Equating supply and demand of labor in a given period yields $1 = L_D(t) + L_R(t)$. Plugging the corresponding values into this equation and solving for $g(t)$ then gives the economy's **resource constraint**:

$$g(t) = \frac{1}{F}\left[1 - b\beta c_P(t) - b(1-\beta)\right] \tag{5}$$

Given that $b > 0$, there is a negative relation between the rate of growth $g(t)$ and the consumption share of the poor $c_p(t)$. The reason for this is that, as the rich always consume one of each of the invented goods in equilibrium, employing more workers in the R&D sector is only possible if less workers are used to produce goods for the poor.[15]

4.2 Innovation and consumption along a balanced growth path

The expected value of an innovation $Z(t)$ equals the expected discounted sum of profit income that accrues to an IPR holder and is given by $Z(t) = \int\limits_{s=t}^{\infty} \pi(s) e^{-\int\limits_{q=t}^{s}(r(q)+\gamma)dq} ds$

Along a balanced growth path (BGP), $N(t)$ and $C_i(t)$ grow at the constant rate $g(t)$, so that the consumption share $c_P(t)$ of the poor is constant over time. Moreover, per period profits $\pi(t) = (1-\beta)(p_R(t) - b) = (p_P(t) - b)$ and the willingness´to pay $z_i(t) = p_i(t)$ are constant. Setting the derivative of $z_i(t) = \frac{e^{R(t,\tau)-\rho(t-\tau)}}{c_i(t)N(t)\mu_i}$ with respect to time equal to zero (note that $\frac{\partial R(t,\tau)}{\partial t} = r(t)$) gives the Euler equation which relates the interest rate to the growth rate:

$$r(t) = \rho + g(t) \tag{6}$$

The rate of interest depends positively on the rate of growth and on the rate of time preference and is constant along the BGP. The expected value of an innovation is therefore given by

$$Z(t) = \frac{\pi}{r+\gamma} = \frac{(1-\beta)(p_R - b)}{\rho + g + \gamma}$$

along the BGP. As IP protection expires with hazard rate γ, this expiration rate acts as an additional discount rate beside the interest rate[16], so that Z decreases in γ.

[15]This negative relation between $g(t)$ and $c_p(t)$ only disappears in the case of a "digital economy" where marginal production costs are zero ($b = 0$).

[16]This can be seen by deriving $Z(t)$ from the arbitrage condition $rZ(t) = \pi - \gamma Z(t)$

Due to free entry into R&D, the value of an innovation Z has to be equal to the (wage) costs of innovating, which are given by F. Therefore, the following free entry condition needs to hold in an equilibrium with positive growth:

$$Z = \frac{(1-\beta)(p_R - b)}{\rho + g + \gamma} = F \tag{7}$$

Multiplying the measure $N(t) - M(t)$ of sectors in which IPRs are protected with the hazard rate γ with which IP protection expires, the absolute increase in the measure $M(t)$ of sectors in which IP protection has expired is given by $\dot{M}(t) = \gamma\,(N(t) - M(t))$. Using the definitions $m(t) = \frac{M(t)}{N(t)}$ and $g(t) = \frac{\dot{N}(t)}{N(t)}$ we can derive $\dot{m}(t) = \gamma\,(1 - m(t)) - m(t)g(t)$. Along the BGP, $\dot{m} = 0$ needs to hold, so that

$$m = \frac{\gamma}{g + \gamma} \tag{8}$$

Given that $c_p > m$ holds (Condition A; Regime A), the consumption expenditures of a poor household in period t are given by

$$\int_{j=0}^{\infty} p(j,t)c_P\,(j,t)\,dj = N(t)\,[m + p_P\,(c_p - m)]$$

along the BGP and those of a rich household by

$$\int_{j=0}^{\infty} p(j,t)c_R\,(j,t)\,dj = N(t)\,[m + p_P(c_P - m) + p_R(1 - c_P)]$$

. While both rich and poor households consume the fraction m of non-IP protected goods and the fraction $c_P - m$ of IP protected goods that are sold at the low price p_P, only the rich consume the fraction $1 - c_P$ of IP protected goods that are sold at the high price p_R. A graphical representation of the equilibrium price structure and consumption shares is given in **Figure 2**. The per period labor income of a poor agent is given by $w(t)l_P = N(t)\vartheta$ and that of a rich agent by $w(t)l_R = N(t)\frac{1-\beta\vartheta}{1-\beta}$. Using the relation $N(t) = N(\tau)e^{g(t-\tau)}$ and (4), the intertemporal budget constraint of a poor household in period $t = \tau$ that equates the discounted sum of consumption expenditures to the discounted sum of wage income plus the value of initial wealth is therefore given by:

$$\frac{N(\tau)\vartheta}{r-g} + V_P(\tau) = \frac{N(\tau)}{r-g}\,[mb + (b\beta + (1-\beta)\,p_R)\,(c_P - m)] \tag{9}$$

For a rich household, the intertemporal budget constraint is given by:

$$\frac{N(\tau)\,(1-\beta\vartheta)}{(r-g)\,(1-\beta)} + V_R(\tau) = \frac{N(\tau)}{r-g}\,[mb + (b\beta + (1-\beta)\,p_R)\,(c_P - m) + p_R\,(1 - c_p)] \tag{10}$$

Multiplying both sides of 9 and 10 by $r - g = \rho$ (see (6)), we obtain the consumption expenditures in period τ on the right hand sides. Expenditures of a poor household in any period $t = \tau$ are therefore equal to $Y_P(\tau) = N(\tau)\vartheta + \rho V_P(\tau)$ and those of a rich household equal to $Y_R(\tau) = N(\tau)\frac{1-\beta\vartheta}{1-\beta} + \rho V_R(\tau) > Y_P(\tau)$. Along the BGP, households therefore spend all their labor income ($w(t)l_i = N(t)l_i$) in each period. While the interest income of a household of type i in period t is given by $rV_i(t)$, only the fraction $(r-g)\,V_i(t) = \rho V_i(t)$ of this income is consumed in the same period and the amount $gV_i(t)$ is saved. This implies that individual wealth and also consumption out of interest income grow at the rate g along a BGP so that $V_i(t) = V_i(\tau)e^{g(t-\tau)}$. As there is a measure $N(t)\,(1-m)$ of sectors in which IP protection has not yet expired and as the value of an IP protected innovation is given by $Z = F$ (see (7)), the total value of initial wealth along the BGP is given by

$$V(t) = N(t)\,(1-m)\,Z = N(t)\,(1-m)\,F \tag{11}$$

When $V(t)$ changes due to a change in m, also $V_R(t)$ and $V_P(t)$ change. As will be shown in Proposition 2, these changes in $V_i(t)$ are in most cases of equal absolute size as both rich and poor households consume the same absolute measure $N(t)m$ of goods on which IP protection has expired and as they are therefore similarly affected by a change in m. Because of that, the following parametrization of the wealth distribution is used:

$$V_R(t) = V_P(t) + XN(t) \tag{12}$$

where $X \geq 0$ is an exogenous parameter[17]. Using (11) and the relation $V(t) = \beta V_P(t) + (1-\beta)\,V_R(t)$, we can derive the individual BGP wealth levels as

$$V_P(t) = (1-m)\,FN(t) - (1-\beta)\,XN(t) \tag{13}$$

and

$$V_R(t) = (1-m)\,FN(t) + \beta XN(t) \tag{14}$$

In order to ensure that $V_P(t) \geq 0$, $X \leq \frac{(1-m)F}{1-\beta}$ (Condition B) therefore needs to hold. Inserting (12) into (10), subtracting (9) from (10) and solving for p_R gives:

$$p_R = \frac{\frac{1-\vartheta}{1-\beta} + \rho X}{1 - c_P} \tag{15}$$

p_R therefore increases in β, X and ρ, decreases in ϑ and increases in c_P. The mechanism behind these results is the following: the entire expenditure difference $Y_R(t) - Y_P(t) = N(t)\,(l_R - l_P) + \rho\,(V_R - V_P) = N(t)\left(\frac{1-\vartheta}{1-\beta} + \rho X\right)$ is used to purchase the measure $N(t)\,(1-c_P)$ of IP protected goods at price p_R. For a given expenditure difference, p_R must therefore increase in c_P as rich households spend all their money on the existing goods. A reduction in ϑ and an increase in β increase the expenditure difference by increasing the difference in labor income $N(t)\,(l_R - l_P)$ and an increase

[17]Because $N(t)$ grows at rate g, both $V_P(t)$ and $V_R(t)$ grow at rate g along the BGP, so that the relative wealth distribution stays constant over time along the BGP and reflects the initial distribution of wealth: $\frac{V_R(t)}{V_P(t)} = \frac{V_R(\tau)}{V_P(\tau)}$. If m changes due to a change in one of the exogenous parameters, the relative wealth distribution, however, changes while the absolute difference in wealth levels is still given by $V_R(t) - V_P(t) = XN(t)$.

in X or ρ increases it by increasing the difference in consumption out of interest income. For c_P given, this increased expenditure difference can only be absorbed if p_R increases.

Inserting (15) into (7) and solving for g gives the following zero-profit condition:

$$g = \left(\frac{\frac{1-\vartheta}{1-\beta} + \rho X}{1 - c_P} - b \right) \frac{1-\beta}{F} - \rho - \gamma \tag{16}$$

The growth rate g depends positively on c_P, X and β and negatively on ϑ, γ and F. The reason for this is the following: an increase in c_P, X and β and a decrease in ϑ increase the value of an innovation by increasing p_R[18] and an increase in γ reduces the value of an innovation. An increase in F, on the other hand increases the costs of innovating. As the value of an innovation decreases if the interest rate r increases due to an increase in the rate of growth (remember that $r = \rho + g$ (6)), the free entry condition, which equates the value of an innovation to the costs of innovating implies a positive relation between g and the value of an innovation and a negative relation between g and F. The effect of ρ on g depends on the size of X: For small values of X, the zero profit condition implies a negative relation between g and ρ which is mainly driven by the fact that an increase in ρ increases r and reduces the value of an innovation through a discounting effect. For large values of X, this discounting effect can, however, be dominated by a positive price effect that results from an increase in the expenditure difference between rich and poor households: As an increase in ρ increases the consumption out of interest income $\rho V_i(t)$, it also increases the expenditure difference and therefore p_R and this effect is stronger the larger X is.

4.3 Properties of the equilibrium and transition dynamics

The zero profit condition (16) together with the resource constraint (5) determine the general equilibrium.

Proposition 1. *An equilibrium (BGP) in regime A exists if* $0 \leq X \leq \widetilde{X}$ *(B),* $b < \frac{1}{1-\beta}$ *(C),* $(1-\beta)\rho X - F(\rho+\gamma) < \vartheta$ *(D),* $\vartheta < (1-\beta)\rho X + \frac{F(\rho+\gamma)(1-b)}{b\beta} + \frac{1-b}{\beta} + b$ *(E), and* $\gamma < \tilde{\gamma}$ *(F), with* $\tilde{X}$ *and* $\tilde{\gamma}$ *defining positive and finite threshold values.*

On the balanced growth path, g *depends negatively on* γ*,* ϑ *and* F *and positively on* β *and* X*, and* c_P *depends positively on* γ *and* ϑ *and negatively on* X*. There is a positive threshold* $\hat{X} < \tilde{X}$ *such that for* $0 \leq X < \hat{X}$*,* g *depends negatively (and* c_P *positively) on* ρ*. For* $\hat{X} < X \leq \tilde{X}$*, an increase in the discount rate* ρ *is associated with an increase in the rate of growth* g *(paradox of thrift) and with a decrease in* c_P*.*

Proof. See Appendix A1 □

An increase in the strength of intellectual property protection (a reduction in γ) makes research more profitable (for a given interest rate $r = \rho + g$) and increases

[18] In the case where β increases, this effect is weakened by a reduction in the relevant market size, $1 - \beta$. However, the value of an innovation still increases as the first effect is stronger than the second one.

the equilibrium rate of growth. At the same time, it reduces the consumption share c_P of poor households, while rich households always consume one of each goods ($c_R = 1$). A reduction in γ reduces the fraction m of sectors in which IP protection has expired and reduces the prices p_P and p_R at which IP protected goods are sold in equilibrium. While both rich and poor households are equally affected by the declines of m and p_P, only the rich households benefit from the reduction in p_R which allows them to purchase the same measure of goods as before. All the additional workers that move into the R&D sector are therefore withdrawn from sectors that initially produce goods for all households and that switch to exclusively serving rich households once IP protection is increased.

An increase in inequality that results from a reduction in ϑ or from an increase in β increases the rate of growth. This result is a generalization of the result established by Föllmi and Zweimüller (2006). They consider the special case in which initial wealth is distributed in the same way as labor endowments ($\frac{V_R}{V_P} = \frac{l_R}{l_p} = \frac{1-\beta\vartheta}{(1-\beta)\vartheta}$) and in which an exogenous change in β or ϑ is accompanied by a corresponding exogenous change in $\frac{V_R}{V_P}$. Moreover, they assume that $\gamma = 0$ (infinite IP protection), but allow for a hierarchy in preferences. If the wealth distribution becomes more unequal due to an increase in X, this increases the rate of growth as it increases the expenditure difference between rich and poor households and allows innovators to raise prices. The paradox of thrift can arise for the following reason: an increase in the discount rate ρ increases the consumption out of interest income and, for $X > 0$, increases the expenditure difference between rich and poor households, which allows innovators to raise prices. If wealth inequality (i.e. X) is large, this effect is stronger than the standard discounting effect due to which a rise in ρ (which increases the interest rate r) discourages innovation, so that an increase in impatience can increase the rate of growth.

Proposition 2. *Assuming that parameters are within the range defined by Proposition 1 so that a BGP in regime A exists. Suppose that the economy is on a BGP and that at date $t = t_0$ there is the following unexpected change in the strength of IP protection: instead of expiring with hazard rate γ like before t_0, IPRs granted after t_0 expire with hazard rate γ_1 while IPRs granted before t_0 that have not expired until t_0 expire with hazard rate γ_0 from t_0 onward. Then, g and c_P (and also r, p_R and p_P) immediately jump to their new BGP values in t_0 while m and V adjust sluggishly. If $X(t) = \frac{V_R(t)-V_P(t)}{N(t)}$ has the (constant) BGP value X_o before t_0, it changes to $X_1 = \frac{\rho+g+\gamma_1}{\rho+g+\gamma_0}X_o$ (with g indicating the new BGP value) in t_0 and stays constant at that level after t_0. If there is an unexpected change in one of the other exogenous parameters (including X_0), g and c_P (and also r, p_R and p_P) immediately jump to their new BGP values and X stays constant after t_0 (and only changes in t_0 if there is an exogenous change in X_0).*

Proof. See Appendix A2. □

If $\gamma_0 = \gamma_1$ so that the strength of IP protection is uniformly changed in t_0 for newly granted and for existing IPRs, the value of initial wealth does not change in t_0. The reason for this is that the fraction $1 - m$ of IP protected goods does not jump in t_0 and that due to the free entry condition the change in γ is exactly offset by a change in p_R and p_P, implying that the value of an IPR stays constant at $Z = \frac{(1-\beta)(p_R-b)}{\rho+g+\gamma} = F$ if $\gamma_0 = \gamma_1$. Therefore, the distribution of initial wealth and X do not change in t_0 in this case. As g and c_p immediately jump to their new BGP values, the same comparative static results as in proposition 1 (including the paradox of thrift) therefore hold if $\gamma_0 = \gamma_1$ (or if parameters other than γ are changed), even if transition dynamics are taken into consideration.

If IP protection is only changed for newly granted IPRs, meaning that $\gamma_1 \neq \gamma_0 = \gamma$, the value of initial wealth and X are multiplied by the factor $\left(\frac{\rho+g+\gamma_1}{\rho+g+\gamma}\right)$ in t_0 (see the proof of Proposition 2). If $\gamma_1 > \gamma$, so that IP protection is decreased in t_0, the value of initial wealth and X therefore increase. The reason for this is that an increase in γ_1 implies an increase in prices (p_R and p_P) as otherwise new innovators could not break even. Holders of previously issued IPRs also benefit from this price increase without being affected by the reduction in the strength of IP protection, so that the value of their IPRs increases. While g and c_p still immediately jump to their new BGP values in t_0, this "valuation effect" implies that this jump might now differ from the comparative statics result derived in Proposition 1 in which X was considered to be exogenous. While the "valuation effect" is negligible for small values of X_0 and therefore does not change the qualitative results of Proposition 1 in this case, things can change if X_0 is sufficiently large.

Corollary 1. *Suppose that $X_0 = \frac{F}{1-\beta}$ (rich households own all initial wealth before t_0), $\vartheta = 1$, $b = \frac{1}{2}$ and $F = \frac{\beta}{2\rho}$. If, starting from a BGP with full IP protection (i.e. $\gamma = 0$; this BGP exists under the above conditions), protection on new IPRs is slightly and unexpectedly reduced in t_0 (so that $\gamma_1 > 0$) while IPRs granted before t_0 that have not expired until t_0 are still fully enforced (i.e. $\gamma_0 = \gamma = 0$), g increases in $t = t_0$ to its new BGP value (and c_P falls).*

Proof. See Appendix A3. □

The explanation for this result is the following: when IP protection is reduced for new innovations but not for old ones the value of initial wealth increases due to the "valuation effect" just explained. As initial wealth is very unequally distributed and as X is large in the case considered in Corollary 1, the absolute increase in X at $t = t_0$ is also large and implies a considerable increase in the expenditure difference between rich and poor households. This increase in inequality rises prices and incentives to innovate more than the reduction in IP protection reduces these incentives so that the rate of growth increases in this particular example[19]. Proposition 2 also allows to study the effects of a policy that unexpectedly "expropriates" owners of IPRs granted before t_0 by setting $\gamma_0 > \gamma = \gamma_1$ in t_0. As the strength of IP protection and therefore prices are unchanged for new innovations, the value of IPRs granted before t_0 decreases due to a faster rate of expiration γ_0. This leads to a one-time reduction of the value of initial wealth and of X in t_0. According to Propositions

[19]If the policy change was already announced to the households in $t_{-1} < t_0$, the valuation effect would already occur in t_{-1} (but would be weaker the earlier t_{-1} was relative to t_0). In this case, a reduction of IP protection in t_0 might therefore lead to an increase in the rate of growth in t_{-1}, but not in t_0.

1 and 2, this reduction of wealth inequality caused by the expropriation leads to a reduction in the rate of growth in t_0.[20]

4.4 Regime B

In this regime, poor households are so poor that they only consume goods the IPRs of which have expired. This means that $C_P(t) < M(t)$ ($c_P(t) \equiv \frac{C_P(t)}{N(t)} < m(t) \equiv \frac{M(t)}{N(t)}$) holds and that Condition A is violated.

Proposition 3. *A BGP in Regime B exists if* $\gamma > \frac{\left(\vartheta+\rho\bar{V}_P\right)\left(1-b(1-\beta)-\beta\left(\vartheta+\rho\bar{V}_P\right)\right)}{F\left(b-\vartheta-\rho\bar{V}_P\right)} > 0$ *(Condition A is violated),* $\vartheta + \rho\bar{V}_P < b$ *and if* $1 - b(1-\beta) - \beta\left(\vartheta + \rho\bar{V}_P\right) > 0$, *where* $\bar{V}_P \equiv \frac{V_P(t)}{N(t)}$ *is constant and exogenous. Along the BGP,* $c_P = \frac{\vartheta+\rho\bar{V}_P}{b}$ *and* $g = \frac{1}{F}\left[1 - b(1-\beta) - \beta\left(\vartheta + \rho\bar{V}_P\right)\right]$. *Therefore, g and c_P are independent of the strength of IP protection γ and higher inequality (lower values of ϑ or $\bar{V}_P$) is associated with a larger rate of growth.*

Suppose that the economy is on a BGP and that at date $t = t_0$ there is an unexpected change in the strength of IP protection like described in Proposition 2. Then, g and c_P (and also r and p_R) immediately jump to their new BGP values while m, V (total wealth) and V_R adjust sluggishly. If $\bar{V}_P(t) = \frac{V_P(t)}{N(t)}$ has the (constant) BGP value $\bar{V}_P^o$ before t_0, it changes to $\overline{V}_P^1 = \frac{\rho+g+\gamma_1}{\rho+g+\gamma_0}\bar{V}_P^o$ (with g indicating the new BGP growth rate) in t_0 and stays constant at that level after t_0. g therefore increases in t_0 if $\gamma_1 < \gamma_0$ holds. If there is an unexpected change in one of the other exogenous parameters (including $\bar{V}_P$) in t_0, g and c_P (and also r and p_R) immediately jump to their new BGP values and $\bar{V}_P$ stays constant after t_0 (and only changes in t_0 if there is an exogenous change in $\bar{V}_P^o$).

Proof. See Appendix A4. □

This regime is therefore more likely to arise the weaker IP protection is (the larger γ is) and the poorer the poor households are (the lower ϑ and $\bar{V}_P$ are). The

[20]The intuition for why in all cases X stays constant along the transition (i.e. for $t > t_0$) is the following: Given a change in an exogenous parameter (for example a fall in γ if $\gamma = \gamma_1 = \gamma_0$) leads to a fall in m during the transition, households anticipate that they need to increase their (normalized) expenditures $\frac{Y_i}{N}$ in the future in order to be able to afford the same (normalized) consumption share $c_i = \frac{C_i}{N}$. The reason for this is that m is the fraction of non-IP protected goods that are sold at the lowest price b and that a fall in m implies that more goods are sold at price $p_P > b$ while still the same measure $1 - c_P$ of goods is supplied at price p_R (note that due to Lemma 2 c_P and prices p_R and p_P are constant along the transition). Households who want to attain constant consumption shares c_i therefore accumulate wealth in the transition phase and the total level of wealth, $V = N(1-m)F$, rises. As both rich and poor households consume the same measure $M = mN$ of cheap non-IP protected goods they need to increase expenditures Y_i by the same absolute amount, which implies that they also need to increase their level of wealth V_i (and the resulting interest income from which they finance the higher future expenditures) by the same absolute amount.

In the case where m rises (for example due to an increase in γ if $\gamma = \gamma_1 = \gamma_0$), both rich and poor households reduce their (normalized) levels of wealth $\frac{V_i}{N}$ by the same absolute amount as they anticipate the same absolute fall in future expenditures. Therefore, $\frac{V_R - V_P}{N} = X$ holds during the whole transition period.

rate of growth in regime B therefore only depends on the strength of IP protection if a change in the latter affects the level of wealth of poor households and therefore the distribution of income and expenditures. If $\gamma_1 < \gamma_0$, either because in t_0 IP protection is increased for new innovations ($\gamma_1 < \gamma = \gamma_0$) or because holders of IPRs issued before t_0 are expropriated after t_0 ($\gamma_1 = \gamma < \gamma_0$), the value of initial wealth held by poor households declines due to the valuation effect described above and their consumption decreases[21]. As rich households always purchase one unit of each good, the total demand for production labor therefore decreases, leaving more labor for R&D (see equation 5) so that the rate of growth increases. If $\gamma_1 = \gamma_0$ there is no valuation effect and a change in IP protection does neither affect the level of initial wealth of poor households and their consumption, nor growth. This can also be understood by looking at the demand side: a reduction in IP protection (i.e. an increase in $\gamma_1 = \gamma_0$) on the one hand reduces the value of an innovation by reducing the expected time span during which an innovator has monopoly power, but on the other hand leads to an increase in the price p_R and in per period monopoly profits. In the case where $\gamma_1 = \gamma_0$ the two effects exactly offset each other, leaving the incentives to innovate and the rate of growth unchanged. While a change in IP protection changes the fraction $1 - m$ of sectors in which IPRs have not expired and the price p_R at which IP protected goods are sold, it does not change the value of initial wealth held by rich households in t_0 if $\gamma_1 = \gamma_0$. As only rich households consume IP protected goods in Regime 2b, such a change in IP protection therefore merely shifts demand across sectors and time but does not affect "total demand" and the profitability of R&D. Because of that, firms find it optimal to hire the same amount of R&D workers as before, leaving the rate of growth unchanged.

If a marginal change in an exogenous parameter leads to a switch between regimes A and B, this does not lead to a discontinuous jump in c_P or g.[22] Therefore, Propositions 2 and 3 cover the whole parameter range in the unconstrained regime and there are no discontinuous effects arising at the threshold where the switch between regimes A and B occurs.

4.5 Welfare analysis

Taking into account that $\int_{j=0}^{\infty} c_R(j,t)\,dj = C_R(t) = N(t)$ and that $\int_{j=0}^{\infty} c_P(j,t)\,dj = C_P(t) = c_P N(t)$, intertemporal utilities (equation 1) along a balanced growth path along which the consumption share c_p ($\equiv \frac{C_P(t)}{N(t)}$) of poor households is constant and

[21]Poor households spend $Y_P(t) = \vartheta N(t) + \rho V_P(t)$ per period and consume the measure $C_P(t) = \frac{Y_P(t)}{b}$ of non-IP protected goods that are sold at price b.

[22]The reason for this is the following: as firms are indifferent about selling an IP protected good at the high price p_R to only rich consumers or at the lower price p_P to both rich and poor consumers, the value of an innovation and of the level of initial wealth do not change in a discontinuous way if poor consumers can either afford to purchase a small amount of IP protected goods (Regime A) or no IP protected goods at all (Regime B). Because of that, there is no discontinuity in the incentives to undertake R&D and in the equilibrium rate of growth g.

along which $N(t)$ grows at the constant rate g can be derived as:

$$U_R(\tau) = \int_{t=\tau}^{\infty} ln\left(N(t)\right) e^{-\rho(t-\tau)} dt = \frac{ln\left(N(\tau)\right)}{\rho} + \frac{g}{\rho^2} \tag{17}$$

$$U_P(\tau) = \int_{t=\tau}^{\infty} ln\left(c_P N(t)\right) e^{-\rho(t-\tau)} dt = \frac{ln\left(N(\tau)\right)}{\rho} + \frac{ln\left(c_P\right)}{\rho} + \frac{g}{\rho^2} \tag{18}$$

As there are no transition dynamics for c_P and g (see Propositions 2 and 3), it suffices to compare intertemporal utilities along balanced growth paths in order to evaluate welfare effects of different policies.

Proposition 4. *In regimes A and B, rich households benefit from any change in IP (or redistributive) policy that increases the rate of growth. Poor households benefit from changes in IP (or redistributive) policies that increase the rate of growth if* $c_P > \frac{F\rho}{b\beta}$ *and are harmed by them if* $c_P < \frac{F\rho}{b\beta}$. *If* $\frac{F\rho}{b\beta} > 1$, $c_P < \frac{F\rho}{b\beta}$ *always holds. If* $\frac{F\rho}{b\beta} < 1$, $c_P < \frac{F\rho}{b\beta}$ *holds under the conditions leading to Regime A if* $\vartheta < \frac{F\rho}{b\beta}\left(1 + F\gamma\right) + \rho X\left(1 - \beta\right) - F\gamma$ *and under the conditions leading to Regime B if* $\vartheta < \frac{F\rho}{\beta} - \rho\overline{V}_P$.

Suppose that the strength of IP protection can only be changed in a uniform way, affecting new and old innovations symmetrically ($\gamma = \gamma_0 = \gamma_1$)*. Then, there is a non-empty range of parameters for which poor households prefer the rate of IP expiration to be set at a positive level equal to* $\gamma_P^* = \frac{\frac{F\rho}{b\beta} - \vartheta + \rho X(1-\beta)}{F\left(1 - \frac{F\rho}{b\beta}\right)} > 0$ *and for which an equilibrium in Regime A results.* γ_P^* *increases in* X *and* ρ *and decreases in* ϑ *and in* β.

Proof. See Appendix A5. □

As rich households always consume one of each of the goods produced with the modern technology ($C_R(t) = N(t)$), they benefit from any policy that increases the rate of growth ($g = \frac{\dot{N}(t)}{N(t)}$) as it increases their future consumption without affecting their current consumption. For poor households, there is, however, a trade-off as a reduction in γ (assuming $\gamma = \gamma_1 = \gamma_0$) or ϑ or an increase in X on the one hand increases g (dynamic effect) but on the other hand reduces their consumption share c_P (static effect). While poor households might benefit from growth-enhancing policies if the equilibrium level of c_P is relatively large[23], they are harmed by them if c_P is relatively low, which can for example be the case if ϑ is sufficiently low or if X is sufficiently high (or $\bar{V}_P$ sufficiently low, when in regime B).

Given that $\gamma = \gamma_1 = \gamma_0$ (so that an increase in γ reduces g and does not affect X) and that an equilibrium in regime A exists in which the strength of IP protection preferred by poor households is interior, the intuition for the comparative statics results is the following: the level of c_P that optimally trades off static and dynamic welfare effects and maximizes the intertemporal utility of poor households is given by

[23] Föllmi and Zweimüller (2006) have already shown that poor households can benefit from a regressive transfer which reduces ϑ (they assume that there is at the same time a proportional increase in wealth inequality).

$c_P = c_P^* = \frac{F\rho}{b\beta}$ and therefore independent of γ, ϑ and X. Given that the strength of IP protection is set in order to attain this outcome, γ_P^* therefore has to increase in X and to decrease in ϑ as c_P increases in γ and ϑ and decreases in X (see Propositions 1 and 2). When ρ increases (agents get more impatient) current consumption becomes more important relative to growth (c_P^* increases) and this effect is so strong that, independently of how ρ affects c_P for a given γ, poor households want to reduce the strength of IP protection (i.e. increase γ) in order to reduce growth. When β increases, c_P^* decreases and poor households prefer a larger rate of growth[24]. While an increase in β already increases g, the first effect is so strong that poor households want to reduce γ in order to increase g further.

Given that $c_P < \frac{F\rho}{b\beta}$, so that (if $\gamma = \gamma_0 = \gamma_1$) poor households want to reduce IP protection (i.e. to increase γ) while rich households want full protection ($\gamma = 0$), there is no efficient strength of IP protection which can make both groups better off, even if income or wealth transfers that affect ϑ and X are permitted. The reason for this is simply that (for β given) rich households can only benefit if g increases, which (due to the resource constraint) implies that c_P has to decrease, making poor households worse off. In order to "buy" the support of poor households for stricter IP protection, rich households would therefore have to agree to transfers that increase ϑ or reduce X so much that the positive growth effect caused by the reduction in γ would be completely offset, destroying any welfare gain for the rich.

Given that $c_P < \frac{F\rho}{b\beta}$, poor households might want to increase γ ($= \gamma_0 = \gamma_1$) and therefore $m = \frac{\gamma}{g+\gamma}$ (note that m increases in γ if g declines in γ) so much that Condition A ($c_P > m$, implying that $\gamma < \tilde{\gamma}$) gets violated, leading to a switch to Regime B. As the economy remains in this regime for any $\gamma > \tilde{\gamma}$, and as g is independent of γ ($= \gamma_0 = \gamma_1$) in this regime, poor households are then indifferent with respect to the exact value of γ.

4.6 Many income groups

Suppose there are k income groups indexed by i with labor endowments l_i, initial wealth V_i and population shares β_i ($\sum_{i=1}^{k} \beta_i = 1$). For simplicity it is assumed that agents in all income groups hold the same (absolute) amount of initial wealth, so that $V_i(t) = V(t)$. In this case, all income groups are affected symmetrically when V changes due to an unexpected change in the strength of IP protection that affects newly granted and previously issued IPRs differently ($\gamma_0 \neq \gamma_1$). This implies that there are no "valuation effects" and that the rate of growth only depends on the hazard rate $\gamma = \gamma_1$ with which newly granted IPRs expire, but not on the rate γ_0, with which previously issued IPRs expire.

Groups are ordered by income (labor endowment), so that group 1 is the poorest and group k the richest ($l_1 < l_2 < ..., < l_{k-1} < l_k$). The size of the population and the

[24]From the resource constraint (equation 5) it can be inferred that the preferred growth rate is given by $g_P^* = \frac{1}{F}\left[1 - b\beta c_P^* - b(1-\beta)\right] = \frac{1}{F}\left[1 - F\rho - b(1-\beta)\right]$ and increases in β. The intuition for this is the following: if β increases, reducing c_P frees more resources that can now be used for R&D. As, contrary to production work, the costs of doing R&D are fixed while the benefits depend on the size of the population, reducing c_P and increasing the rate of growth is therefore more likely beneficial for the poor the larger their population share β is.

total labor endowment is normalized to one ($\sum_{i=1}^{k} l_i = 1$). Agents belonging to the richest income group again consume one of each of the invented goods in equilibrium, so that $C_k(t) = N(t)$. Agents belonging to a poorer group i only consume the fraction $c_i(t) = \frac{C_i(t)}{N(t)}$ of the goods, with $c_i(t)$ increasing in i. In equilibrium, IP protected goods are again sold at different prices which are inversely related to the number of income groups that purchase a particular good.

Proposition 5. *a) Given that $V_i = V$ and that a BGP in the unconstrained regime exists in which even the poorest income group consumes some IP protected goods, so that $c_1 > m$ holds*[25]*. Then, an increase (a decrease) in the strength of IP protection resulting from a reduction in $\gamma = \gamma_1$ increases (decreases) the rate of growth g and reduces (increases) the consumption shares $c_i(t) = \frac{C_i(t)}{N(t)}$ of all but the richest income groups. The poorer a group is, the larger is the absolute reduction (increase) in c_i. $g(t)$ and $c_i(t)$ immediately jump to the new BGP values when γ is changed.* [26]

b) The richest income group always prefers full IP protection ($\gamma_k^* = 0$). Suppose that the intertemporal utility of income group j ($1 < j < k$) is maximal for a positive IP expiration rate γ_j^* that is compatible with the existence of an equilibrium. Then, all poorer income groups ($i < j$) benefit from an increase of γ above γ_j^* (i.e. from a reduction in IP protection) and are harmed by a reduction of γ below γ_j^* while the opposite holds for all richer income groups ($i > j$).

Proof. See Appendix A6. □

The intuition behind these results is the following: if IP protection increases, m falls and prices p_i of IP protected goods fall (p_i denotes the price of goods that are still consumed by households of group i but not by poorer households[27]). While all households consume all the $M(t) = m(t)N(t)$ goods on which IP protection has expired and - taking expenditures as given - suffer from a reduction in m, the effects of the fall in prices p_i affect poor and rich households differently: For a given expenditure difference (which does not depend on γ if $V_i = V$), the consumption difference between a richer and a poorer household increases as the richer one benefits from a reduction in prices of IP protected goods that the poorer one cannot afford to buy. Taking into account that the fall in p_i is sufficiently large to allow households in the richest income group to still consume one of each of the invented goods in equilibrium, this implies that the reduction in consumption is largest (in absolute terms) for the poorest group ($i = 1$) that only benefits from the reduction in p_1 but not from the reduction in any other prices p_i with $i > 1$. Consequently, poorer households are more likely to benefit from a reduction in IP protection and prefer a weaker level of protection (i.e. a higher γ), while households from the richest income group always prefer full IP protection ($\gamma = 0$).

Given that $\gamma = \gamma_1 = \gamma_0$, so that there are no valuation effects when the strength of IP protection is changed, no household belonging to income group i ever wants to

[25] For such a BGP to exist, γ needs to be sufficiently small (see the proof)

[26] If income is redistributed in such a way that $l_{j+1} - l_j$ increases for some $i = j$ and does not decrease for any other $i \neq j$, equilibrium growth g increases (see Appendix A7).

[27] A fall in p_i therefore only implies that prices of IP protected goods fall conditional on demand, but not that prices for any IP protected good fall as an increase in IP protection implies that some goods are sold at a higher price than before to less households.

increase γ beyond the point where m (the fraction of goods on which IP protection has expired) starts to exceed the own consumption share c_i. The reason for this is that (absent valuation effects) households with $c_i < m$ always benefit from an increase in IP protection (i.e. a reduction in γ) as it increases the rate g with which their wages grow[28] without reducing their own consumption share. The result that poorer households prefer weaker IP protection does therefore only hold for "middle class" households that consume some IP protected goods, but not for very poor households that do not consume any of these goods[29].

4.6.1 International Context

The above analysis can readily be applied to an international context where countries differ with respect to their intellectual property policies. Let us for simplicity assume that labor productivities in the final good production and the R&D sectors are the same across countries and that R&D leads to global knowledge spillovers that increase these productivities. The value of an innovation is then given by the discounted sum of profit incomes derived from an IPR, summed up over all countries. If there are no restrictions to (parallel) trade and if trade is costless, the price of a given differentiated good has to be the same in all countries and the analysis is very similar as above. Then, expiration of an IPR in one country implies that the good has to be priced at marginal cost in all other countries as producers in this country can supply the corresponding good at marginal cost and therefore force all other producers in the rest of the world to also charge marginal cost prices. For this to happen, it is not necessary that there is actually trade between the countries as the mere threat to start exporting cheap goods suffices to reduce prices in the globally competitive market[30]. Because of that, the fraction of non-IP protected goods is the same in all countries and is driven by the IP policy of the country in which IP protection expires first. When $\frac{1}{\gamma}$ is taken as a proxy of the length of IP protection, the global effective length of protection is therefore equal to the shortest of all lengths of protection in all countries.

If countries differ with respect to their incomes (that means their labor endowments) and also the distribution of these incomes within the country, the model therefore predicts that relatively poorer countries or countries in which the politically most important income group is poorer, prefer a weaker global strength of IP

[28] While a change in γ can only affect g through valuation effects if there is only one income group that consumes IP protected goods (see Proposition 3 studying Regime B in the case of two income groups), a reduction in $\gamma = \gamma_1 = \gamma_0$ always increases g if IP protected goods are bought by more than one income group.

[29] The result that very poor households benefit from an increase in IP protection, arises due to the assumption that R&D leads to knowledge spillovers that increase the productivity of labor. This assumption was made in order to allow for balanced growth (see footnote 7) but might not be very realistic. The result that poorer "middle class" households with $c_i > m$ prefer weaker IP protection is, however, more robust as it is mainly driven by the change in the price structure that occurs when the fraction of IP protected and therefore monopolistically supplied goods is changed. In fact, these results also hold in a "digital economy" where $b = 0$ and where there are no productivity spillovers or in the case where the rate of growth is exogenously given.

[30] If a "pirating" country is very small and not able to supply huge amounts of cheap non patented goods to the rest of the world, global prices might however fall less in response (as competitors do not take the threat of entry very seriously).

protection[31]. If IP policies are set non-cooperatively by the different countries, the global effective strength of IP protection is then determined by the preferences of the poorest country which finds it optimal to grant the lowest level of protection. Such an outcome is clearly not efficient as the poorest country does not take into account that its "piracy" harms all other countries by reducing the (global) rate of growth. Therefore, an international agreement that sets minimal standards for the protection of intellectual property rights (like TRIPS) can lead to a Pareto-improvement if the poorest countries are compensated appropriately.

If trade barriers are high or international agreements prevent the export of "pirated" goods to countries in which these goods are protected by IPRs (this is what the Paris Convention tries to achieve), a given good can be sold at different prices in different countries. Let us for simplicity take the case in which IP holders can completely price discriminate between countries, but not between rich and poor people within a country. In this case, countries can differ with respect to the strength of IP protection. Given that the richest income group in a country is not too poor, markups are then such that agents belonging to this group can consume one of each goods in equilibrium, while poorer people in the same country just consume a fraction of the goods.

As the (global) rate of growth enters linearly into the intertemporal utility function, the preferred policies within a country do not depend on how much the patent protection in other countries contributes to an increased rate of growth and the above model can be applied to any given country. Therefore, richer people prefer a stronger protection of patents in their country than poorer people, while it is not clear whether poor people in a poor country prefer weaker patents than poor people in rich countries, as they pay lower prices in equilibrium. Again, setting IP policies in a non-cooperative way is likely to lead to inefficiently little R&D as countries do not take into account that all other countries also benefit from an increase in the rate of growth if they increase the level of IP protection in their country. The extent of IP protection in a given country is then more likely to be strict if the rich in a country decide about the IP policy or if they compensate the poor majority if they agree to increase the level of IP protection. Global effective IP protection and R&D incentives might therefore be larger if the rich decide about IP policies in many countries or if there are well functioning compensation schemes in many countries.

If a country is so poor that even the richest income group cannot afford to buy all existing goods at marginal cost, IP holders cannot earn any profits in this country as competition between producers of differentiated goods drives their prices down to marginal cost. In the case of perfect price discrimination, such a "very poor" country can therefore only benefit from stronger global IP protection which increases the rate of growth and productivity spillovers without leading to increased markups.

[31] As in the main model, the reason for that is that they consume less differentiated goods than richer countries and not that they do less R&D and get less profit income.

5 Conclusion

This paper has studied the effects that intellectual property rights have on inequality and growth in a model in which, in accordance with empirical evidence, rich households consume a larger variety of goods than poorer ones. Two main results come out of this analysis:

Although rich households consume more IP protected goods than poorer ones, they are harmed less from an increase in IP protection as they benefit from the fact that all markups fall if there is a larger proportion of sectors in which producers have monopoly power. As a consequence, poorer households prefer weaker IP protection. This is the first main result of the analysis. It should be noted that this result is derived in a model in which relative labor productivities of rich and poor households are exogenously given and in which, due to free entry, agents are indifferent between working in the R&D or the production sector, so that R&D workers do not benefit more from an increase in IP protection than workers in other sectors. Moreover, the result is not driven by the fact that rich households own more wealth in the form of IPRs than poorer ones. The study therefore provides a new reason for why (relatively) poor households or countries might loose if the strength of (global) IP protection is, like possibly in the TRIPS agreement, set at a level preferred by rich households or countries.

The second main result is that the effect that strengthening IP protection has on innovation and growth can depend on the extent of inequality in the economy and can even become negative when wealth is very unequally distributed. Therefore, how income and wealth are distributed might even matter for policymakers who only care about aggregate growth but not about the distribution of income per se.

An evaluation of the quantitative importance of these new mechanisms is left for future research.

6 Appendix

A1 Proof of Proposition 1

Proof. In the following, the shapes of the zero profit (ZP,16) and the resource constraint (RC,5) curves are analyzed in g - c_P space (with the latter variable on the horizontal axis) and the comparative static results are derived. A graphical representation is given in **Figure 3**. The RC curve is continuous, linear and downward sloping (for $b > 0$) and crosses the g - axis at $g^{RC}(c_P = 0) = \frac{1-b(1-\beta)}{F}$ and the c_P - axis at $c_P^{RC}(g = 0) = \frac{1-b(1-\beta)}{b\beta}$[32]. $b < \frac{1}{1-\beta}$ (Condition C) therefore needs to hold in order to ensure that $g > 0$ and $c_P > 0$ can hold. For $c_P < 1$, the ZP curve is continuous and upward sloping. It crosses the c_P - axis at $c_P^{ZP}(g = 0) = 1 - \frac{1-\vartheta+(1-\beta)\rho X}{F(\rho+\gamma)+(1-\beta)b} < 1$ and the g - axis at $g^{ZP}(c_P = 0) = \left(\frac{1-\vartheta+(1-\beta)(\rho X-b)}{F}\right) - \rho - \gamma$ and approaches the asymptote $c_P = 1$ ($\lim\limits_{g\to\infty} c_P^{ZP} = 1$). Therefore, $c_P < 1$ always holds. Given Condition C holds, the two curves cross in the positive quadrant if $g^{ZP}(c_P = 0) < g^{RC}(c_P = 0)$ (D) and if $c_P^{ZP}(g = 0) < c_P^{RC}(g = 0)$ (E). These conditions are satisfied if $\vartheta > (1-\beta)\,\rho X - F\,(\rho+\gamma)$ (D) and if $\vartheta < (1-\beta)\,\rho X + \frac{F(\rho+\gamma)(1-b)}{b\beta} + \frac{1-b}{\beta} + b$ (E) holds. In any equilibrium with positive growth, $m = \frac{\gamma}{g+\gamma} < 1$ must hold so that $\tilde{X} \equiv \frac{(1-m)F}{1-\beta} > 0$. Therefore, Condition B ($X \leq \frac{(1-m)F}{1-\beta}$) that ensures that $V_P(t) > 0$ holds is satisfied if $X \leq \tilde{X} > 0$. In order to obtain an equilibrium in regime A, Condition A ($c_P > m$) has to hold. For any fixed value of $g > 0$, $m = \frac{\gamma}{g+\gamma}$ continuously increases from 0 to 1 if γ increases from 0 to ∞. As condition D implies that $c_P > 0$ holds (even if $\gamma = 0$), there is always a positive threshold $\tilde{\gamma}$, such that for $0 \leq \gamma < \tilde{\gamma}$, $c_P > m$ holds in equilibrium.

The comparative static results can be derived by analyzing how a change in an exogenous parameter shifts the RC and the ZP curves: an increase in γ or ϑ and a decrease in X shift the ZP curve downwards and leaves the RC unaffected, implying a reduction in g and an increase in c_P. An increase in F and a reduction in β shift both the RC and the ZP curves downward and leads to a reduction in g. Deriving the ZP condition (16) with respect to ρ gives $\left.\frac{\partial g}{\partial \rho}\right|_{ZP} = \frac{X(1-\beta)}{F(1-c_P)} - 1$. Therefore, $\left.\frac{\partial g}{\partial \rho}\right|_{ZP} > 0$ holds if $X > \frac{(1-c_P)F}{1-\beta} \equiv \hat{X} > 0$. This condition is compatible with Condition B ($X \leq \frac{(1-m)F}{1-\beta} \equiv \tilde{X}$) due to Condition A ($c_P > m$). Therefore, an increase in ρ shifts the ZP curve upward if $\hat{X} < X \leq \tilde{X}$, while it shifts it downward if $0 \leq X < \hat{X}$. As the RC curve does not depend on X, the equilibrium growth rate g therefore increases in ρ if $\hat{X} < X \leq \tilde{X}$ and decreases in ρ if $0 \leq X < \hat{X}$. □

A2 Proof of Proposition 2

Proof. In the following it is shown that all equilibrium conditions are satisfied if g and c_P, r, p_R and p_P immediately jump to their new BGP values and if X immediately jumps from X_0 to X_1 when an unexpected change in IPR policy or in one of the other exogenous parameters occurs in $t = t_0$. The following notation is

[32] the notation $x^y(z = k)$ indicates the value x given by equation (curve) y if z has value k.

used: old BGP values are labeled with the subscript $_o$ while new BGP values are not labeled. As neither the resource constraint (5) nor the zero profit condition (16 with X replaced by $X_1 = \frac{\rho+g+\gamma_1}{\rho+g+\gamma_0} X_o$) directly depend on the sluggishly adjusting state variables V and m it suffices to show that the intertemporal budget constraints, in which both V and m enter, are still fulfilled with equality if g, c_P, r, p_R, p_P and X immediately jump to their new BGP values while m and V adjust slowly. As p_R (equation 15) is derived by subtracting the budget constraint of a poor household from that of a rich household, the latter is always satisfied if p_R is constant, so that it suffices to only check whether the budget constraint of a poor household is satisfied with equality. At point in time $t = t_0$ the number of competitively supplied goods is given by the old BGP value $M(t_0) = m_o N(t_0)$. When prices immediately jump to their new steady state value in t_0, p_R can be derived as

$$p_R = \frac{F\left(\rho + g + \gamma_1\right)}{1 - \beta} + b$$

from the free entry condition (7) that is forward looking and satisfied at each instant of time. Taking into account that the stock of existing IPRs expires at rate γ_0 after t_0, so that the value of one of these IPRs is given by $Z_o = \frac{(1-\beta)(p_R - b)}{\rho+g+\gamma_0} = F\left(\frac{\rho+g+\gamma_1}{\rho+g+\gamma_0}\right)$, the total value of initial wealth at point in time $t = t_0$ is given by

$$V(t_0) = N(t_0)\left(1 - m_o\right) Z_o = N(t_0)\left(1 - m_o\right) F\left(\frac{\rho + g + \gamma_1}{\rho + g + \gamma_0}\right)$$

If $\gamma_1 = \gamma_2$, the value of initial wealth therefore does not change in t_0, even if the strength of IPR protection is changed (i.e. if $\gamma \neq \gamma_1 = \gamma_2$). The initial wealth of a poor household if given by

$$V_P(t_0) = N(t_0)\left(\frac{\rho + g + \gamma_1}{\rho + g + \gamma_0}\right)\left[\left(1 - m_o\right) F - \left(1 - \beta\right) X_o\right]$$

Inserting this expression into the intertemporal budget constraint of a poor household (9), replacing $\frac{N(t_0)m}{r-g}$ by $\int_{t=t_0}^{\infty} M(t) e^{-(\rho+g)(t-t_0)} dt$ to account for the fact that $m(t)$ changes during the transition, and taking into account that $p_P(t) = z_P(t) = \beta b + (1-\beta) p_R(t)$ (equation4), this budget constraint at point in time t_0 can be written as

$$\frac{N\left(t_0\right)}{p_P - b}\left[\frac{p_P c_P - \vartheta}{\rho} + \left(\frac{\rho + g + \gamma_1}{\rho + g + \gamma_0}\right)\left[\left(1 - \beta\right) X_o - \left(1 - m_o\right) F\right]\right] = \int_{t=t_0}^{\infty} M(t) e^{-(\rho+g)(t-t_0)} dt \tag{19}$$

In order to replace the integral on the right hand side, $M(t)$ has to be determined. Defining the stocks of IPRs granted before and after t_0 by $P^0(t)$ and $P^1(t)$, the differential equation

$$\frac{\partial M(t)}{\partial t} = \gamma_0 P^0(t) + \gamma_1 P^1(t)$$

determines the evolution of $M(t)$ for $t > t_0$ as the measure $M(t)$ of competitively supplied goods increases due to the expiration of both stocks of IPRs. As $\frac{\partial P^0(t)}{\partial t} = -\gamma_0 P^0(t)$ and as $P^0(t_0) = N(t_0)(1 - m_0)$, $P^0(t)$ is given by

$$P^0(t) = N(t_0)(1 - m_0) e^{-\gamma_0 (t - t_0)}$$

From the differential equation $\frac{\partial P^1(t)}{\partial t} = \frac{\partial N(t)}{\partial t} - \gamma_1 P^1(t) = gN(t) - \gamma_1 P^1(t)$ and the initial condition $P^1(t_0) = 0$, the evolution of the stock $P^1(t)$ of IPRs granted after t_0 can be derived as

$$P^1(t) = \frac{gN(t_0)}{g+\gamma_1}\left[e^{g(t-t_0)} - e^{-\gamma_1(t-t_0)}\right]$$

Therefore,

$$\frac{\partial M(t)}{\partial t} = \gamma_0 P^0(t) + \gamma_1 P^1(t) = \gamma_0 N(t_0)(1-m_0)e^{-\gamma_0(t-t_0)} + \gamma_1 \frac{gN(t_0)}{g+\gamma_1}\left[e^{g(t-t_0)} - e^{-\gamma_1(t-t_0)}\right]$$

Taking into account that $M(t_0) = N(t_0)m_o$, we can derive

$$M(t) = N(t_0)\left[\frac{\gamma_1}{g+\gamma_1}e^{g(t-t_0)} + \frac{g}{g+\gamma_1}e^{-\gamma_1(t-t_0)} - (1-m_0)\,e^{-\gamma_0(t-t_0)}\right]$$

so that

$$\int_{t=t_0}^{\infty} M(t)e^{-(\rho+g)(t-t_0)}dt = N(t_0)\left[\frac{\gamma_1}{(g+\gamma_1)\,\rho} + \frac{g}{(g+\gamma_1)\,(\rho+g+\gamma_1)} - \frac{1-m_0}{\rho+g+\gamma_0}\right] \quad (20)$$

Inserting this, $p_P = F\,(\rho+g+\gamma_1) + b$ (from equations 4 and 7 with γ replaced by γ_1) and $c_P = \frac{1}{b\beta}\,(1 - gF - b\,(1-\beta))$ (from equation 5) into equation 19 gives the following cubic equation that determines g

$$-g^3F^2 + g^2F\,[1 + b\beta - 2b - F\,(2\rho+\gamma_0+\gamma_1)] + gb\,[1 - \vartheta\beta - b\,(1-\beta) + \beta\,(1-\beta)\,X_0\rho] +$$
$$gF\,[2\rho+\gamma_0+\gamma_1 - F\,(\rho+\gamma_0)\,(\rho+\gamma_1) - b\,(\rho\,(3-\beta) + \gamma_0\,(2-\beta) + \gamma_1)] +$$
$$(\rho+\gamma_0)\,[F\,(\rho+\gamma_1)\,(1-b) - b\,(\vartheta\beta - 1 + b\,(1-\beta))] + b\beta\,(1-\beta)\,X_0\rho\,(\rho+\gamma_1) = 0$$

This equation is independent of m_0 and the same as the one that results when the new BGP value of g is determined by inserting the zero profit condition (equation 16, with X_0 replaced by $X_1 = \frac{\rho+g+\gamma_1}{\rho+g+\gamma_0}X_o$) into the resource constraint (equation 5). Therefore, this budget constraint equation is satisfied if g, c_P, r, p_R, p_P and X immediate jump to the new BGP values in t_0 while m (and V) adjust sluggishly. □

A3 Proof of Corollary 1

Proof. While the resource constraint is always given by

$$g(t) = \frac{1}{F}\left[1 - b\beta c_P(t) - b(1-\beta)\right]$$

(5) the zero profit condition is given by

$$g^o = \left(\frac{\frac{1-\vartheta}{1-\beta} + \rho X_o}{1-c_P^o} - b\right)\frac{1-\beta}{F} - \rho$$

before t_0 and by

$$g=\left(\frac{\frac{1-\vartheta}{1-\beta}+\rho\left(\frac{\rho+g+\gamma_1}{\rho+g}\right)X_o}{1-c_P}-b\right)\frac{1-\beta}{F}-\rho-\gamma_1$$

after t_0 (see16and 2 with $\gamma_0=0$). The new BGP value g therefore lies above the old one g^o if the new zero profit curve lies above the old one (i.e. if for given value of c_p, the value of g is larger in the new ZP condition). This is the case if

$$X_0>\frac{F\left(1-c_P\right)\left(\rho+g\right)}{\left(1-\beta\right)\rho}$$

holds for both the old and the new BGP values of g and c_P. Inserting the maximal value $X_o=\frac{F}{1-\beta}$ for which $V_P^o=0$ (see equation 13 with $m=0$; this coincides with the case where $X=\widetilde{X}$ in Proposition 1, implying that Condition B is satisfied with equality), this condition becomes

$$c_P>1-\frac{\rho}{\rho+g}$$

(**Condition** G). As only a marginal increase in γ_1 is analyzed and as for $\gamma_1=0$ the new ZP condition coincides with the old one, it suffices to show that Condition G is satisfied for the old BGP values g^o and c_p^o. Inserting the resource constraint into the zero profit condition gives a quadratic equation with the interior solution

$$c_P^o=\frac{1+b\beta+F\rho-\sqrt{\left(1+b\beta+F\rho\right)^2-4b\beta\vartheta}}{2b\beta}$$

Inserting this into the resource constraint gives

$$g^o=\frac{1}{2F}\left[1+b\beta-2b-F\rho+\sqrt{\left(1+b\beta+F\rho\right)^2-4b\beta\vartheta}\right]$$

Inserting these values into Condition G leads to the inequality

$$\sqrt{\left(1+b\beta+F\rho\right)^2-4b\beta\vartheta}\left(\frac{1}{\beta}-1\right)-\left[\frac{1+b\beta+F\rho}{\beta}+1+b\beta-2b-\rho F-2\vartheta\right]>0$$

If $\vartheta=1$, $b=\frac{1}{2}$ and $F=\frac{\beta}{2\rho}$, this inequality is satisfied and an interior solution ($0<c_P^o<1$ and $g^o>0$) is obtained. Therefore g increases in t_0 in this case[33]. □

[33]During the transition period, m increases and the normalized value of wealth $\frac{V(t)}{N(t)}$ decreases. As X is constant during the transition (see the proof of Proposition 2) this implies that the value of initial wealth of a poor household, $V_P(t)=V(t)-(1-\beta)\,XN(t)$ falls below zero in the long run because it is equal to zero at t_0. In the example analyzed in Corollary 1, the poor therefore end up with a small amount of debt in the long ($V_P<0$), a feature that was excluded by assumption in the previous analysis. The qualitative result of Corollary 1 does, however, not depend on this feature: As the relation between X, g and c_P is continuous and as only a marginal reduction in IP protection is analyzed (γ_1 is small), the qualitative result of Corollary 1 can still be obtained in cases where the poor start with a slightly positive amount of initial wealth (i.e. if $X_0=\frac{F}{1-\beta}-\epsilon$ with ϵ small) and never accumulate debt so that $V_P>0$ always holds.

A4: Proof of Proposition 3

Proof. As the prices of the competitively supplied goods are equal to the marginal cost b the expenditures $Y_P(t)$ of a poor household that only purchases these kind of goods are equal to $Y_P(t) = bC_P(t)$ in period t. Along a balanced growth path on which $C_P(t)$ $(= c_P(t)N(t))$ and the labor income $\vartheta N(t)$ of a poor household grow at rate g, the intertemporal budget constraint of such a household can (like in equation ??) be derived as

$$\frac{N(\tau)\vartheta}{r-g} + V_P(\tau) = \frac{N(\tau)}{r-g} c_P b$$

Taking into consideration that $r = \rho + g$ (equation 6) holds along a BGP and using the definition $\bar{V}_P \equiv \frac{V_P(\tau)}{N(\tau)}$, this equation can be solved for

$$c_P = \frac{\vartheta + \rho \bar{V}_P}{b} \tag{21}$$

In order to get an interior solution $c_P < 1$, $\vartheta + \rho \bar{V}_P < b$ has to hold. As rich households still consume one of each invented goods in every period ($C_R(t) = N(t)$) the resource constraint is still given by equation 5. Inserting equation 21 into this resource constraint allows to derive the BGP growth rate as

$$g = \frac{1}{F}\left[1 - b(1-\beta) - \beta\left(\vartheta + \rho \bar{V}_P\right)\right] \tag{22}$$

In order for a BGP with $g > 0$ to exist, $1 - b(1-\beta) - \beta\left(\vartheta + \rho \bar{V}_P\right) > 0$ therefore has to hold. Moreover, $c_P < m$ needs to hold in regime B (Condition A must be violated). Inserting equation 21 and $m = \frac{\gamma}{g+\gamma}$ (which can again be derived like in equation 8) with g replaced by its equilibrium value into this condition and solving the resulting inequality for γ gives the inequality $\gamma > \frac{\left(\vartheta+\rho\bar{V}_P\right)\left(1-b(1-\beta)-\beta\left(\vartheta+\rho\bar{V}_P\right)\right)}{F\left(b-\vartheta-\rho\bar{V}_P\right)}$ under which Condition A is violated. Given that $g > 0$ and $c_P < 1$, both the numerator and the denominator of the right hand side of this inequality are positive, so that $\gamma > 0$ has to hold.

Like in the proof of Proposition 2 it is now shown that all equilibrium conditions are satisfied if g, c_P, r and p_R immediately jump to their new BGP values and if $\bar{V}_P$ immediately jumps from $\bar{V}_P^o$ to $\bar{V}_P^1$ when an unexpected change in IPR policy or in one of the other exogenous parameters occurs in $t = t_0$. Old BGP values are again labeled with the subscript $_o$ while new BGP values are not labeled. As poor households do not consume IP protected goods and are therefore not confronted with changing consumption prices, they spend all their labor income $\vartheta N(t)$ and consume the amount $(r-g)\bar{V}_P N(t) = \rho \bar{V}_P N(t)$ every period, implying that $V_P(t)$ grows at rate g and that $\bar{V}_P$ is constant after t_0. What remains to be shown is that the intertemporal budget constraint of a rich household is satisfied with equality if the transition of m is taken into account. When prices immediately jump to their new steady state value in t_0, p_R (the price at which IP protected goods are sold to rich households) can be derived as

$$p_R = \frac{F(\rho + g + \gamma_1)}{1-\beta} + b$$

from the free entry condition (7). As the stock of existing IPRs expires at rate γ_0 after t_0, so that the value of one of these IPRs is given by $Z_o = \frac{(1-\beta)(p_R-b)}{\rho+g+\gamma_0} = F\left(\frac{\rho+g+\gamma_1}{\rho+g+\gamma_0}\right)$, the total value of initial wealth at point in time $t = t_0$ is given by

$$V(t_0) = N(t_0)\,(1-m_o)\,Z_o = N(t_0)\,(1-m_o)\,F\left(\frac{\rho+g+\gamma_1}{\rho+g+\gamma_0}\right)$$

The initial wealth of a poor household therefore changes from $\bar{V}_P^o$ to $\bar{V}_P^1 = \left(\frac{\rho+g+\gamma_1}{\rho+g+\gamma_0}\right)\bar{V}_P^o$ in t_0. As $V = \beta V_P + (1-\beta)\,V_R$ and as $\bar{V}_P^o = \frac{V_P^o}{N}$, the stock of initial wealth held by a rich household in t_0 can be derived as

$$V_R(t_0) = \frac{N(t_0)}{1-\beta}\left[\frac{\rho+g+\gamma_1}{\rho+g+\gamma_0}\right]\left((1-m_o)\,F - \beta\bar{V}_P^o\right)$$

The intertemporal budget constraint of a rich household is given by

$$\frac{N(t_0)}{(r-g)}\frac{(1-\beta\vartheta)}{(1-\beta)} + V_R(t_0) = \int\limits_{t=t_0}^{\infty}\left[M(t)b + (N(t) - M(t)p_R\right]e^{-r(t-t_0)}dt$$

as rich households buy the measure $M(t)$ of competitively supplied goods sold at price b and the measure $N(t)-M(t)$ of IP protected goods sold at price p_R in period t. Replacing r, p_R and $V_R(t_0)$, this budget constraint can be written as

$$N(t_0)\left[\frac{1}{\rho} - \frac{1-\beta\vartheta-b\,(1-\beta)}{F\rho\,(\rho+g+\gamma_1)} - \frac{(1-m_o)\,F-\beta\bar{V}_P^o}{F\,(\rho+g+\gamma_0)}\right] = \int\limits_{t=t_0}^{\infty}M(t)e^{-r(t-t_0)}dt$$

Replacing

$$\int_{t=t_0}^{\infty}M(t)e^{-r(t-t_0)}dt = N(t_0)\left[\frac{\gamma_1}{(g+\gamma_1)\,\rho} + \frac{g}{(g+\gamma_1)\,(\rho+g+\gamma_1)} - \frac{1-m_0}{\rho+g+\gamma_0}\right]$$

which has already been derived as equation 20 in the proof of Proposition 2, the budget constraint can be rewritten as:

$$g = \frac{1}{F}\left[1 - b\,(1-\beta) - \beta\vartheta - \beta\rho\bar{V}_P^o\left(\frac{\rho+g+\gamma_1}{\rho+g+\gamma_0}\right)\right]$$

As this is the same equation as equation 22 with $\bar{V}_P$ replaced by $\bar{V}_P^1 = \bar{V}_P^o\left(\frac{\rho+g+\gamma_1}{\rho+g+\gamma_0}\right)$, the intertemporal budget constraints of rich households are satisfied with equality when g, c_P, p_R, r and $\bar{V}_P$ immediately jump to their new BGP values while m and $\frac{V_R(t)}{N(t)}$ adjust sluggishly. □

A5: Proof of Proposition 4

Proof. The result that rich households benefit from any policy that increases g directly follows from the fact that their intertemporal utility (equation 17) increases in g. Inserting the resource constraint (equation 5) which has to be satisfied in both regimes into equation 18 gives the intertemporal utility of a poor household as a continuous and concave function of the endogenous variable c_P. As this intertemporal utility is maximal for $c_P = \frac{F\rho}{b\beta}$, poor households benefit from policies that reduce c_P and therefore (taking the resource constraint into account) increase the rate of growth if $c_P > \frac{F\rho}{b\beta}$ and are harmed by them if $c_P < \frac{F\rho}{b\beta}$. As $c_P < 1$, poor households therefore always want to reduce g if $\frac{F\rho}{b\beta} > 1$ (there is a non-empty range of parameters for which this condition is compatible with the existence of an equilibrium in both Regime A and B). In Regime B, $c_P = \frac{\vartheta + \rho \bar{V}_P}{b} < \frac{F\rho}{b\beta}$ holds if $\vartheta < \frac{F\rho}{\beta} - \rho \bar{V}_P$ and there is a non-empty range of parameters for which an equilibrium exists and for which this condition in addition to the condition $\frac{F\rho}{b\beta} < 1$ is satisfied. In Regime A, the (interior) consumption share of poor households can be derived from equations 5 and 16 as

$$c_P = \frac{1 + F\rho + b\beta - \sqrt{(1 + F\rho + b\beta)^2 - 4b\beta \left(F\rho + \vartheta - \rho X (1 - \beta)\right)}}{2b\beta} \tag{23}$$

If $\frac{F\rho}{b\beta} < 1$, $c_P < \frac{F\rho}{b\beta}$ requires that $\vartheta < \frac{F\rho}{b\beta}(1 + F\gamma) + \rho X (1 - \beta) - F\gamma$ holds in addition to the other existence conditions from Proposition 1. (there is a non-empty range of parameters that fulfills all conditions). Assuming that $\frac{F\rho}{b\beta} < 1$, $c_P = \frac{F\rho}{b\beta}$ holds if $\gamma = \frac{\frac{F\rho}{b\beta} - \vartheta + \rho X (1-\beta)}{F\left(1 - \frac{F\rho}{b\beta}\right)}$. Given that this value of γ is compatible with the existence of an equilibrium in Regime A, poor households therefore want to set the IP expiration rate at this value as it implements the level of c_P for which their intertemporal utility is maximized. For the welfare analysis it suffices to compare BGP values holding X constant as in the case where $\gamma = \gamma_1 = \gamma_0$ a change in γ leads to an immediate jump to the new BGP without affecting X (see Proposition 2). It is now shown that there are parameter constellations for which $0 < \gamma_P^* < \tilde{\gamma}$ is an interior solution in Regime A: Suppose that $\gamma_P^* = \frac{\frac{F\rho}{b\beta} - \vartheta + \rho X (1-\beta)}{F\left(1 - \frac{F\rho}{b\beta}\right)}$ is only slightly positive. Then, Condition D from Proposition 3reduces to $(1 - \beta)\rho X - F\rho < \vartheta$ and Condition E to $\vartheta < (1 - \beta)\rho X + \frac{F\rho(1-b)}{b\beta} + \frac{1-b}{\beta} + b$ (the parameter range defined by these two conditions is nonempty under Condition C). γ_P^* is positive if $\vartheta < \frac{F\rho}{b\beta} + \rho X (1 - \beta)$ and if $\frac{F\rho}{b\beta} < 1$ and there is a non-empty range of parameters for which all of the above conditions are satisfied. Given that γ_P^* is only slightly positive, $m = \frac{\gamma}{g+\gamma}$ (see equation 8) is close to zero in an equilibrium with positive growth, so that $c_P > m$ (Condition A) and therefore $\gamma < \tilde{\gamma}$ (Condition F) holds if $c_P > 0$. The latter is always satisfied as the term $(F\rho + \vartheta - \rho X (1 - \beta))$ that appears under the square root in equation 23 is positive due to Condition E. Therefore, an interior solution exists under these conditions.

Deriving γ_P^* with respect to the different parameters gives $\frac{\partial \gamma_P^*}{\partial \vartheta} < 0$, $\frac{\partial \gamma_P^*}{\partial \rho} > 0$, $\frac{\partial \gamma_P^*}{\partial X} > 0$ and $sign\left(\frac{\partial \gamma_P^*}{\partial \beta}\right) = sign\left(-F(1 - \vartheta) - X\left(b\beta^2 + F\rho(1 - 2\beta)\right)\right) < 0$, where the last inequality holds due to the fact that $\frac{F\rho}{b\beta} < 1$ and therefore $b\beta > F\rho$ needs to be satisfied in order to obtain an interior solution. □

A6: Proof of Proposition 5

Proof. a): Let us denote the price of an IP protected good that is sold to all but the poorest $i-1$ income groups by p_i and the resulting profits by π_i. Then, $\pi_i = (p_i - b)\left(1 - \sum_{s=1}^{i-1} \beta_s\right)$ if $2 \leq i \leq k$, while profits of a firm selling to all income groups ($i = 1$) are given by $\pi_1 = p_1 - b$. As goods are symmetric, profits derived from each non-expired IPR must be the same in equilibrium, independently of the number of income groups to which a good is sold (i.e., $\pi_i = \pi$ must hold). This "equal profit condition" allows to express all prices p_i as a positive function of p_k, the price charged by firms that sell exclusively to the richest income group:

$$p_i = b + \frac{(p_k - b)\left(1 - \sum_{s=1}^{k-1} \beta_s\right)}{1 - \sum_{s=1}^{i-1} \beta_s} \tag{24}$$

Prices increase in i and are therefore larger for goods that are sold to less income groups. Moreover, all prices depend positively on p_k and do not directly depend on income differences between groups, but only on population shares.

Using the same notation as above (with $m(t) = \frac{M(t)}{N(t)}$ denoting the measure of goods on which IP protection has expired) and assuming that the economy is on a BGP, the intertemporal budget constraints of agents belonging to income group i are given by:

BC($i = 1$): $l_1 + (r - g)\frac{V(t)}{N(t)} = bm + p_1(c_1 - m)$

BC($i = 2$): $l_2 + (r - g)\frac{V(t)}{N(t)} = bm + p_1(c_1 - m) + p_2(c_2 - c_1)$

BC(general): $l_i = l_{i-1} + p_i(c_i - c_{i-1})$, for $2 \leq i \leq k$

BC($i = k$): $l_k = l_{k-1} + p_k(1 - c_{k-1})$

The simplification in the third line is obtained by substituting the budget constraint of group $i-1$ and the one in the fourth line by substituting $c_k = \frac{C_k(t)}{N(t)} = 1$. From the fourth line, we obtain $p_k = \frac{l_k - l_{k-1}}{1 - c_{k-1}}$ so that the profits of firms selling exclusively to the richest income group are given by $\pi_k = \left(\frac{l_k - l_{k-1}}{1 - c_{k-1}} - b\right)\left(1 - \sum_{i=1}^{k-1} \beta_i\right)$. Due to free entry, $\frac{\pi}{r+\gamma} = F$ must hold along a BGP, which, together with the familiar Euler equation $r = \rho + g$ (6) gives the zero profit condition

$$\frac{\left(\frac{l_k - l_{k-1}}{1 - c_{k-1}} - b\right)\left(1 - \sum_{i=1}^{k-1} \beta_i\right)}{\rho + g + \gamma} = F \tag{25}$$

This conditions can be plotted as an upward sloping ZP curve in g - c_{k-1} - space (with g on the vertical axis) which shifts up if γ is decreased (the notation $\gamma = \gamma_1$ is used throughout).

Rewriting the intertemporal budget constraint of group i and inserting the equal profit condition (equation 24) with p_k replaced by $p_k = \frac{l_k - l_{k-1}}{1 - c_{k-1}}$ gives

$$c_i = c_{i+1} - \frac{l_{i+1} - l_i}{p_{i+1}} = c_{i+1} - \frac{l_{i+1} - l_i}{b + \left(\frac{l_k - l_{k-1}}{1 - c_{k-1}} - b\right)\frac{\left(1 - \sum_{s=1}^{k-1} \beta_s\right)}{\left(1 - \sum_{s=1}^{i} \beta_s\right)}} \tag{26}$$

This equation determines the different consumption shares c_i (with $1 \leq i \leq k-2$) as a function of c_{k-1} and of the labor endowments and population shares. As p_{i+1}

depends positively on c_{k-1}, $c_{i+1} - c_i = \frac{l_{i+1}-l_i}{p_{i+1}}$ decreases if c_{k-1} increases. This implies that, for l_i and β_i given, all consumption shares c_i depend positively on c_{k-1} and that a decrease in c_{k-1} leads to an even larger decrease in c_i (if $i < k-1$). Indeed, the fall in c_i caused by the fall in c_{k-1} is the larger in absolute terms the smaller i is, that means the poorer an income group is.

Taking into consideration that the total stock of labor can be either used for R&D or for the production of final goods, the Resource constraint can be derived as:

$$g = \frac{1}{F}\left[1 - b\sum_{i=1}^{k} c_i\beta_i\right] \tag{27}$$

This equation implies a negative relation between g and c_{k-1}, as all consumption shares c_i (with $1 \leq i \leq k-2$) depend positively on c_{k-1} (due to equation 26). The resource constraint therefore defines a downward-sloping curve in g - c_{k-1} - space which is independent of γ. Given an equilibrium exists, the equilibrium values of g and c_{k-1} are determined by the zero profit condition (equation 25) and by the resource constraint (27), i.e. by the intersection of the ZP and the RC curve. As the ZP curve shifts up when γ declines and as the RC curve is independent of γ, a decrease in γ (an increase in the strength of IP protection) therefore increases g and reduces c_{k-1}. Due to equation 26 this decrease in c_{k-1} implies a decrease in all other consumption shares c_i that is greater in absolute value the poorer a group is (the lower i is). For an equilibrium to exist, $m < c_1$ must hold. These conditions can only be satisfied if γ is sufficiently small (remember that $m = \frac{\gamma}{g+\gamma}$ as in equation 8).

As the state variables m and V do not enter into equations 25, 27 and 26, there are no transition dynamics and p_i, c_i and g immediately jump to their new BGP values[34]. Given that $V_i = V$, an unexpected change in the strength of IP protection on previously issued IPRs (i.e. a change in γ_0) which affects the value of initial wealth $V_i = V$ affects all households symmetrically. As such a change does not affect the absolute expenditure differences between income groups (which are given by $l_i - l_{i-1}$, as can be inferred form the intertemporal budget constraints) and therefore does not affect prices p_i and profits, it has no impact on the zero profit condition (equation 25). As neither the resource constraint (equation 27) depends on γ_0, g and c_i therefore only depend on the hazard rate $\gamma_1 = \gamma$ with which newly issued IPRs expire and not on γ_0. This is a generalization of the result that was formally derived in the case of two income groups where $V_i = V$ holds if $X = 0$ (see Proposition 2).

b) Intertemporal utilities along the balanced growth path are given by $U_i(\tau) = \int_{t=\tau}^{\infty} ln\left(c_i N(t)\right) e^{-\rho(t-\tau)}dt = \frac{ln(N(\tau))}{\rho} + \frac{ln(c_i)}{\rho} + \frac{g}{\rho^2}$. As there are no transition dynamics for the variables c_i and g, an agent in income group i therefore likes to (unexpectedly) reduce IP protection (to increase γ) if $\frac{\partial U_i}{\partial \gamma} = \frac{1}{\rho}\left[\frac{1}{c_i}\frac{\partial c_i}{\partial \gamma} + \frac{1}{\rho}\frac{\partial g}{\partial \gamma}\right] > 0$ and wants increase

[34]For such an instantaneous transition to be feasible, the budget constraint of the poorest income group (BC($i = 1$)) which contains the state variables V and m must be satisfied during the whole transition phase. This is not verified here and would require an analysis similar to the one undertaken in the case of two income groups (see the proof of proposition 2).

IP protection if $\frac{\partial U_i}{\partial \gamma} < 0$. $\frac{\partial U_i}{\partial \gamma} > 0$ holds if $\frac{1}{c_i}\frac{\partial c_i}{\partial \gamma} > -\frac{1}{\rho}\frac{\partial g}{\partial \gamma} > 0$, which is more likely satisfied the smaller i and the poorer an income group is as c_i increases in i and as $\frac{\partial c_i}{\partial \gamma}$ (> 0) decreases in i (see above)[35]. As $c_k = 1$ and $\frac{\partial c_k}{\partial \gamma} = 0$, $\frac{\partial U_k}{\partial \gamma} < 0$ always holds, so that households in the richest income group always prefer full IP protection, i.e. $\gamma_k^* = 0$. Suppose that there exists a value $\gamma_j^* > 0$ for which the utility of an income group j (with $1 < j < k$) is maximal and for which an equilibrium in the unconstrained regime exists. Then, $\frac{\partial U_j}{\partial \gamma}\left(\gamma = \gamma_j^*\right) = 0$ must hold as U_i is continuous and differentiable in γ in the unconstrained regime. As $\frac{\partial U_i}{\partial \gamma}$ decreases in i, poorer households $(i < j)$ then benefit from a marginal increase in γ above the level γ_j^* (as $\frac{\partial U_i}{\partial \gamma}\left(\gamma = \gamma_j^*\right) > 0$ for $i < j$) while richer households $(i > j)$ benefit from a marginal reduction in γ below the level γ_j^* (as $\frac{\partial U_i}{\partial \gamma}\left(\gamma = \gamma_j^*\right) < 0$ for $i > j$). In order to avoid messy calculations, it is not shown here that $\frac{\partial^2 U_i(\gamma)}{\partial \gamma^2} < 0$ globally holds. Therefore, it is unclear whether there might in fact be several local maxima for which $\frac{\partial U_i}{\partial \gamma} = 0$ holds. Given that γ_j^* is the global maximum for group j (out of several local maxima) this, however, does not imply that households with $i < j$ might actually benefit from a non-marginal reduction in γ below the level γ_j^*. The reason for this is the following: as $\frac{\partial U_i}{\partial \gamma}$ decreases in i, we can write $U_j = U_i + f_{ij}(\gamma)$ with $f_{ij}(\gamma) > 0$ if $i < j$ and $\frac{\partial f_{ij}(\gamma)}{\partial \gamma} < 0$. Given that U_j is maximal for the value γ_j^*, $U_i = U_j - f_{ij}(\gamma)$ can therefore not be maximal for a value $\tilde{\gamma} < \gamma_j^*$ (which might be a local maximum) as $f_{ij}(\tilde{\gamma}) > f_{ij}(\gamma_j^*)$. A symmetric argument can be used to show that it is not possible either that households with $i > j$ can benefit from a non-marginal increase in γ above the level γ_j^*. Therefore, an income group $i < j$ prefers a weaker level of IP protection than group j (i.e. $\gamma_i^* > \gamma_j^*$) and an income group $i > j$ prefers a stronger level of IP protection than group j (i.e. $\gamma_i^* < \gamma_j^*$).

The analysis conducted so far was based on the assumption that an interior solution $\gamma_j^* > 0$ which is compatible with the existence of an equilibrium exists. There is indeed a non-empty range of parameters for which such a solution exists: As has been shown in Proposition 4, there are conditions under which an interior solution exists in the case of two income groups, R and P. Suppose now that more income groups i are added and that the per capita incomes of these groups lie below the one of group R and that their population shares β_i are very small. This does not affect the resource constraint (27) or the zero profit condition (25, solved for as a function of c_P and g) in a discontinuous way, so that the consumption share of group P, c_P, and the rate of growth g does not change discontinuously. Therefore, an interior solution $\gamma_P^* > 0$ exists if parameters lie in the range derived in the proof of Proposition 4 and if the additional groups i are only endowed with a small fraction of the total labor endowment. Then $\gamma_i^* > \gamma_P^*$ holds if group i is poorer than group P and $\gamma_i^* < \gamma_P^*$ if group i is richer than group P. □

[35] Even if the consumption shares c_i of all income groups were reduced by the same absolute amount if γ increased (i.e. if $\frac{\partial c_i}{\partial \gamma}$ was independent of i), poorer households would still be more likely to suffer from such a policy change. The reason for this is that intertemporal utility is concave in current consumption so that richer households are willing to reduce their consumption by a larger absolute amount than poorer households in order to obtain a certain increase in the rate of growth.

A7: The effect of inequality on growth

Suppose that labor endowments (or wage incomes, that are equal to $l_i N(t)$) are redistributed from poor to rich households in such a way that $l_{j+1} - l_j$ increases for income group $j < k-1$ and does not change for any other group $i \neq j$[36]. This does not affect the ZP condition 25 as $l_k - l_{k-1}$ is constant, but it shifts the resource constraint curve (equation 27) outward. The latter can be seen from equation 26: $c_i = c_{i+1} - \frac{l_{i+1}-l_i}{p_{i+1}}$ which indicates that - for c_{k-1} given - resources are freed for R&D and growth as the consumption shares c_i of groups with index $i \leq j$ are reduced but those of groups with index $i > j$ do not change. Therefore, such a redistribution from poor to rich households increases the rate of growth g and increases c_{k-1}. If $l_k - l_{k-1}$ increases but all other $l_{i+1} - l_i$ are unchanged, the ZP curve shifts up in g - c_{k-1} - space while the RC curve is unchanged so that g increases and the consumption shares of all but the richest group k decrease. Generalizing the argument, we obtain:

Proposition 6. *Suppose that $V_i(\tau) = V(\tau)$ and that an equilibrium exists in which $c_1 > m$ holds. If income is redistributed in such a way that $l_{j+1} - l_j$ increases for some $i = j$ and does not decrease for any other $i \neq j$, equilibrium growth g increases. If $l_{i+1} - l_i$ increases for $i = j$ but does not change for any other $i \neq j$, g increases and the consumption shares c_i of all groups $i \leq j$ decrease while the consumption shares of the richer groups $j < i < k$ increase ($c_k = 1$ always holds).*

The intuition for this result is the following: if for given prices income is redistributed from a poor to a rich household, the consumption share c_i of the poor household decreases more than that of the rich increases as the rich needs to pay higher prices for additional goods than the poor. Therefore, profits and the incentives to innovate increase (or, put differently: less labor in the production sector is needed which can now be used in the R&D sector).

Given that the consumption share of a poor income group has been reduced due to such an increase in inequality, this group now prefers a weaker level of IP protection than before as increasing g by increasing IP protection becomes more costly in terms of current consumption if c_i is lower (because intertemporal utility is linear in g but concave in c_i; see the proof of Proposition 5). If the median voter decides about the strength of IP protection (and no transfer payments are used) an increase in inequality that reduced the consumption share c_m of the median voter will therefore be accompanied by a reduction in the strength of IP protection.

A8: Extending the scope of IP protection

It is now assumed that previous innovators can charge licensing fees from future innovators who build on the knowledge created by them. For simplicity, the case of full IP protection ($\gamma = 0$) is considered. As all previously invented goods are symmetric, each holder of an IPR that was granted in the past obtains the same licensing payment from an innovator who enters the market. Let us assume that an innovator is required to pay total licensing fees equal to Q upon entry. As the mass of

[36] This implies that the incomes of groups with index $i \leq j$ ($> j$) all decrease (increase) by the same absolute amount.

inventors in period t is given by $N(t)g$, each of the $N(t)$ holders of previously granted patents receives licensing income equal to gQ in any period t. As per period profits π from the sale of a good are constant along a balanced growth path, the value of an innovation is given by $V = \frac{\pi}{r} + \frac{gQ}{r} - Q$. Inserting $r = \rho + g > g$, one clearly sees that the value of an innovation decreases in the size of the licensing fee Q. Given that $Q < \frac{\pi}{\rho}$, the zero-profit curve (that is defined by the equation $V = \frac{\pi}{r} + \frac{gQ}{r} - Q = F$) is upward-sloping in g-n- space (note that π depends positively on n) and moves to the right if Q increases so that the equilibrium rate of growth declines in the size of the licensing fee. The reason why an increase in Q decreases the rate of growth is that it makes profits for innovators more backloaded. Therefore, Q would need to rise faster than the rate of interest in order to increase innovation incentives. But this is not possible in the long run as licensing payments have to be paid out of total profits which grow at a rate that is lower than the rate of interest.

7 Graphs

Figure 1: Market demand of a monopolistic firm

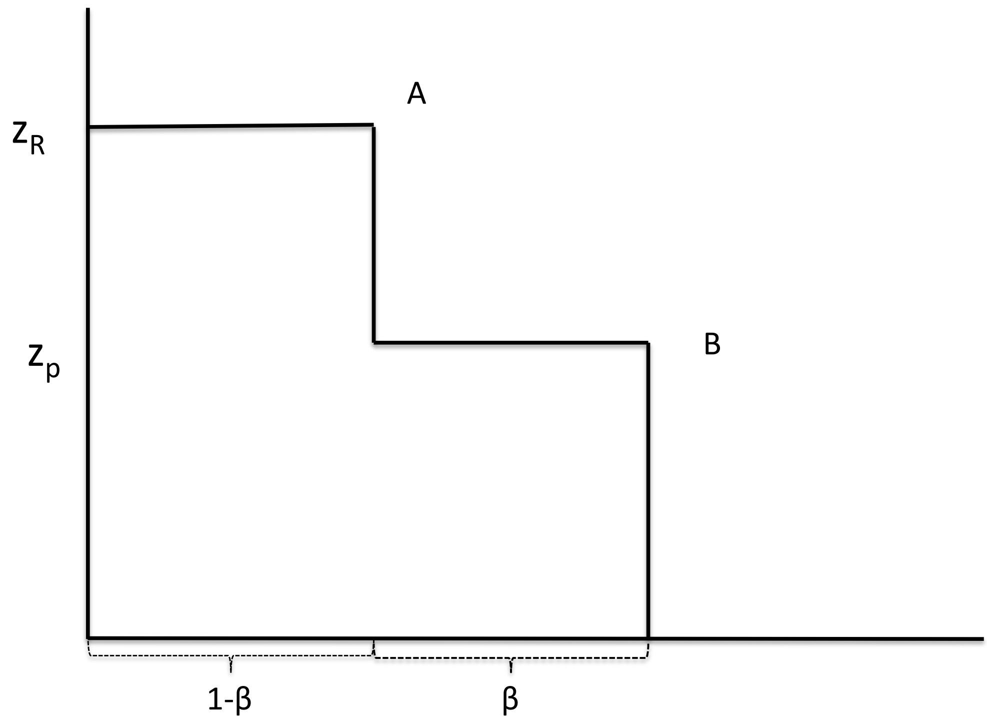


Figure 2: Equilibrium price structure and consumption shares

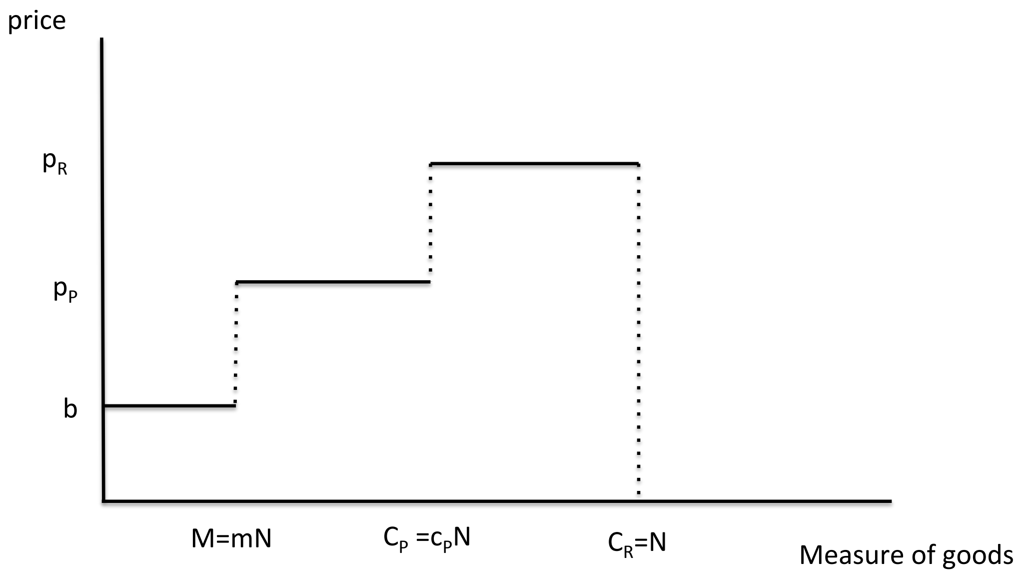
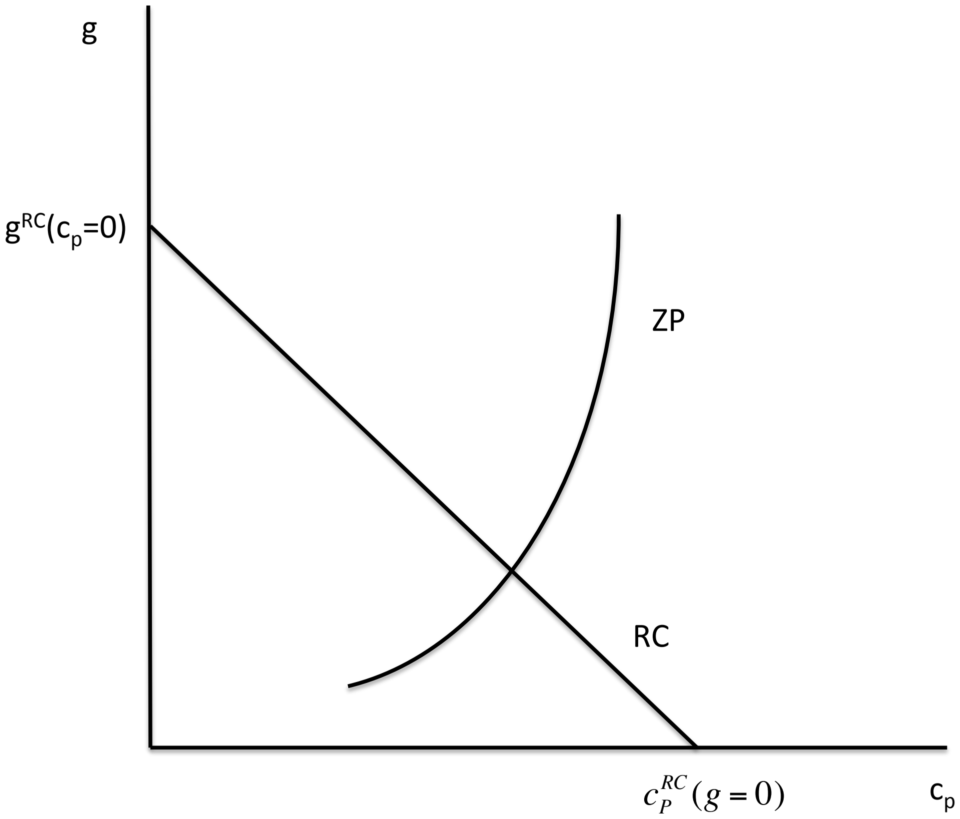


Figure 3: The general equilibrium



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