Intellectual Property Rights in a Quality-Ladder Model with Persistent Leadership

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Abstract

This article analyzes the effects of intellectual property rights in a quality-ladder model of endogenous growth in which incumbent firms preemptively innovate in order to keep their position of leadership. Unlike in models with leapfrogging, granting forward protection and imposing a non-obviousness requirement reduces growth. Under free entry, infinite protection against imitation, granted independently of the size of the lead, maximizes growth. If entrants have to engage in costly catch up before they can undertake frontier R&D, growth is maximal for a finite (expected) length of protection against imitation. (JEL L40, O31, O34)

Keywords: Intellectual property rights, cumulative innovation, persistent leadership, forward protection, non-obviousness requirement

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1 Introduction

This article studies the effects of different intellectual property right (IPR) policies in a quality-ladder model of endogenous growth. In contrast to most models of this type, where entrants undertake all R&D, so that there is leapfrogging, this article analyzes the case in which incumbent firms innovate preemptively in order to prevent being replaced by entrants, and in which there is persistence in leadership.

Empirical studies show that incumbent firms carry out a considerable number of innovations and remain the industry leaders for a sustained period of time. Camerani and Malerba (2007) study patents granted by the European patent office to firms from 6 countries (France, Italy, Germany, Sweden, USA and UK) in 13 sectors between 1990 and 2003. **Table 1** (see Appendix A1) shows the fraction of patents that were granted to technological entrants who patented for the first time in a given sector. Those fractions are relatively small in all sectors and periods (ranging from 0.1 to 0.27 in 2002-03), implying that firms that innovate persistently within one sector obtain a disproportionate share of all patents.

Córcoles and Triguero (2013) study the persistence of innovation using firm level panel data on R&D expenditures and innovation of Spanish manufacturing firms for the period 1990 - 2008. They find that on average 72.6 % of the innovators in one year also innovated in the subsequent year and that 58.1 % also innovated five years later. Persistence was even higher in R&D decisions: on average 87.56 % of the firms that undertook R&D in one year also did so in the following year and 78.55 % also undertook R&D five years later. These and other transition probabilities between (non-) R&D performing and (non-) innovating firms for one year ($t-1$ to t) and five year ($t-5$ to t) transitions are shown in **Table 2** (Appendix A1).

Numerous other empirical studies find high levels of firm innovation persistence when measured with patent, innovation or R&D data.[1]

Garcia-Macia, Hsieh and Klenow (2015) study U.S. manufacturing data from 1963 to 2002. Using a growth model they indirectly infer that most growth resulted from innovation by incumbents and from improvements of existing varieties, carried out

[1] Using an innovation panel data set on German manufacturing and service firms for the period 1994 – 2002, Peters (2009) finds that in manufacturing (services) about 89% (about 70 %) of the innovators in one year also innovated in the next year. Jang, Tsai and Chen (2008) calculate transition probabilities using patent data for the period 1990 - 2003. They find that for the U.S, Japan, South Korea and Taiwan, the probabilities that firms that applied for more than 6 patents in one year also applied for more than 6 patents two years later were in the range between 70 and 80 %.

Foster, Grim and Zolas (2013) find that there is high persistence among the top 200 R&D performing firms in the US. They document that the firms that were among the top 200 in 1976 still made up more than half of the total R&D expenditures of the top 200 firms in 2003 and more than 35% of total US R&D expenditures in 2003.

Le Bas and Scellato (2014) provide an overview over 33 recent empirical papers on firm innovation persistence and classify 25 (2) of them as finding high (low) innovation persistence.

mostly by the incumbents who had previously produced them. Given that intellectual property rights are used to stimulate innovation, it is therefore important to understand how they should be designed in order to encourage cumulative innovation in a context where a large fraction of innovations are carried out by incumbent firms.

The following model setup is used in order to generate persistent leadership: Innovation is assumed to increase the quality of different intermediate goods that are used to produce a final consumption good. There is free entry into R&D in each intermediate good sector, and the R&D productivity of entrants and incumbents is the same. Contrary to most leapfrogging models, it is assumed that the R&D technology is characterized by decreasing returns at the industry level and that incumbents move first in the R&D game.[2] By increasing her own R&D effort, an incumbent can therefore decrease the profitability of R&D undertaken by entrants in her sector and preempt entry. The analysis mainly focuses on the case of quasi-drastic innovations in which an incumbent who has undertaken two successive innovations (is two steps ahead) can charge the unconstrained monopoly price and earns larger profits than an entrant who has to compete with the previous incumbent in an intermediate good market. As incremental profits are lower for an incumbent than for entrants due to the Arrow replacement effect, incumbents have less incentives to innovate than entrants if they do not face any entry pressure. Under free entry, incumbents, however, find it profitable to preempt entry as they value not being replaced and keeping their (two step) lead more than entrants value entry (which brings a one step lead). Therefore, incumbents carry out all the R&D in equilibrium and there is persistent leadership.

The analysis builds on Denicolò (2001), who introduces the preemption mechanism of Gilbert and Newbery (1982) into a quality-ladder growth model, but does not study the role of intellectual property rights. In equilibrium, the amount of R&D that an incumbent undertakes in order to prevent being replaced depends positively on the value of an innovation for an entrant expecting to become the new industry leader upon entry. The prediction that incumbents invest more in R&D if entry pressure increases is supported by an empirical study of Czarnitzki, Etro and Kraft (2014), who use a dataset and a survey from the German manufacturing sector.

Within this setting of persistent leadership, the following IPR policies are analyzed: (i) forward protection (new innovations infringe the IPRs (patents) of previous innovators), (ii) a non-obviousness requirement (minimal inventive step), and (iii) protection against imitation (duration until rivals can copy a protected technology). The analysis generates three main results:

First, granting forward protection reduces growth. Under forward protection, en-

[2] Even without the latter assumption, a similar Walrasian equilibrium can be considered, as explained in footnote 17.

try is discouraged as incumbents can block entrants' innovations. Even though it is assumed that forward protection allows incumbents and entrants to collude in prices, entrants never obtain larger profits than without forward protection and without collusion. Therefore, granting forward protection unambiguously reduces entry pressure, the amount of R&D undertaken by incumbents, and growth. This result differs from that obtained by O'Donoghue and Zweimüller (2004), who study forward protection in the leapfrogging case in which patents for any two succeeding innovations are held by different firms. In their setting, forward protection also discourages innovation by leading to more backloaded profit flows. As entrants, however, benefit more from the collusion effect that allows to increase joint profits than in the case of forward protection, the overall effect that forward protection has on growth can be positive.

The *second* main result is that imposing a non-obviousness (patentability) requirement reduces growth. The reason for this is that such a requirement reduces entry pressure and therefore the amount of R&D that incumbents need to undertake in order to preempt entry and that, for given R&D spending, incumbents find it profitable to select the growth-maximizing step size themselves. This result differs from those obtained in the context of leapfrogging, where imposing a non-obviousness requirement can increase innovation by reducing the threat that innovators lose their monopoly power due to "creative destruction" by marginal follow-on innovations (see O'Donoghue (1998), O'Donoghue and Zweimüller (2004), and Hunt (2004)).

Finally, it is shown that full protection against imitation for any size of the lead maximizes growth. This is the *third* main result. In order to allow for less than full protection, it is assumed that with a certain hazard rate, competitors can copy a given invention and compete away all profits. As in Acemoglu and Akcigit (2012), the general case of state-dependent intellectual property (IP) protection is considered in which the rate at which protection against imitation expires can depend on the size of the lead of an IP holding firm. Unlike in Acemoglu and Akcigit (2012), however, growth is not maximal under state dependent IP protection, but rather in the case of full uniform protection, i.e. if protection never expires.

The last result, however, needs to be qualified: If there is *trade secrecy* in the sense that entrants have to incur some fixed catch-up costs before they can use the state-of-the-art R&D technology, it is shown that incumbents can preempt entry without doing any R&D. In this case, no innovation takes place in equilibrium under full IP protection against imitation[3]. If, however, IP protection expires with a positive probability, so that incumbents lose their lead from time to time, this regularly creates a neck and neck situation with positive innovation incentives where firms try to become the next leader

[3]In order to obtain this result, simplifying assumptions about the form of the R&D technology are made.

in an industry. Because of that, the rate of growth is maximal for an intermediate probability of IP expiration (i.e. if patents are of finite duration). This result differs from that obtained in the case of leapfrogging, where full protection against imitation maximizes growth, even if there are catch-up costs due to trade secrecy.

In the following, the related literature is discussed in more detail.

The question of how antitrust policies should be designed in innovative industries where entrants expect to become the next incumbents is analyzed by Segal and Whinston (2007), who find that entrants should be well protected against incumbents in most cases in order to guarantee that profit flows for successful innovators do not become too backloaded. They therefore identify the same effect as O'Donoghue and Zweimüller (2004), who show that innovation increases if profit flows become more frontloaded. Chu (2009) studies a generalized version of the model of O'Donoghue and Zweimüller (2004) and quantitatively estimates the effect of blocking patents on R&D using US data. O'Donoghue, Scotchmer and Thisse (1998) analyze the role of forward protection in a quality-ladder model with leapfrogging and compare the case of short and broad to long and narrow patents, assuming that forward protection allows firms to collude and that investment opportunities arrive at an exogenous rate.

Bessen and Maskin (2009) and Llanes and Trento (2012) analyze models of sequential innovation in which patents grant blocking power over follow-on innovations and in which licensing is inefficient due to asymmetric information. Assuming that firms can appropriate some surplus in the final goods markets even in the absence of IP protection, they show that innovation can be larger when there is no (forward) IP protection. In both articles, it is assumed that innovation does not lead to the replacement of the previous technology, so that issues like the Arrow replacement effect or the possibility of preemption that arise in a quality-ladder context are not considered.

The main difference between the above-mentioned and the present article is, that the former do not study the case in which incumbents undertake R&D.[4] While several articles have analyzed the conditions under which (some) persistence in leadership can arise in quality-ladder models[5], the role of IP protection has only received little

[4]Segal and Whinston (2007) also study the case of innovation by leaders but do not analyze patent policies in this context.

[5]In most quality-ladder growth models (like Aghion and Howitt (1992)), the case of leapfrogging is analyzed, although incumbents are actually indifferent about their share in total R&D if the R&D sector is competitive and markets are Walrasian, so that there might as well be some persistence in leadership (see Cozzi (2007)). In many continuous-time patent race models (like Reinganum (1983 and 1985)) where marginal R&D costs are increasing at the firm-, but not at the industry level, preemption is not possible and incumbents invest less in R&D than challengers in the standard case with simultaneous moves and drastic innovations. In a similar setup with fixed costs of entering the R&D sector, Etro (2004) finds that in the case where there is free entry and industry leaders move first in the R&D game (are Stackelberg leaders), they do more R&D than entrants so that there is some persistence in leadership. Denicolò (2001), on which the current article builds, analyzes the case where preemption is possible due to decreasing R&D productivity at the industry level and finds that

attention in settings in which incumbent firms also innovate.

Acemoglu and Akcigit (2012) analyze a model of step-by-step innovation in which there is a race between two firms in each sector and where the laggard first has to catch up through duplicative (but non-infringing) R&D before he can undertake frontier R&D. They argue that IP protection should be stronger for firms that have a larger technological lead over their rivals. In their model, IP protection affects innovation by affecting the incremental profits that the two incumbent firms obtain from moving one step ahead.[6] Reducing IP protection solely for firms with a smaller lead can therefore increase innovation incentives for laggards and the rate of growth by increasing incremental profits. In contrast, there is free entry into (frontier) R&D in the model analyzed here, and while reducing IP protection in a state-dependent way can induce firms with a lower lead to innovate at a faster rate, it comes at the cost of reducing the value of entering and the amount of R&D incumbents need to undertake in order to preempt entry. This entry-discouraging effect is so strong that, contrary to Acemoglu and Akcigit (2012), reducing IP protection in a state-dependent way always reduces growth compared to the case of full uniform IP protection. While this result accords with the findings of first-generation quality ladder models with free entry (based on Aghion and Howitt, 1992), it is derived in a setup where, unlike in these models, innovation rates can be lead dependent, giving rise composition effects that otherwise only occur in step-by step models and are absent in the case of leapfrogging.

Denicolò and Zanchettin (2012) study a growth model with non-drastic innovations and constant returns to R&D in which incumbents and entrants simultaneously decide about the sizes of their R&D investments and in which there is no preemption. Unlike the present article, they assume that the R&D productivity of incumbents is larger than that of entrants so that incumbents might find it profitable to innovate, in spite of the Arrow replacement effect. They find that there can be stochastic leadership cycles, meaning that incumbents are not replaced immediately, but undertake some R&D and (on average) advance to a certain lead before they are replaced. The authors show that requiring successful outsiders to pay a licensing fee to the previous incumbent in this setting can increase the rate of growth, even if such forward protection does not facilitate collusion. Introducing state-dependent patent breadth (modeled as varying price caps) is shown to affect the share of R&D the incumbent undertakes, but it has no effect on the rate of growth.[7]

there is persistent leadership if innovations are non-drastic and incumbents move first. Fudenberg, Gilbert, Stiglitz and Tirole (1983) analyze the conditions under which preemption is possible and when there can be competition in patent races.

[6] It is assumed that parameters are such that the laggard never stops innovating and never drops out of the market.

[7] Other articles in which (some) persistence in leadership results because incumbents are assumed to be more productive in doing R&D than entrants are Segerstrom and Zolnierek (1999) and Segerstrom

The article is structured as follows: In *Section* 2, the model is introduced and in *Section 3* the equilibrium is derived. *Section 4* studies the effects of different IPR policies like forward protection (*Section 4.1*), a non-obviousness requirement (*Section 4.2*) and the case of expiring IP protection (*Sections 4.3* and *4.4*). *Section 5* concludes. Proofs are collected in *Appendix A* and *B*.

2 Model Setup and Equilibrium

This analysis is based on the quality-ladder growth model of Barro and Sala-i-Martin (2004). While Denicolò (2001) analyzes a one-sector version of this model, there is a continuum of sectors in the version of the model studied here. This allows to have non-stochastic growth, but does not affect the qualitative results.

2.1 Preferences

There is a measure one of identical, infinitely lived individuals. Time is continuous and intertemporal preferences are given by

$$U(\tau) = \int_{t=\tau}^{\infty} c\,(t)\, e^{-r(t-\tau)} dt \tag{1}$$

with $c(t)$ denoting consumption at time t. As intertemporal preferences are linear, the equilibrium rate of interest is constant and equal to the rate of time preference r. Moreover, individuals are risk neutral, so that all assets must yield the same instantaneous expected net rate of return r in equilibrium[8].

2.2 Goods Production

There is a final good y which can be consumed, used for research or used to produce intermediate goods, of which there is a continuum of measure one. The quality of each intermediate good ω can be increased step-wise through innovation. It is assumed that each innovation increases the quality by the factor $q > 1$. Normalizing the initial quality of all intermediate goods at time 0 to one, the highest available quality of an intermediate good ω at time t is given by $q^{k(\omega,t)}$, with $k(\omega, t)$ indicating the number of innovations that have been achieved in industry ω by time t.

(2007). Acemoglu and Cao (2010) assume that only incumbents are capable of undertaking incremental innovations, while only entrants can attain radical innovations. Athey and Schmutzler (2001) analyze conditions under which dominance of an incumbent firm arises if there is ongoing firm-specific investment, but no entry into frontier R&D.

[8] The qualitative results of the paper are robust to the introduction of more general intertemporal preferences (see the end of Appendix A2).

The final good is produced using labour L and intermediate goods of vintages $v \in \{0, 1, ..., k(\omega, t)\}$ and quantities $x(v, \omega, t)$ according to the following constant returns production function:

$$y(t) = L^{1-\alpha} \int_{\omega=0}^{1} \left[\sum_{v=0}^{k(\omega,t)} q^v x(v, \omega, t) \right]^{\alpha} d\omega, 0 < \alpha < 1$$

In order to produce one unit of any type of intermediate good, one unit of the final good is needed as an input. The final good and also labour are supplied in perfectly competitive markets while intermediate goods can be protected by intellectual property rights (IPRs). If more than one firm supplies intermediate goods in sector ω, there is price competition. Since different vintages of intermediate goods are perfect substitutes within their sector, only the newest vintage $v = k(\omega, t)$ with the highest available quality is used in equilibrium. Each individual inelastically supplies one unit of labour at each point in time so that the total labour supply is given by $L = 1$. Therefore, the final good production function reduces to

$$y(t) = \int_{\omega=0}^{1} \left[q^{k(\omega,t)} x(k, \omega, t) \right]^{\alpha} d\omega \tag{2}$$

The final good is used as the numeraire.

2.3 The R&D Sector

Given k is the newest vintage of the intermediate good in sector ω, the next vintage $k+1$ can be invented if R&D is undertaken. The instantaneous arrival rate $\phi_\omega(k+1)$ with which the innovation occurs is given by

$$\phi_\omega(k+1) = min \left\{ \left(\frac{n_\omega}{cg^k} \right)^{\frac{1}{1+\epsilon}}, \bar{\phi} \right\} \tag{3}$$

where n_ω denotes the total amount of the final good used for R&D in sector ω and $g \equiv q^{\frac{\alpha}{1-\alpha}} > 1$. $c > 0$ is an inverse measure of R&D productivity and $\epsilon > 0$ indicates the extent of decreasing returns to R&D at the industry level. $\bar{\phi} > 0$ is an upper bound that the innovation arrival rate cannot exceed due to technological reasons. This upper bound only becomes relevant in *Sections 4.3* and *4.4* of the analysis where, for reasons of tractability, the limit case of constant returns to R&D ($\epsilon \to 0$) is considered. The total R&D costs in terms of the final good are therefore given by

$$C(\phi_\omega(k+1)) = \begin{cases} cg^k \phi_\omega^{1+\epsilon} \text{ if } \phi_\omega < \bar{\phi} \\ \infty \text{ if } \phi_\omega > \bar{\phi} \end{cases} \tag{4}$$

In order to obtain a balanced growth path, these costs increase by the factor g if the vintage (quality-level) k of the good increases by one step due to an innovation[9]. There is free entry into frontier R&D, meaning that all firms have access to the same R&D technology and are capable of undertaking frontier R&D in all sectors without having to duplicate previous innovations of other firms first.

As $\epsilon > 0$, marginal and average R&D costs increase in ϕ_ω. If more than one firm does R&D, it is assumed that firm i obtains the innovation with arrival rate $\phi_{\omega i} = \beta_{\omega i}\phi_\omega$ if its share in total R&D costs in sector ω is given by $\beta_{\omega i}$. The R&D costs of firm i are therefore given by $C_i(\phi_{\omega i}(k+1)) = cg^k\phi_{\omega i}\phi_\omega^\epsilon$. A firm therefore increases the R&D costs of all other firms if it increases the (industry) arrival rate ϕ_ω by increasing its own R&D spending. A reason for such decreasing returns might be that it is impossible to perfectly coordinate all R&D activities, implying that the probability of duplicative research (even within one firm) increases if more R&D is undertaken in a given sector[10].

It is assumed that incumbent firms have a first-mover advantage in R&D due to their visibility, meaning that entrants at each instant of time first observe the R&D effort undertaken by an incumbent before they decide about their own R&D effort.

2.4 Intellectual Property (IP) Protection

It is assumed that an inventor of a new vintage of a good gets IP protection on it, allowing her to exclude other firms from producing her generation of the good. In the first part of the analysis, IP protection is assumed to be of infinite duration (e.g. patents are infinitely lived). The effects of forward protection (inventors infringe on previously issued IPRs), of a non-obviousness requirement and of expiring IP protection are discussed in separate sections.

2.5 Equilibrium Concept and Steady State

It is assumed that the perfectly competitive final good and labour markets clear at any point in time. In the imperfectly competitive intermediate goods markets, firms behave strategically in the races for new innovations and Markov Perfect Equilibria (MPE) are analyzed in which strategies only depend on payoff-relevant state variables[11].

The analysis focuses on steady states. While productivity-enhancing innovations

[9] As will be shown later on, every innovation increases profits by the factor g. In order to obtain a balanced growth path on which ϕ_ω is constant, R&D costs therefore have to increase by the same factor.

[10] In the case of duplication, an increase in the R&D effort exerted by one firm increases the expected R&D costs per obtained IPR (patent) of rivals as either only one firm gets IP protection on the innovation or as the profits derived from the IPR have to be split among several innovators.

[11] Non-stationary endogenous cycle equilibria that can occur in quality-ladder models are not considered.

arrive at random intervals within a given intermediate good industry, the economy grows smoothly as there is a continuum of intermediate good industries, implying that there is no randomness in the production of the final good due to the law of large numbers.

In a steady state, the output of the final and of all intermediate goods, consumption, the wage rate and total R&D expenditures all grow at a constant rate. In the case where IP protection expires, the fraction of industries in which the industry leader leads by a certain step size and the average innovation rate within an industry are constant.

3 General Equilibrium

3.1 Goods Market Equilibrium

Profit maximization of perfectly competitive firms in the final goods sectors implies that the demand for the latest vintage $k(\omega, t)$ of an intermediate good of type ω as a function of its price $p(k, \omega, t)$ is given by

$$x(k, \omega, t) = \alpha^{\frac{1}{1-\alpha}} g^k p(k, \omega, t)^{-\frac{1}{1-\alpha}} \tag{5}$$

The demand for intermediate good ω is independent of the prices of intermediate goods in other sectors. The demand function has the constant elasticity $\frac{1}{1-\alpha}$ and shifts up by the factor $g \equiv q^{\frac{\alpha}{1-\alpha}}$ if an innovation, which increases k by one step, occurs. Producers of different vintages of intermediate goods within the same sector can be treated as if they were supplying the same good, measured in efficiency units, but at a different cost (in terms of the final good) per unit: the costs of vintage $k - l$ are therefore q^l times larger than those of vintage k.

The firm who supplies the newest vintage k of the intermediate good in industry ω maximizes profits under the constraint that the price has to be so low that no rival firm in the same industry finds it profitable to enter. If the industry leader has obtained l successive IP protected innovations and therefore has a lead of l steps, entry is deterred if $p(k, \omega, t) \leq q^l$ holds[12].

Profits, that are given by $\pi(k, \omega, t) = (p(k, \omega, t) - 1)\, x(k, \omega, t)$, with $x(k, \omega, t)$ taken from equation 5, are therefore maximized for the unconstrained monopoly price $p_m = \frac{1}{\alpha}$ if $\frac{1}{\alpha} \leq q^l$ and for the limit price $p = q^l$ if $q^l < \frac{1}{\alpha}$. Let s denote the minimal lead size which allows a leading firm to engage in (unconstrained) monopoly pricing. Formally,

[12]The marginal costs of producing an efficiency-equivalent to one unit of a good of vintage k (the marginal costs of which are one) are equal to q^l for producers of vintage $k - l$, so that they cannot break even if the price lies below these marginal costs.

s is an integer implicitly defined by the following condition:

$$q^{s-1} < \frac{1}{\alpha} \leq q^s$$

Equilibrium profits can be derived as:

$$\pi_m(k, \omega, t) = (1-\alpha)\, \alpha^{[(1+\alpha)/(1-\alpha)]} g^{k(\omega,t)} \equiv \pi_m g^{k(\omega,t)}, l \geq s \tag{6}$$

$$\pi_l(k, \omega, t) = \left(q^l - 1\right) \alpha^{1/(1-\alpha)} q^{-\frac{l}{1-\alpha}} g^{k(\omega,t)} \equiv \pi_l g^{k(\omega,t)}, l < s \tag{7}$$

Profits increase in the vintage k of the good and increase in the lead l relative to the closest competitor if $l < s$, but are independent of l if $l \geq s$, i.e. if firms are unconstrained monopolists. In order to simplify the notation, the definitions $\pi_m \equiv (1-\alpha)\, \alpha^{\frac{1+\alpha}{1-\alpha}}$ and $\pi_l \equiv \left(q^l - 1\right) \alpha^{\frac{1}{1-\alpha}} q^{-\frac{l}{1-\alpha}}$ are used in the following and referred to as "normalized profits".

Incremental Profits

In order to understand a firm´s incentive to innovate, one needs to know how undertaking an innovation changes the profits of a firm depending on the size of the firm´s initial lead. Taking into account that an innovation increases the vintage and also the lead l by one step, **incremental profits** are given by

$$\triangle\pi(k, l, \omega, t) = \pi(k+1, l+1, \omega, t) - \pi(k, l, \omega, t)$$

The following Lemma shows how, for a given vintage of an intermediate good, incremental profits depend on the initial lead of a firm in the industry.

Lemma 1. ***a****): If $q < \frac{1}{\alpha} \leq q^2$ (**Condition A**), incremental profits fall in the initial lead l if $0 \leq l \leq 2$. For $l \geq 2$, firms are unconstrained monopolists and incremental profits are independent of l.*

b*): Given that $q^s = \frac{1}{\alpha}$ $(s \geq 1)$, incremental profits decrease in l if $0 \leq l \leq s$. For $l \geq s$, incremental profits are independent of l.*

c*): For any step size $q > 1$, incremental profits are larger for entrants $(l = 0)$ than for unconstrained monopolists (this also holds if $q \geq \frac{1}{\alpha}$, in which case innovations are drastic and a one step lead suffices to become an unconstrained monopolist).*

Proof. See Appendix B1 □

Given that $q < \frac{1}{\alpha} \leq q^2$ (Condition A) holds, innovations are **quasi-drastic**. This means that each single innovation is non-drastic so that firms with a one step lead engage in limit pricing, while a firm with a two or more step lead is an unconstrained monopolist. In this case, incremental profits are largest for entrants ($l = 0$), lower for incumbents with a one step lead ($l = 1$) and even lower for unconstrained monopolists with an initial lead of two or more steps ($l \geq 2$). Part b) of the Lemma (used in *Section 4.1)* shows that the result that undertaking an innovation increases profits less for firms with a larger initial lead l also holds if the step size is such that firms become unconstrained monopolists with a lead of exactly s steps. Moreover, Part c) of Lemma 1 (used in *Sections 4.1* and *4.2*) shows that for any step size, incremental profits are larger for entrants than for unconstrained monopolists (incumbents who have already reached the maximal lead).

The feature that incremental profits decrease in the size of the lead arises in many contexts and more general settings and is the source of the "Arrow replacement effect" (Arrow (1962)) and the "escape competition effect" (Aghion et al. (2001)), which are widely discussed in the literature[13] .

In order to keep the derivations an the exposition simple, the following analysis focuses (unless otherwise stated) on the case of quasi-drastic innovations which arises under Condition A.

3.2 Innovation Race

The expected value of being the leading firm that supplies vintage k of the intermediate good in industry ω is denoted by $V_1(\omega, k, t)$ if the firm has a one step lead and by $V_2(\omega, k, t)$ if the firm has a lead of two or more steps. $V_1(\omega, k, t) < V_2(\omega, k, t)$ must hold as firms with a one step lead have lower profits in period t ($\pi_1(\omega, k, t) = \pi_1 g^{k(\omega,t)} < \pi_2(\omega, k, t) = \pi_m(\omega, k, t) = \pi_m g^{k(\omega,t)}$) and can only catch up and obtain a more profitable two step lead if they undertake an innovation in the future. Along a balanced growth path, the expected value of being the leading firm with an l step lead increases by the factor g if an innovation takes place, so that $V_l(\omega, k+1, t) = gV_l(\omega, k, t)$.

In the following, the equilibrium innovation behavior in an intermediate good industry ω is studied. The indices ω and t are dropped for convenience as there are no interactions between industries and as time only affects payoffs in an indirect way by affecting the newest vintage k.

Suppose that k is the newest vintage of an intermediate good in an industry. Then, the expected value that an entrant obtains from introducing generation $k+1$ is given by $V_1(k+1) = gV_1(k)$ as it brings a one step lead over the previous incumbent.

An entrant i chooses the innovation arrival rate ϕ_i in order to maximize expected

[13] Denicolò (2001), on which this article builds, however, does not prove that incremental profits fall in the lead in the setting analyzed here.

discounted profits

$$V_0(k) = \phi_i V_1(k+1) - cg^k \phi_i \phi^\epsilon$$

It is therefore profitable for entrants to do R&D and to select $\phi_i > 0$ as long as the average (industry) R&D costs, that are given by $a(\phi) = cg^k\phi^\epsilon$ ($= \frac{C(\phi)}{\phi}$, see equation 4), do not exceed $V_1(k+1)$[14]. As there is free entry into R&D, $V_1(k+1) \leq cg^k\phi^\epsilon$ therefore has to hold as entrants would otherwise find it profitable to do more R&D and to increase ϕ.

The case where $V_1(k+1) < cg^k\phi^\epsilon$ can only hold if the incumbent finds it profitable to do all of the R&D in the industry and to increase the innovation rate ϕ beyond the point where the average R&D costs start to exceed the value of an innovation for an entrant and where entrants drop out of the innovation race. This would require that, absent any threat of entry, the incumbent values obtaining the next innovation more than entrants do. This is, however, not the case as, due to the replacement effect, incremental profits are lower for the incumbent than for entrants, independently of whether the incumbent has an initial lead of one or two steps (see Lemma 1)[15]. Therefore $V_1(k+1) < cg^k\phi^\epsilon$ does not hold in the case studied here[16] and the following free entry condition needs to hold with equality.

$$V_1(k+1) = cg^k(\phi^*)^\epsilon \tag{8}$$

Rearranging this condition gives $\phi^* = \left(\frac{V_1(k+1)}{cg^k}\right)^{\frac{1}{\epsilon}}$, implying that the equilibrium innovation rate ϕ^* in an industry is an increasing function of the value of an innovation for an entrant, $V_1(k+1)$. But the fact that $V_1(k+1)$ determines ϕ^* does not imply that entrants actually carry out R&D in equilibrium.

Persistent Leadership

As it is assumed that incumbents move first (are Stackelberg leaders) in the innovation race, entrants adjust their R&D spending after observing the level of R&D that the incumbent undertakes. As equation 8 always holds, one unit of R&D undertaken by the incumbent crowds out one unit undertaken by entrants, leaving the industry

[14]The average and not the marginal costs in the corresponding industry ω matter as a single firm does not take into account that by increasing its own R&D effort it increases the R&D costs of all other firms, so that it still finds entry profitable if the marginal (industry) costs are higher than $V_1(k+1)$, as long as the average costs do not exceed $V_1(k+1)$.

[15]Absent any threat of entry, the incumbent takes into account that by increasing the innovation rate, she not only increases the chance of discovering the next vintage of the good but also the chance of displacing her currently supplied vintage from the market, which makes her less willing to invest in R&D than entrants. A formal proof of this argument is provided in Appendix B2.

[16]In *Sections 4.3* and *4.4*, cases are discussed in which incumbents with a one step lead have higher incentive to innovate than entrants.

innovation rate ϕ^* unchanged. An incumbent can therefore reduce the instantaneous probability of replacement from ϕ^* to $(1-\beta)\phi^*$ if she bears the costs $\beta c g^k (\phi^*)^{1+\epsilon}$ and if she innovates herself with arrival rate $\beta\phi^*$.

While the R&D costs are the same for entrants and the incumbent, the incumbent values not being replaced and obtaining the next innovation herself more than an entrant values entry: while successfully innovating entrants have to compete with the previous incumbent in the intermediate good market and only get the value $V_1(k+1)$ upon entry, the incumbent gets the larger value $V_2(k+1)$ in the case of a successful innovation as it guarantees her unconstrained monopoly power.

With the total innovation rate ϕ^* pinned down by the free entry condition, incumbents lose their current value of incumbency, which is given by $V_2(k)$ in the case of a two step lead and by $V_1(k)$ in the case of a one step lead with the hazard rate ϕ^*. As this replacement rate is independent of the share of R&D and of the innovation rate $\beta\phi^*$ of the incumbent, the incumbent has incentives to increase her R&D effort as long as $\beta < 1$ as she values obtaining the next innovation herself more than entrants do. Because of this "efficiency effect" (that was first analyzed by Gilbert and Newbery (1982) and introduced in the growth model presented here by Denicolò (2001)), the incumbent therefore fully preempts entry by doing as much R&D as needed to push average R&D costs up to the value of an innovation for an entrant.

Lemma 2. *Given that the incumbent moves first in the innovation race, she undertakes exactly as much R&D as needed to discourage R&D of entrants in her industry. Consequently, only the incumbent innovates and there is persistent leadership*[17]

Proof. A formal proof is provided in Appendix B2 □

As the same reasoning applies when incumbents need a lead of more than two steps in order to obtain unconstrained monopoly power ($s > 2$), there is also persistent leadership in this more general case.

3.3 Equilibrium Innovation and Growth

In order to derive the equilibrium innovation rate ϕ^* from the free entry condition $\phi^* = \left(\frac{V_1(k+1)}{cg^k}\right)^{\frac{1}{\epsilon}}$, we need to first derive $V_1(k+1)$, the value of an innovation for

[17]The equilibrium analyzed here can either be interpreted as one where the incumbent is a Stackelberg leader in the R&D game or as a Walrasian equilibrium where the total demand for R&D inputs is equal to the supply and in which the auctioneer allocates all R&D to the incumbent who is willing to pay at least as much for it as the entrants. As the entrants get zero profits in equilibrium, they are indifferent about the amount of R&D that they undertake and, if an incumbent would do less R&D than the equilibrium level, the Walrasian auctioneer would simply assign a larger amount of R&D to entrants in order to obtain the equilibrium (see Cozzi (2008) for a more detailed discussion).

A similar equilibrium might also be obtained as the outcome of an auction in which incumbents and entrants simultaneously bid for R&D inputs.

an entrant. As successful entrants anticipate to become the new incumbents and to extend their lead to two and more steps, this value again depends on the value $V_2(k+2) = g^2 V_2(k)$ of having a lead of two or more steps. $V_2(k)$ can be derived from the following arbitrage condition:

$$rV_2(k) = \pi_m g^k - cg^k \left(\phi^*\right)^{1+\epsilon} - \phi^* V_2(k) + \phi^* V_2(k+1)$$

The first term on the right hand side indicates per period profits derived from supplying vintage k of the intermediate good and the second term the R&D costs of the incumbent who does the entire R&D. The third and fourth terms on the right hand side indicate that, if an innovation occurs with the arrival rate ϕ^*, the firm stops supplying vintage k and loses the value $V_2(k)$, but at the same time starts supplying vintage $k+1$ of the intermediate good and gains the value $V_2(k+1)$. Replacing $V_2(k+1) = gV_2(k)$ and solving for $V_2(k)$ gives

$$V_2(k) = \frac{\pi_m g^k - cg^k \left(\phi^*\right)^{1+\epsilon}}{r - \phi^* \left(g-1\right)} \tag{9}$$

In the following, it is assumed that $r > \phi^*(g-1)$ (**Condition B**) holds in order for $V_2(k)$ to be bounded[18]. The value of being an incumbent with a one step lead supplying vintage k of the good can be derived from the arbitrage condition

$$rV_1(k) = \pi_1 g^k - cg^k \left(\phi^*\right)^{1+\epsilon} - \phi^* V_1(k) + \phi^* V_2(k+1)$$

which takes into account that this incumbent initially gets the lower profits $\pi_1 g^k$ and then advances to a two step lead with the next innovation. This arbitrage condition can be solved for

$$V_1(k) = \frac{\pi_1 g^k - cg^k \left(\phi^*\right)^{1+\epsilon} + \phi^* V_2(k+1)}{r + \phi^*}$$

Inserting $V_2(k+1) = gV_2(k)$ from equation 9 into this condition, plugging $V_1(k+1) = gV_1(k)$ into Equation 8 and dividing by g^k, the free entry condition can be written as:

$$\frac{g\left(\pi_1 - c\left(\phi^*\right)^{1+\epsilon}\right)}{r+\phi^*} + \frac{\phi^*}{r+\phi^*} \frac{g^2\left(\pi_m - c\left(\phi^*\right)^{1+\epsilon}\right)}{r - \phi^*\left(g-1\right)} = c\left(\phi^*\right)^{\epsilon} \tag{10}$$

This equation implicitly defines the equilibrium innovation rate ϕ^* in a given sector.

Proposition 1. *The equilibrium innovation rate ϕ^* is the same in all sectors and the equilibrium rate of growth G^* of the economy is given by $G^* = (g-1)\,\phi^*$.*

Both ϕ^* and G^* increase in π_1, π_m and g and decrease in r and c. In the case where

[18] If this condition is violated, the interest rate is lower than the rate of growth of the economy and utility becomes unbounded (see the proof of Proposition 1).

incumbents need a lead of $s > 2$ steps in order to obtain unconstrained monopoly power, ϕ^* and G^*, moreover, increase in the (normalized) profits π_l that incumbents obtain for lead sizes $1 < l < s$. In order for an equilibrium to exist, $r > G^*$ (Condition B) must hold, which is always satisfied if r is sufficiently large.

Proof. See Appendix A2 □

The intuition for these results is straightforward: Absent the threat of entry incumbents have less incentives to innovate than entrants due to the replacement effect and only conduct as much R&D as needed to preempt entry. This preemptive R&D level depends positively on entry pressure, i.e. on the incentives of entrants to undertake R&D in order to enter and to become the next incumbent. Entry pressure decreases in the R&D costs c and increases in the value of an innovation for an entrant which depends positively on (normalized) profits π_1 (or more generally π_l) and π_m and negatively on the rate of interest r due to a discounting effect. When g increases, entry pressure increases as each innovation then leads to a larger increase in profits. Taking into account that $g \equiv q^{\frac{\alpha}{1-\alpha}}$ and that π_1 (and also π_l for $l < s$) depend positively on q, ϕ^* therefore increases in the innovation step size q. As the equilibrium rate of growth of output is given by $G^* = (g-1)\phi^*$, all parameters have the same qualitative effect on G^* that they have on ϕ^*.

In the following sections, the effects that different IP policies have on the rate of growth are analyzed.

4 The Effects of different IPR Policies

4.1 Forward Protection

It is now assumed that an entrant´s IPR that covers vintage $k+1$ of a certain intermediate good infringes on the IPR of the previous vintage k of the same good (but not on vintage $k-1$), so that the incumbent can block the production of vintage $k+1$ of the good by an entrant. An IPR therefore grants protection against future superior innovations, labeled "forward protection". It is assumed that there is still a research exemption so that entrants are permitted to undertake frontier R&D as long as they do not produce the invented goods (for an analysis of the case where there is no research exemption, see *Section 4.4.2*). Moreover, it is assumed that the antitrust authority allows entrants and incumbents to collude in prices if they negotiate a licensing agreement in the case of blocking IPRs but that collusion is prohibited if there is no forward protection. Incumbents are assumed to always collude with entrants if they find it profitable to do so once entry has occurred. Therefore, they cannot ex ante commit not to collude ex post.

Proposition 2. *Forward protection reduces growth. This result generalizes to the case where $s > 2$ innovation steps are needed in order to obtain unconstrained monopoly power and to the case where $q^s = \frac{1}{\alpha}$, $s \geq 2$ and where forward protection allows to block $R > 1$ future innovations.*

Proof. See Appendix A3 □

This is the first main result of the article and the intuition for the simple case is the following: while absent forward protection entrants get the profits $\pi_1 g^{k+1}$ if they obtain a one step lead, they maximally get the incremental profits $\pi_m(g^{k+1} - g^k)$ by which their innovation increases monopoly profits in the case of forward protection. This is because the incumbent has the possibility to keep producing her old vintage and earning profits $\pi_m g^k$ (until the entrant obtains a two-step lead) if she blocks the entrant from producing vintage $k+1$ and is therefore only willing to accept a collusive licensing agreement with the entrant if it gives her at least the same, leaving less than $\pi_m(g^{k+1} - g^k)$ per period for the entrant. As $\pi_m(g^{k+1} - g^k) < \pi_1 g^{k+1}$ holds due to declining incremental profits (Lemma 1), forward protection therefore reduces profits of entrants in the period where they have a one step lead. Once they have obtained a two step lead, profits are again given by $\pi_m g^{k+1}$, like in the case without forward protection. Therefore, forward protection reduces the value of an innovation for an entrant, entry pressure and growth.

This result differs from the ones obtained in models where there is leapfrogging, like O'Donoghue and Zweimüller (2004) and O'Donoghue, Scotchmer and Thisse (1998). In these models, firms never lead by more than one step so that, absent forward protection, profits for entrants are given by $\pi_1 g^{k+1}$ until they are replaced by the next innovation, $k+2$. Forward protection then has two effects: by giving blocking power to the incumbent, it forces the entrant to pay licensing fees to the previous incumbent, but allows him to recoup licensing fees from the next entrant. For constant markups, blocking power then leads to a more backloaded (and therefore more heavily discounted) profit flow for an entrant and can be shown to reduce the incentives to innovate. At the same time, forward protection allows firms to collude and to increase the joint profits that accrue to entrants and replaced incumbents who still get licensing revenues. If the backloading effect is not too large (because the interest rate is low or because entrants are strong bargainers), forward protection can therefore increase the value of an innovation for an entrant and the equilibrium innovation rate.

In the case of persistent leadership, this cannot happen for the following reasons: forward protection then also has a blocking effect as entrants have to pay licensing fees to previous incumbents, but do not expect to obtain any licensing fees in the future as they plan to become the new industry leaders and to preempt all entry in the future[19].

[19]Forward protection, however, also brings delayed rewards in the form of reduced R&D costs as

Moreover, collusion only allows to increase joint profits in the case where an entrant has obtained a one step lead, but not anymore when he has advanced to a two step lead, as he then obtains unconstrained monopoly profits even absent forward protection. As an incumbent with blocking power never leaves a share of the collusive profits that exceeds $\pi_1 g^{k+1}$ to the entrant when $l = 1$, entry is unambiguously discouraged under forward protection as the entrant does not reap any of the collusive gains.

It should be noted that simply permitting entrants and incumbents to collude in prices (or to sell their IPRs or exclusive licenses to each other) without granting forward protection unambiguously increases growth in both the case of persistent leadership and of leapfrogging[20]. The reason for this is that by granting blocking power to incumbents, forward protection greatly discourages entry by increasing the incumbents' bargaining power.

The analysis is restrictive in the sense that it only focuses on cumulative innovations along given quality ladders without allowing for product innovations that increase the set of goods the quality of which can be increased. If also the inventions of the first vintages ($k = 1$) in each industry were endogenized, forward protection would encourage such product innovations as it increases profits for first innovators by giving them blocking power over follow-on innovations. Then, a trade-off would arise as forward protection would encourage product innovation while it would discourage quality-improving follow-on innovations (see Chu, Cozzi and Galli (2012) and also Denicolò (2002)). If growth is mainly driven by improvements of existing varieties as Garcia-Macia, Klenow and Hsieh (2015) argue, such effects that intellectual property policy might have on product innovations might, however, be relatively small.

4.2 Non-Obviousness Requirement

So far it has been assumed that firms cannot influence the size of the quality improvement q that an innovation brings and that q is such that a two (or s) step lead allows to charge the unconstrained monopoly price $\frac{1}{\alpha}$. This section looks at the case where firms are capable of targeting different innovation step sizes $\mu > 1$ at different costs. Unconstrained monopoly power (a quality difference larger than $\frac{1}{\alpha}$ relative to the closest competitor) can therefore be attained by either undertaking many little

it reduces the entry-deterring innovation rate ϕ^*. The blocking effect of forward protection is similar to the effect that imposing entry fees would have: It is simple to show that such fees that entrants have to pay upon entry into a product market lead to reduced entry pressure and growth.

[20]In the case of persistent leadership collusion allows to avoid the phase of competition when entrants have a one step lead and to increase their profits above the level $\pi_1 g^{k+1}$ that they can obtain absent collusion. This increases entry pressure and growth. As entry no longer reduces joint profits in the case of collusion, the incumbent values not being replaced and obtaining the next innovation equally much as the entrants value entering, as she still gets a share of the surplus in the case of entry. Therefore, the incumbent is indifferent about her share in total R&D and there need not be persistent leadership anymore.

steps (small μ) or a few large steps (large μ). Incremental profits resulting from an innovation are now endogenous and increase in the step size μ. In this setting, there is an additional instrument that intellectual property policy can use: a non-obviousness requirement that sets a lower bound $\underline{\mu}$ on the inventive step below which an inventor cannot obtain IP protection.

It is assumed that the R&D technology is the same in all intermediate good sectors ω. The R&D costs in terms of the final good for a firm $i = 1$ that wants to improve upon the currently highest quality $\overline{q}_\omega(t)$ of an intermediate good by the factor μ_1 (> 1) and that targets the arrival rate ϕ_1 are given by

$$C_i(\omega, t) = c \left(\overline{q}_\omega(t)\right)^{\left(\frac{\alpha}{1-\alpha}\right)} \phi_1 \lambda(\mu_1) \left(\sum \phi_i \lambda\left(\mu_i\right)\right)^{\epsilon}$$

with $\frac{\partial \lambda(\mu)}{\partial \mu} > 0$ and with $\sum \phi_i \lambda\left(\mu_i\right)$ indicating the overall R&D effort in sector ω. For given overall R&D spending, targeting a larger inventive step μ_i therefore implies a lower hazard rate ϕ_i. Given that $\epsilon > 0$, R&D costs for firm i increase in the overall R&D effort in the industry. Due to this assumption, an incumbent can again preempt entry by undertaking a large enough amount of R&D in order to increase the entrants' R&D costs[21]. In order to allow for balanced growth, the growth factor $\left(\overline{q}_\omega(t)\right)^{\left(\frac{\alpha}{1-\alpha}\right)}$ is included due to which R&D costs increase in line with profits when the quality $\bar{q}_\omega$ of the newest vintage of an intermediate good ω increases. It is assumed that it is prohibitively costly to target a drastic innovation which would give an entrant unconstrained monopoly power in a single step.

In this more general setting, the efficiency effect is present again: as for any size of the lead an incumbent earns larger profits due to larger monopoly power, she again values not being replaced and obtaining the next innovation herself more than an entrant values entry. While entrants and incumbents might find it optimal to select different step sizes for their innovations, it is never more costly for incumbents to innovate than for entrants as they always have the option to select the same step size as entrants. Therefore, incumbents again find it profitable to preempt entry and there is persistent leadership (as in Denicolò (2001)).

Due to the fact that for any step size, incremental profits are lower for unconstrained monopolists than for entrants (see Lemma 1, Part c)), incumbents who have advanced to a lead that is large enough to earn unconstrained monopoly profits never

[21] It is therefore assumed that only the total R&D effort of the incumbent matters for the entrants' R&D costs, but not the combination of step size and innovation rate that the incumbent chooses. This assumption seems plausible in settings where the price of (uniform) R&D inputs increases if demand for them increases and where it is possible to preempt entry by simply buying enough of these inputs in order to increase their price. If, however, the incumbent´s R&D activity affected the entrants' R&D profitability by increasing the risk of duplication, also the combination of step size and innovation rate would matter.

want to do more R&D than needed to preempt entry (as, whatever step size is most profitable for incumbents, incremental profits for entrants are larger if they select the same step size).

In order to simplify the notation, the indices ω and t are omitted in the following. Expected profits of an entrant who targets step size μ_e and innovation rate ϕ_e in order to obtain a one step lead over the initial quality level $\bar{q}$ are given by

$$V_0(\bar{q}) = \phi_e V_1(\bar{q}, \mu_e) - c\bar{q}^{\left(\frac{\alpha}{1-\alpha}\right)} \phi_e \lambda(\mu_e) \left(\phi_e \lambda(\mu_e) + \sum_{i \neq e} \phi_i \lambda(\mu_i) \right)^{\epsilon}$$

with $V_1(\bar{q}, \mu_e)$ denoting the maximal value that the entrant can derive from obtaining a one step lead. $V_1(\bar{q}, \mu_e)$ not only depends on the size μ_e of the first inventive step, but on the whole path of inventive steps and innovation rates that the entrant finds optimal to choose in the future in order to advance his lead and to preempt entry of others. The free entry condition ($V_0(\bar{q}) = 0$) that must be satisfied in an industry where the incumbent is an unconstrained monopolist and undertakes all the R&D is therefore given by

$$V_1(\bar{q}, \mu_e) = c\bar{q}^{\left(\frac{\alpha}{1-\alpha}\right)} \lambda(\mu_e) \left(\phi_m \lambda(\mu_m)\right)^{\epsilon} \tag{11}$$

This condition pins down the R&D effort $\phi_m \lambda(\mu_m)$ that the incumbent undertakes in order to preempt entry as an increasing function of the maximized and normalized value that an innovation has for an entrant, $\tilde{V}_1 \equiv \frac{V_1(\bar{q}, \mu_e)}{\bar{q}^{\frac{\alpha}{1-\alpha}} \lambda(\mu_e)}$.

Without any non-obviousness requirement, entrants choose the sequence and sizes of inventive steps (including the first one, μ_e) that maximize the present discounted value of their R&D activity (and therefore also $\tilde{V}_1$). Therefore, any binding non-obviousness requirement that restricts the R&D decisions of entrants and of incumbents who have not yet reached a lead large enough to be unconstrained monopolists decreases the maximized value $\tilde{V}_1$ that entrants derive from undertaking R&D, and therefore reduces entry pressure and the total R&D effort $\phi_m \lambda(\mu_m)$ of incumbents who are unconstrained monopolists. Moreover, the following can be shown:

Lemma 3. *For a given R&D effort $\phi_m \lambda(\mu_m)$, unconstrained monopolists find it optimal to select the growth-maximizing innovation step size $\mu_m^* = \mu^*$.*

Proof. See Appendix A4 □

Lemma 3 therefore implies that imposing a non-obviousness requirement cannot increase growth by increasing the innovation step size that unconstrained monopolists choose. As such a requirement at the same time reduces the R&D effort of industry leaders, we can state the following:

Proposition 3. *Imposing a non-obviousness requirement reduces equilibrium growth.*

This result differs from the previous literature. O'Donoghue and Zweimüller (2004) and Hunt (2004) show that, in the case of leapfrogging, imposing a patentability (non-obviousness) requirement can increase the rate of innovation, growth and welfare. O'Donoghue (1998) reaches the same conclusion in a model where even incumbent firms do some R&D in equilibrium, but in which preemption is not possible, so that there is no persistent leadership. The following mechanism is at work in these models: when an entrant selects the profit-maximizing combination of step size and innovation arrival rate for a given R&D spending, he does not take into account that increasing the latter (but not the former) reduces the expected profits of the previous incumbent by leading to a faster rate of "creative destruction". Imposing a non-obviousness requirement can therefore reduce the threat that innovators lose their monopoly power due to marginal (low step size) follow-on innovations and can, ceteris paribus, prolong the duration of monopoly power. This effect turns out to be stronger than the direct innovation-discouraging effect stemming from increased R&D costs (which is partially offset by larger markups and profits resulting from a larger step size), so that imposing a (weak) non-obviousness requirement can increase innovation in the case of leapfrogging.

In the case of persistent leadership, allowing entrants to target small innovation step sizes and thereby increasing the risk of creative destruction for the incumbent has different effects: as the incumbent wants to keep her unconstrained monopoly power, she responds to increased entry pressure by increasing her own R&D spending. As entry does not occur in equilibrium and as incumbents find it profitable to target the growth-maximizing step sizes, marginal innovations on which firms could obtain IP protection are never carried out and industry profits are never reduced below the monopoly level[22].

Expiring Intellectual Property Protection

In order to make the model more tractable, the limit case in which there are constant returns to R&D ($\epsilon = 0$) is studied in the following sections. As for $\epsilon = 0$, the equilibrium innovation rate ϕ_2^* in an industry is undetermined if the free entry condition $V_1(k+1) = cg^k$ is satisfied with equality, it is assumed that the equilibrium is selected that results as the limit if $\epsilon \to 0$.[23]

[22] Chu and Pan (2013) study the effects of forward protection in a model with leapfrogging with endogenous innovation step sizes. They show that (marginally) increasing the blocking power of incumbent firms can push entrants to pursue larger innovation step sizes and can thereby increase innovation. In the case of persistent leadership, increasing the blocking power of incumbents cannot have such a positive effect as it either reduces entry pressure by reducing profits for entrants that infringe on the incumbent's IPRs (see Section 4.1) or by pushing entrants to target larger step sizes than they would find optimal absent forward protection (due to the argument just presented).

[23] While the value of an innovation for an entrant, $V_1(k+1)$, depends on the expected future innovation rate ϕ_2^*, it is independent of the level of current R&D if $\epsilon = 0$. For arbitrarily small values of $\epsilon > 0$, the innovation rate ϕ_2^* is, however, uniquely determined by the free entry condition so

There can be cases in which the equilibrium innovation rate is not pinned down by the free entry condition and where with constant returns to R&D some firms find it profitable to increase their R&D effort infinitely. Because of this, it is assumed that there is an upper bound $\bar{\phi}$ (like specified in equations 3 and 4) that the innovation rate cannot surpass and that is selected in these cases.

In order to avoid the uninteresting cases where, absent the threat of future entry, not even entrants find it profitable to undertake R&D or where even incumbents with a two step lead want want to achieve the maximal innovation rate $\bar{\phi}$, the following condition is assumed to be satisfied[24] (**Condition C**):

$$\frac{\pi_m\left(g^{k+1}-g^k\right)}{r} < cg^k < \frac{\pi_1 g^{k+1}}{r}$$

4.3 State-dependent Intellectual Property Protection

This section analyzes the effects of IP expiration in the basic model in which IPRs protect against imitation, but in which there is no forward protection and no collusion and in which the step size is constant and exogenous. Like in Acemoglu and Akcigit (2012), the general case of "state-dependent intellectual property protection" is analyzed in which the probability of IP expiration can depend on whether a firm has a one step or a two (or more) step lead over its rivals. The IPRs of a firm with a one step lead are assumed to expire with the instantaneous Poisson arrival rate γ_1 and those of a firm with a two step lead with the instantaneous arrival rate γ_2. When IP protection expires in an industry, the newest vintage of the intermediate good falls in the public domain, allowing competitors to copy it and to fully catch up[25]. Profits therefore fall to zero in the case of IP expiration.

In the case of persistent leadership, $V_1(k)$ (that means the value of being one step ahead[26]) and the value $V_2\left(k\right)$ of being two (or more) steps ahead can be derived from the following arbitrage conditions, where ϕ_l denotes the innovation rate chosen by an incumbent with an l step lead:

that the preemption equilibrium results in which the current innovation rate is equal to the expected future one.

[24]Absent the threat of future entry and with infinite IP protection, the value of an innovation for an entrant is given by $\triangle V = V_1(k+1) = \frac{\pi_1 g^{k+1}}{r}$ and that of an incumbent with a two step lead by $\triangle V = V_2(k+1) - V_2(k) = \frac{\pi_m\left(g^{k+1}-g^k\right)}{r}$. As the marginal (and average) R&D costs are given by cg^k, firms that maximize expected profits $\phi(\triangle V - cg^k)$ therefore find it optimal to increase ϕ if $\triangle V > cg^k$ and to not undertake any R&D if $\triangle V < cg^k$.

[25]This specification implicitly assumes that IP protection on second-newest vintages never expires. The case in which firms with a two step lead can lose this lead for a one step lead (because their IPR on the second-newest vintage expires) is not considered.

[26]Note that absent collusion this value is independent of whether the IPR of the previous generation of the good has expired and also independent of the size of the previous incumbent´s lead.

$$rV_2(k) = \pi_m g^k - cg^k\phi_2 - \gamma_2 V_2(k) + \phi_2 \left(V_2(k+1) - V_2(k)\right)$$

$$rV_1(k) = \pi_1 g^k - cg^k\phi_1 - \gamma_1 V_1(k) - \phi_1 V_1(k) + \phi_1 V_2(k+1)$$

Taking into account that $V_l(k+1) = gV_l(k)$ along a balanced growth path, we can solve for:

$$V_1(k+1) = g^{k+1}\frac{\pi_1 - c\phi_1}{r+\gamma_1+\phi_1} + \frac{\phi_1}{r+\gamma_1+\phi_1} g^{k+2}\frac{\pi_m - c\phi_2}{r+\gamma_2-\phi_2\,(g-1)}$$

A firm with a one step lead sets its innovation rate ϕ_1 as a function of the IP expiration rates γ_1 and γ_2 which determine the relative profitability of a two step lead compared to a one step lead.

Lemma 4. *Given that* $\frac{\pi_m(g-1)}{r} < c < \frac{\pi_1 g}{r}$ *(Condition C) and that* $\bar{\phi}$ *is sufficiently large, the following cases arise:*

i)*: If* $\gamma_1 > \frac{g\pi_1}{c} - r$ *and if* $\gamma_2 < \frac{g^2\pi_m}{(g+1)c} - r$*, a firm with a one step lead selects innovation rate* $\phi_1^* = \bar{\phi}$

ii)*: If* $\gamma_1 \leq \frac{g\pi_1}{c} - r$ *and if* $\gamma_2 \leq \gamma_1 + \frac{g(\pi_m-\pi_1)}{c}$*, a firm with a one step lead selects the preemptive innovation rate* $\phi_1^* = \phi_2^* < \bar{\phi}$

iii)*: If* $\gamma_1 \leq \frac{g\pi_1}{c} - r$ *and if* $\gamma_2 > \gamma_1 + \frac{g(\pi_m-\pi_1)}{c}$*, a firm with a one step lead does not do any R&D (*$\phi_1^* = 0$*), so that there is leapfrogging.*

Proof. See Appendix B4 □

As $\pi_1 g > \pi_2 g - \pi_1$ (Lemma 1), the conditions in Regime (i) imply that $\gamma_1 > \gamma_2$ needs to hold, meaning that IP protection needs to expire sufficiently more quickly in the case of a one step lead than in the case of a two step lead in this regime. This makes a two step lead relatively more profitable, so that firms with a one step lead do the maximal amount of R&D, $\bar{\phi}$, in order to reach a two step lead as quickly as possible.

In Regime (ii), firms with a one step lead value not being replaced and reaching a two step lead more than entrants value entry, but they do not want to do more R&D than necessary to preempt entry so that $\phi_1^* = \phi_2^*$. In Regime (iii), IPRs expire so much faster in the case of a two step lead than in the case of a one step lead that firms with a one step lead do not find reaching a two step lead worthwhile and stop innovating. In Regimes (i) and (ii), the innovation rate in the case where the IPR on the currently newest vintage of an intermediate good has expired ("zero step lead") is given by $\phi_0^* = \phi_2^*$. While entrants might undertake R&D in this case, the innovation rate is the same as the preemptive level ϕ_2^* that incumbents select in order to prevent entry, as it is pinned down by the same free entry condition $V_1(k+1) = cg^k$. In Regime (ii), the innovation rate is therefore independent of the size of the lead, while it is higher in industries with a one step lead than in industries with a zero or a two step lead in Regime (i). In Regime (iii), entrants undertake all R&D, and the innovation rate is given by ϕ^*, independently of whether in a given sector IP protection on the currently newest vintage of the good has expired or not.

We can now analyze the effect of IP expiration on equilibrium growth G^* and derive the third main result of the article.

Proposition 4. *Given a change in γ_i does not lead to a switch between regimes, the equilibrium rate of growth G^* decreases in γ_1 in all Regimes and decreases in γ_2 in Regimes (i) and (ii). If a marginal shift in parameters leads to a switch from Regime (ii) to Regime (i), there is a discontinuous increase in G^*, while there is no discontinuity in the case of a switch between Regimes (ii) and (iii). G^* is maximal if $\gamma_1 = \gamma_2 = 0$, that means under full uniform IP protection.*

Proof. See Appendix B5 □

Reducing IP protection by increasing γ_1 or γ_2 decreases the value of an innovation for entrants and entry pressure in all regimes (with the exception that γ_2 does not affect outcomes in Regime iii)). This directly implies a reduction in the equilibrium innovation and growth rates in Regimes iii) and ii), and also a decrease in the innovation rates $\phi_2^* = \phi_0^*$ in the cases of a two and a zero step lead in Regime (i). In Regime (i), there is, however, also a "composition effect" which goes in the opposite direction: increasing γ_2 can now increase the fraction of industries in which there is a one step lead and in which the innovation rate is maximal.[27] However, this composition effect is not strong enough to overcompensate the negative effect that an increase in γ_2 has on $\phi_2^* = \phi_0^*$, so that an increase in γ_2 still leads to a reduction in the equilibrium rate of growth G_i^* in Regime i). G_i^* is therefore maximal if $\gamma_2 = 0$, in which case the composition effect disappears as leaders never lose a two step lead. But given that $\gamma_2 = 0$, the growth rate decreases in γ_1, as increasing the rate with which IP protection for firms with a one step lead expires merely reduces the value of an innovation for entrants and therefore the amount of R&D that incumbents need to undertake in order to preempt entry. Consequently, growth is maximal under full uniform IP protection and, unlike in Acemoglu and Akcigit (2012), cannot be increased by reducing IP protection in a state-dependent way (i.e. by increasing γ_1 more than γ_2).

However, given that γ_1 and γ_2 are already positive and at a certain threshold value, a small increase in γ_1 can lead to an increase in growth by inducing a switch from Regime (ii) to Regime (i). The reason for this is that such a decrease in IP protection

[27] Increasing γ_1 always decreases this fraction (see Appendix B5).

for firms with a one step lead can induce them to switch their strategy from only undertaking the amount of R&D that is necessary to preempt entry to undertaking the maximal amount of R&D in order to reach a two step lead as quickly as possible. There is therefore a nonlinear relation between γ_1 and the rate of growth so that the "incentive effect" identified by Acemoglu and Akcigit (2012) is at work at a specific threshold value of γ_1.

4.4 Potential Perils of strong Protection against Imitation

In the previous section it was shown that under free entry, reducing the strength with which intellectual property rights protect against imitation (that means increasing γ_1 or γ_2) cannot increase the rate of growth through a composition effect, i.e. by increasing the fraction of industries in which R&D incentives are higher due to a lower lead. In the following subsections, cases are analyzed in which this is different and where reducing IP protection can increase growth through such a composition effect. It is again assumed that there is no forward protection and that the step size is constant and exogenous.

4.4.1 Trade Secrecy

So far it has been assumed that there is free access to the R&D sectors and that entrants have complete knowledge about the currently newest technologies and can directly start doing frontier R&D. While, ideally, a firm is only granted IP (patent) protection when it discloses the functioning of its innovation, firms often succeed in keeping part of their knowledge secret so that competing firms first have to engage in some duplicative catch-up R&D before they can start conducting frontier R&D. It is therefore assumed that entrants first have to spend the fixed catch-up costs Rg^k before they can undertake frontier R&D directed at inventing vintage $k+1$ in a given industry[28]. It is assumed that the incumbent can observe an entrant's catch-up and thus can adjust her R&D spending after observing entry into the R&D sector. IP protection expires with the constant hazard rate γ and, in the case of expiration in a given industry, the newest available vintage of the intermediate good falls in the public domain and is supplied at the marginal cost of one[29]. In this case, profits fall to zero, and it is assumed for simplicity that all knowledge about the currently newest vintage

[28]In Appendix B6, the somewhat related problem where two R&D stages must be completed in order to obtain a marketable final innovation is analyzed. It is shown that in such a case, growth is maximal if entrants are allowed to obtain IP protection on the intermediate R&D inputs generated in the first R&D stage while incumbents are not.

[29]This specification is chosen for reasons of simplicity, and neglects the case where firms with a two step lead lose this lead for a one step lead because their IPR on the second-newest vintage expires. It is therefore implicitly assumed that IPRs on second-newest vintages never expire.

of the good falls in the public domain.

The equilibrium can be derived through backward induction. Given the incumbent in a certain industry has a lead of one or two steps, and entry into the corresponding R&D sector has occurred, the preemption equilibrium results in which the incumbent does enough R&D to completely discourage R&D by the entrant and in which the entrant makes zero profits. Expecting this, no entrant finds it profitable to pay the catch-up costs Rg^k, so that there is no entry into the R&D sector in an industry in which the currently newest vintage of the good is protected by IPRs. This, however, implies that there is no entry pressure and that incumbents do not need to undertake any R&D in order to preempt entry. Due to Condition C, incumbents thus stop innovating after having obtained a two step lead and might even find it profitable to stop innovating after having obtained a one step lead.

If IPRs have expired and if the currently newest vintage of an intermediate good is in the public domain, firms might, however, find it profitable to undertake R&D in order to become the next leader, given that IP protection does not expire too quickly. Denoting the the equilibrium innovation rates in the case of an l step lead by ϕ_l^* and the innovation rate in the case where IPRs have expired by ϕ_0^*, the following proposition holds:

Proposition 5. *Given that* $\frac{\pi_m(g-1)}{r} < c < \frac{\pi_1 g}{r}$ *(Condition C) holds, three regimes arise:*

Regime A*): if* $0 \leq \gamma \leq \frac{\pi_m g - \pi_1}{c} - r$, $\phi_0^* = \phi_1^* = \bar{\phi}$ *and* $\phi_2^* = 0$

Regime B*): if* $\frac{\pi_m g - \pi_1}{c} - r < \gamma \leq \frac{\pi_1 g}{c} - r$, $\phi_0^* = \bar{\phi}$ *and* $\phi_1^* = \phi_2^* = 0$

Regime C*): if* $\gamma > \frac{\pi_1 g}{c} - r$, $\phi_0^* = \phi_1^* = \phi_2^* = 0$

The equilibrium rate of growth is given by:

$$G^* = \begin{cases} (g-1)\,\frac{\gamma^3 + 2\gamma^2\bar{\phi}}{(\bar{\phi}+\gamma)^2} \text{ in Regime A} \\ (g-1)\,\frac{\bar{\phi}\gamma}{\bar{\phi}+\gamma} \text{ in Regime B} \\ 0 \text{ in Regime C} \end{cases}$$

Given that $\frac{\pi_m g - \pi_1}{c} - r > 0$ and $\bar{\phi}(\frac{2g\pi_m - (2+g)\pi_1}{c} - r) + \left(\frac{\pi_m g - \pi_1}{c} - r\right)^2 > (<) 0$, G^* is maximal if $\gamma = \frac{\pi_m g - \pi_1}{c} - r$ $\left(\gamma = \frac{\pi_1 g}{c} - r\right)$. If $\frac{\pi_m g - \pi_1}{c} - r < 0$, G^* is maximal if $\gamma = \frac{\pi_1 g}{c} - r$.

Proof. See Appendix A5 □

There is therefore an inverted-U relation between the rate of growth and the strength of IP protection[30]. If IPRs are fully protected (the sub-case of Regime A

[30] It should be noted that this result holds for any size of R, as long as incumbents can observe catch-up and entry and can readjust their R&D effort ex post. There is therefore a discontinuity in

where $\gamma = 0$), the growth rate is zero as there is no entry pressure and as monopolists rest on their laurels. Reducing the strength of IP protection then increases the fraction of industries in which due to expired IPRs no firm has a lead over its rivals and in which innovation incentives are maximal, but also the fraction of industries in which the incumbent only has a one step lead and might (in Regime A) find it profitable to do R&D in order to obtain a two step lead. If, however, IP protection is too weak (Regime C), the growth rate is again zero, as appropriability is so low that firms do not find it profitable to do R&D, even if the currently newest vintage of an intermediate good is in the public domain.

It should be noted that this result only arises if incumbents can (and want to) preempt entry, but not in the case of leapfrogging when only entrants innovate. Then, full protection against imitation ($\gamma = 0$) always maximizes innovation and growth as it encourages firms to be the first to overcome the fixed catch-up costs and to then win the innovation race in which they cannot be preempted by an incumbent anymore.

4.4.2 Lack of Research Exemption and different Forms of Preemption

Even if entrants are capable of undertaking frontier R&D without having to catch up first, broad intellectual property protection granted to previous innovators and the lack of a research exemption might make it illegal for entrants to do frontier R&D without first negotiating licensing contracts with previous innovators. If these contracts specify a positive up front licensing fee and if the incumbent can observe licensing, such a fee, however small it might be, plays the same role as the catch-up costs in the section above and leads to a complete blockage of entry. Therefore, imposing a research exemption for entrants seems to be particularly important to encourage cumulative innovation when incumbents can innovate preemptively.

Proposition 5 holds more generally in any situation where an incumbent (or several incumbents colluding on R&D decisions) does not face any entry pressure[31] and it is in line with the findings of Horowitz and Lai (1996) and Cadot and Lippman (1995) who assume that only one incumbent firm is capable of doing R&D[32]

the effect that IP protection has on growth when R falls from $\epsilon > 0$ to 0. This discontinuity, however, disappears if incumbents are not capable of observing the catch-up (which might be plausible if catch-up costs Rg^k are low): then, they actually have to undertake R&D in order to preempt entry, so that the analysis is again very similar to that in the previous sections. The fixed catch-up costs then only decrease entry pressure while stronger IP protection (a lower γ) again encourages innovation.

[31] Given the constant-return R&D technology with the upper bound ϕ_m, the innovation rate ϕ_0 in the case where the currently newest version of the good is supplied competitively is the same in the cases where one or several firms are capable of doing R&D.

[32] If only a single firm is capable of doing R&D in a given industry, the result even extends to the case where innovations are drastic (i.e. where $q > \frac{1}{\alpha}$). If $\frac{\pi_m(g-1)}{r} < c < \frac{\pi_m g}{r}$, such a firm only has incentives to innovate when IP protection has expired. Then, growth is maximal for the expiration rate $\gamma = \frac{\pi_m g}{c} - r$ at which firms still find it profitable to set $\phi_0^* = \bar{\phi}$ and that maximizes the fraction of industries in which IP protection has expired and in which innovation takes place.

It was assumed in the analysis above that incumbents can only discourage entrants' R&D by doing R&D themselves. There might, however, be cases in which incumbents can increase the R&D costs of potential entrants, or decrease their expected benefits of innovating, without doing R&D themselves. If there is an upward-sloping supply curve for R&D labour in an industry, incumbents might for example hire a certain amount of R&D labour in order to increase the wages that rivals need to pay, but employ the researchers in other areas than R&D. In the case where knowledge is to some extent tacit and where only researchers who were involved in past R&D are capable of doing frontier R&D, incumbents can simply offer them long term contracts or make them sign non-compete clauses in order to prevent them from doing research for entrants. While preemption is not costless in these cases, the result that incumbents stop innovating once they have obtained a sufficient lead is therefore the same. Because of that, the rate or growth is again zero if IPRs are fully protected and might increase if IP protection is decreased.

5 Conclusion

While most of the literature analyzing the role of intellectual property rights in models of cumulative innovation focuses on the case of leapfrogging, this article studies the case of persistent leadership. This is relevant from an empirical perspective, since a considerable amount of R&D is undertaken by firms that innovate on a permanent basis. Three main results come out of this analysis: *first*, that forward protection reduces growth and *second*, that imposing a non-obviousness requirement reduces growth. Both these results differ from the ones obtained in models with leapfrogging. The *third* main result is that full uniform protection against imitation, and not state-dependent IP protection, maximizes growth. Moreover, the article analyzes cases in which reducing the (expected) length during which IPRs protect against imitation can increase growth.

The model generates persistent leadership by assuming that innovations are non-drastic and that incumbents are capable of preempting entry, but - unlike other models - it does not rely on the assumption that incumbents are more productive in undertaking R&D than entrants. As within this setting, entry can occur if IP protection has expired or if entrants and incumbents can collude or sell their IPRs, the model is also consistent with cases in which there is less than perfect persistence. If, however, entry occurs because innovations are drastic, because entrants' R&D productivity is larger than that of incumbents, or because preemption is not possible (or not an optimal strategy for incumbents) due to technological or informational reasons, the mechanisms that arise in leapfrogging models become more relevant. Therefore, it might be beneficial to use different IPR policies in sectors that differ substantially in the persistence of innovative activities. In order to study such questions, it would be interesting to analyze a more general model in which entry plays a more prominent and realistic role. This extension is left for future research.

In quality-ladder models like the one studied here, welfare does not always increase if growth increases[33]. If equilibrium growth is excessive, it is, however, not recommendable to reduce it by introducing forward protection or a non-obviousness requirement if it is possible to instead reduce growth by reducing the maximal price that unconstrained incumbents can charge[34]. The reason for this is that only the latter policy reduces monopoly distortions while the other policies mainly reduce entry pressure without affecting the price setting power of unconstrained incumbents, who supply all intermediate goods in equilibrium[35].

[33]For a detailed analysis, see Denicolò and Zanchettin (2014)

[34]This can for example be done by imposing price regulations or by reducing the breadth of IP protection, forcing incumbents to engage in limit pricing in order to prevent entry of competitors supplying non-infringing imitates of inferior quality

[35]A formal welfare analysis is provided in Appendix B7

Appendix A

A1: Tables

Table 1

Technological entrants share per sector and period (patents)

Sector	**1990-91**	**1992-93**	**1994-95**	**1996-97**	**1998-99**	**2000-01**	**2002-03**
Audiovisual technology	0.16	0.18	0.16	0.23	0.19	0.21	0.17
Biotechnologies	0.14	0.15	0.15	0.13	0.16	0.16	0.19
Information technology	0.13	0.14	0.17	0.21	0.26	0.28	0.21
Machine tools	0.35	0.31	0.30	0.32	0.31	0.30	0.27
Macromolecular chemistry	0.07	0.09	0.08	0.10	0.10	0.11	0.13
Materials; Metallurgy	0.21	0.22	0.21	0.25	0.23	0.24	0.24
Medical engineering	0.27	0.22	0.21	0.22	0.23	0.22	0.20
Optics	0.10	0.09	0.11	0.14	0.16	0.15	0.18
Pharmaceuticals; Cosmetics	0.15	0.15	0.14	0.14	0.16	0.17	0.17
Semiconductors	0.11	0.10	0.10	0.12	0.13	0.15	0.15
Space technology; Weapons	0.17	0.18	0.18	0.22	0.25	0.23	0.22
Control/Measures/Analysis	0.23	0.22	0.23	0.24	0.25	0.24	0.24
Telecommunications	0.11	0.10	0.11	0.11	0.10	0.13	0.10

Source: Camerani and Malerba (2007), Table 6

Table 2

Transition probability matrices

		Time t			
		Non-innovators	Innovators	Non-R&D	R&D
One year transition					
Time t-1	Non-innovators	79.95	20.05	78.68	21.32
	Innovators	27.40	**72.60**	42.75	57.25
	Non-R&D	71.49	28.51	93.18	6.82
	R&D	33.89	66.11	12.44	**87.56**
Five year transition					
Time t-5	Non-innovators	72.23	27.77	75.88	24.12
	Innovators	41.90	**58.10**	44.31	55.69
	Non-R&D	71.22	28.78	86.46	13.54
	R&D	37.89	62.11	21.45	**78.55**

Source: Córcoles and Triguero (2013), Table 2

A2: Proof of Proposition 1

Proof. The general case is considered in which a lead $l \geq s \geq 2$ guarantees unconstrained monopoly power. When incremental profits fall in the size of the lead, incumbents only conduct as much R&D as necessary to preempt entry and the innovation rate is given by ϕ^* for any size of the lead. In order to also cover cases in which incremental profits can locally rise in l (see *Sections 4.1* and *4.3*), the possibility that $\phi_l > \phi^*$ might hold for some $l < s$ is also considered. Proceeding as in the main part of the text, the value of an incumbent with less than the maximal lead size ($l < s$) satisfies the arbitrage condition

$$V_l(k) = \frac{1}{r + \phi_l} \left[\pi_l g^k - c g^k \left(\phi_l\right)^{1+\epsilon} + \phi_l V_{l+1}(k+1) \right] \tag{12}$$

with either $\phi_l = \phi^*$ if $\left.\frac{\partial V_l(k)}{\partial \phi_l}\right|_{\phi_l = \phi^*} \leq 0$ or $\phi_l > \phi^*$ if $\left.\frac{\partial V_l(k)}{\partial \phi_l}\right|_{\phi_l = \phi^*} > 0$. Like in equation 9 the value of an incumbent with unconstrained monopoly power ($l \geq s$) is given by

$$V_m(k) = \frac{\pi_m g^k - c g^k \left(\phi^*\right)^{1+\epsilon}}{r - \phi^* \left(g - 1\right)}$$

Iterating equation 12 and replacing $V_m(k)$ allows to calculate the value of an innovation for an entrant as

$$V_1(k+1) = g^{k+1} \frac{\pi_1 - c\phi_1^{1+\epsilon}}{r + \phi_1} + \sum_{i=1}^{s-2} \frac{\prod_{j=1}^{i} \phi_j}{\prod_{k=1}^{i+1} \left(r + \phi_k\right)} \left[\pi_{i+1} - c\phi_{i+1}^{1+\epsilon}\right] g^{k+1+i}$$

$$+ \frac{\prod_{j=1}^{s-1} \phi_j}{\prod_{k=1}^{s-1} \left(r + \phi_k\right)} \left[\frac{\pi_m - c\left(\phi^*\right)^{1+\epsilon}}{r - \phi^* \left(g - 1\right)} \right] g^{k+s} \tag{13}$$

Suppose that $\phi^* > 0$ holds, which will be shown later on. As all R&D is undertaken by incumbents, all of them eventually obtain a lead $l \geq s$ which guarantees unconstrained monopoly power so that the (long run) equilibrium innovation rate ϕ^* is the same in all sectors and determined by the free entry condition $V_1(k+1) = c g^k \left(\phi^*\right)^{\epsilon}$ (equation 8 with $V_1(k+1)$ defined by equation 13). The comparative static results can be derived graphically by looking at the intersection of the left hand and the right hand sides of equation 8. If $\phi_l = \phi^*$ for all lead sizes l, the left hand side, $V_1(k+1)$, clearly increases in π_l, π_m and g and decreases in r, c and ϕ^*[36]. These results are unchanged if for some lead sizes incumbents prefer to undertake more R&D than needed to preempt entry, i.e. if $\phi_l > \phi^*$ with ϕ_l pinned down by the first order condition $\frac{\partial V_l}{\partial \phi_l} = 0$ (note that V_l is concave in ϕ_l). The reason for this is that $\frac{\partial V_l}{\partial \phi_l} = 0$ implies that also $\frac{\partial V_1}{\partial \phi_l} = 0$ needs

[36] The latter relation holds as the value of an innovation for an entrant is reduced if entrants anticipate that they have to innovate at a higher rate ϕ^* in the future in order to preempt entry.

to hold (this becomes clear by looking at equation 12), so that $\frac{\partial V_1}{\partial \phi_l}\frac{\partial \phi_l}{\partial Q} = 0$ (with Q standing for either π_l, π_m, g, r, c or ϕ^*) holds, implying that any indirect effect that a change in variable Q might have on V_1 through changing some value ϕ_l is of second order[37]. Given that $\epsilon > 0$ and that $r > \phi^* (g-1)$ (Condition B) an equilibrium in which $\phi^* > 0$ holds always exists as the right hand side of the free entry condition continuously increases from zero to infinity if ϕ^* increases from zero to infinity, while the left hand side (equation 13) is initially positive and then decreases continuously. The curve characterized by the left hand side shifts upward in π_l, π_m and g and downward in r and c while the curve characterized by the right hand side shifts up in c (ϕ^* is assumed to be on the horizontal axis). Consequently, ϕ^* increases in π_l, π_m and g and decreases in r and c. As ϕ^* decreases in r, Condition B ($r > \phi^* (g-1)$) is always satisfied if r is sufficiently large.

In a long run equilibrium, incumbent firms in all intermediate good industries have advanced to a lead that allows them to charge the unconstrained monopoly price $p_m = \frac{1}{\alpha}$. The corresponding demand for intermediate goods of sector ω can then be derived from equation 5. Plugging this into the final good production function (equation 2) and taking into consideration that $g = q^{\frac{\alpha}{1-\alpha}}$, the production of the final good in period t can be derived as

$$y(t) = \alpha^{\frac{2\alpha}{1-\alpha}} \int_{\omega=0}^{1} g^{k(\omega,t)} d\omega$$

Since innovations that increase the vintage k by one step arrive with the same hazard rate ϕ^* in all sectors, the rate at which $y(t)$ grows in equilibrium is given by

$$G^* = \frac{\dot{y}(t)}{y(t)} = \frac{\int_{\omega=0}^{1} \left(g^{k(\omega,t)+1} - g^{k(\omega,t)}\right) \phi^* d\omega}{\int_{\omega=0}^{1} g^{k(\omega,t)} d\omega} = (g-1)\,\phi^*$$

Therefore, the rate of growth increases if ϕ^* increases and also if g increases. As consumption also grows at rate G^* along a balanced growth path, $r > G^*$ (Condition B) needs to hold in order for intertemporal utility (equation 1) to be bounded.

For more general intertemporal preferences, the interest rate r is endogenous and depends positively on G through the Euler equation. The free entry condition, that gives a negative relationship between r and G and the Euler equation can then be simultaneously solved in order to obtain the equilibrium rate of growth G^*. Proposition 1 then still holds □

[37] In section 4.1 the case is studied where profits π_l decrease in a non-marginal way for firms with a low lead. The result that $V_1(k+1)$ decreases in π_l holds also in the case of such a non-marginal change: if, after the reduction in π_l, $V_1(k+1)$ could be increased above the previous level by changing some ϕ_l, then it is impossible that the initial values of ϕ_l were chosen in an optimal way.

A3: Proof of Proposition 2

Proof. As forward protection allows entrants and incumbents to collude in prices, it is not a priori clear whether the incumbent still (due to an efficiency effect) values not being replaced and obtaining the next innovation herself more than an entrant values entry and whether there is still persistent leadership. As the possibility to collude can maximally make the incument indifferent about her share in total R&D, the free entry condition and the equilibrium values of ϕ^* and G^* are, however, always the same as those derived under the assumption of persistent leadership. Therefore, the effects that forward protection has on ϕ^* and on G^* are in the following derived under the assumption that there is persistent leadership in the case of forward protection, but would be the same if entrants would instead undertake a share (or all) of the R&D in the case of forward protection. Conditions under which there is persistent leadership under forward protection are analyzed in Appendix B3.

In the case of persistent leadership, an entrant who has, out of equilibrium, invented vintage $k+1$ of a certain intermediate good does all the follow-on R&D. The profits that he obtains are only affected by the presence of leading breadth if $l \leq R$, i.e. if he has not yet advanced to a lead l that is large enough to avoid infringing the IPRs of the previous incumbent. Let us first consider the case where $R = 1$ and where $s > 1$. Then, forward protection only affects $V_1(k+1)$ through affecting the profits $\tilde{\pi}_1 g^{k+1}$ that the entrant earns when $l = 1$, which again depend on the licensing agreement that he negotiates with the previous incumbent.

Let us assume that the entrant and the incumbent are allowed to collude over their whole jointly owned product range if the innovation of the entrant infringes the latest IPR of the previous incumbent and if they negotiate a licensing agreement[38]. If the previous incumbent already possessed unconstrained monopoly power when supplying vintage k, a licensing agreement therefor allows both parties to also obtain unconstrained monopoly power when they supply vintage $k+1$, increasing joint profits to $\pi_m g^{k+1}$. As the incumbent can still obtain the previous profits $\pi_m g^k$ if she does not permit the entrant to produce vintage $k+1$, the maximal compensation in terms of profits per period that the incumbent is willing to give the entrant until he advances to a two step lead is therefore given by $\tilde{\pi}_1^{max} g^{k+1} = \pi_m(g^{k+1} - g^k)$. Due to Lemma 1 c), $\pi_m(g^{k+1} - g^k) < \pi_1 g^{k+1}$ holds, implying that an entrant with a one step lead earns less profits under forward protection than in the case where there is no forward protection. Given that incumbents only do as much R&D as needed to preempt entry,

[38]Suppose that instead the incumbent and the entrant were only allowed to collude over the subset of infringing IPRs and that the incumbent was only allowed to license to the entrant if she at the same time put some old vintages in the public domain, reducing her price setting power. Then the incumbent would be even less willing to license to the entrant (and might prefer to simply block entry instead) and forward protection would reduce entry pressure even more.

Proposition 1 that states that ϕ^* and G^* depend positively on π_1 therefore implies that forward protection reduces growth. [39].

Suppose now that $R > 1$, $q^s = \frac{1}{\alpha}$ and $s > 1$ hold. Then, the previous incumbent is maximally willing to leave per period profits $\tilde{\pi}_l^{max} g^{k+1} = g^k \left(g^l - 1\right) \pi_m$ to the entrant if he has advanced to a lead of $l \leq R$ steps (i.e. if he has invented vintage $k + l$) as this is the amount by which collusion allows her to increases per period profits when compared to the case where she blocks the production of all infringing follow-on innovations. Without forward protection, an entrant obtains profits $g^{k+l}\pi_l$ when the lead is equal to l steps. Due to Lemma 1 b), the following inequality must hold:

$$g^k \left(g^l - 1\right) \pi_m = g^k \left[\left(g - 1\right) \pi_m + \left(g^2 - g\right) \pi_m + ... + \left(g^l - g^{l-1}\right) \pi_m\right]$$

$$< g^k \left[\left(\pi_1 g\right) + \left(\pi_2 g^2 - \pi_1 g\right) + ... + \left(\pi_l g^l - \pi_{l-1} g^{l-1}\right)\right] = g^{k+l}\pi_l$$

The reason for this is that Lemma 1 b) implies that $\left(g - 1\right) \pi_m < \pi_1 g$ and that $\left(g^l - g^{l-1}\right) \pi_m \leq \pi_l g^l - \pi_{l-1} g^{l-1}$, where the latter is a strict inequality when $l \leq s$. When $s > 1$ and $R > 1$, at least the two first terms in the squared bracket on the left hand side are therefore smaller than the two first terms in the squared bracket on the right hand side, while the (potentially) remaining terms on the left hand side are either smaller or of equal size as those on the right hand side. Therefore, $\tilde{\pi}_l^{max} g^{k+1} = g^k \left(g^l - 1\right) \pi_m < g^{k+l}\pi_l$ holds and entrants obtain lower profits $\tilde{\pi}_l g^{k+l}$ under forward protection for any lead size $l \leq R$. Given that incumbents only do as much R&D as needed to preempt entry, Proposition 1, that states that ϕ^* and G^* increase in π_l, therefore implies that forward protection also reduces growth in the case where $R > 1$, $s > 1$ and where $q^s = \frac{1}{\alpha}$ (the latter assumption is needed in order to apply Lemma 1b)).

If entrants are weak bargainers or if the breadth of forward protection, R is sufficiently large, it is possible that $V_1(k + 1) < V_m(k + 1) - V_m(k)$ holds[40], implying that the value of an innovation for an entrant is so low that incumbents with unconstrained monopoly power end up having larger incentives to innovate than entrants and, in equilibrium, undertake more R&D than needed to preempt entry. In such cases, forward protection clearly reduces growth by eliminating entry pressure. □

[39] Also in the case where the incumbent who has IP protection on vintage k only has a one step lead, the entrant with vintage $k + 1$ gets less than $\pi_1 g^{k+1}$ for the following reason: the incumbent can either block the entrant and collude with the firm that holds the IPR on vintage $k - 1$, obtaining joint profits $\pi_2 g^k$, or collude with the entrant and push vintage $k - 1$ out of the market, obtaining joint profits $\pi_2 g^{k+1}$. By making firm $k - 1$ and the entrant compete for offers, the incumbent (firm k) can therefore extract the whole surplus $\pi_2 g^k$ that results from colluding with firm $k - 1$, leaving a maximum of $\pi_2(g^{k+1} - g^k) < \pi_1 g^{k+1}$ for the entrant (the inequality holds due to Lemma 1).

[40] This inequality does not hold if entrants have all the bargaining power as they then get the same incremental profits as incumbents when $l \leq R$, but larger incremental profits when $l > R$.

A4: Proof of Lemma 3

Proof. The value $V_m(\bar{q})$ of being an unconstrained monopolist supplying an intermediate good of quality $\bar{q}$ can be derived from the arbitrage condition

$$rV_m(\bar{q}) = \pi_m \bar{q}^{\left(\frac{\alpha}{1-\alpha}\right)} - c\bar{q}^{\left(\frac{\alpha}{1-\alpha}\right)} \left(\phi_m \lambda(\mu_m)\right)^{1+\epsilon} - \phi_m V_m(\bar{q}) + \phi_m V_m(\bar{q} + \mu_m)$$

Inserting $(\phi_m \lambda(\mu_m))^{1+\epsilon} = \left(\frac{\tilde{V}_1}{c}\right)^{\frac{1+\epsilon}{\epsilon}}$ and $\phi_m = \frac{\left(\frac{\tilde{V}_1}{c}\right)^{\frac{1}{\epsilon}}}{\lambda(\mu_m)}$ from the free entry condition 11 (where $\tilde{V}_1 \equiv \frac{V_1(\bar{q},\mu_e)}{\bar{q}^{\frac{\alpha}{1-\alpha}} \lambda(\mu_e)}$ is constant along a balanced growth path) and taking into account that $V_m(\bar{q} + \mu_m) = V_m(\bar{q}) \mu_m^{\frac{\alpha}{1-\alpha}}$ along a balanced growth path, we obtain:

$$V_m(\bar{q}) = \bar{q}^{\frac{\alpha}{1-\alpha}} \left(\frac{\pi_m - c\left(\frac{\tilde{V}_1}{c}\right)^{\frac{1+\epsilon}{\epsilon}}}{r - \left(\frac{\tilde{V}_1}{c}\right)^{\frac{1}{\epsilon}} \left(\frac{\mu_m^{\frac{\alpha}{1-\alpha}} - 1}{\lambda(\mu_m)}\right)} \right)$$

In order to maximize $V_m(\bar{q})$, the monopolist therefore selects the step size μ_m^* that maximizes the term $\frac{\mu_m^{\frac{\alpha}{1-\alpha}} - 1}{\lambda(\mu_m)}$. Given that $\lambda(\mu_m)$ is such that an interior solution $1 < \mu_m^* < \frac{1}{\alpha}$ exists, and given that r is large enough in order to get a positive denominator $r - \left(\frac{\tilde{V}_1}{c}\right)^{\frac{1}{\epsilon}} \left(\frac{(\mu_m^*)^{\frac{\alpha}{1-\alpha}} - 1}{\lambda(\mu_m)}\right)$, the equilibrium innovation rate can be derived as $\phi_m^* = \frac{\left(\frac{\tilde{V}_1}{c}\right)^{\frac{1}{\epsilon}}}{\lambda(\mu_m^*)}$ from the free entry condition (equation 11). The equilibrium rate of growth is then given by

$$G^* = \left((\mu_m^*)^{\frac{\alpha}{1-\alpha}} - 1\right) \phi_m^* = \left(\frac{\tilde{V}_1}{c}\right)^{\frac{1}{\epsilon}} \left(\frac{(\mu_m^*)^{\frac{\alpha}{1-\alpha}} - 1}{\lambda(\mu_m^*)}\right)$$

As incumbents choose μ_m^* in order to maximize the term in the second bracket on the right hand side, the preferred step size of the incumbent is equivalent to the step size μ^* that maximizes the rate of growth. □

A5: Proof of Proposition 5

Proof. Given that $\frac{\pi_m(g-1)}{r} < c$ (Condition C), an incumbent with a two step lead who can preempt entry without doing R&D herself does not find it profitable to do any R&D, even if IPRs are fully protected ($\gamma = 0$). Therefore, the innovation rate is given by $\phi_2^* = 0$ in the case of a two step lead. Consequently, the value of having a two step lead is given by $V_2(k) = \frac{\pi_m g^k}{r+\gamma}$ and that of a one step lead by

$$V_1(k) = \frac{\pi_1 g^k - \phi_1 c g^k + \phi_1 V_2(k+1)}{r + \phi_1 + \gamma}$$

with $V_2(k+1) = \frac{\pi_m g^{k+1}}{r+\gamma}$. An incumbent with a one step lead who does not face any entry pressure finds it profitable to do R&D and sets $\phi_1^* = \bar{\phi}$ if $\frac{\partial V_1(k)}{\partial \phi_1} > 0$, which is the case if $\gamma < \frac{\pi_m g - \pi_1}{c} - r$ (Regime A) and does not do any R&D and sets $\phi_1^* = 0$ if $\gamma > \frac{\pi_m g - \pi_1}{c} - r$ (Regime B).

In the case where IP protection has expired in an industry and where the currently newest vintage of the intermediate good is in the public domain, the R&D incentives depend on $V_1(k+1)$, the value of obtaining a one step lead. If $V_1(k+1) > cg^k$, the marginal benefits of doing R&D exceed the marginal and average costs if $\phi < \bar{\phi}$, and firms find it profitable to increase R&D until the upper bound of the innovation rate is reached[41], so that $\phi_0^* = \bar{\phi}$. This condition is always satisfied if $\frac{\pi_1 g^{k+1}}{r+\gamma} > cg^k$, i.e. if $\gamma < \frac{\pi_1 g}{c} - r$ (note that $\frac{\pi_1 g}{c} - r > \frac{\pi_m g - \pi_1}{c} - r$ due to Lemma 1), as it then pays off to undertake R&D if IP protection has expired even in Regime B where firms anticipate to stop innovating after having obtained a one step lead. If $\gamma \leq \frac{\pi_m g - \pi_1}{c} - r$ (Regime A), $\phi_0^* = \phi_1^* = \bar{\phi}$ therefore holds, meaning that the innovation rate is at the upper bound in industries where IP protection has expired and also in industries where the incumbent has a one step lead.

If $\frac{\pi_m g - \pi_1}{c} - r < \gamma \leq \frac{\pi_1 g}{c} - r$ (Regime B), we get $\phi_1^* = 0$ and $\phi_0^* = \bar{\phi}$, so that incumbents with a one step lead do not invest in R&D, while firms undertake R&D if IPRs have expired. If $\gamma > \frac{\pi_1 g}{c} - r$ (Regime C), IP protection is so weak that no firm finds it profitable to do R&D even if the newest version of a good is in the public domain, so that $\phi_0^* = \phi_1^* = 0$.

Denoting the fraction of intermediate good industries in which the leading firm has an l-step lead (is in "state l") by σ_l ($l = 0$ means that IP protection has expired) the rate of growth of the economy is given by $G = (g-1)(\sigma_0\phi_0 + \sigma_1\phi_1 + \sigma_2\phi_2)$. Along a balanced growth path, the fractions σ_l are constant, implying that the expected inflow into a "state" must be equal to the expected outflow. The following three equations, that have expected inflows on the left hand sides and expected outflows on the right hand sides therefore have to hold. As there is a continuum of sectors, uncertainty washes out and the fractions σ_l are constant.

$l = 0$: $\gamma(\sigma_1 + \sigma_2) = \sigma_0\phi_0^*$

$l = 1$: $\sigma_0\phi_0^* = \sigma_1\phi_1^* + \sigma_1\gamma$

$l = 2$: $\sigma_1\phi_1^* = \sigma_2\gamma = (1 - \sigma_0 - \sigma_1)\gamma$

Solving the system of equations for the equilibrium values σ_k^* and deriving the rate of growth, we obtain $G_A^* = (g-1)\frac{\bar{\phi}(\gamma^2 + 2\gamma\bar{\phi})}{(\bar{\phi}+\gamma)^2}$ in Regime A and $G_B^* = (g-1)\frac{\bar{\phi}\gamma}{\bar{\phi}+\gamma}$

[41]As marginal R&D costs go to infinity if $\phi = \bar{\phi}$, average R&D costs rise in total R&D spending and there is entry up to the point where these average costs are equal to $V_1(k+1)$.

Even though there is the upper bound on the total arrival rate $\bar{\phi}$, the free entry condition can be satisfied with equality if firms in the aggregate undertake more R&D than necessary to obtain $\bar{\phi}$, so that average costs increase above cg^k. This can happen if the individual probability of obtaining an IPR (patent) depends on the ratio between individual and total R&D spending.

in Regime B. In both Regimes, the equilibrium rate of growth G^* increases in γ, so that G^* is maximal for the maximal level of γ that still lies within the admissible parameter range, that means for $\gamma_a = \frac{\pi_m g - \pi_1}{c} - r$ in Regime A if $\frac{\pi_m g - \pi_1}{c} - r > 0$ and for $\gamma_b = \frac{\pi_1 g}{c} - r > \gamma_a$ in Regime B (note that $\gamma_B > 0$ due to Condition C).

If $\frac{\pi_m g - \pi_1}{c} - r > 0$, $G_a^*(\gamma_a) > (<)\, G_b^*(\gamma_b)$ if $\phi_m(2\gamma_a - \gamma_b) + \gamma_a^2 > (<)\, 0$, that means if

$$\bar{\phi}(\frac{2g\pi_m - (2+g)\pi_1}{c} - r) + \left(\frac{\pi_m g - \pi_1}{c} - r\right)^2 > (<)0$$

If $\frac{\pi_m g - \pi_1}{c} - r < 0$, $\phi_1^* = 0$, so that the rate of growth is maximal if $\gamma = \gamma_b = \frac{\pi_1 g}{c} - r$. □

Appendix B

B1: Proof of Lemma 1

Proof. As part of the proof of Part b) is used to prove Part a), the proof of Part b) is presented first. While k depends on the industry ω and on time t, the notation k instead of $k(\omega, t)$ is used for simplicity.

Part b): The proof is divided into three sections showing that: **(b1)** $\triangle\pi(k,l)$ is independent of l if $l \geq s$, **(b2)** $\triangle\pi(k,l)$ falls in l if $0 \leq l \leq s-1$ (this case is only relevant if $s \geq 2$), **(b3)** $\triangle\pi(k,l)$ falls in l if $s-1 \leq l \leq s$.

b1): For $l \geq s$, firms are unconstrained monopolists and profits are given by $\pi_m(k,l) = \alpha^{\frac{1+\alpha}{1-\alpha}} (1-\alpha)\, g^k$ (see equation 6). Incremental profits are therefore given by $\triangle\pi(k,l) = \pi_m(k)\,(g-1)$ and are independent of the size of the lead l.

b2): For $0 \leq l \leq s-1$, there is limit pricing and profits are given by $\pi(k,l) = \left(q^l - 1\right) \alpha^{\frac{1}{1-\alpha}} g^k q^{-\frac{l}{1-\alpha}}$ (see equation 7). Incremental profits obtained by introducing generation $k+1$ of the good are therefore given by

$$\triangle\pi(k,l) = \alpha^{\frac{1}{1-\alpha}} g^k q^{-\frac{l}{1-\alpha}} \left[\left(q^{l+1} - 1\right) g q^{-\frac{1}{1-\alpha}} - q^l + 1\right]$$

These incremental profits fall in l if $\triangle\pi(k,l) < \triangle\pi(k,l-1)$ (starting from $l \geq 1$), which holds if

$$\left(q^{l+1} - 1\right) g q^{-\frac{1}{1-\alpha}} - q^l + 1 < q^{\frac{1}{1-\alpha}} \left[\left(q^l - 1\right) g q^{-\frac{1}{1-\alpha}} - q^{l-1} + 1\right]$$

Replacing $g = q^{\frac{\alpha}{1-\alpha}}$, multiplying both sides by q and rearranging terms, this gives $q - 1 < q^{\frac{1}{1-\alpha}} (q-1)$. As $q > 1$ and $\alpha < 1$, this inequality is always satisfied. Therefore, incremental profits fall in l if $0 \leq l \leq s-1$ (note that for $l = s-1$, incremental profits can still be derived from the limit pricing formula as, due to the assumption that $q^s = \frac{1}{\alpha}$, the unconstrained monopoly price is equal to the limit price if $l = s$).

b3): For $l = s$, incremental profits are given by

$$\triangle\pi(k, l = s) = \pi_m(k)\,(g-1)$$

and for $l = s-1$ by

$$\triangle\pi(k, l = s-1) = \pi_m(k) g - \pi(k, s-1)$$

with $\pi(k, s-1) = \left(q^{s-1} - 1\right) \alpha^{\frac{1}{1-\alpha}} g^k q^{-\frac{s-1}{1-\alpha}}$ (see equation 7).

As $\pi(k, s-1) < \pi_m(k)$, $\triangle\pi(k, l = s-1) > \triangle\pi(k, l \geq s)$ holds so that incremental profits decrease if the lead increases from $l = s-1$ to $l = s$.

Part a): Firms with a two or more step lead ($l \geq 2$) are unconstrained monopolists, so that, like in Section b1 of the proof, incremental profits are independent of the size of the lead. Using the reasoning from Section b3, incremental profits decrease if the lead increases from $l = 1$ to $l = 2$. Therefore, it remains to be shown that incremental profits are higher for entrants ($l = 0$) than for firms with a one step lead ($l = 1$): (**Condition 1**):

$$\triangle\pi(k,0) = \pi(k+1,1) > \triangle\pi(k,1) = \pi_m(k+1) - \pi(k,1)$$

Inserting the corresponding values and rearranging terms, Condition 1 holds if

$$Q \equiv \left((q-1)\,\alpha^{\frac{1}{1-\alpha}} q^{-\frac{1}{1-\alpha}}\right)\frac{(g+1)}{g} > \alpha^{\frac{1+\alpha}{1-\alpha}}\,(1-\alpha)$$

From Section b3 we know that Condition 1 holds if $q = \frac{1}{\alpha}$ (in which case firms with a one step lead are unconstrained monopolists) and from Section b2 that Condition 1 also holds if $q = \frac{1}{\sqrt{\alpha}}$ (i.e. if $q^2 = \frac{1}{\alpha}$, in which case exactly two steps are needed to become an unconstrained monopolist). Therefore, Condition 1 also holds for all values $\frac{1}{\sqrt{\alpha}} < q < \frac{1}{\alpha}$ (this is Condition A: $q < \frac{1}{\alpha} < q^2$) if Q lies above its boundary values ($Q(q = \frac{1}{\alpha})$ and $Q\left(q = \frac{1}{\alpha^2}\right)$) in this intermediate range of q. A sufficient condition for this to be the case is that Q is a continuous and concave function in this range. Replacing $g = q^{\frac{\alpha}{1-\alpha}}$, it can be seen that Q is continuous and that for $\frac{1}{\sqrt{\alpha}} < q < \frac{1}{\alpha}$

$$\frac{\partial^2 Q}{\partial q^2} = \frac{\alpha^{\frac{1}{1-\alpha}}}{(1-\alpha)^2} q^{-2-\frac{1}{1-\alpha}}\left[\alpha\,(q+1) - 2 + 2q^{-\frac{\alpha}{1-\alpha}}\,(q\alpha\,(1+\alpha) - 1 - \alpha)\right] < 0$$

Therefore, incremental profits decrease if the lead increases from one to two steps and if the size of the quality improvement lies in the range $q < \frac{1}{\alpha} < q^2$.

Part c): Incremental profits of unconstrained monopolists are given by $\triangle\pi(k, l \geq 1) = \pi_m(k)\,(g-1)$ (see Section b1). If $q \geq \frac{1}{\alpha}$, a one-step lead suffices to become an unconstrained monopolist and incremental profits for entrants are given by $\triangle\pi(k, l = 0) = \pi_m(k)g$ and are therefore larger than those of unconstrained monopolists. If $1 < q < \frac{1}{\alpha}$, incremental profits of entrants are given by (see Section b2 with $l = 0$)

$$\triangle\pi(k, l=0) = \alpha^{\frac{1}{1-\alpha}} g^k \left[(q-1)\,gq^{-\frac{1}{1-\alpha}}\right]$$

Replacing $g = q^{\frac{\alpha}{1-\alpha}}$ and $\pi_m(k) = \alpha^{\frac{1+\alpha}{1-\alpha}}\,(1-\alpha)\,g^k$, it can be shown that incremental profits for entrants are larger than those of unconstrained monopolists if

$$\frac{q-1}{q\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)} > \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)$$

(**Condition 2**) holds. Because (due to the argument just made) this inequality holds if $q = \frac{1}{\alpha}$, it also holds for the whole range $1 < q < \frac{1}{\alpha}$ if the left hand side (LHS) of Condition 2 decreases in q in this range. $\frac{\partial LHS}{\partial q} < 0$ holds if $1 - \alpha > q^{\frac{\alpha}{1-\alpha}} - \alpha q^{\frac{1}{1-\alpha}} \equiv R$ (**Condition 3**). As $\frac{\partial R}{\partial q} < 0$, Condition 3 always holds as it is satisfied with equality for the smallest feasible value of q, i.e. for $q = 1$. Therefore, Condition 2 holds and incremental profits are larger for entrants than for unconstrained monopolists also in the range where $1 < q < \frac{1}{\alpha}$ □

B2: Proof of Lemma 2

Proof. Given the incumbent bears the fraction β of the total R&D costs in an industry, the value of being an incumbent with a two step lead supplying vintage k of the good is defined by the following arbitrage condition

$$rV_2(k) = \pi_m g^k - \beta c g^k \left(\phi^*\right)^{1+\epsilon} - \phi^* V_2(k) + \beta \phi^* V_2(k+1)$$

The first term on the right hand side indicates per period profits, the second term the R&D costs of the incumbent, the third term accounts for the probability of losing the current market due to an innovation and the fourth term for the probability of achieving the next innovation. Replacing $V_2(k+1) = gV_2(k)$ and solving for $V_2(k)$ gives

$$V_2(k) = \frac{\pi_m g^k - \beta c g^k \left(\phi^*\right)^{1+\epsilon}}{r - \phi^* \left(\beta g - 1\right)}$$

In the following, it is assumed that $r > \phi^*(g-1)$ holds (Condition B) so that $V_2(k)$ is bounded. Incumbents want to increase their share in the total R&D expenditures, β, if $\frac{\partial V_2(k)}{\partial \beta} > 0$. We obtain

$$sign\left\{\frac{\partial V_2(k)}{\partial \beta}\right\} = sign\left\{g\pi_m - c\left(\phi^*\right)^{\epsilon}\left(r + \phi^*\right)\right\}$$

Inserting $\left(\phi^*\right)^{\epsilon} = \frac{V_1(k+1)}{cg^k} = \frac{V_1(k)}{cg^{k-1}}$ from the free entry condition (equation 8), $\frac{\partial V_2(k)}{\partial \beta} > 0$ therefore holds if $V_1(k) < \frac{g^k \pi_m}{r+\phi^*}$ (**Condition 4**). We know that

$$V_1(k) < V_2(k) = \frac{\pi_m g^k - \beta c g^k \left(\phi^*\right)^{1+\epsilon}}{r - \phi^* \left(\beta g - 1\right)}$$

Inserting $\left(\phi^*\right)^{\epsilon} = \frac{V_1(k)}{cg^{k-1}}$ and rearranging again gives Condition 4. Therefore, the inequality $V_1(k) < V_2(k)$ implies that $\frac{\partial V_2(k)}{\partial \beta} > 0$ holds, so that incumbents with a two step lead find it profitable to increase β to one and to undertake all the R&D.

The value of being an incumbent with a one step lead supplying vintage k of the good is defined by the arbitrage condition

$$rV_1(k) = \pi_1 g^k - \beta_1 c g^k (\phi^*)^{1+\epsilon} - \phi^* V_1(k) + \beta_1 \phi^* V_2(k+1)$$

with β_1 denoting the fraction of the total R&D undertaken by the incumbent. This arbitrage condition can be solved for

$$V_1(k) = \frac{\pi_1 g^k - \beta_1 c g^k (\phi^*)^{1+\epsilon} + \beta_1 \phi^* V_2(k+1)}{r + \phi^*}$$

Taking into account that $V_2(k+1)$ is independent of β_1, we get

$$\frac{\partial V_1(k)}{\partial \beta_1} = \frac{\phi^*}{r+\phi^*}\left[V_2(k+1) - c g^k (\phi^*)^{\epsilon}\right]$$

Inserting $(\phi^*)^{\epsilon} = \frac{V_1(k)}{cg^{k-1}}$ from Equation 8 and replacing $V_2(k+1) = gV_2(k)$, we obtain

$$\frac{\partial V_1(k)}{\partial \beta_1} = \frac{g\phi^*}{r+\phi^*}\left[V_2(k) - V_1(k)\right] > 0$$

Therefore, also incumbents with a one step lead find it profitable to set $\beta_1 = 1$ and to do all the R&D.

As incumbents do all the R&D, there is persistence in leadership.

As a next step, it is shown that the fact that incremental profits fall in a firm's lead (Lemma 1) indeed implies that incumbents with a one or two step lead have lower stand-alone innovation incentives than entrants (with a zero step lead) and therefore only want to do as much R&D as needed to preempt entry. The value of a firm with an l step lead that undertakes all the R&D in its industry and innovates at rate ϕ_l is given by the arbitrage condition

$$rV_l(k) = \pi_l g^k - c g^k (\phi_l)^{1+\epsilon} + \phi_l \left[V_{l+1}(k+1) - V_l(k)\right]$$

Therefore, the firm wants to increase the innovation rate if $\frac{\partial (V_l)}{\partial \phi_l} > 0$, which holds if

$$\left[V_{l+1}(k+1) - V_l(k)\right] - c g^k (1+\epsilon) \phi_l^{\epsilon} > 0$$

(**Condition 5**). Suppose that $V_{l+1}(k+1) - V_l(k)$ falls in l so that it is maximal for $l = 0$. Then, Condition 5 cannot hold if the incumbent already innovates at the entry preempting rate ϕ^* as $\left.\frac{\partial (V_l)}{\partial \phi_l}\right|_{\phi_l = \phi^*} < 0$ holds due to the free entry condition (equation 8) which implies that $V_o(k) = 0$ and that $V_1(k+1) = c g^k (\phi^*)^{\epsilon}$.[42] Therefore, a sufficient

[42]The free entry condition equates the value of an innovation to the average R&D costs while, absent free entry, a firm with a zero-step lead would want to only equate it to the marginal R&D costs and therefore to undertake less R&D.

condition that guarantees that incumbents never want to increase the innovation rate above the level ϕ^* is that $V_{l+1}(k+1) - V_l(k)$ falls in l (**Condition 6**) if $\phi = \phi^*$. It is now shown that Condition 6 is satisfied for any innovation rate $\tilde{\phi} > 0$ if incumbents are the only innovators and if they choose this rate for any lead size. In the case of quasi-drastic innovations ($s = 2$) Condition 6 then consists of the following two inequalities

$$V_2(k+1,\tilde{\phi}) - V_2(k,\tilde{\phi}) < V_2(k+1,\tilde{\phi}) - V_1(k,\tilde{\phi})$$

$$V_2(k+1,\tilde{\phi}) - V_1(k,\tilde{\phi}) < V_1(k+1,\tilde{\phi}) - V_0(k,\tilde{\phi})$$

The first inequality is satisfied because $V_1(k,\tilde{\phi}) < V_2(k,\tilde{\phi})$ holds due to the fact that $\pi_1 < \pi_2 = \pi_m$. Inserting the corresponding values into the second inequality[43], simplifying and rearranging terms leads to the inequality $r\left(g\pi_2 - g\pi_1 - \pi_1\right) < \phi\pi_1$. This inequality is satisfied for any value $\tilde{\phi} \geq 0$ if $g\pi_1 > g\pi_2 - \pi_1$. The latter inequality (implying that incremental profits are larger for entrants than for incumbents with a one step lead) holds as due to Lemma 1. Therefore, $V_{l+1}(k+1,\tilde{\phi}) - V_l(k,\tilde{\phi})$ falls in l, implying that $\left.\frac{\partial(V_l)}{\partial\phi_l}\right|_{\phi_l=\phi^*} < 0$ holds. Consequently, incumbents never want to increase the innovation rate above the entry-preempting level ϕ^*. □

B3: Persistent Leadership under Forward Protection

While there is clearly persistent leadership under forward protection if the latter eliminates all entry pressure, this is less clear in the remaining cases. A preemption equilibrium results when the incumbent values not being replaced and obtaining the next innovation herself more than an entrant values entry. This holds if entry leads to a reduction in the total surplus (efficiency effect). The latter is always the case if $1 \leq R < s-1$ holds, as profits are then reduced below the unconstrained monopoly level once the entrant obtains the lead $l = R+1 < s$ at which he is not infringing the IPRs of the previous incumbent anymore (implying that collusion is inhibited) and at which he has not yet reached unconstrained monopoly power himself.

But even if $R \geq s-1$, so that collusion allows to jointly earn unconstrained monopoly profits for any lead size l, $\phi_l > \phi^*$ might hold for some lead sizes $l < s$, implying that joint net profits are also reduced when compared to the case without entry where the previous incumbent would have kept innovating at the lower (and less expensive) entry-preempting rate ϕ^*. Therefore, a new efficiency effect arises in such a case and the incumbent has again incentives to do all the R&D and to preempt entry, implying that there is persistent leadership[44]. The condition $\phi_l > \phi^*$ can hold if a

[43] $V_2(\tilde{\phi})$ and $V_1(\tilde{\phi})$ are the same as in the main part of the text with ϕ^* replaced by $\tilde{\phi}$. Note that $V_0(k,\tilde{\phi}) = -cg^k\tilde{\phi}^{1+\epsilon} + \frac{\tilde{\phi}}{(r+\tilde{\phi})}V_1(k+1,\tilde{\phi})$ need not be equal to zero for an arbitrary $\tilde{\phi}$.

[44] It should be noted that the result that forward protection reduces growth was derived under the

(recent) entrant with an l step lead values obtaining the next innovation sufficiently more than entrants with a zero step lead do[45]. This condition is most likely satisfied for step size $l = R$, as the next innovation then "frees" the innovator from paying royalties to the previous incumbent. From Lemma 4 i) it can be inferred that $\phi_1 > \phi^*$ indeed holds if $R = 1$, $s = 2$, $\epsilon \to 0$, $\frac{\pi_m(g-1)}{r} < c < \frac{\pi_1 g}{r}$ (Condition C) and if $\frac{g^2\pi_m}{g+1} > rc$[46].

It, however, cannot be ruled out that in certain cases the efficiency effect disappears under forward protection, implying that incumbents are indifferent about their share in total R&D so that there might not be persistent leadership anymore.

B4: Proof of Lemma 4

Proof. **Regime i)**: A firm with a one step lead selects R&D effort $\phi_1 = \bar{\phi}$ if $\frac{\partial V_1(k)}{\partial \phi_1} > 0$, which holds if (**Condition** 7):

$$\phi_2^* < \frac{(r+\gamma_1)\left(g\pi_m - c\left(r+\gamma_2\right)\right) - \pi_1(r+\gamma_2)}{(r+\gamma_1)\,c - (g-1)\,\pi_1}$$

Given $\phi_1 = \bar{\phi}$, we can solve the free entry condition ($V_1(k+1) = cg^k$) for ϕ_2^* to get:

$$\phi_2^* = \frac{\bar{\phi}g^2\pi_m - (r+\gamma_2)\left(\bar{\phi}c\,(g+1) + c\,(r+\gamma_1) - g\pi_1\right)}{\bar{\phi}c - (g-1)\left(c\,(r+\gamma_1) - g\pi_1\right)}$$

The condition $\bar{\phi} > \phi_2^* > 0$ must be satisfied in order to have an equilibrium. As $\lim_{\bar{\phi} \to \infty} \phi_2^* = \frac{g^2\pi_m}{c} - (r+\gamma_2)\,(g+1)$, $\bar{\phi} > \phi_2^* > 0$ holds if $\bar{\phi}$ is sufficiently large and if $\frac{g^2\pi_m}{c} > (r+\gamma_2)\,(g+1)$ (**Condition** 8). Plugging $\lim_{\bar{\phi} \to \infty} \phi_2^*$ into Condition 7 and taking into account that

$$(r+\gamma_1)\,c - (g-1)\,\pi_1 > rc - (g-1)\,\pi_m > 0$$

assumption that forward protection only allows the entrant and the incumbent to collude in prices but not to coordinate their joint R&D expenditures. If forward protection also allows for the latter and if under forward protection $\phi_l > \phi^*$ holds for some l, the incumbent is willing to compensate the entrant if he reduces the innovation rate from ϕ_l to ϕ^*. In a previous version of the paper that studies a simplified version of the model it is shown that forward protection might even increase entry pressure and growth in such a case if entrants have all the bargaining power.

[45] In order to induce a firm with an l step lead to increase ϕ_l above the level ϕ^* that is pinned down by the free entry condition, $V_{l+1}(k+1) - V_l(k)$ needs to be sufficiently larger than $V_1(k+1)$ as such a firm wants to equate $V_{l+1}(k+1) - V_l(k)$ to the marginal R&D costs while entrants with a zero step lead "over-invest" in R&D and equate $V_1(k+1)$ to the average R&D costs.

[46] This can be shown by verifying that the inequalities defining case i) in Lemma 4 are satisfied if $\gamma_1 = \gamma_2 = 0$ (no IPR expiration), if $\pi_1 = \tilde{\pi}_1^{max} = \pi_m(g-1)$, i.e. if the entrant obtains the maximal possible profits under forward protection if $l = 1$, and if $\frac{g^2\pi_m}{g+1} > rc$.

due to Condition C ($\frac{\pi_m(g-1)}{r} < c$), the following condition results:

$$\pi_m (g-1)(c(r+\gamma_1) - g\pi_1) < c(r+\gamma_2)(c(r+\gamma_1) - g\pi_1)$$

As $\pi_m (g-1) < cr < c(r+\gamma_2)$ due to Condition C, this condition can only hold if$c(r+\gamma_1) - g\pi_1 > 0$, i.e. if $\gamma_1 > \frac{g\pi_1}{c} - r$ (**Condition** 9). Given that $\bar{\phi}$ is sufficiently large, a firm with a one step lead therefore selects R&D effort $\phi_1 = \bar{\phi}$ if Condition 9 holds and $\phi_2^* > 0$ holds if $\gamma_2 < \frac{g^2\pi_m}{(g+1)c} - r$, that means if Condition 8 is satisfied.

Regimes ii) and **iii)**: Given that $\gamma_1 \leq \frac{g\pi_1}{c} - r$ (**Condition** 10), so that Condition 9 does not hold, a firm with a one step lead either undertakes the preemptive R&D effort $\phi_1 = \phi_2^*$ (Regime (ii)) or does not do any R&D ($\phi_1 = 0$, Regime (iii)), in which case there is leapfrogging. In Regime (ii), the value of an innovation for an entrant is given by

$$V_1^{ii}(k+1) = g^{k+1}\frac{\pi_1 - c\phi_2}{r+\gamma_1+\phi_2} + \frac{\phi_2}{r+\gamma_1+\phi_2}g^{k+2}\frac{\pi_m - c\phi_2}{r+\gamma_2-\phi_2(g-1)}$$

and in Regime (iii) by

$$V_1^{iii}(k+1) = g^{k+1}\frac{\pi_1}{r+\gamma_1+\phi_2}$$

Therefore, $V_1^{ii}(k+1) > V_1^{iii}(k+1)$ if $\phi_2 < \frac{g\pi_m}{c} - r - \gamma_2$ (**Condition 11**). At the point of indifference where $V_1^{ii}(k+1) = V_1^{iii}(k+1)$, the free entry condition $V_1^{iii}(k+1) = cg^k$ determines the equilibrium innovation rate as (**Equation D**):

$$\phi^* = \frac{g\pi_1}{c} - r - \gamma_1$$

Inserting equation D into Condition 11, we find that in order to be in Regime (ii) where a firm with a one step lead finds it optimal to do all the follow-on R&D and to set $\phi_1 = \phi_2^*$, the condition $\gamma_2 \leq \gamma_1 + \frac{g(\pi_m - \pi_1)}{c}$ must be satisfied. If $\gamma_2 > \gamma_1 + \frac{g(\pi_m - \pi_1)}{c}$, the economy is in Regime (iii) where the rate of innovation ϕ^* is given by equation D. If Condition 10 holds, the equilibrium innovation rates ϕ^* in regime (iii) and ϕ_2^* in Regime (ii) are positive (which can be inferred from equation D and the fact that $V_E^{ii} > V_E^{iii}$ in Regime (ii)). □

B5: Proof of Proposition 4

Proof. In **Regime i)** where $\phi_1^* = \bar{\phi} > \phi_2^* = \phi_0^*$, the arrival rate depends on the size of the lead, which changes stochastically over time within each industry. Along a balanced growth path, the fractions of industries σ_k in which the lead is equal to k steps ($k \in \{0; 1; 2\}$, with $k = 0$ denoting the case where IP protection has expired in an industry) are, however, constant. This implies that the expected entry into a certain state, i.e. into an industry with a certain lead k must be equal to the expected exit. As there is a continuum of sectors, there is no uncertainty in the aggregate. The following conditions therefore need to be satisfied along a balanced growth path in Regime (i):

$k=0$: $\sigma_o \phi_2^* = \gamma_1\sigma_1 + \gamma_2\sigma_2$

$k=1$: $\sigma_1(\bar{\phi}+\gamma_1) = \sigma_0\phi_2^*$

$k=2$: $\sigma_2\gamma_2 = \sigma_1\bar{\phi}$

Looking at the measure of industries in which the lead is given by a certain step size k, the left hand sides of these conditions indicate how many firms lose this step size by switching to another lead and the right hand sides indicate how many firms newly obtain this lead at a given point in time. Taking as an example the case where $k=0$, the measure of industries that lose a zero step lead at a certain point in time is given by the arrival rate of an innovation $\phi_0^* = \phi_2^*$ in industries with a zero step lead times the fraction of industries σ_0 in which IP protection has expired and in which there is a zero step lead. Due to the expiration of IP protection, the measure of industries that switch from a two or a one step lead to a zero step lead is given by the arrival rate of IP expiration in the case of a one step lead times the fraction of industries with a one step lead ($\gamma_1\sigma_1$), plus the corresponding expression for $k=2$ ($\gamma_2\sigma_2$). Using the condition $\sigma_0+\sigma_1+\sigma_2=1$ and these three equations, we can compute the equilibrium industry shares:

$$\sigma_0^* = \frac{\left(\bar{\phi}+\gamma_1\right)\gamma_2}{\left(\bar{\phi}+\gamma_1\right)\gamma_2 + \phi_2^*\left(\gamma_2+\bar{\phi}\right)}$$

$$\sigma_1^* = \frac{\phi_2^*\gamma_2}{\left(\bar{\phi}+\gamma_1\right)\gamma_2 + \phi_2^*\left(\gamma_2+\bar{\phi}\right)}$$

$$\sigma_2^* = \frac{\phi_2^*\bar{\phi}\left(\bar{\phi}+\gamma_1\right)\gamma_2}{\left(\bar{\phi}+\gamma_1\right)\gamma_2\left(\left(\bar{\phi}+\gamma_1\right)\gamma_2 + \phi_2^*\left(\gamma_2+\bar{\phi}\right)\right)}$$

The equilibrium rate of growth G_i^* in Regime i) is given by the growth factor $(g-1)$ multiplied by the weighted sum of innovation arrival rates ϕ_k, with the weights given by the equilibrium industry shares σ_k^*:

$$G_i = (g-1)\left(\bar{\phi}\sigma_1^* + \phi_2^*(\sigma_0^*+\sigma_2^*)\right) = (g-1)\,\frac{\phi_2^*\left(\gamma_2\left(\bar{\phi}+\gamma_1\right)+\bar{\phi}(\phi_2^*+\gamma_2)\right)}{\left(\bar{\phi}+\gamma_1\right)\gamma_2+\phi_2^*\left(\gamma_2+\bar{\phi}\right)}$$

The equilibrium growth rates in **Regimes ii)** and **iii)** are simply given by $G_{ii}^* = (g-1)\,\phi_2^*$ and $G_{iii}^* = (g-1)\,\phi^*$.

Now, the effects of IPR expiration can be analyzed in the different regimes:

Regime iii): the growth rate is given by $G_{iii}^* = (g-1)\left(\frac{g\pi_1}{c} - r - \gamma_1\right)$ (with ϕ^* given by equation B) and clearly decreases in γ_1.

In **Regime ii)**, ϕ_2^* and therefore also $G_{ii}^* = (g-1)\,\phi_2^*$ decrease in γ_i. This can be inferred from the free entry condition $V_1^{ii}(k+1) = cg^k$ as

$$V_1^{ii}(k+1) = g^{k+1}\frac{\pi_1 - c\phi_2}{r+\gamma_1+\phi_2} + \frac{\phi_2}{r+\gamma_1+\phi_2} g^{k+2} \frac{\pi_m - c\phi_2}{r+\gamma_2-\phi_2(g-1)}$$

decreases in γ_i and also in ϕ_2 as incumbents only undertake as much R&D as needed to preempt entry.

For the parameter constellation $\gamma_2 = \gamma_1 + \frac{g(\pi_m - \pi_1)}{c}$ where the switch between Regimes ii) and iii) occurs, we have $V_1^{ii}(k+1) = V_1^{iii}(k+1)$, and as in both regimes the innovation rate does not depend on the size of the lead and is determined by the free entry condition $V_1(k+1) = cg^k$, there is no discontinuous jump in the innovation rate and in the rate of growth at this point.

Regime i): Solving the free entry condition $V_1(k+1) = cg^k$ for ϕ_2^*, we get:

$$\phi_2^* = \frac{\bar{\phi} g^2 \pi_m - (r+\gamma_2)\left(\bar{\phi}c\,(g+1) + c\,(r+\gamma_1) - g\pi_1\right)}{\bar{\phi}c - (g-1)\,(c\,(r+\gamma_1) - g\pi_1)}$$

Deriving with respect to the expiration rates, we obtain

$$sign\frac{\partial \phi_2^*}{\partial \gamma_1} = sign\left[\frac{\pi_m\,(g-1)}{r+\gamma_2} - c\right] < 0$$

due to Condition C and

$$\frac{\partial \phi_2^*}{\partial \gamma_2} = -\frac{\left(\bar{\phi}c\,(g+1) + c\,(r+\gamma_1) - g\pi_1\right)}{\bar{\phi}c - (g-1)\,(c\,(r+\gamma_1) - g\pi_1)} < 0$$

The last inequality holds as $c(r+\gamma_1) - g\pi_1 > 0$ (Condition 9 from *Appendix* B4) and as the denominator is positive given that $\bar{\phi}$ is sufficiently large. The equilibrium rate of growth is given by:

$$G_i^* = (g-1)\left(\bar{\phi}\sigma_1^* + \phi_2^*(\sigma_0^* + \sigma_2^*)\right) = (g-1)\left(\bar{\phi}\sigma_1^* + \phi_2^*(1-\sigma_1^*)\right)$$

Taking into account that $\sigma_1^* = \frac{\phi_2^*\gamma_2}{(\bar{\phi}+\gamma_1)\gamma_2 + \phi_2^*(\gamma_2+\bar{\phi})}$, we obtain

$$sign\frac{\partial \sigma_1^*}{\partial \gamma_1} = sign\left\{\frac{\partial \phi_2^*}{\partial \gamma_1}\left(\bar{\phi}+\gamma_1\right)\gamma_2 - \phi_2^*\gamma_2\right\} < 0$$

so that

$$\frac{\partial G_i^*}{\partial \gamma_1} = (g-1)\left(\frac{\partial \sigma_1^*}{\partial \gamma_1}(\bar{\phi} - \phi_2^*) + \frac{\partial \phi_2^*}{\partial \gamma_1}(1-\sigma_1^*)\right) < 0$$

as $\bar{\phi} > \phi_2^*$ and as $\sigma_1^* < 1$.

Deriving the growth rate

$$G_i^* = (g-1)\left(\frac{\phi_2^*\left(\gamma_2\left(\bar{\phi}+\gamma_1\right)+\bar{\phi}(\phi_2^*+\gamma_2)\right)}{\left(\bar{\phi}+\gamma_1\right)\gamma_2+\phi_2^*\left(\gamma_2+\bar{\phi}\right)}\right)$$

with respect to γ_2 gives

$$sign\frac{\partial G^*}{\partial \gamma_2} =$$

$$sign\left\{\frac{\partial \phi_2^*}{\partial \gamma_2}\left[\left(\gamma_2\left(\bar{\phi}+\gamma_1\right)+\bar{\phi}(\phi_2^*+\gamma_2)\right)\gamma_2(\bar{\phi}+\gamma_1)\ +\phi_2^*\bar{\phi}\left(\gamma_2(\bar{\phi}+\gamma_1)+\phi_2^*(\gamma_2+\bar{\phi})\right)\right]\right.$$

$$\left.+\bar{\phi}(\phi_2^*)^2(\bar{\phi}-\phi_2^*)\right\} < 0$$

This derivative is negative as $\frac{\partial \phi_2^*}{\partial \gamma_2} < -g-1$ under Condition 9 and for $\bar{\phi}$ sufficiently large, implying that the positive term $\bar{\phi}^2(\phi_2^*)^2$ in the lowest line is dominated by the negative term $\frac{\partial \phi_2^*}{\partial \gamma_2}\bar{\phi}^2(\phi_2^*)^2$ that is part of the middle line. Therefore, setting $\gamma_2 = 0$ maximizes G_i^*. Given that $\gamma_2 = 0$, an incumbent who has obtained a two step lead never loses it, so that the rate of growth is simply given by $G_i^* = \phi_2^*$, as $\sigma_0^* = \sigma_1^* = 0$ in this case. But as ϕ_2^* decreases in γ_1 (because increasing γ_1 reduces the value of an innovation for an entrant $V_1(k+1)$ in all regimes and therefore the amount of R&D the incumbent needs to undertake in order to preempt entry), equilibrium growth is maximal if $\gamma_1 = \gamma_2 = 0$, that means under full uniform IPR protection.

At the switching point between **Regimes i)** and **ii)** (where Condition 9 holds with equality), there is no discontinuity in the value of an innovation for an entrant and $V_1^i(k+1) = V_1^{ii}(k+1)$ holds independently of whether entrants set $\phi_1 = \bar{\phi}$ or $\phi_1 = \phi_2^*$. Therefore, the innovation rates ϕ_0^* and ϕ_2^* are also the same in both cases. However, the growth rate G^* is higher if firms with a one step lead choose $\phi_1 = \bar{\phi}$, so that there is a discontinuous increase in G^* when a slight shift in parameters leads so a switch from Regime ii) to Regime i). □

B6: Intermediate R&D Inputs

Let us now look at the case where two R&D stages have to be completed in order to improve the quality of an intermediate good by one step. In the first stage, an intermediate R&D input (which might be thought of as an idea) has to be invented and this input is used in the second stage to develop an improved version of the intermediate good. The two stages can also be interpreted as a research and a development stage. The R&D technology is again stochastic and given by equation 3 for both stages that are needed for the invention of a vintage $k+1$. Therefore, preemption is possible at both stages. It is assumed that there is full IP protection against imitation in the intermediate goods markets ($\gamma = 0$). In the case where IP protection is granted on an intermediate R&D input, it allows to prevent other firms from using this input, and therefore from developing a better version of the corresponding intermediate good.

Looking again at the case of constant returns to R&D ($\epsilon \to 0$) and assuming that Condition C ($\frac{\pi_2\left(g^{k+1}-g^k\right)}{r} < cg^k < \frac{\pi_1 g^{k+1}}{r}$) holds, three cases are considered:

1): *No IP protection is granted on intermediate R&D inputs*

In this case, there is no growth as no firm has incentives to invent such an input.[47]

2): *Both entrants and incumbents can obtain IP protection on intermediate R&D inputs*

Innovation and growth also come to a halt in this case as incumbents always use the possibility of obtaining IP protection on the newest inputs in order to block entry and to eliminate the threat of being replaced by an entrant.[48]

3): *Only entrants can obtain IP protection on intermediate R&D inputs, but are not allowed to license to incumbents*

In this case, sustained innovation and growth is possible, as an incumbent has incentives to preempt R&D of entrants at each stage, without ever being able to block future entry completely. If an input is invented by an incumbent and freely accessible to entrants, the incumbent has incentives to preempt entry by exerting a large enough effort in the race for the second R&D stage. Expecting this, she finds inventing the intermediate input worthwhile (even if she does not obtain IP protection on it), as it prevents entrants from inventing and obtaining IP protection on it, which would allow them to replace her in the future. Once the second R&D stage is completed and the incumbent has developed the next vintage of an intermediate good, the whole process starts again.

[47] Once the input is invented, there is free entry into the second stage development race, so that expected profits for entrants are zero in this race. Expecting this, no entrant has incentives to spend money on inventing such an intermediate input in the first place. But neither does an incumbent who has already obtained a two step lead and, due to Condition C, does not find it profitable to continue innovating if there is no threat of entry.

[48] Given that an incumbent has obtained a two step lead in an industry and has also obtained an IPR on the R&D input which is needed to develop the next vintage of the good, she uses it to block follow-on R&D by entrants. If, instead, the entrant has IP protection on the newest version of the input, he finds it profitable to license it to the incumbent as he values doing follow-on R&D and entering the market with an improved vintage of the good less than the leader values blocking follow-on R&D. Even if entrants are not allowed to license to incumbents, the incumbent can prevent an entrant from obtaining IP protection on the intermediate R&D input by undertaking a sufficient amount of research effort in the race for the input.

B7: Welfare

Suppose that that IP protection does not expire and that incumbents with a two-step lead can maximally charge the price $\bar{p} < \frac{1}{\alpha}$, implying that their profits π_2 rise in $\bar{p}$. Once incumbents in all industries have obtained the maximal lead and charge price $\bar{p}$, the innovation rate is the same in all sectors ($\phi_\omega = \phi^*$) and consumption in period t is given by

$$c(t) = y(t) - \int\limits_{\omega=0}^{1} x\left(k, \omega, t\right) d\omega - \int\limits_{\omega=0}^{1} c g^{k(\omega,t)} \left(\phi^*\right)^{1+\epsilon} d\omega$$

Inserting $y(t)$ from equation 2 and $x(k, \omega, t)$ from equation 5 gives

$$c(t) = \left(\alpha^{\frac{\alpha}{1-\alpha}} \bar{p}^{\left(-\frac{\alpha}{1-\alpha}\right)} - \alpha^{\frac{1}{1-\alpha}} \bar{p}^{\left(-\frac{1}{1-\alpha}\right)} - c\left(\phi^*\right)^{1+\epsilon}\right) \int\limits_{\omega=0}^{1} g^{k(\omega,t)} d\omega$$

For $\bar{p} > 1$, $c(t)$ falls in $\bar{p}$.

Along a balanced growth path, $c(t)$ grows at the rate $G^* = (g-1)\phi^*$ at which the aggregate quality index $\int_{\omega=0}^{1} g^{k(\omega,t)} d\omega$ grows (see Appendix A2) and falls in π_m and therefore in π_2 and in $\bar{p}$ (see Proposition 1). Intertemporal utility (welfare) is then given by

$$U(\tau) = \int\limits_{t=\tau}^{\infty} c(t) e^{-r(t-\tau)} dt = \frac{c\left(\tau\right)}{r - G^*}$$

and increases in $c(\tau)$ and in G^*. If a certain innovation rate ϕ^* and growth rate G^* can be attained with either a high $\bar{p}$ and low entry pressure (e.g. a low π_1 resulting from forward protection) or high entry pressure and a low level of $\bar{p}$, the latter option is therefore preferable from a welfare perspective as it increases consumption (for a given rate of growth).

References

Acemoglu, Daron and Akcigit, Ufuk (2012); "Intellectual Property Rights Policy, Competition and Innovation", Journal of the European Economic Association, 10 (1), pp. 1-42

Acemoglu, Daron and Dan Cao (2010); "Innovation by Entrants and Incumbents", working paper, September 12, 2010

Aghion, Philippe and Peter Howitt (1992); "A model of Growth through Creative Destruction", Econometrica 60 (2): 323- 351

Aghion, Philippe, C. Harris, P. Howitt and J. Vickers (2001); "Competition, Imitation and Growth with Step-by-Step Innovation", Review of Economic Studies, 68

(3): 467-492

Arrow, Kenneth (1962); "Economic Welfare and the Allocation of Resources for Invention", in National Bureau of Economic Research, *The Rate and Direction of Inventive Activity.* Princeton, NJ: Princeton University Press

Athey, S. and A. Schmutzler (2001); “Investment and Market Dominance”, The RAND Journal of Economics, Vol. 32, No. 1, pp. 1-26

Barro, R.J. and Sala- i- Martin, X. (2004); "Economic Growth", The MIT Press

Bartelsman, Eric J. and Mark Doms (2000); "Understanding Productivity: Lessons from Longitudinal Microdata", Journal of Economic Literature vol. 38 (3), pp. 569-594

Bessen, James and Eric Maskin (2009); “Sequential Innovation, Patents, and Imitation”, Rand Journal of Economics, vol. 40, issue 4, pp. 611 - 635

Cadot, O. and Lippman, S. (1995); "Barriers to Imitation and the Incentive to Innovate", INSEAD working paper

Camerani, R. and F. Malerba (2007); “Patterns of Technological Entry in Different Fields: An Analysis of Patent Data”; Report to the European Commission, KEINS, Project no. 506022

Chu, Angus C. (2009); "Effects of Blocking Patents on R&D: A Quantitative DGE Analysis", Journal of Economic Growth, 2009, vol. 14, pp. 55-78.

Chu, Angus C., Cozzi, Guido and Silvia Galli (2012); "Does intellectual monopoly stimulate or stifle innovation?" European Economic Review, vol. 56 (4), pp. 727-746

Chu, Angus C. and S. Pan (2013); “The escape-infringement effect of blocking patents on innovation and economic growth”, Macroeconomic Dynamics, 17, pp. 955-969

Córcoles, D. and Á. Triguero (2013); “Understanding innovation: An analysis of persistence for Spanish manufacturing firms”; Research Policy, 42, 340 - 352

Cozzi, Guido (2007); "The Arrow Effect under Competitive R&D", The B.E. Journal of Macroeconomics, Volume 7, Issue 1

Czarnitzki, Dirk, Federico Etro, and Kornelius Kraft (2014); "Endogenous Market Structures and Innovation by Leaders: an Empirical Test", Economica , Vol. 81, 321 (January), pp. 117-139

Denicolò, Vincenzo (2001); "Growth with non-drastic innovations and the persistence of leadership", European Economic Review, vol. 45(8), pp. 1399-1413

Denicolò, Vincenzo (2002); "Sequential innovation and the patent-antitrust conflict", Oxford Economic Papers, pp. 1-20

Denicolò, Vincenzo and Piercarlo Zanchettin (2012); "Leadership Cycles in a Quality-Ladder Model of Endogenous Growth", The Economic Journal, Vol. 122, 561 (June), pp. 618-650

Denicolò, Vincenzo and Piercarlo Zanchettin (2014); “What Causes Over-investment in R&D in Endogenous Growth Models?”. The Economic Journal. doi: 10.1111/ecoj.12132

Etro, F. (2004); "Innovation by Leaders", The Economic Journal, Vol. 114, 495 (April), pp. 281-310

Foster, Lucia, Cheryl Grim and Nikolas Zolas (2013); "Tracing the Evolution and Characteristics of Top R&D Performing U.S. Firms”; working paper

Fudenberg, Drew, Richard Gilbert, Joseph Stiglitz and Jean Tirole (1983); “Preemption, leapfrogging and competition in patent races”, European Economic Review 22, 3-31

Garcia-Macia, Hsieh and Klenow (2015); “How Destructive is Innovation?”, Working Paper, January 30

Gilbert, Richard J. & Newbery, David M. G., (1982); "Preemptive Patenting and the Persistence of Monopoly", American Economic Review, vol. 72(3), pages 514-26

Horowitz, Andrew W. and Lai, Edwin L.- C. (1996); "Patent Length and the Rate of Innovation", International Economic Review vol. 37 (4), pages 785-801

Hunt, Robert M. (2004); "Patentability, Industry Structure, and Innovation", The Journal of Industrial Economics, Vol. 52, No. 3, pp. 401-425

Jang, S.-L.,Y.-T. Tsai & J. Chen (2008); “Persistent innovation: a cross-country study of output and diversity over time”, Applied Economics Letters, 15:4, 323-326

Le Bas, Christian and Giuseppe Scellato (2014); “Firm innovation persistence: a fresh look at the frameworks of analysis”; Economics of Innovation and New Technology, 23:5-6, 423-446

Llanes, G. and S. Trento (2012); "Patent Policy, Patent Pools, and the Accumulation of Claims in Sequential Innovation"; Economic Theory, 50 (3), pp. 703-725

O’Donoghue, Ted (1998); "A patentability requirement for sequential innovation", RAND Journal of Economics Vol. 29, No. 4, pp. 654-679

O’Donoghue, Ted, Suzanne Scotchmer and Jacques François Thisse (1998); “Patent Breadth, Patent Life and the Pace of Technological Progress”, Journal of Economics and Management Strategy, 7, 1-32.

O’Donoghue, Ted and Joseph Zweimüller (2004); “Patents in a Model of Endogenous Growth”, Journal of Economic Growth, 9, 81-123.

Peters, Bettina (2009); "Persistence of innovation: stylised facts and panel data evidence", Journal of Technology Transfer, 34: 226-243

Reinganum, Jennifer (1983); "Uncertain Innovation and the Persistence of Monopoly", American Economic Review, Vol. 73, No. 4

Reinganum, Jennifer (1985); "Innovation and Industry Evolution", The Quarterly Journal of Economics, Vol. 100, No. 1

Segal, Ilya, and Michael D. Whinston (2007); "Antitrust in Innovative Industries", American Economic Review, 97(5): 1703–1730.

Segerstrom, Paul (2007); "Intel Economics", International Economic Review, 48: 247-280

Segerstrom, Paul and James Zolnierek (1999); "The R&D incentives of industry leaders", International Economic Review, 40: 745-766