Optimal Patent Strength under Financial Constraints

Christian Kiedaisch*
University of Zurich
December 30, 2013

Abstract

This paper analyzes how the optimal strength of patents on basic inventions is affected by financial constraints on the side of either inventors or developers. The lower the net wealth of a developer is, the more difficult it becomes for an inventor to license her invention to him as she has to rely more heavily on royalties that are only paid in the case of success and discourage the developer from exerting costly effort. Because of this, the distortions arising from patents on inventions are larger if more developers are financially constrained and it is optimal to reduce patent protection in this case. If the inventor is financially constrained, it is, however, optimal to grant stronger patent protection as inventions become more costly due to additional agency costs. (JEL: L24, O31, O34)

Keywords: patent protection, licensing agreements, financial constraints

*Contact: christian.kiedaisch@econ.uzh.ch.
I gratefully acknowledge financial support by the Swiss National Science Foundation
I thank Gilles Saint - Paul, Klaus Schmidt, Armin Schmutzler, Georg von Graevenitz and seminar/conference participants in Bern, Zurich, Wildbad Kreuth and Stockholm for helpful comments.

1 Introduction

Investments in research and development are an important driver of economic growth. The financing of these investments is often difficult because outcomes are uncertain and because there are informational asymmetries between investors, researchers and developers[1]. In many industries, the patent system is used in order to prevent imitation and in order to help innovators to cover their fixed R&D costs. This paper analyzes interactions between financial constraints and the effects that patent policy has on innovation. This is done in a setup in which an inventor (she) can come up with a basic invention that can then be used by one or more developers (he) that can improve upon this invention and develop marketable goods out of it. The paper analyzes how financial constraints of either the inventor or the developers affect the design of licensing contracts and studies the implications that this has for the optimal strength of patent protection on basic inventions.

Once an invention has been made, the probability with which a developer can profitably use it is assumed to depend on the effort undertaken by him, which cannot be observed by the inventor or a third party. If an invention is protected by a patent, the inventor licenses it to the developers while they have free access to the invention if the patent is not enforced. The revenue-maximizing licensing contract relies more heavily on royalties that are only paid in the case of success and distort the developer´s effort choice downwards if a developer is endowed with a lower level of net wealth and cannot afford to pay a large fixed fee. If a developer is not financially constrained so that he only pays a fixed licensing fee, or if he has free access to the invention because the patent is not enforced, he always exerts a high level of effort. However, the inventor induces him to exert an inefficiently low level of effort in the case where he is financially constrained as this allows the inventor to extract more rents. While a reduction in the probability of patent enforcement reduces the expected licensing revenues of the inventor and therefore ex ante invention incentives, it also reduces ex post inefficiencies in the case where developers are financially constrained. Because of this, the optimal strength of

[1]For empirical evidence on the importance of financial constraints for innovation, see the summary article by Hall and Lerner (2010).

patent protection is the lower the more developers are financially constrained. If it is possible to have industry - (or region -) specific patent protection, it is optimal to have lower protection in industries (regions) in which more developers are financially constrained. Applied to the global patent system, this suggests that it might be optimal to reduce patent protection in developing countries where local firms that obtain licenses from big multinational firms are often financially constrained.

If the inventor is financially constrained and needs to get funding from an outside investor who cannot observe the effort that she undertakes, the results are different: now, the investor needs to give some rents to the inventor in order to induce a high level of effort and is only willing to finance the same amount of R&D investment if patent protection is increased. Consequently, it is optimal to grant stronger patent protection if the inventor is financially constrained. Under some conditions, this result also holds if there is uncertainty about the research costs of the inventor or if there are multiple inventors with different research costs for whom the strength of patent protection is the same. Then, an increase in patent protection induces more firms to invent but also increases the deadweight losses for all inframarginal inventors that are already active. As less inventors break even if there are financial constraints, the second effect is weaker in this case, making stronger patent protection optimal.

In *Appendix B* the case is analyzed where the (unconstrained) inventor cannot observe the net wealth of the developer and only knows its conditional distribution. She can then offer a menu of contracts to the developer and in order to give incentives for him to self-select into the desired contract without giving away too much rents, the inventor might find it optimal to distort the effort choice of the developer with a positive probability even though she would not do so if she could observe his net wealth. In the case where nonpledgeable income and not unobservable effort is the source of the agency problem, the inventor might (if she cannot observe net wealth) find it profitable to charge such a high fixed fee that the developer is excluded from getting a license if his net wealth is low. In these cases, the inefficiencies induced by patent protection increase and the optimal strength of patent protection decreases.

2 Related literature

This paper builds on a very simple and standard model of financial constraints which is broadly used in the literature and the new feature is the analysis of patent policy within this setup.

Aghion and Tirole (1994a, 1994b) analyze a similar setup where the probability of innovating is a positive function of the levels of effort exerted by a research unit and by a customer who derives value from the invention. Using an incomplete contracts approach, they analyze the incentives of both parties to invest depending on the allocation of property rights on any forthcoming invention. As both parties cannot be made residual claimants at the same time, investment is in general inefficient and the authors analyze how the two parties want to allocate property rights ex ante depending on the relative R&D efficiencies, the relative ex ante bargaining power and potential cash constraints. Aghion and Tirole assume that it is not possible to contract for delivery of an invention and that investment decisions occur simultaneously, while the structure in my model is sequential so that the inventor first has to innovate and can then negotiate a licensing agreement with a firm which might consist of "selling" the invention to the firm by only charging a fixed fee. Aghion and Tirole (1994a and b) mention that property rights might not be allocated in a way that maximizes social surplus if one of the agents is financially constrained and cannot appropriately compensate the other party ex ante in order to obtain the property right. This is the same mechanism as in my model where the inventor might want to distort the effort level of the firm in an inefficient way by charging a success - based royalty if the firm cannot afford to pay a sufficient fixed fee. The main difference in my model is therefore that I introduce the strength of patent protection as a policy variable which allows to affect the allocation of property rights and that I analyze how the optimal strength of patent protection is affected by financial constraints. Aghion and Tirole (1994a and b) focus on the case where investments are substitutable so that the invention can be obtained even if only the customer and not the research unit exerts effort and they compare the two cases where either the research unit or the customer get the property right. By introducing a variable strength of patent enforcement that - after an invention has occurred - grants the property right to the inventor with a certain probability, I allow for intermediate allocations of property rights which might be preferable from a social point of view. While I explicitly model the agency problem that gives rise to financial constraints and derive the optimal licensing contract as a function of net wealth, Aghion and Tirole (1994a and b) give an example where they assume that financial constraints increase the investment costs of the customer. They find that it can become optimal to allocate the property right to the research unit instead of the customer if the latter becomes cash constrained as allocating the property right to her encourages research relatively less in this case. In my model, giving stronger patent protection to the inventor increases the probability that he induces distortive royalties on the firm so that it is optimal to reduce patent protection if the firm is financially constrained. I therefore reach the opposite conclusion.

Llobet and Suarez (2013) analyze the effect of financial constraints on the optimal extent of patent protection in a quality-ladder model of sequential innovation. In their model the presence of financial constraints increases R&D costs and reduces the rate of innovation in equilibrium. Llobet and Suarez (2013) study the case where patents at the same time provide protection against imitation and also against replacement by future innovators who want to enter the market with improved versions of goods. While the protection against imitation increases the value of an innovation and R&D incentives, protection against replacement decreases it as it leads to more backloaded profit flows for successful innovators. The optimal strength of patent protection therefore optimally balances these two opposing effects. If financial frictions increase and equilibrium growth decreases, the rate of replacement is reduced so that also the negative effect that patent protection has by protecting incumbents against replacement becomes weaker. As increasing protection against imitation still encourages R&D as before, it therefore becomes optimal to increase patent protection if R&D costs are increased due to financial constraints.

Chatelain, Kirsten and Amable (2010) study an endogenous growth model with financially constrained innovators who can use their previously obtained patents as collateral for loans. They analyze the effects of policies that reduce the uncertainty surrounding the use of patents as collateral for lenders and also the effects of tax policies, but assume that patents are always fully protected. Contrary to that, this paper analyzes how the presence of financial constraints on the side of innovators affects the trade-off between ex ante innovation incentives and ex post monopoly distortions.

Schroth and Szalay (2010) analyze the implications of financial constraints on the R&D decisions of firms in a patent race. Their model predicts that a firm is more likely to win the race if its wealth is larger and if the wealth of the rival is lower and they find empirical support for these predictions.

Gallini and Wright (1990) and Martimort, Poudou and Sand - Zantman (2009) analyze models in which there is asymmetric information about the quality of an invention and in which inventors want to charge success - (or output -) based royalties instead of only fixed fees in order to signal the quality of their invention. I do not consider this possibility in my model where royalties are never used if developers are not financially constrained. My conjecture is, however, that the qualitative results that I obtain about the effect of financial constraints on the optimal strength of patent protection would be the same in a more complex model: if inventors want to use royalties in order to signal the quality of their invention, financial constraints of developers will probably still push them to impose larger royalties and lower fixed fees than they would impose in the case where developers are not financially constrained.

Using licensing data from biotech firms, Hegde (2010) tests hidden quality and unobservable effort theories about contractual payment schemes. Among other things, he finds that fixed fees are positively correlated with the „inventive age" of licensees which is measured as the difference between the license date and the application date of the licensees earliest patent application. As financial constraints are likely to decrease with inventive age, this might be interpreted as empirical support for the model prediction that fixed fees are larger if inventions are licensed to firms that are less financially constrained. However, Hegde (2010) does not find any statistically significant relation between licensees inventive age and the size of royalty payments so that it is not clear whether royalties are in fact higher if licensees are more financially constrained as predicted by the model.

In an empirical study using a firm level panel from Belgium, Czarnitzki, Hottenrott and Thorwarth (2010) find that research investments of firms are more sensitive to firms´operating liquidity than development investments. This indicates that financial constraints are larger for inventors than for developers in this case and that it might be optimal to grant strong patent protection on basic inventions. It is, however, questionable whether this result also holds in cases where large companies in developed countries undertake the research, while small companies in developing countries adapt and develop the inventions to the needs of local markets.

3 The model setup

There is an inventor who has access to an R&D technology that allows her to make an invention with probability x if she incurs R&D costs $C(x) = \frac{1}{k}x^k$, where $k > 1$ is assumed to hold. The invention - which might be thought of as an idea - can be used as an input in the production process of different developer firms (indexed by $i \in \{1; n\}$) that operate in independent markets. Given these firms have access to the invention they can develop a marketable goods out of it[2]. The probability of successfully developing such a good in market i is given by p_i^H if developer i exerts a high level of effort and by p_i^L $(< p_i^H)$ if the level of effort is low. Exerting the high level of effort costs e_i while exerting the low level of effort is costless. If a good is developed, developer i derives profits π_i from selling it while profits are zero if no good is developed. All agents are risk neutral and it is assumed that it is efficient to exert the high level of effort so that $p_i^H \pi_i - e_i > p_i^L \pi_i$ (**Condition 1**).

Developer firms are endowed with net assets I_i and their liability is restricted to these assets plus their profit income. Agents external to a developer firm cannot observe the level of effort exerted by the firm so that contracts between developers and an external agent like the inventor cannot be made conditional on effort. If an invention occurs, the inventor is granted a patent that is enforced with probability Δ. An enforced patent allows to exclude developers from using the invention and also guarantees that licensing contracts are enforced. If a patent is not enforced, all developers have costless access to the invention but it is assumed that they can

[2]The innovation is a nonrival good and for simplicity it is assumed that it is costless to make it accessible to developers.

still earn the same profits π_i given they successfully develop a good[3]. All agents are risk neutral and maximize expected profits. It is assumed that the inventor knows the values of p_i^H, p_i^L, e_i and π_i and can also observe I_i (in *Appendix B*, the case where I_i is not observable is analyzed). No restrictions are imposed on private contracting so that licensing contracts can depend on all these parameters and might therefore be developer - specific.

The timing of the game is the following: in period $t = 0$, the strength of patent protection is set. In $t = 1$, the inventor decides how much to invest in R&D and in period $t = 2$, an invention either occurs or does not occur. In period $t = 3$, it is decided whether an invention receives patent protection and if it does, the inventor and the developers agree to the terms of the licensing contracts. In period $t = 4$, developers decide whether to exert effort and in period $t = 5$ the success of the development is realized and developers pay licensing fees if the patent is enforced. There is no discounting.

4 The optimal licensing contract

In the following section it is assumed that the inventor is not financially constrained and has all the bargaining power. Given an invention has occurred and the patent is enforced, the expected licensing revenue that the inventor gets from developer i is denoted by W_i. Without loss of generality, it is assumed that licensing contracts specify a royalty R_i that has to be paid in the case where the developer successfully develops good i and a fixed fee F_i which has to be paid independently of the development success. Due to limited liability, the (LL) constraints $F_i \leq I_i$ and $R_i + F_i \leq I_i + \pi_i$ need to be satisfied. As developers operate in independent markets, total expected profits of the inventor are maximized if she offers each developer the licensing contract that maximizes the expected licensing revenue extracted from him. This optimal (from the point of view of the inventor) licensing contract can be derived as the solution to a very standard moral hazard problem and is given as follows:

[3] The extent to which a developer who is the first to develop final good i can appropriate the social surplus created by his development is therefore taken as given and assumed to be independent of Δ.

Lemma 1. *Defining* $z_i \equiv e_i \frac{p_i^H}{p_i^H - p_i^L} - \pi_i(p_i^H - p_i^L)$*, an optimal licensing contract* $F_i^*(I_i); R_i^*(I_i)$ *for developer* i *as a function of* I_i *is given as follows:*

Case a): if $z_i \leq I_i$, $F_i^* = \min\left\{I_i; \frac{p^L e_i}{p^H - p^L}\right\}$ *and* $R_i^* = \pi_i - \frac{e_i}{p_i^H - p_i^L}$

Case b): if $0 \leq I_i < z_i$, $F_i^* = 0$ *and* $R_i^* = \pi_i$

In case (a), the developer exerts the high level of effort while he exerts the low level of effort in case (b)

Proof. A developer is induced to exert the high level of effort if the incentive compatibility constraint $p_i^H(\pi_i - R_i) - e_i \geq p_i^L(\pi_i - R_i)$ (IC) is met, that means if $R_i \leq \pi_i - \frac{e_i}{p_i^H - p_i^L}$. Given (IC) holds and the inventor has all the bargaining power, developer i accepts a licensing contract if the expected payoff is nonnegative, that means if the participation constraint $p_i^H(\pi_i - R_i) - F_i - e_i \geq 0$ (PC_H) is satisfied. If the IC constraint is violated so that the developer exerts the low level of effort, the participation constraint is given by $p_i^L(\pi_i - R_i) - F_i \geq 0$ (PC_L). In order to determine the optimal licensing contract, the expected licensing revenues in the cases where the high and the low effort levels are implemented have to be compared: If the high level of effort is implemented, the maximization problem is given as:

$\max_{R_i, F_i} W_i = p_i^H R_i + F_i$ s.t. $R_i \leq \pi_i - \frac{e_i}{p_i^H - p_i^L}$ (IC); $p_i^H(\pi_i - R_i) - F_i - e_i \geq 0$ (PC_H); $F_i \leq I_i$ and $R_i + F_i \leq I_i + \pi_i$ (LL).

Setting the royalty at the highest level compatible with the IC ($R_i = \pi_i - \frac{e_i}{p_i^H - p_i^L}$), W_i is maximal given the PC and the LL constraints if $F_i = \min\left\{I_i; \frac{p^L e_i}{p^H - p^L}\right\}$. If $I_i \geq \frac{p_i^L e_i}{p_i^H - p_i^L}$, licensing revenues are then given by $W_i^H = p_i^H \pi_i - e_i$, so that the inventor extracts all the rents from the developer. If, however, $I_i < \frac{p_i^L e_i}{p_i^H - p_i^L}$, the developer obtains rents equal to $\frac{p_i^L e_i}{p_i^H - p_i^L} - I_i$ and $W_i^H = p_i^H \pi_i - \frac{p_i^H e_i}{p_i^H - p_i^L} + I_i < p_i^H \pi_i - e_i$. If the inventor implements the low level of effort, her maximization problem is given as:

$\max_{R_i, F_i} W_i = p_i^L R_i + F_i$ s.t. $p_i^L(\pi_i - R_i) - F_i \geq 0$ (PC_L); $F_i \leq I_i$ and $R_i + F_i \leq I_i + \pi_i$ (LL)

A solution to this problem is given by $R_i^L = \pi_i$ and $F_i^L = 0$ and the expected licensing revenues that the inventor receives from developer i are then given as $W_i^L = p_i^L \pi_i$, implying that no rents are given to the developer. Comparing the licensing revenues in the cases where the high and the low level of effort are implemented, we find that - using **Condition 1** - $W_i^H > W_i^L$ if $I_i > e_i \frac{p_i^H}{p_i^H - p_i^L} - \pi_i (p_i^H - p_i^L) \equiv z_i$, so that it is optimal to use the contract that implements the high level of effort if $I_i > z_i$ and the one implementing the low level of effort if $0 \leq I_i < z_i$. □

In the following it is assumed that $z_i > 0$ (**Condition 2**), as otherwise inventors would always want to implement the high level of effort, independently of how low the net wealth of a developer is.

The main result is therefore that the optimal licensing contract relies more heavily on distortive royalties and less on fixed fees if net wealth falls below a certain level[4]. The expected licensing revenue that the inventor obtains from developer i weakly increases in I_i. For low levels of I_i, the inventor chooses a licensing contract that induces the developer to exert the low level of effort. While this is not efficient from a social point of view, it is optimal for the inventor as inducing the high level of effort would require her to leave a lot of rents to the developer. The main problem is therefore that limited net wealth of developers restricts the hight of the fixed fee that they can be charged so that inventors have to earn licensing revenues through royalties which, if they become large, discourage the developer from exerting effort. The threshold z_i depends positively on e_i and p_i^L and negatively on π_i and p_i^H, so that it is more likely that the inventor implements the low level of effort if the agency problem becomes more pronounced, that means if e_i or p_i^L increase or if π_i or p_i^H decrease[5].

5 Optimal patent strength

An inventor anticipates to obtain the expected licensing revenue $Q \equiv \sum_{i=1}^{n} W_i(I_i)$ if she makes an invention and if her patent is enforced. Expected profits of undertaking R&D are therefore given as $\Pi = x\Delta \sum_{i=1}^{n} W_i(I_i) - \frac{1}{k}x^k$ and the research

[4]It should be noted that the derived licensing contract is not the only possible solution to the maximization problem and that, in the case where the high level of effort is implemented, the inventor could as well reduce the royalty and increase the fixed fee if the net wealth of a developer becomes larger. But this would not have any effect on the expected licensing revenues and the threshold level z_i below which the low level of effort is implemented.

[5]A reduction in appropriability in the final good market which decreases π_i would therefore make it more likely that the inventor implements the low level of effort.

intensity x^* that maximizes these expected profits is given by

(1) $$x^* = \left(\Delta \sum_{i=1}^{n} W_i(I_i)\right)^{\frac{1}{k-1}}$$

Note that x^* increases in patent strength Δ and weakly increases in I_i. If the patent is not enforced, developers have free access to the invention and always exert the high level of effort as this maximizes their expected payoff if they are residual claimants (due to **Condition 1**). There is therefore a trade - off associated with patent policy: stronger patent protection increases the incentives to invent but might create inefficiencies ex post if financially constrained developers are induced to exert the low level of effort. In the following, the optimal patent strength that maximizes the total expected surplus $S(\Delta)$ is derived:

Given an invention has occurred and developer i has access to it and can extract the whole surplus from consumers, the expected total surplus created by him is given by $V_i^H = p_i^H \pi_i - e_i$ if he exerts the high level of effort and by $V_i^L = p_i^L \pi_i$ ($< V_i^H$) if he exerts the low level of effort. If there are h developers which are induced to exert the low level of effort (if the patent is enforced) because their net wealth lies below the critical level z_i, the expected social surplus is given by:

(2) $$S(\Delta) = x^* \left(\Delta \left(\sum_{i=1}^{h} V_i^L + \sum_{i=h+1}^{n} V_i^H\right) + (1-\Delta) \sum_{i=1}^{n} V_i^H\right) - \frac{x^{*k}}{k}.$$

Replacing x^* by the value given in equation (1) and maximizing with respect to Δ gives the optimal patent strength Δ^* as:

(3) $$\Delta^* = \min\left\{1; \frac{\sum_{i=1}^{n} V_i^H}{\sum_{i=1}^{n} W_i(I_i) + k \sum_{i=1}^{h} \left(V_i^H - V_i^L\right)}\right\}.$$

Proposition 2. *Denoting the number of developers for which $I_i < z_i$ by h, we obtain $\Delta^* = 1$ if $h = 0$. The optimal strength of patent protection Δ^* (weakly) decreases in h.*

Proof. If $I_i \geq z_i$ for all i, so that $h = 0$, the second term in the denominator of equation (3) disappears. If $I_i \geq e_i \frac{p_i^L}{p_i^H - p_i^L} > z_i$ for all i, $W_i(I_i) = V_i^H = p_i^H \pi_i - e_i$ and the denominator in (3) becomes equal to $\sum_{i=1}^{n} V_i^H$ so that $\Delta^* = 1$. And also if $I_i \geq z_i$ for all i and $I_i < e_i \frac{p_i^L}{p_i^H - p_i^L}$ for at least one i, full patent protection ($\Delta^* = 1$) is optimal as $\left.\frac{\partial S(\Delta)}{\partial \Delta}\right|_{\Delta=1} > 0$ in this case. If net wealth I_j of developer j drops from a value $I_j > z_j$ to a value $I_j < z_j$, so that h increases by one, licensing revenues

$W_j(I_j)$ coming from developer j and therefore the first term in the denominator of equation (3) maximally decrease by the amount $V_i^H - V_i^L$. At the same time, the second term in the denominator of (3) increases by $k\left(V_i^H - V_i^L\right)$ and as $k > 1$, this increase in the second term is larger than the decrease of the first term so that the denominator increases. Δ^* therefore (weakly) decreases in h as the denominator in equation (3) increases in h (if $e_i \frac{p_i^L}{p_i^H - p_i^L} > I_i > z_i$ for some of the unconstrained developers, the corner solution $\Delta^* = 1$ is still possible even if $h > 0$, so that Δ^* does not depend on h in this range). □

If all developers are endowed with a sufficient amount of net wealth $I_i \geq z_i$ and can pay large enough fixed fees so that none of them is induced to choose the low level of effort, there are no ex post inefficiencies arising from patent protection and full protection is optimal as it allows the inventor to appropriate the maximal possible share of the social surplus stemming from her invention and therefore gives the best possible invention incentives[6]. The situation however changes if the net wealth I_i of some developers lies below the threshold z_i so that the inventor imposes such a high royalty on them that they exert the low level of effort if patents are enforced. In this case, a reduction in patent protection again reduces invention incentives and therefore the probability x that an invention occurs, but at the same time reduces the ex post inefficiencies arising from a distorted effort choice. And if more developers are financially constrained, this second effect becomes more important so that a weaker protection of patents is optimal.

If it is possible to make the strength of patent enforcement sector - specific, that means to allow developer i to have free access to the invention with probability $1 - \Delta_i$, it is optimal to grant full protection in sectors in which developers are not financially constrained and to only reduce patent protection in sectors where developers are financially constrained. If there are enough unconstrained sectors in which patents are enforced, it might even be optimal not to grant any protection in a sector where developers face severe financial constraints as the benefits that patent protection in this sector brings in the form of increased invention incentives

[6]If $e_i \frac{p_i^L}{p_i^H - p_i^L} > I_i > z_i$, the inventor cannot appropriate the whole surplus stemming from her innovation as she has to give rents to the developer. Because of that, there is underinvestment in R&D even in the case of full patent protection.

might be outweighted by the costs in terms of a lower ex - post efficiency. For a more formal discussion of this point see *Appendix A*.

6 Financially constrained inventor

While the previous sections focused on the case of financially constrained developers, this section analyzes the implications that financial constraints on the side of the inventor have for the optimal strength of patent enforcement. For simplicity it is assumed that there is only one developer with net wealth $I < z$, so that the inventor wants to induce him to exert the low level of effort by offering the contract $R = \pi$ and $F = 0$ if the patent is enforced. There is therefore an ex post inefficiency arising from patent protection[7] and the expected revenue of a successful inventor is given by $W = \Delta p^L \pi$.

In order to create the simplest possible moral hazard problem on the side of the inventor, it is assumed that the inventor needs to spend the fixed amount Y in order to be able to invent and makes an invention with probability x^H if she exerts a high level of effort and with probability x^L $(< x^H)$ if she exerts a low level of effort. Exerting the high level of effort costs f while exerting the low level of effort is assumed to be costless. If the inventor has enough funds to finance her own R&D, she prefers to exert the high level of effort if $x^H \Delta p^L \pi - f \geq x^L \Delta p^L \pi$, that means if $\Delta > \frac{f}{p^L \pi (x^H - x^L)}$. And given that the inventor exerts the high level of effort, she breaks even if $x^H \Delta p^L \pi - f - Y > 0$. The minimal patent strength which allows the inventor to break even is large enough to induce the high level of effort if $\frac{Y+f}{x^H p^L \pi} > \frac{f}{p^L \pi (x^H - x^L)}$, which is the case if $Y > \frac{x^L f}{x^H - x^L}$ (**Condition 3**) and this condition is assumed to hold in the following. Strengthening the probability of patent protection beyond the minimal level $\underline{\Delta} = \frac{Y+f}{x^H p^L \pi}$ which is required to make the inventor break even increases ex post inefficiencies but has no effect on the probability of successfully inventing and developing a good so that the optimal patent strength in the case where the inventor is not financially constrained is given by

[7] If this was not the case, full protection would always be optimal, as financial constraints on the side of the inventor tend to make stronger patent protection optimal as will become clear later on.

(4) $\Delta^* = \underline{\Delta} = \frac{Y+f}{x^H p^L \pi} < 1$

In the following it is assumed that $\Delta^* < 1$ (which always holds if π is large enough). Let us now consider the case where the inventor has no own funds to pay for the R&D costs[8], so that she needs to get funds equal to Y from an external investor in order to be able to undertake R&D. Given that the external investor (he) is risk neutral and cannot observe the level of effort exerted by the inventor, he can only recoup his investment if he charges a royalty T which the inventor has to pay out of her licensing revenue that she obtains if she successfully innovates, if her patent is enforced and if the developer develops the good. The inventor only exerts the high level of effort if the incentive compatibility constraint $x^H \Delta p^L (\pi - T) - f \geq x^L \Delta p^L (\pi - T)$ is satisfied, that means if $T \leq \pi - \frac{f}{\Delta p^L (x^H - x^L)}$ (IC^I). Given that the investor has all the bargaining power, he therefore either implements the high level of effort, sets the royalty equal to $T = \pi - \frac{f}{\Delta p^L (x^H - x^L)}$ and earns expected profits $\Omega^H = x^H \Delta p^L \pi - \frac{x^H f}{x^H - x^L} - Y$, or implements the low level of effort, charges the maximal royalty $T = \pi$ and earns expected profits $\Omega^L = x^L \Delta p^L \pi - Y$. Implementing the high level of effort yields larger expected profits ($\Omega^H > \Omega^L$) if $\Delta > \frac{x^H f}{\pi p^L (x^H - x^L)^2}$ (**Condition 4**), that means if patent strength is above a certain threshold. If $\Delta < \frac{x^H f}{\pi p^L (x^H - x^L)^2}$ so that the investor wants to implement the low level of effort, he cannot break even if $Y > \frac{x^H x^L f}{(x^H - x^L)^2}$. Therefore, there only exists a nonempty range of patent strengths $\frac{Y}{x^L p^L \pi} < \Delta < \frac{x^H f}{\pi p^L (x^H - x^L)^2}$ for which the investor wants to implement the low level of effort and makes nonnegative profits if $Y < \frac{x^H x^L f}{(x^H - x^L)^2}$ (**Condition 5**)[9].

Proposition 3. *Given $Y > \frac{x^H x^L f}{(x^H - x^L)^2}$ so that **Condition 5** is violated, the optimal patent strength is given by $\Delta^{**} = \frac{f}{p^L \pi (x^H - x^L)} + \frac{Y}{x^H p^L \pi} < 1$ if the net wealth of the inventor is zero. This patent strength is larger than the optimal patent strength in the case where the inventor is not financially constrained (which is given by $\Delta^* = \frac{Y+f}{x^H p^L \pi}$).*

Proof. If **Condition 5** is violated, the investor always prefers to implement the high level of effort given that Δ is large enough so that he makes nonnegative

[8]However, the inventor can always exert the high level of effort and pay the costs f by herself so that these costs might be interpreted as psychological costs.

[9]This condition is compatible with **Condition 3** as $\frac{x^H x^L f}{(x^H - x^L)^2} > \frac{x^L f}{x^H - x^L}$

profits. As an increase in patent strength does not increase the probability with which a good is invented and developed if the investor already finances the R&D costs but at the same time increases ex post inefficiencies, the optimal strength of patent protection Δ^{**} is given by the minimal level of Δ which allows the investor to break even (e.g., for which $\Omega^H \geq 0$), so that $\Delta^{**} = \frac{f}{p^L\pi(x^H - x^L)} + \frac{Y}{x^H p^L \pi}$. Given π is large enough, $\Delta^{**} < 1$ always holds and given **Condition 5** is violated, one obtains $\Delta^{**} > \Delta^*$ □

The intuition for this result is that the investor needs to give some rents to the inventor in order to induce the high level of effort, so that the patent strength that makes the investor break even is larger than the one that is needed to induce the inventor to undertake R&D and to exert effort if she is not financially constrained.

Proposition 4. *If the inventor has zero net wealth and if* $\frac{x^L f}{x^H - x^L} < Y < \frac{x^H x^L f}{(x^H - x^L)^2}$ *so that* ***Conditions 3*** *and* ***5*** *are satisfied, the optimal patent strength is given by* $\Delta_H^* \equiv \frac{x^H f}{\pi p^L (x^H - x^L)^2}$ *if*
$\left(x^H - x^L\right)\left(p^H\pi - e\right) - f - \frac{p^H\pi - e - p^L\pi}{p^L\pi}\left(\frac{\left(x^H\right)^2 f}{(x^H - x^L)^2} - Y\right) > 0$ *(**Condition 6**) and by* $\Delta_L^* \equiv \frac{Y}{x^L p^L \pi}$ *if* ***Condition 6*** *is violated. The relation* $\Delta_H^* > \Delta_L^* > \Delta^*$ *holds.*

Proof. If **Condition 5** holds, the investor can select between implementing the high or the low level of effort (of the inventor) and only chooses the high level if **Condition 4** is satisfied. However, this condition requires a larger level of patent protection than necessary to make the investor break even ($\frac{x^H f}{\pi p^L (x^H - x^L)^2} > \Delta^{**} = \frac{f}{p^L\pi(x^H - x^L)} + \frac{Y}{x^H p^L \pi}$ if $Y < \frac{x^H x^L f}{(x^H - x^L)^2}$) so that patent strength has to be at least equal to $\Delta_H^* \equiv \frac{x^H f}{\pi p^L (x^H - x^L)^2}$ in order to implement the high level of effort. Given **Condition 4** is violated so that the investor implements the low level of effort, Δ has to be at least equal to $\Delta_L^* \equiv \frac{Y}{x^L p^L \pi}$ in order to guarantee that the investor breaks even (e.g. that $\Omega^L > 0$). Given **Condition 5** holds, the optimal patent strength that minimizes ex post inefficiencies for a given rate of invention is therefore either given by Δ_H^* or by Δ_L^*, where $\Delta_H^* > \Delta_L^*$ due to **Condition 5**. Ex ante social welfare in the case where the high level of effort is implemented by setting $\Delta = \Delta_H^*$ is given by $S_H = x^H\left(\Delta_H^*\left(p^L\pi\right) + (1 - \Delta_H^*)(p^H\pi - e)\right) - f - Y$ and if the low level of effort is implemented by setting $\Delta = \Delta_L^*$, we get $S_L = x^L\left(\Delta_L^*\left(p^L\pi\right) + (1 - \Delta_L^*)(p^H\pi - e)\right) - Y$. Inserting the corresponding values, one finds that $S_H > S_L$ if $\left(x^H - x^L\right)\left(p^H \pi - e\right) - f - \frac{p^H \pi - e - p^L \pi}{p^L \pi}\left(\frac{\left(x^H\right)^2 f}{\left(x^H - x^L\right)^2} - Y\right) > 0$ (**Condition 6**). Comparing Δ_H^* with the patent strength Δ^* that is optimal in the case where the inventor is not financially constrained, one obtains that $\Delta_H^* > \Delta^*$ if $Y < \frac{f\left(2x^H x^L - \left(x^L\right)^2\right)}{\left(x^H - x^L\right)^2}$, which is always satisfied if **Condition 5** holds. Moreover, $\Delta_L^* > \Delta^*$ also always holds if Y lies in the range $\frac{x^L f}{x^H - x^L} < Y < \frac{x^H x^L f}{\left(x^H - x^L\right)^2}$ which is given by **Conditions 3** and **5**. □

The optimal strength of patent protection is therefore always larger in the case where the inventor is financially constrained than in the case where she is not. Given the inventor is financially constrained, the optimal strength of patent protection is larger if **Condition 6** holds, in which case it is optimal to induce him to exert the high level of effort. Let us now look at this condition in more detail and rewrite it in the following way:

$$(5) \quad \left(x^H - x^L\right)\left(p^H \pi - e\right) - f > \frac{p^H \pi - e - p^L \pi}{p^L \pi}\left(\frac{\left(x^H\right)^2 f}{\left(x^H - x^L\right)^2} - Y\right)$$

The condition $\Delta^* = \frac{Y+f}{x^H p^L \pi} \leq 1$ together with **Condition 3** implies that $\left(x^H - x^L\right) p^L \pi - f > 0$, so that the left hand side of equation (5) is positive because $p^H \pi - e > p^L \pi$ due to **Condition 1**. Due to **Conditions 1** and **5**, also the right hand side is positive, so that it is not a priori clear whether **Condition 6** is satisfied. If the inefficiencies resulting from patent protection are small because $p^H \pi - e - p^L \pi$ is small, the right hand side becomes negligibly small and we get $S_H > S_L$, so that the optimal patent strength is given by Δ_H^*. If however p^L becomes very small, implying that the ex post inefficiencies resulting from patent protection are large, the right hand side of inequality (5) becomes very large so that **Condition 6** is violated and we get $S_H < S_L$, so that the optimal patent strength is given by Δ_L^*. It is therefore only optimal to grant stronger patent protection in order to induce the inventor to exert the high level of effort if the increase in ex post inefficiency caused by stronger patent protection is not too large. The reason why the optimal strength of patent protection is larger when the inventor is financially constrained is the following: if the high level of effort is implemented, the investor needs to give some rents to the inventor in order to induce a high level of effort and is only willing to finance the same amount of R&D investment if patent protection is increased. And if the low level of effort is implemented, patents still need to be stronger than in the case without financial constraints as the expected total payoff from innovation (which is extracted entirely by the investor) is lower than in the case where the developer exerts effort (due to **Condition 1**).

It should be noted that the analysis and the conclusions would be qualitatively the same if a setup was used where the inventor directly sells its invention to consumers but where there are inefficiencies associated with patent protection due to monopoly pricing.

6.1 Varying invention costs

When there is a single inventor with known invention costs Y like assumed above, there are no benefits in extending the level of patent protection beyond the level which induces the inventor to exert effort and which allows him to break even. In the following, the case is considered where a uniform strength of patent protection applies to different inventors that operate in separate markets and have different values of Y (but the same values for all other parameters). In this case, the analysis is more involved as reducing patent protection always has the effect of discouraging some inventors with high values of Y. This setting can also be interpreted as one in which there is uncertainly about the value of Y of a single inventor.

In order to save space, the notations $V^H = p^H\pi - e$ and $V^L = p^L\pi$ $(< V^H)$ are used. It is assumed that Y is continuously distributed with density $q(Y)$ in the interval between $\underline{Y}$ and $\bar{Y}$. Only the case is considered in which $\underline{Y} > \frac{x^H x^L f}{(x^H - x^L)^2}$, so that **Condition 5** is violated while **Condition 3** holds, implying that an inventor with $Y > \underline{Y}$ (independently of whether she is financially constrained or not) always exerts the high level of effort given that Δ is large enough to allow her to break even. Moreover, it is assumed that $\bar{Y} \geq x^H V^L - f$, implying that it is only possible that all inventors that are not financially constrained break even at the same time if patents are fully protected (that means if $\Delta = 1$). A financially constrained inventor breaks even if $Y \leq \tilde{Y}_c \equiv \Delta x^H V^L - f\frac{x^H}{x^H - x^L}$, while an inventor who is not financially constrained breaks even if $Y \leq \tilde{Y}_u \equiv \Delta x^H V^L - f$, meaning if Y lies below a critical level that is higher than $\tilde{Y}_c$. Assuming that $\bar{Y} > x^H V^L - f$ holds therefore guarantees that increasing Δ always increases the number of inventors that manage to break even, given that there is a positive number of them. In order to guarantee that a financially constrained inventor breaks even at least in the case where $\Delta = 1$ and $Y = \underline{Y}$, it needs to be assumed that $\frac{fx^H}{(x^H - x^L)^2} < V^L = p^L \pi$ holds (**Condition 7**). Given the (probability) distribution of Y, the (ex ante) optimal patent strength can now be derived in the case where the inventors are never (for no realization of Y) financially constrained and in the case where they are always (for any realization of Y) financially constrained.

Proposition 5. *Given that Y is continuously distributed with density $q(Y)$ in the interval $\left[\underline{Y}, \bar{Y}\right]$ with $\underline{Y} > \frac{x^H x^L f}{(x^H - x^L)^2}$ and $\bar{Y} > x^H V^L - f$, that $\frac{fx^H}{(x^H - x^L)^2} < V^L$ (**Condition 7**) and that $\frac{\partial q(Y)}{\partial Y} \leq 0$, the optimal strength of patent protection is larger in the case where the inventors are financially constrained than in the case where they are not financially constrained ($\Delta_c^* > \Delta_u^*$)*

Proof. Social welfare in the case where the inventors are not financially constrained is given by $S_u = \int_{Y=\underline{Y}}^{\tilde{Y}_u} \left(x^H \left(\Delta V^L + (1-\Delta) V^H\right) - f - Y\right) q(Y) dY$ and by $S_c = \int_{Y=\underline{Y}}^{\tilde{Y}_c} \left(x^H \left(\Delta V^L + (1-\Delta) V^H\right) - f - Y\right) q(Y) dY$ in the case where the inventors are financially constrained. Because $\tilde{Y}_u = \Delta x^H V^L - f > \tilde{Y}_c = \Delta x^H V^L - f \frac{x^H}{x^H - x^L}$, welfare is higher in the unconstrained case. Patent policy faces the trade-off that increasing Δ reduces the surplus $x^H \left(\Delta V^L + (1-\Delta) V^H\right) - f - Y$ generated by inventors that break even, but at the same time increases the critical value $\tilde{Y}$, allowing more inventors to break even. Deriving social welfare with respect to Δ gives $\frac{\partial S_u}{\partial \Delta} = x^H \left(x^H (1-\Delta) V^H V^L q(\tilde{Y}_u) - \int_{\underline{Y}}^{\tilde{Y}_u} \left(V^H - V^L\right) q(Y) dY\right)$ and

$\frac{\partial S_c}{\partial \Delta} = x^H \left(\left(x^H (1-\Delta) V^H V^L + \frac{x^L}{x^H - x^L} V^L f\right) q(\tilde{Y}_c) - \int_{\underline{Y}}^{\tilde{Y}_c} \left(V^H - V^L\right) q(Y) dY\right)$.

Therefore, $\frac{\partial S_c}{\partial \Delta} - \frac{\partial S_u}{\partial \Delta} =$

$x^H \left(x^H (1-\Delta) V^H V^L \left(q(\tilde{Y}_c) - q(\tilde{Y}_u)\right) + \frac{x^L}{x^H - x^L} V^L f q(\tilde{Y}_c) + \int_{\tilde{Y}_c}^{\tilde{Y}_u} \left(V^H - V^L\right) q(Y) dY\right)$.

As the last two terms in the big bracket are positive, a sufficient condition for this expression to be positive is that $q(\tilde{Y}_c) - q(\tilde{Y}_u) \geq 0$. This is satisfied if $q(Y)$ is non-increasing in Y, that means if $\frac{\partial (q(Y))}{\partial Y} \leq 0$. Given S_u is maximized for a unique interior value Δ_u^*, $\Delta_c^* > \Delta_u^*$ therefore holds if $q(Y)$ is non-increasing in Y. As $sign\left(\frac{\partial^2 S_u}{\partial \Delta^2}\right) = sign\left(-\left(2V^H - V^L\right) q(\tilde{Y}_u) + V^H V^L x^H (1-\Delta) \frac{\partial q(\tilde{Y}_u)}{\partial \tilde{Y}_u}\right)$, S_u is concave in Δ if $\frac{\partial (q(Y))}{\partial Y} \leq 0$. Because $\frac{\partial S_u}{\partial \Delta}\Big|_{\Delta=1} < 0$, there is a unique interior solution $\Delta_u^* < 1$. Therefore, $\Delta_c^* > \Delta_u^*$ holds. □

The intuition behind this result is the following: an increase in patent protection on the one hand reduces the surplus generated by each active inventor due to increased deadweight losses originating from distortive licensing contracts, but on the other hand increases the number of inventors for which inventing becomes profitable. As in the case of financial constraints less inventors break even for any strength of patent protection, increasing protection does not increase deadweight losses as much as it does in the case without financial constraints where more inventors break even. Given the assumptions made about the distribution of firm types, it is not possible that an increase in patent protection increases the number of active inventors more in the case where inventors are not financially constrained than in the case where they are. Starting from the same level of patent protection, increasing protection is therefore more beneficial in the case where inventors are financially constrained.

7 Discussion

An important question is clearly whether there are other means beside a change in the strength of patent protection to deal with the problem of financial constraints. As long as agents like venture capitalists cannot observe the effort exerted by a developer (or the inventor) they face the same agency problem as the inventor (or the investor) and cannot improve upon the situation. And even if the inventor can buy a firm that develops his invention (or if the investor can buy the inventor), she (he) might still not be able to perfectly monitor the development (R&D) activities and might have to give rents to the managers in order to induce them to exert a high level of effort[10]. Another possibility might be that the government increases the net wealth of financially constrained agents by redistributing tax revenue. However, there might be considerable shadow costs of public funds and the government might lack the information needed in order to only give the funds to the most financially constrained firms.

While the paper only analyzes the case where a patent is either enforced or

[10]Vertical integration might only be jointly profitable to both parties if patents are very weak and if it can be used as a substitute for patent protection (as it might facilitage trade secrecy). Then, a developer might be willing to compensate the inventor in order to be the only one getting access to the invention.

not enforced, it might also be possible to restrict the set of licensing contracts that inventors are allowed to use. So, an upper bound on the royalty could be introduced in order to induce firms to exert a high level of effort or fixed fees could be restricted in order to reduce the probability that a firm is excluded from getting a license and these policies (in combination with full enforcement) might be preferable to a policy where patents are only enforced with a certain probability but that does not impose any restrictions on licensing contracts. However, such policies might be more difficult to enforce.

In the analysis it is assumed that the inventor has all the bargaining power so that there are no inefficiencies if neither of the parties is financially constrained and if the developers can appropriate the entire social surplus. If the developers would have some bargaining power, they might negotiate smaller licensing fees, but the mechanism that licensing contracts would rely more heavily on distortive royalties and less on fixed fees if developers are endowed with less net wealth would be the same, so that weaker patent protection would again be optimal if more developers were financially constrained. However, in such a case inventors would have insufficient ex ante invention incentives as they could not appropriate the entire social surplus created by their invention even if developers were not financially constrained. This would tend to increase the optimal strength of patent protection.

The paper studies the effects of granting patent protection on inventions that are inputs into the production and development processes of developers, taking the extent to which developers can appropriate surplus from consumers as given. It might, however, be possible to use separate patent or antitrust policies in order to determine to which extent developers that are the first to develop a certain final good should be protected against a possible threat of imitation in the final goods market. If the strength of patent protection in the final goods markets is set in order to optimally solve the trade - off between ex ante invention incentives and ex post monopoly distortions, the optimal strength might be higher in the case where developers are financially constrained. The reason for that would be that stronger patent protection in the final good sector eases the moral hazard problem and tends to induce developers to still select the high level of effort even if royalties are increased due to financial constraints. An increase in patent protection in the final goods sectors might therefore lead to a larger spur in invention incentives in the case where developers are financially constrained (and would choose the low level of effort for weaker patent protection) so that it might be optimal to increase this protection even if this also leads to larger deadweight losses[11]. If the strengths of patent protection for the initial invention and for the final goods sectors are jointly set in order to maximize the expected surplus, it would still be optimal to fully enforce patents on the initial invention if developers are not financially constrained[12]. In the case where developers are financially constrained , it might however again be optimal to reduce patent protection on the initial invention, even if patent protection in the final goods sectors is increased at the same time.

In the model it is assumed that a given developer only licenses from one inventor. In the more realistic case where a developer licenses from different inventors, additional complications arise as the licensing contract offered by one inventor affects the remaining net wealth of the firm and its incentives to exert effort and therefore also the optimal choice of licensing contracts by other inventors and their return. The basic feature that licensing contracts (if somehow aggregated) rely more heavily on success - based distortionary royalties if the net wealth of a firm is lower, should, however, be the same.

Given the mechanisms highlighted by the model are of relevance in real world licensing agreements, some **policy implications** can be derived:

The optimal strength of patent protection should depend positively on the extent of financial constraints on the side of inventors and negatively on the prevalence of financial constraints on the side of developers who license inventions.

Applied to the global patent system, the model suggests that it is optimal (from a global point of view) to have weaker patent protection in countries where many developing firms that obtain licenses are financially constrained. This might in particular be the case in less developed countries or emerging economies where

[11]The argument is therefore the same as in the case where the inventor is financially constrained.

[12]Decreasing the protection on the initial innovation would only reduce innovation incentives without reducing ex post inefficiencies. However, it might be optimal to grant less than full protection in the final goods market in order to reduce deadweight losses arising from monopoly pricing.

not much invention takes place but which license a lot of inventions from firms in developed countries, for which financial constraints might be less important. However, if there are possibilities for international arbitrage (through parallel trade), it might not be possible or desirable to make the extent of patent protection very heterogeneous across countries.

8 References

Aghion, Philippe and Jean Tirole (1994a); "The Management of invention"; The Quarterly Journal of Economics, Vol. 109, No. 4 (Nov., 1994), pp. 1185-1209

Aghion, Philippe and Jean Tirole (1994b); "Opening the black box of invention"; European Economic Review 38, pp. 701 - 710

Amable B., J.B. Chatelain & K. Ralf (2010); “Patents as Collateral”; Journal of Economic Dynamics and Control. Vol. 34 Issue 6, 1092-1104

Czarnitzki, Hottenrott and Thorwarth (2010); “Industrial research versus development investment: the implications of financial constraints”; Cambridge Journal of Economics, 2010, 1 of 18

Gallini, Nancy and Brian Davern Wright (1990); "Technology Transfer under Asymmetric Information"; RAND Journal of Economics, vol. 21, issue 1, pages 147-160

Hall, B. H. and Lerner, J. “The financing of R&D and Innovation”; Handbook of the Economics of Innovation (eds. Hall, B. H. and N. Rosenberg), Elsevier-North Holland

Hegde, Deepak (January 31, 2010); "Imperfect information and contracts in the market for ideas: evidence from the licensing of biomedical inventions"; working paper

Llobet, Gerard and Javier Suarez (2013); “Entrepreneurial Innovation, Patent Protection and Industry Dynamics”, Working Paper

Martimort, David, Jean-Christophe Poudou and Wilfried Sand-Zantman (2009); "Contracting for an invention under Bilateral Asymmetric Information"; The Journal of Industrial Economics

Schroth, Enrique and Dezsö Szalay (2010); "Cash Breeds Success: The Role of Financing Constraints in Patent Races"; Review of Finance 14(1), 73-118

9 Appendix A: Sector - specific patent protection

If it is possible to condition the strength of patent enforcement on specific sectors, that means to allow developer i to have free access to the invention with probability $1-\Delta_i$, the profit maximizing research intensity is given by

(6) $x^* = \left(\sum_{i=1}^{n} \Delta_i W_i(I_i)\right)^{\frac{1}{k-1}}$

The expected social surplus as a function of the different patent strengths can now be derived as:

(7) $S = x^* \left(\sum_{i=1}^{n} V_i^H - \sum_{i=1}^{h} \Delta_i \left(V_i^H - V_i^L\right)\right) - \frac{x^{*k}}{k}$

Inserting (6) into (7) and deriving with respect to Δ_j allows to determine the optimal patent strength Δ_j^* in sector j. If $I_j > z_j$ so that the firm in sector j exerts the high level of effort if the patent is enforced we get[13]:

$$sign \frac{\partial S}{\partial \Delta_j} = sign \left\{\sum_{i=1}^{n} V_i^H - \sum_{i=1}^{h} \Delta_i \left(V_i^H - V_i^L\right) - \sum_{i=1}^{n-1} \Delta_i W_i(I_i) - \Delta_j W_j(I_j)\right\}$$

This sign is nonnegative as even in the case where $\Delta_i = 1$ for all i the expression inside the brackets is given by $V_j^H - \Delta_j W_j(I_j) = V_j^H(1-\Delta_j) \geq 0$. Therefore, $\Delta_j^* = 1$ given that $I_j > z_j$, so that full patent protection is optimal in sectors where the developers are not financially constrained. The reason for this is again that there are no inefficiencies associated with patent protection in sector j so that full protection in this sector is optimal as it allows the inventor to appropriate the whole surplus created by his invention in this sector.

In the case where $I_j < z_j$, so that the developer in sector j exerts the low level of effort if the patent is enforced, the optimal patent strength in this sector can be derived as[14]:

(8) $\Delta_j^* = \frac{1}{k\left(V_j^H - V_j^L\right) + V_j^L} \left(\sum_{i=1}^{n} V_i^H - \sum_{i=2}^{h} \Delta_i \left(V_i^H - V_i^L\right) - \left(1 + \frac{(k-1)\left(V_j^H - V_j^L\right)}{V_j^L}\right) \sum_{i=2}^{n} \Delta_i W_i(I_i)\right)$

This optimal patent strength, which is a function of the extent of patent protection in all the other sectors, can lie in the whole range between zero and one: If the patent is for example not protected in any other sector (so that $\Delta_i = 0$ for all $i \neq j$) and if n is large enough, we get $\Delta_j^* = 1$. Even though there are inefficiencies associated with patent protection in sector j, full protection in this sector is optimal in this case (given that the extent of patent protection in other sectors cannot

[13] The notation choosen is such that the index j corresponds to n.

[14] The notation choosen is such that the index j corresponds to 1.

be changed) as this encourages invention from which other sectors also benefit. If however $\Delta_i = 1$ for all $i \neq j$ and if $h = 1$ so that only sector j is financially constrained, $\Delta_j^* = 0$ is optimal given that $\sum_{i=1}^{n} V_i^H - \left(1 + \frac{(k-1)\left(V_j^H - V_j^L\right)}{V_j^L}\right) \sum_{i=2}^{n} V_i^H \leq 0$ (which is more likely to hold if k, n and/ or V_j^H are large and/ or V_j^L is small). In this case, the benefits of patent protection in sector j stemming from increased invention incentives are outweighed by the costs caused by an inefficiently low level of effort so that it is optimal not to protect the patent in sector j. In other cases, an intermediate strength of patent protection in sector j can be optimal.

Compared to the case of uniform protection, the benefit of sector-specific protection is clearly that patent enforcement can be reduced in financially constrained sectors without at the same time reducing it in unconstrained sectors which would lead to a greater reduction in invention incentives.

10 Appendix B: Non - observable net wealth

Taking the setup from above in which the inventor is not financially constrained, it is now assumed that the inventor cannot observe the net wealth I_i of a developer and only knows its conditional distribution. A reason for this might be that developers endowed with a lot of assets can hide them or outsource their development programs to firms endowed with less assets. In this case, the inventor can offer a menu of contracts to developer i which is conditional on the known parameters π_i, p_i^H, p_i^L and e_i. In order to simplify notation and to clarify the analysis, the case of a single developer is analyzed in the following.

For any distribution for which there is a positive probability that $I < z$, it is optimal to include the contract $F^L = 0$ and $R^L = \pi$ in a menu of contracts offered to the developer as it allows to obtain the maximal expected licensing revenue in the case where $I < z$, but does not leave any rents to the developer so that it does not induce him to choose this low-effort contract if his net wealth is large enough to choose a high-effort contract (if this gives nonnegative rents). If there is a positive probability that $z \leq I < e\frac{p^L}{p^H - p^L}$, the developer can be induced to exert the high level of effort if his net wealth I lies in this interval if the contract $F \leq I$ and $R = \pi - \frac{e}{p^H - p^L}$ is included in the menu of contracts. However, this contract gives rents to the developer (see proof of Lemma 1), implying that he always prefers this contract compared to another one that extracts more rents by setting a larger fixed fee or royalty, even if he has enough net wealth to pay a higher fixed fee. The inventor can therefore not increase expected revenues by offering a menu of contracts that all implement the high level of effort and by offering a single contract that induces the high level of effort, he faces the trade-off that extracting more rents by setting a higher fixed fee F reduces the probability that the firm selects this contract as the firm might not have enough net wealth to pay this fixed fee. Depending on the distribution of I, the inventor might therefore include such a high fixed fee in the contract which induces the high level of effort that there is a positive probability that the developer cannot afford to pay this fee and that he selects the low-effort contract although he would have been offered a high-effort contract if his net wealth was observable. This becomes clear by looking at the following example: let us assume that the developer has net wealth $I = I^H > e\frac{p^L}{p^H - p^L}$ with probability β and net wealth $I = I^L$ with probability $1 - \beta$ and that $z \leq I^L < e\frac{p^L}{p^H - p^L}$. If wealth was observable, the developer would therefore always be offered a contract that induces the high level of effort and this contract would give him some rents in the case where $I = I^L$ while the inventor would extract all rents in the case where $I = I^H$ (see proof of Lemma 1). If net wealth is unobservable for the inventor, she can either induce the high level of effort in both cases by offering the contract $F = I^L$ and $R = \pi - \frac{e}{p^H - p^L}$ (pooling) in which case she has to give some rents to the developer, or she can extract all rents by offering the menu of contracts $F = \frac{p^L e}{p^H - p^L} \left(> I^L\right)$; $R = \pi - \frac{e}{p^H - p^L}$ (high effort) and $F = 0$; $R = \pi$ (low effort) in which case the developer selects the high (low) effort contract if his net wealth is given by $I = I^H$ $(I = I^L)$[15]. Expected licensing revenues in the case of pooling are given by $W^P = p^H \pi - e\frac{p^H}{p^H - p^L} + I^L$ and in the case where the menu of contracts is offered by $W^M = \beta(p^H \pi - e) + (1 - \beta)p^L \pi$, so that the inventor prefers to offer the menu of contracts to pooling if $I^L < e\frac{p^H}{p^H - p^L} - \pi\left(p^H - p^L\right)(1 - \beta) - \beta e \equiv g$. This threshold g is equal to z (which constitutes the lower bound for I^L) if $\beta = 0$ and equal to $e\frac{p^L}{p^H - p^L}$ (which is the

[15]In the case where $I = I^H$ it is assumed that the developer always selects the high-effort contract although he is indifferent between the two choices. Alternatively, it could be assumed that the fixed fee in the high-effort contract is a little bit smaller so that the developer strictly prefers this contract as it gives (arbitrarily small) rents.

upper bound for I^L) if $\beta = 1$ so that it is more likely that the inventor prefers to offer the menu of contracts if the probability β that the net wealth of the developer lies in the range where no rents have to be given to him ($I^H > e\frac{p^L}{p^H - p^L}$) becomes bigger. Given the inventor chooses to offer the menu of contracts, this implies that the developer exerts the low level of effort with probability $1 - \beta$ although the inventor would prefer to always induce the high level of effort if she could observe net wealth. The fact that net wealth cannot be observed therefore increases the probability that patent protection induces developers to select the low level of effort and as this is inefficient from a social point of view, it is again optimal to reduce the extent of patent protection in this case: If $I^L > g$ so that the inventor prefers to offer the pooling contract, full patent protection is optimal as it allows the inventor to appropriate a larger part of the surplus created by her invention without leading to inefficiencies due to a distorted effort choice. In the case where $I^L < g$, so that the inventor offers the menu of contracts, the expected social surplus is given as:

(9) $S = x^* \left[\Delta \left(\beta \left(p^H \pi - e\right) + (1 - \beta)\, p^L \pi\right) + (1 - \Delta) \left(p^H \pi - e\right)\right] - \frac{x^{*k}}{k}$

Taking the case of a quadratic R&D cost function ($k = 2$), we obtain $x^* = \Delta W^M$ and the optimal patent strength Δ^* that maximizes S is given by

(10) $\Delta^* = \frac{p^H \pi - e}{p^H \pi - e + (1-\beta)(p^H \pi - e - p^L \pi)} < 1$

Δ^* is lower than one as $p^H \pi - e - p^L \pi > 0$ (due to **Condition 1)** and it increases in β.

Summing up: an increase in the probability β that the developer is endowed with the low level of net wealth makes it more likely that the inventor prefers to offer a menu of contracts, in which case less than full patent protection is optimal. But at the same time, an increase in β increases the optimal protection in the case where a menu of contracts is offered as it implies that the probability that a developer selects the (inefficient) low effort contract in the case where the patent is enforced becomes smaller.

10.1 Non - pledgeable income

This section analyzes a very similar setup in which the source of the moral hazard problem is not that a developer i has to exert (unobservable) effort but that, once he has access to the invention, he can select between an efficient and an inefficient development technology, where the latter allows to derive some private nonpledgeable income or benefits of size B_i. The efficient (inefficient) technology leads to the development of a good with probability p_i^H (p_i^L) and is efficient because the relation $p_i^H \pi_i > p_i^L \pi_i + B_i$ (**Condition 7**) is assumed to hold. Otherwise, the setup is the same as above. If the inventor wants to induce developer i to choose the efficient technology, the maximization program is given as:

$\max\limits_{R_i,F_i} W_i^H = p_i^H R_i + F_i$

s.t.: $p_i^H(\pi_i - R_i) \geq p_i^L(\pi_i - R_i) + B_i$ (IC); $p_i^H(\pi_i - R_i) - F_i \geq 0$ (PC$_H$); $F_i \leq I_i$ and $R_i + F_i \leq I_i + \pi_i$ (LL)

If the inventor wants to induce developer i to select the inefficient technology, she solves the program:

$\max\limits_{R_i^L,F_i^L} W_i^L = p_i^L R_i^L + F_i^L$

s.t.: $p_i^L(\pi_i - R_i^L) + B_i - F_i^L \geq 0$ (PC$_L$); $F_i^L \leq I_i$ and $R_i^L + F_i^L \leq I_i + \pi_i$ (LL)

Proceeding in a similar way as above allows to derive the optimal licensing contract for firm i as a function of I_i. Assuming that $B_i > \frac{\pi_i\left(p_i^H - p_i^L\right)^2}{p_i^H}$ (**Condition 8**)[16] and defining $q_i \equiv B_i\left(\frac{2p_i^H - p_i^L}{p_i^H - p_i^L}\right) - \pi_i\left(p_i^H - p_i^L\right)$, an optimal contract is given as:

if $\frac{p_i^H B_i}{p_i^H - p_i^L} \leq I_i$ (case a), $F_i^* = \frac{p_i^H B_i}{p_i^H - p_i^L}$, $R_i^* = \pi_i - \frac{B_i}{p_i^H - p_i^L}$ and $W_i = p_i^H \pi_i$, high effort, no rents

if $q_i \leq I_i < \frac{p_i^H B_i}{p_i^H - p_i^L}$ (b), $F_i^* = I_i$, $R_i^* = \pi_i - \frac{B_i}{p_i^H - p_i^L}$ and $W_i = p_i^H \pi_i - \frac{p_i^H B_i}{p_i^H - p_i^L} + I_i$, high effort, positive rents ($= \frac{p_i^H B_i}{p_i^H - p_i^L} - I_i$)

if[17] $B_i \leq I_i < q_i$ (c), $F_i^* = B_i$, $R_i^* = \pi_i$ and $W_i = p_i^L \pi_i + B_i$, low effort, no rents

if $0 \leq I_i < B_i$ (d), $F_i^* = I_i$, $R_i^* = \pi_i$ and $W_i = p_i^L \pi_i + I_i$, low effort, positive rents ($= B_i - I_i$)

In the case where $I_i < q_i$, the optimal licensing contract therefore induces the developer to select the socially inefficient technology. Using the same analysis as in the previous section, it can again be shown that full patent protection is optimal if no developer is financially constrained (that means if $I_i \geq q_i$ for all i) but that

[16] If this condition does not hold, the inventor always (independently of I_i) wants to induce the firm to use the efficient technology

[17] Under **Condition 8**, we always have $B_i < q_i$

the optimal patent strength decreases in the number of firms that are financially constrained (that means for which $I_i < q_i$).

10.1.1 Non - observable net wealth

In the case where the inventor cannot observe net wealth I_i and only knows its conditional distribution, the analysis is however not the same as in the case of unobservable effort as now rents have to be given to developers with the lowest level of net wealth $I_i < B_i$. In the following, subscripts are dropped in order to simplify the notation and the analysis again focuses on the case of a single developer. Offering a menu of contracts in an incentive compatible way requires that the developer wants to select a "high-wealth contract" if his wealth is high although he can also pick a "low-wealth contract" with a lower fixed fee. Therefore, nonnegative rents have to be given to the developer in order to induce him to select a "high-wealth contract" if his net wealth is high. Because of that, a contract targeting the developer if his wealth lies in the parameter range (a) has to be modified in order to give higher rents to the developer if it is offered in a menu with a contract for the range (d), that gives him some rents and requires a lower fixed fee. Moreover, expected revenues cannot be increased by offering menus of contracts covering the ranges (b) and (a) or the ranges (c) and (d) as the induced level of effort is the same in these ranges and the developer would always select the contract that gives the highest rents.

Looking at the case where the developer has net wealth $I = I^H \geq \frac{p^H B}{p^H - p^L}$ (case a) with probability γ and net wealth $I = I^L < B$ (case d) with probability $1-\gamma$, the inventor can offer the menu of contracts $F^L = I^L$; $R^L = \pi$ and $F^H = \frac{p^H B}{p^H - p^L} - B + I^L$; $R^H = \pi - \frac{B}{p^H - p^L}$, where the first (second) one induces the low (high) level of effort and is chosen if the developer has the low (high) level of net wealth and both give rents equal to $B - I^L$ to him. Expected licensing revenues are then given as $W^M = \gamma(p^H \pi - B + I^L) + (1-\gamma)(p^L \pi + I^L)$. However, the inventor can also only offer the contract $F^E = \frac{p^H B}{p^H - p^L}$; $R^E = \pi - \frac{B}{p^H - p^L}$ (exclusion, like case a)) which extracts all the rents but which the developer only accepts in the case where $I = I^H$ and which therefore excludes him from getting access to the invention in the case where $I = I^L$[18]. Expected licensing revenues under exclusion are given by $W^E = \gamma p^H \pi$ and are larger than the revenues W^M in the case where the menu of contracts is offered if $\gamma > \frac{p^L\pi + I^L}{p^L\pi + B}(< 1)$ so that exclusion is more likely to be the preferred choice of the inventor if γ is large and/ or if I^L is low.

If the patent is not enforced, the firm always (independently of I) gets access to the invention and selects the efficient technology. Taking the case of a quadratic R&D cost function ($k = 2$), expected social surplus in the case where the inventor prefers to offer only the exclusive contract is given as:

$S^E = x^* \left(\Delta\gamma p^H\pi + (1 - \Delta)p^H\pi\right) - \frac{x^{*2}}{2}$

Inserting $x^* = \Delta W^E = \Delta\gamma p^H\pi$ and deriving with respect to Δ gives:

$\Delta^{*E} = \frac{p^H\pi}{2(1-\gamma)\pi p^H + W^E} = \frac{1}{2-\gamma}$, which is smaller than one for $\gamma < 1$ and increases in γ (decreases in $1 - \gamma$).

In the case where the inventor prefers to offer the menu of contracts, the expected social surplus is given as:

$S^M = x^* \left(\Delta\left[\gamma p^H\pi + (1 - \gamma)(p^L\pi + B)\right] + (1 - \Delta)p^H\pi\right) - \frac{x^{*2}}{2}$

Inserting $x^* = \Delta W^M = \Delta\left(p^L\pi + I^L + \gamma(p^H\pi - p^L\pi - B)\right)$ and deriving with respect to Δ gives:

$\Delta^{*M} = \frac{p^H\pi}{2(1-\gamma)(\pi(p^H-p^L)-B)+W^M} = \frac{p^H\pi}{(2-\gamma)(\pi(p^H-p^L)-B)+p^L\pi+I^L}$, which again increases in γ.

Comparing the optimal patent strength in the two cases, we get that $\Delta^{*M} > \Delta^{*E}$ if $W^M - W^E < 2(1 - \gamma)\left(p^L\pi + B\right)$ (or, replacing W^M and W^E with their corresponding values, if $I^L < B(2 - \gamma) + p^L\pi(1 - \gamma)$). If $W^E = W^M$, so that the inventor is indifferent between both strategies - but also if $I^L$is sufficiently low or γ sufficiently large - this inequality is satisfied[19] and the optimal strength of patent protection is therefore lower in the case of exclusion than in the case where the inventor offers a menu of contracts. The reason for this is that exclusion implies larger inefficiencies as it prevents the developer from using the invention in the case where $I = I^L$ while the only inefficiency arising in this case if a menu of contracts is offered is that he selects the inefficient technology.

[18]It is never optimal to only offer the contract $F = I$, $R = \pi$ which induces the low level of effort and which the developer always accepts as expected revenues W^P in this case are lower than if the menu of contracts is offered: $W^P = p^L\pi + I^L < W^M = p^L\pi + I^L + \gamma(p^H\pi - p^L\pi - B)$ due to **Condition 7**.

[19]The inequality holds if γ is sufficiently large as it is assumed that $I^L < B$.

Summing up, an increase in the probability γ that the developer´s net wealth is large makes it more likely that there is exclusion and the optimal strength of patent protection under exclusion tends to be lower than that if a menu of contracts is offered. In both cases, the optimal strength of patent protection increases in γ as the probability that patent protection leads to inefficiencies is given by $1-\gamma$.

Compared to the case where unobservable effort is the source of the agency problem, the possibility of exclusion constitutes a new form of inefficiency that only arises in the setup with nonpledgeable income and implies that a developer might be prevented from accessing the invention and not only induced to select an inefficiently low level of effort if patents are enforced.