# The saturation of spending diversity and the truth about Mr Brown and Mrs Jones

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**Abstract**

Several cross country studies find that rising household income leads to consumption spending being spread more evenly across different spending categories (Clements et al., 2006). We argue that this result is likely due to aggregation. Using more disaggregated UK household level spending data, we show that the spending diversity of households only rises up to a certain income level and then starts to decline as households concentrate more of their spending on particular expenditure categories that differ across households. It is precisely because of this growing heterogeneity of spending patterns on the household level that the average spending diversity of the population can nevertheless always rise in income. We build a model to capture these observed patterns and use it to show that ignoring preference heterogeneity across households and focusing on a model with representative households leads to an underestimation of the value of product variety.

*JEL classification:* D12, C14, O33.

*Keywords:* Demand for variety, Engel's Law, Spending Diversity.

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‘The preference hypothesis only acquires prima facie plausibility when it is applied to the *statistical average*. To assume that the representative consumer acts like an ideal consumers is a hypothesis worth testing; to assume that an actual person, the Mr. Brown or Mrs. Jones, who lives around the corner, does in fact act in such a way does not deserve a moment’s consideration.’
*J.R. Hicks- A Revision of Demand Theory (1956)* -

# 1 Introduction

One of the most salient features of developed economies is the wide range of goods and services produced and consumed in the economy. The growth in the range of goods consumed is widely recognized to have vital implications for a range of economic issues: when the demand for different final goods changes with the level of income, this can lead to changes in the industrial composition and structural change (Pasinetti, 1981; Saviotti, 2001; Metcalfe et al., 2006; Foellmi and Zweimuller, 2008), impact the incentives to innovate (Foellmi and Zweimuller, 2006), as well as influence the realization of economies of scale (Bresnahan and Gambardella, 1998; Lipsey et al., 2008) and international trade flows (Hallak, 2010).

While much has been said about how firm behavior generates product variety, less has been said about how demand may also contribute to this phenomenon (Gronau and Hamermesh, 2008). In standard product variety models with homothetic preferences (Dixit and Stiglitz, 1977), the demand for variety is independent of income in the sense that the expenditure shares on particular consumption items are the same for rich and poor households. In random utility models that incorporate heterogeneity in consumer preferences (McFadden, 1984; Calvet and Common, 2003), this heterogeneity usually does not depend on economic factors like household income, either.

We argue that demand side factors could also be responsible for the growth of variety as the spending patterns of the rich tend to quite distinct from those of the poor. For example, it is a well established fact that the budget share dedicated to food spending tends to decline as households income rises (Engel’s Law). This suggests that households tend to diversify their spending as their affluence grows. Indeed, a number of studies have found evidence that the demand for variety increases in household income (Prais, 1952; Theil, 1967; Theil and Finke, 1983; Jackson, 1984; Falkinger and Zweimüller, 1996; Bils and Klenow, 2001).

Several have further argued that consumers have a taste for diversity in the sense that they seek to allocate their expenditures more smoothly across different goods and services when their income grows. To support this conjecture, studies used entropy measures to measure the dispersion of household spending across different expenditure categories, which we dub the 'diversity of spending' (Theil and Finke, 1983; Clements and Chen, 1996; Clements et al., 2006). These cross-country studies of spending patterns suggest that this diversity always increases when income rises. In other words, as their income grows, consumers appear to spread their spending more evenly across all available goods and services.

We argue that this literature has ignored the possibility that data on aggregated consumption might not reflect the behavior of individual households as aggregation across heterogeneous households might mask systematic patterns that are present at the household level. Many recognize that it is crucial to study the precise relationship between aggregate and individuals behavior (Grandmont, 1987, 1992; Hildenbrand, 1994; Quah, 1997; Blundell and Stoker, 2005). A number of researchers have begun considering how behavioural heterogeneity can be modelled (Calvet and Common, 2003; Beckert and Blundell , 2008). This represents a departure from the main paradigm of postwar demand analysis that has concentrated on studying aggregates to verify representative agent models of behavior, even though these aggregates may not reflect actual household behavior. This paradigm is reflected in the above quote by J. R. Hicks, who argued that rather than attempting to account for actual household behavior, scholars should restrict their focus on *average* household behavior. It also underpins many commonly used models of demand analysis such as AIDS (Deaton and Muellbauer, 1980).

In the case of spending diversity, whether to focus on average rather than individual behavior turns out to be particularly important. In this paper, we argue that as households shift their spending from basic necessities towards more discretionary categories, heterogeneity in spending patterns is likely to increase in income as consumers concentrate their spending into different consumption areas once incomes are sufficiently large. It is a well known fact that among the poorest, spending patterns are highly homogeneous across households as food spending tends to dominate household outlays (Banerjee and Duflo, 2007; Clements et al., 2006). The notion that heterogeneity of spending grows with income is consistent with evidence that Engel curves are highly heteroskedastic (Blundell and Stoker, 2005; Lewbel, 2008). Using UK household level spending data, we find evidence suggesting that the diversity of household spending tends to fall at high income levels and that the overall differences in household spending patterns tend to grow for high income levels. In other words, the truth about Mr. Brown and Mrs. Jones is that they not only possess different spending patterns, but that the differences in these patterns increase in income when they are sufficiently rich.

The tendency of rising income to magnify consumption heterogeneity is worth taking into account. We develop a model that accounts for the fact that demand heterogeneity increases in income at high income levels and that can explain why there can be a hump shaped relation between spending diversity and income at the individual level and a positive relation at the aggregate level. The key characteristic of the model that differences between household spending patterns increase in income for high income levels does not arise in previous models (Jackson, 1984; Theil and Finke, 1983; Gronau and Hamermesh, 2008) that study variety demand using the representative consumer approach.

Within this model setup, we analyze how much an increase in product variety is valued by individual households and by representative households the preferences of which are such that the resulting aggregate demand for each good is the same as in the case of consumer heterogeneity. We find that the representative households value an increase in product variety less than individual households with heterogeneous tastes do. As it is widely believed that the welfare effects of increasing product variety are substantial, this finding therefore calls for more sophisticated welfare analyses that take individual heterogeneity into account.

In terms of methodology, this paper studies the relationship between income and variety demand using cross sectional data. It may be tempting to study household spending patterns over time. However, the main obstacle in doing so is that one cannot control for exogenous changes in variety demand over time. There has been rapid growth in the number of good available over time, which fundamentally affect the measurement of spending diversity. For this reason the main focus of this paper is on cross sectional results.

## 2 Stylized facts about spending diversity

We begin by reporting some stylized facts about how households diversify their spending across different goods, and about how this diversity changes with income. We do this by estimating the relationship between spending diversity and income, both at the household level and at a more aggregated level for groups of households that possess similar incomes. This allows us to derive "Engel curves for spending diversity" at the household and aggregate level.

We use the following notation in our analysis: There are $n$ households indexed by $i$ and $k$ expenditure categories (or goods) indexed by $j$. Total expenditures on all $k$ categories by household $i$ are denoted by $x_i$ (and also referred to as income). The expenditure share of household $i$ on good $j$ is denoted by $s_{ij}$, so that $s_i = (s_{i1}, s_{i2}..., s_{ik})$ denotes the vector of expenditure shares for household $i$. The overall expenditures of household $i$ on a good $j$ are consequently given by $x_i \times s_{ij}$.

We measure the diversity of household spending across expenditure categories by using an entropy measure. While there exist a number of different diversity measures that can be use for this purposes[1], we follow (Theil, 1967; Theil and Finke, 1983; Clements et al., 2006) and use the following entropy measure of

[1]There are several other measures of spending diversity, like the Hirschmann-Herfindahl or the Gini index. In a preliminary study that employs the same data, we show that using such alternative measures does not affect the shape of the Engel curve for spending diversity, i.e. the qualitative relation between spending diversity and income (Chai et al. (2015)).

the expenditure shares:

$$E_i = -\sum_{j=1}^{k} \phi(s_{ij}) \qquad \begin{cases} \phi(s_{ij}) = s_{ij} \ln s_{ij} & s_{ij} > 0 \\ \phi(s_{ij}) = 0 & s_{ij} = 0 \end{cases} \tag{1}$$

The spending entropy $E_i$ is a number that measures the extent to which spending of household $i$ is dispersed across expenditure categories. It takes on a value of zero when all the expenditure is concentrated on a single item, and is equal to $\ln(k)$ $(> 0)$ when the expenditure shares on all items are equal. We use this measure to estimate the cross-sectional household level Engel curve for spending diversity, i.e. the relationship between $E_i$ and $x_i$.

In order to replicate the cross-country studies cited above that use more aggregated spending data, it is necessary to investigate the shape of the Engel curve for spending diversity on the aggregate level. For this purpose, we order our sample of households according to their expenditure levels $(x_1 < x_2 < ...x_n)$ and partition them into 50 income groups.[2] The expenditure shares are then averaged within these groups in order to derive a measure of the diversity of aggregated spending at the group level. To do so, the average expenditure shares at the group level are denoted by $\hat{s}_{jd} = [50/n] \sum_{i \in d} s_{ij}$, where $d$ is the group under consideration. The entropy $\hat{E}$ of these shares $\hat{E}_d(\hat{s}_{jd})$ is then calculated as a function of the average income level of households within a group and denoted as the spending diversity of aggregated income. From this measure, the Engel curve for spending diversity on the aggregate level, i.e. the relationship between $\hat{E}$ and $x$, can be derived.[3] As the expenditure distributions within the richer (poorer) groups are likely to be similar to the distributions of aggregate expenditures in richer (poorer) countries, we can compare our results to those derived in the cross country studies cited above.

To depict the Engel curves for spending diversity on the individual and group (aggregated) level, we use kernel regressions based on Nadarya (1964) and

[2]Figure 11 shows the results when households are instead partitioned into 20 or 10 groups

[3]As we consider households' equivalence-scale-adjusted expenditures $x_i$ and not their actual expenditures, we base our analysis on the average of the budget shares of all households within a group (i.e. on $\hat{s}_{jd} = [50/n] \sum_{i \in d} s_{ij}$), instead of basing it on the share of the total (non-equivalence-scale-adjusted) expenditures on good $j$ by households falling into group $d$.

Watson (1964). These are non-parametric regressions for which it is not necessary to assume a specific functional form for the relationship between $E$ and $x$. We use second order polynomial terms and choose the bandwidth that minimizes the mean integrated squared error.[4]

In terms of data, we use annual household data sourced from the UK Family Expenditure Survey (FES) from 1990 to 2000. Over this time period the classification method for expenditure categories has been subject to some change. To ensure consistency across sample periods, the classification method specified by the Office of National Statistics in 2000 featuring $k = 12$ categories (see Table 1) was selected and retrospectively applied to the data. In addition, we also study the case of three goods in which the 12 categories are aggregated into 'Food', 'Goods' and 'Services', and the case of 200+ aggregation categories in which no aggregation procedure is used.

We exclude certain housing expenditures because of well-known problems with this data (Tanner, 1999; Blow et al., 2004). Savings are also excluded as we focus on consumption expenditures. We censor the data by removing Northern Ireland and households with more than two adults, but keep all households with two adults and any number of children.[5] In order to control for different sizes and compositions of the households, OECD equivalence scales are used.

Household spending on major durable spending items (e.g. automobile purchases) is converted into weekly expenditure equivalents as provided by the UK Office for statistics. Inflation is accounted for by using the Retail Price Index (RPI) percentage change over 12 months.[6] In terms of the growth rate of total expenditure, our data is broadly consistent with other data sets devised by Blow et al. (2004) and the UK National Accounts. Some differences are likely due to the fact that we have dropped households with more than two adults and excluded recall categories from 1986. Across the thirty year

[4] This is done using the **lpoly** command in Stata

[5] This reduces the number of share houses and households co-inhabited by extended family in the sample

[6] This is calculated using data from the UK Office of National statistics on all consumption items except for housing and mortgage payments (CDKG).

period, the average annual sample size is about 6000 observations but drops to 5000 between 1998 to 2000. The years 1990, 1995 and 2000 were selected in order to study spending patterns across a decade. Due to substantial changes in the UK Family expenditure survey in 2001, later years were not included in the analysis.[7] Prior to 1990, major changes in the family expenditure survey also took place in 1987 with the introduction of credit card purchases and recall categories (Blow et al. (2004)).

As most household expenditure surveys have less observations at high levels of household income, a common problem is sample selection bias. However, Tanner (1999) finds that the ratio of non-housing total expenditure in the FES to non-housing total expenditure in the National Accounts was around 90 per cent between 1974 and 1992. This instills us with some confidence that the magnitude of the sample selection bias is not too large as the FES expenditure match the National accounts relatively well in this earlier period. We moreover remove all households with incomes more than three standard deviations above the average household income.

Table 2 provides an overview of how the average subgroup budget shares $\hat{s}_{jd}$ for the three broad categories food, goods, and services evolve across income levels and time (for simplicity, the case of 10 subgroups is considered). Income $x$ is measured by real weekly total expenditure. This table reveals a relatively stable pattern that is consistent with Engel's law: as household income rises, the average budget share dedicated to food declines. Also consistent with other studies is the fact that poor households on average spend a considerable fraction of their budget on food (Banerjee and Duflo, 2007; Clements et al., 2006), while spending on average tends to become more widely dispersed across different expenditure categories when income rises.

***FIGURE 1 ABOUT HERE***

Figure 1 depicts the estimated Engel curves for spending diversity observed on the individual (household) level (left hand side), as well as on the group

[7]From 2001, the both the FES and the National Food Survey (NFS) were replaced by a new survey, the Expenditure and Food Survey (EFS)

level (right hand side). Note that income is measured by real weekly total household expenditure. The first row depicts the case where consumption items are aggregated into three broad categories (food, goods and services), the middle row depicts the 12 good aggregation (See Table 1), and the last row the case where goods are highly disaggregated (200+ expenditure categories). Each figure contains curves for three years: 1990, 1995 and 2000.[8]

From these results, a number of stylized facts can be observed:

- **Stylized fact 1**: The Engel curve for individual spending diversity is inverse-U shaped, i.e. there exists an inverse-U relationship between spending diversity observed on the household level, $E_i$, and household income $x$.

At low income levels, spending diversity $E_i$ rises in income as households allocate their spending more evenly across goods. At high income levels, $E_i$ tends to fall in income as the opposite is the case: households tend to concentrate their spending on particular consumption categories as their income grows.

- **Stylized fact 2**: On the more aggregated group level, the Engel curve for spending diversity is either upward sloping or has an inverse-U shape. There is therefore either a positive relation or an inverse-U relation between the diversity (entropy) $\hat{E}$ of (aggregated) group spending and average group income $x$.

Interestingly, entropies fall more rapidly in income at high income levels in the case of 200+ aggregation categories.

***FIGURES 2, 5, and 8 ABOUT HERE***

Beyond differences in the shapes of Engel curves for spending diversity at the household and at the aggregate level, there are also important differences in the levels of spending diversity across different levels of aggregation. This can be seen in Figure 2 where $E_i$ and $\hat{E}$ are depicted together for the years

[8] The choice of years does not seem to affect the results. We conducted tests in other years between 1987 and 2000 and found similar results. Due to major changes in the expenditure categories used by the UK Family expenditure survey, years after 2001 are not used.

1990 (left), 1995 (middle) and 2000 (right) in the case of three expenditure categories. From this figure, as well as from Figures 5 and 8 that consider the case of 12 and 200+ expenditure categories, the following stylized fact emerges:

- **Stylized fact 3**: The Engel curve for spending diversity on the aggregate (group) level is always situated above the Engel curve for spending diversity on the individual (household) level. In other words, $\hat{E}$ exceeds $E_i$ for each level of household income $x$.

Spending diversity on the group level is therefore greater than spending diversity observed on the individual level across all income levels. This suggests that the process of aggregating household expenditure leads to an increase in diversity. If each household with a certain income $x$ would spend its income in exactly the same fashion, then $\hat{E} = E_i$ would hold. The observed pattern must therefore stem from the fact that different households belonging to the same income groups allocate their spending differently across different goods. As such, these observed differences between $\hat{E}$ and $E_i$ represent a measure of differences in household spending patterns.

An interesting pattern of the data is that the entropy $\hat{E}$ of aggregated spending appears to keep rising in income at income levels at which individual entropy $E_i$ already falls in income, and that $\hat{E}$ consequently reaches its maximum (in case there is one) at higher levels of income than $E_i$ does. This pattern can also be inferred more directly:

***FIGURES 3, 6, and 9 ABOUT HERE***

Figure 3 shows the calculated difference $\hat{E} - E_i$ between aggregate and household level spending diversities for the case of three consumption categories in each year. Figures 6 and 9 show the same for the cases of 12 and 200+ categories. Are these differences statistically significant? Figures 4, 7, and 10 depicts each estimated Engel curve with 95% confidence intervals. These figures confirm that these differences increase in income when income is large and that this pattern is statistically significant.

***FIGURES 2, 5, and 8 ABOUT HERE***

From these figures, we obtain our last stylized fact:

- **Stylized fact 4**: The difference $\hat{E} - E_i$ between the spending diversities on the (aggregated) group and the household level tends to either rise in income or to first fall and to then rise in income (U-relation).

This suggests that the heterogeneity in variety demand across households belonging to the same income group depends on the level of income and that it rises in income when income is sufficiently large. As can be inferred from figures 3, 6, and 9, this stylized fact results from the following shapes of the entropy curves: at low income levels, both $\hat{E}$ and $E_i$ rise and $\hat{E} - E_i$ can either rise or fall. At high levels of income, household spending diversity $E_i$ falls, while $\hat{E}$ either rises or falls less strongly, implying that $\hat{E} - E_i$ increases.

It should be noted that, unlike in Figure 1, the $E_i$ curves are shortened to the length of the $\hat{E}$ curves in Figures 2 to 10. In these figures, both curves therefore begin at the average income of the poorest of the 50 income groups and end at the average income of the richest of these groups as those are the values for which the $\hat{E}$ curve is properly defined.[9] In Figure 11, the Engel curves for spending diversity on the aggregate level (i.e. $\hat{E}$ as a function of $x$) are plotted for the cases where households are grouped into 10 groups (left), 20 groups (middle), and 50 groups (right) and where averages are formed within these larger groups (the case of three consumption categories is considered). The Engel curves for spending diversity can then only be plotted for a smaller income range, but their shapes do not change much within this range.

# 3 Model setup

We now turn to introduce a model that can account for the stylized facts relating to the Engel curves for spending diversity on the individual (household)

[9]We refrain from artificially extending this curve to lower and higher values of $x$ in order to avoid that for the lowest and highest values of $x$, $E_i$ "mechanically" falls short of $\hat{E}$ simply due to the fact that $E_i$ rises (falls) in $x$ when $x$ is small (large) and that this trend is averaged out in the $\hat{E}$ curve.

and aggregated (group) level and to the observed differences between these two curves. We then use the model in order to undertake a welfare analysis.

The utility of household $i$ is given by the generalized Stone Geary form:

$$U_i = \left[ \sum_{j=1}^{k} \beta_{ij}^{\frac{1}{\varepsilon}} \left( q_{ij} - \gamma_j \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2}$$

The terms $q_{ij} \geq 0$ denote the quantity of good $j$ consumed by household $i$ and $\gamma_j \gtreqless 0$ the "subsistence consumption" level of good $j$. This utility function is only defined if $q_{ij} \geq \gamma_j$ holds, i.e. if the household is rich enough to consume the subsistence level of all goods with $\gamma_j > 0$. The parameter $\varepsilon > 0$ determines the degree of substitutability between goods: when $\varepsilon \to 0$, goods become perfectly complementary (utility is then given by $\underset{\varepsilon \to 0}{lim} U_i = \underset{j}{min} \left\{ \beta_{ij}(q_{ij} - \gamma_j) \right\}$), while they become perfectly substitutable when $\varepsilon \to \infty$. When $\varepsilon = 1$, utility is given by the standard Stone-Geary form: $U_i = \prod_{j=1}^{k} \left( q_{ij} - \gamma_j \right)^{\beta_{ij}}$. The degree of substitutability therefore increases in $\varepsilon$.

It is assumed that $\sum_{j=1}^{k} \beta_{ij} = 1$ holds. In order to explain the empirically observed heterogeneity of consumption patterns of households with similar incomes, some preference heterogeneity is introduced: It turns out that all patterns observed in the data can be explained by assuming that only the parameters $\beta_{ij} \geq 0$ can vary across households while the parametes $\gamma_j$ are the same for all of them. It is therefore assumed that the subsistence consumption levels $\gamma_j$ are the same for all households (as they might reflect "biological" needs for food, shelter etc.), while households might differ with respect to the relative importance that they attribute to consumption exceeding these levels (that is reflected by the size of the parameters $\beta_{ij}$).

Total income (or expenditures) of household $i$ is denoted by $x_i$ and the price of one unit of good $j$ by $p_j$. The budget constraint of household $i$ is therefore given by[10]:

$$x_i = \sum_{j=1}^{k} p_j q_{ij} \tag{3}$$

[10]As utility is strictly increasing in $q_{ij}$, this budget constraint is always satisfied with equality

The following analysis focuses on the case in which income of household $i$ lies (weakly) above the threshold income level $\underline{x}$ which is required to purchase positive quantities of all goods ($q_{ij} > 0$), i.e. in which $x_i \geq \underline{x}$ (**Condition A**) holds[11]. Setting up the Lagrangian $L_i = U_i + \lambda_i \left[x_i - \sum_{j=1}^{k} p_j q_{ij}\right]$ and deriving with respect to $q_{ij}$ gives the first order conditions:

$$\frac{\partial L_i}{\partial q_{ij}} = U_i^{\frac{1}{\varepsilon-1}} \beta_{ij}^{\frac{1}{\varepsilon}} \left(q_{ij} - \gamma_j\right)^{-\frac{1}{\varepsilon}} - \lambda_i p_j = 0 \tag{4}$$

Dividing the first order conditions for goods $j$ and $l \neq j$ by each other gives the equation:

$$\frac{q_{ij} - \gamma_j}{(q_{il} - \gamma_l)} = \frac{\beta_{ij}}{\beta_{il}} \left(\frac{p_l}{p_j}\right)^{\varepsilon} \tag{5}$$

Combining equations 5 and 3 then allows us to solve for the optimal quantity $q_{ij}^*$ of good $j$ by household $i$:[12]

$$q_{ij}^* = \frac{x_i - \sum_{l \neq j} \left[p_l \gamma_l - p_l \gamma_j \frac{\beta_{il}}{\beta_{ij}} \left(\frac{p_j}{p_l}\right)^{\varepsilon}\right]}{p_j + \sum_{l \neq j} p_l \frac{\beta_{il}}{\beta_{ij}} \left(\frac{p_j}{p_l}\right)^{\varepsilon}} \tag{6}$$

The optimal quantity $q_{ij}^*$ is a linear function of income $x_i$, implying linear Engel curves. As $q_{ij}$ increases by $\frac{1}{p_j + \sum_{l \neq j} p_l \frac{\beta_{il}}{\beta_{ij}} \left(\frac{p_j}{p_l}\right)^{\varepsilon}}$ ($= \frac{\beta_{ij}}{p_j}$ if $\varepsilon = 1$) units for each unit that $x_i$ increases, the slope of the Engel curve for good $j$ increases in $\beta_{ij}$ and decreases in $p_j$. Differences in the taste parameters $\beta_{ij}$ across households can therefore generate the heteroskedasticity of Engel curves that is observed in the data[13].

The Engel curve of household $i$ for good $j$ shifts up when $\gamma_j$ increases and the size of this shift does not depend on $x_i$ or $\gamma_l$ ($l \neq j$)[14]. The income elasticity

[11] In the case where $\gamma_j \geq 0 \; \forall j$, Condition A is given by $x_i > \sum_{j=1}^{k} p_j \gamma_j = \underline{x}$.

[12] Equation 5 can be rewritten as $q_{il} = \gamma_l + \frac{\beta_{il}}{\beta_{ij}} \left(\frac{p_j}{p_l}\right)^{\varepsilon} (q_{ij} - \gamma_j)$ and equation 3 as $q_{ij} = \frac{x_i - \sum_{l \neq j} p_l q_{il}}{p_j}$. Inserting the first into the latter then gives the result.

[13] Another way to generate such heteroskedasticity within the model setup would be to assume that different households face different prices $p_{ij}$ for the same goods $j$. We, however, focus on the case of preference heterogeneity which we believe to be an important driver of this empirically observed heteroskedasticity.

[14] While the Engel curve of household $i$ is only defined for $x_i \geq \underline{x}$ (i.e. when $q_{ij}^* \geq 0$), they all "originate" at the values $q_{ij} = \gamma_j \gtreqless 0$ that are reached when $x_i = \sum_{j=1}^{k} p_j \gamma_j$ holds. See Figure 15 (right hand side)

of demand for good $j$ by individual $i$ is given by

$$\epsilon_{jx}(i) = \frac{\partial q^*_{ij}}{\partial x_i}\frac{x_i}{q^*_{ij}} = \frac{x_i}{x_i - \sum_{l\neq j}\left[p_l\gamma_l - p_l\gamma_j\frac{\beta_{il}}{\beta_{ij}}\left(\frac{p_j}{p_l}\right)^{\varepsilon}\right]} > 0 \tag{7}$$

and therefore decreases if $\gamma_j$ increases. Goods with a high value $\gamma_j$ therefore represent basic need goods on which poor households concentrate their expenditures, while goods with a lower (or even negative) value $\gamma_j$ are more luxurious and are only purchased in substantive amounts by rich households. The share of the budget that household $i$ allocates to good $j$ is given by $s_{ij} = \frac{q^*_{ij}p_j}{x_i}$ and increases in $\beta_{ij}$ as $q^*_{ij}$ increases in $\beta_{ij}$ if Condition A holds (as a strict inequality).

## 3.1 An example with three goods

In order to show which mechanisms can generate the four stylized facts observed in the data, the following simple example is considered: There are three goods consisting of one basic need good $j = 1$ for which $\gamma_1 > 0$ holds and two more luxurious goods $j = 2$ and $j = 3$, for which $\gamma_2 = \gamma_3 \gtreqless 0$ holds. While the price of good 1 is normalized to one ($p_1 = 1$), the prices of goods 2 and 3 are given by $p_2 = p_3 = p$. While $\beta_{i1}$ (the welfare weight on good 1) is assumed to be the same for all households and equal to the constant $\beta_{i1} = 1 - \bar{\beta}$, the degree to which household $i$ prefers good 2 over good 3 is allowed to vary within the range in which the household still purchases positive quantities of all available goods and in which $\beta_{i2} + \beta_{i3} = \bar{\beta}$ holds.

From equation 6 we can infer that $q_{i1}$ and also the sum $q_{i2} + q_{i3}$ then only depend on on the aggregate welfare weight $\bar{\beta}$ for goods 2 and 3, but not on how much goods 2 and 3 are liked by a particular household. This allows to study the role of individual heterogeneity in the following simple setup:

There are two households: the household of Mr Brown ($i = B$) and the household of Mrs Jones ($i = J$). Both households are assumed to have the same income $x_i = x$, but have opposing preferences with respect to the otherwise

for the case of three goods.

identical goods 2 and 3, so that $\beta_{B2} = \beta_{J3}$ and $\beta_{B3} = \beta_{J2}$ holds in addition to $\beta_{i2} + \beta_{i3} = \bar{\beta}$ (implying that $\beta_{B2} + \beta_{J2} = \beta_{B3} + \beta_{J3} = \bar{\beta}$). The aggregated demand $Q_j = q_{Bj} + q_{Jj}$ for goods $j = 1$, $j = 2$ and $j = 3$ then only depends on $x$, $\gamma_j$, $p$ and $\bar{\beta}$, but not on the individual values $\beta_{i2}$ and $\beta_{i3}$ as individual preference heterogeneity washes out in the aggregate. Aggregated demand for good $j$ and also the elasticity of aggregated demand with respect to the relative price $p$ is therefore the same as in the case where both households value both goods equally ($\beta_{i2} = \beta_{i3} = \frac{\bar{\beta}}{2}$) and can also be derived from the utility maximization problem of two "average" households with preference parameters $\beta_{a1} = 1 - \bar{\beta}$ and $\beta_{a2} = \beta_{a3} = \frac{\bar{\beta}}{2}$ and (per household) expenditures $x_a = x$.[15]

Using equation 6 and the parameter assumptions from above, the optimal budget shares can be derived as

$$s_{B1}(x) = s_{J1}(x) = \frac{q^*_{i1}(x)}{x} = \frac{\left(1-\bar{\beta}\right)\left(x - 2p\gamma_2\right)p^{\varepsilon} + \gamma_1\bar{\beta}p}{x\left(p\bar{\beta} + \left(1-\bar{\beta}\right)p^{\varepsilon}\right)} \tag{8}$$

$$s_{B2}(x) = s_{J3}(x) = \frac{pq^*_{B2}(x)}{x} = \frac{p\left[\beta_{B2}\left(x - \gamma_1 - 2p\gamma_2\right) + \left(1-\bar{\beta}\right)\gamma_2 p^{\varepsilon} + \bar{\beta}p\gamma_2\right]}{x\left(p\bar{\beta} + \left(1-\bar{\beta}\right)p^{\varepsilon}\right)} \tag{9}$$

$$s_{B3}(x) = s_{J2}(x) = \frac{pq^*_{B3}(x)}{x} = \frac{p\left[\left(\bar{\beta} - \beta_{B2}\right)\left(x - \gamma_1 - 2p\gamma_2\right) + \left(1-\bar{\beta}\right)\gamma_2 p^{\varepsilon} + \bar{\beta}p\gamma_2\right]}{x\left(p\bar{\beta} + \left(1-\bar{\beta}\right)p^{\varepsilon}\right)} \tag{10}$$

Without loss of generality, it is assumed that $\beta_{B2} = \beta_{J3} > \beta_{B3} = \beta_{J2}$ holds, implying that $s_{B2}(x) = s_{J3}(x) > s_{B3}(x) = s_{J2}(x)$ if $x > \underline{x}$, i.e. that household Brown prefers good 2 over good 3, while household Jones prefers good 3 over good 2. A graphical representation of the Engel curves resulting in this 3 good example is given in **Figure 15**.

The levels of household spending diversities $E_i(x)$ measured by the entropies

[15] The analysis would be similar in a setting with more than two households as long as there was an equal number of households of each type. As households of the same preferency type would then have the same linear Engel curves (assuming that $x_i > \underline{x}$ holds), only the total income of each group would then matter for aggregated demand and not its distribution across households of the same preference type.

of consumption spending of households are given by

$$E_B(x) = -s_{B1}\ln s_{B1} - s_{B2}\ln s_{B2} - s_{B3}\ln s_{B3} = E_J(x) = -s_{J1}\ln s_{J1} - s_{J2}\ln s_{J2} - s_{J3}\ln s_{J3} \tag{11}$$

For a given income level $x$, these entropies are therefore the same for both households as their consumption shares coincide for good 1, and are simply reversed for goods 2 and 3.

When aggregated consumption is considered, the share $\hat{s}_1(x) = s_{B1}(x) = s_{J1}(x)$ of the aggregated income (which equals $2x$) is spent on good 1 and the shares $\hat{s}_2(x) = \hat{s}_3(x) = \frac{p\left(q^*_{B2}(x)+q^*_{J2}(x)\right)}{2x} = \frac{s_{B2}(x)+s_{J2}(x)}{2} = \frac{s_{B2}(x)+s_{B3}(x)}{2}$ on goods 2 and 3. These shares are of equal size as the heterogeneity of individual consumption washes out in the aggregate. The entropy of aggregated consumption spending when the spending of each of the two households is equal to $x$ is therefore given by

$$\hat{E}(x) = -\hat{s}_1\ln\hat{s}_1 - \hat{s}_2\ln\hat{s}_2 - \hat{s}_3\ln\hat{s}_3 = -\hat{s}_1\ln\hat{s}_1 - 2\hat{s}_2\ln\hat{s}_2 \tag{12}$$

**Lemma 1.** *Suppose that $\gamma_1 > \frac{2\gamma_2\left(1-\bar{\beta}\right)p^{\varepsilon}}{\bar{\beta}}$ (**Condition B**) holds, implying that the spending shares on the basic need good 1 fall in income $x$ (i.e. that $\frac{\partial(s_{i1}(x))}{\partial x} = \frac{\partial(\hat{s}_1(x))}{\partial x} < 0$ holds). Then, the entropy of aggregated consumption spending $\hat{E}$ continuously rises in $x$ when $\bar{\beta} < \frac{2}{p^{1-\varepsilon}+2}$ holds (Case i), while it first rises in $x$ (for $\underline{x} \leq x < \check{x}$) and then falls in $x$ (for $\check{x} < x < \infty$) when $\bar{\beta} > \frac{2}{p^{1-\varepsilon}+2}$ holds and when $\gamma_1$ is sufficiently large (Case ii).*
*(In Case ii, $\gamma_1$ is sufficiently large if $\gamma_1 > p\gamma_2$ and $\gamma_2 \geq 0$ (**Condition C1**) or if $\gamma_1 > \frac{-\gamma_2\left(p\left(2\beta_{B2}-\bar{\beta}\right)-\left(1-\bar{\beta}\right)(3-p^{\varepsilon})\right)}{2\left(\bar{\beta}-\beta_{B2}\right)}$ and $\gamma_2 < 0$ hold (**Condition C2**)).*

*Proof.* See Appendix A1. □

The parameter conditions in this Lemma guarantee that poor households (for which $x$ is close to $\underline{x}$) spend more than one third of their budget on the basic need good 1 and that the budget share of this good falls as income grows, implying that the shares $\hat{s}_2(x) = \hat{s}_3(x)$ rise in $x$. At low levels of income, an increase in income therefore always leads to a rise in the entropy of aggregated consumption spending $\hat{E}$. This is due to the fact that it leads to a smoother allocation of consumption spending over the three goods (note that entropy is maximal if one third of the budget is spent on each of the goods). If the budget share of good 1 still exceeds one third when income becomes infinitely large (Case i), $\hat{E}$ therefore always rises in $x$. When the budget share of good 1 falls below one third at a finite income threshold $\breve{x} > \underline{x}$, there is an inverse-U relation between $\hat{E}$ and $x$. $\hat{E}$ then first rises in $x$, but falls in $x$ once $x > \breve{x}$ holds. For the described parameter values, the model can therefore generate **Stylized fact 2** concerning the shape of the Engel curve for spending diversity on the group level.

While the model is not designed to exactly fit the data in the case of three goods, but to rather provide qualitative insights that can also be applied to the case with more than three goods, the assumptions about the shares of the aggregated expenditures $\hat{s}_j$ made in Lemma 1 do indeed match the data quite well in the case of three goods: Table 2 shows that in this case, the average budget share of food (partitioning the population into income deciles) exceeds one third for all but the richest income decile and that it falls as income rises (Engel's law). Moreover, the average budget shares for goods and services initially lie below one third and tend to rise in income[16]. Figure 1 (the top right figure) shows that the entropy $\hat{E}$ of aggregated group consumption tends to always rise in income in 1995, and only falls in income for high income levels in 1990 and in 2000. This pattern is therefore in line with Lemma 1.

When $x > \underline{x}$ holds, the entropy $E_i$ of individual consumption spending falls short of that of aggregated consumption spending (i.e. $E_i < \hat{E}$ holds) as, due to the heterogeneity of preferences, the budget shares are more unequal at the individual level, implying a lower entropy and therefore consumption diversity at this level[17]. This is in line with **Stylized fact 3** that the Engel curve for spending diversity on the aggregate level is situated above the Engel

[16]Unlike in the stylized modelling example, these shares are, however, not of equal size and tend to be larger for goods than for services. By assuming that the subsistence consumption level for food ($\gamma_2$) exceeds that for services ($\gamma_3$), the modelling example could be extended to also account for this feature.

[17]Given that $s_{B2}(x) > s_{B3}(x)$ and $s_{B2}(x) + s_{B3}(x) = 2\hat{s}_2(x)$, the term $-s_{B2}(x)\ln s_{B2}(x) - s_{B3}(x)\ln s_{B3}(x)$ falls in $s_{B2}(x)$ and is therefore maximal if $s_{B2}(x) = s_{B3}(x)$ holds. This implies that $E_i$ is maximal and that $E_i = \hat{E}$ holds if $s_{B2}(x) = s_{B3}(x)$.

curve for spending diversity on the individual level. The following proposition analyzes the relation between $E_i$ and $\hat{E}$:

**Proposition 1.** *Suppose that $\gamma_1 > -2\gamma_2 p$ (**Condition D**) and that the conditions from Lemma 1 (leading to either Case i or ii) are satisfied, implying that $\frac{\partial(s_{i1}(x))}{\partial x} < 0$, $\frac{\partial(s_{B2}(x))}{\partial x} > 0$ and that $\frac{\partial(s_{B2}(x))}{\partial x} > \frac{\partial(s_{B3}(x))}{\partial x}$ hold. Then, the (non-negative) difference $\hat{E} - E_i$ between the individual and the aggregated consumption entropy increases in income $x$ if $\gamma_2 > 0$, while it first decreases and then increases in income when $\gamma_2 < 0$.*

*Formally, $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ when $\gamma_2 > 0$, while $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} < 0$ for $\underline{x} \leq x < \tilde{x}$ and $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ for $\tilde{x} < x < \infty$ when $\gamma_2 < 0$ (when $\gamma_2 = 0$, $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ $(= 0)$ holds for $x > \underline{x}$ $(x = \underline{x})$).*

*Proof.* See Appendix A2. □

This proposition shows under which conditions the model can generate **Stylized fact 4**. Whether the entropy difference $\hat{E} - E_i$ continuously rises in $x$ or is U-shaped in $x$ therefore depends on whether $\gamma_2$ is positive or negative. In the following, both cases are discussed separately:

When $\gamma_2 \geq 0$, $E_i(x) = \hat{E}(x)$ holds at the minimal income level $x = \underline{x}$ as all households then consume the same quantities $q_{ij} = \gamma_j$ (See Figure 15. A graphical representation of the Entropy curves is given in **Figure 16**). When income exceeds the level $\underline{x}$, individual households allocate their spending in more uneven ways across goods 2 and 3 than households do on average. $\hat{E}$ then exceeds $E_i$, and the more so the more heterogeneous individual tastes are, i.e. the more $\beta_{B2} = \beta_{J3}$ exceed the value $\frac{\bar{\beta}}{2}$ of the average consumer. As the consumption of individual households becomes more specialized when income rises, $\hat{E} - E_i$ then continuously rises in $x$. The heterogeneity of demand is therefore emergent in the sense that differences in spending patterns between different household types grow when household income rises.

Due to the fact that individual consumption patterns closely reflect average consumption patterns at low levels of income, the assumptions (from Lemma 1) that guarantee that $\frac{\partial \hat{E}}{\partial x} > 0$ holds for low income levels also guarantee that $E_i$ rises in $x$ when $x$ is low. $E_i$ can, however, fall as $x$ rises when $x$ is sufficiently high and when the share of the budget that a household allocates to either good 2 or 3 becomes disproportionally large. There can therefore be an inverse-U relationship between individual consumption entropy $E_i$ and $x$ as we have found in the data (**Stylized fact 1** on the shape of the Engel curve for spending diversity on the individual level). Given that the model generates Stylized fact 1, the finding that $\hat{E} - E_i$ rises in $x$ when $\gamma_2 > 0$ holds implies that there can be the following relations between $\hat{E}$ and $x$ in this case: $\hat{E}$ either continuously rises in $x$ (see Figure 16 on the left hand side), or that there is also an inverse-U relation between $\hat{E}$ and $x$ and $\hat{E}$ reaches its maximum for larger values of $x$ than $E_i$ does (see Figure 16 on the right hand side).

When $\gamma_2 < 0$ holds, $\hat{E} > E_i$ holds even at the minimal income level $x = \underline{x}$, as individual households then do not purchase any units of either good 2 or 3 (i.e. as $s_{B3} = s_{J2} = 0$ holds), while the aggregate spending shares $\hat{s}_2 = \hat{s}_3$ are positive for these goods (a graphical representation is given in **Figure 17**). As $\frac{\partial E_i}{\partial x} = -\frac{\partial s_{i1}}{\partial x}(\ln s_{i1} + 1) - \frac{\partial s_{i2}}{\partial x}(\ln s_{i2} + 1) - \frac{\partial s_{i3}}{\partial x}(\ln s_{i3} + 1)$ and as $\frac{\partial s_{i2}}{\partial x}$ and $\frac{\partial s_{i3}}{\partial x}$ are positive in the case considered in proposition 1 when $\gamma_2 < 0$ holds (this is shown at the beginning of the proof of Proposition 1) the derivative $\frac{\partial E_i}{\partial x}$ gets infinitely large when $s_{B3}$ or $s_{J2}$ go to zero. This implies that the spending diversity of a household increases substantially when it starts consuming positive quantities of an good that it has not consumed before at lower levels of income. When $x$ is close to $\underline{x}$, $\hat{E} - E_i$ therefore falls in $x$ when $\gamma_2 < 0$ holds as $\frac{\partial E_i}{\partial x}$ exceeds the value of $\frac{\partial \hat{E}}{\partial x}$ which is finite even at the point where $x = \underline{x}$[18].

When income is so large that all consumption shares are sufficiently distinct from zero, the mechanisms that are already at work in the case where $\gamma_2 \geq 0$ become dominant again and a further increase in income induces households to devote an ever increasing share of their budget towards their preferred consumption good. This reduces individual consumption entropy relative to aggregated consumption entropy as consumption heterogeneity washes out in

[18]This argument can be generalized to the case of more than three consumption goods.

the aggregate. Consequently, $\hat{E}-E_i$ again rises in $x$ when $x$ is sufficiently large (i.e. when $\tilde{x} < x < \infty$) and increasing spending diversity at the aggregate level can again go along with declining diversity at the household level[19]. Given that parameters are such that there is an inverse-U relation between $E_i$ and $x$ (Stylized fact 1), the fact that there is a U-shaped relation between the entropy difference $\hat{E} - E_i$ and $x$ when $\gamma_2 < 0$ holds therefore implies that the relation between $\hat{E}$ and $x$ can again be of two forms in this case: $\hat{E}$ either continuously rises in $x$ (see Figure 17 on the left), or there is an inverse-U relation between $\hat{E}$ and $x$ and the inverse-U of $\hat{E}$ reaches its maximal level at a higher level of income than the inverse-U of $E_i$ (see Figure 17 on the right).

As Figures 3, 4, 6, 7, 9, and 10 show that $\hat{E}-E_i$ can either rise or be U-shaped in $x$ in our data, both the case where $\gamma_2 > 0$ holds and the case where $\gamma_2 < 0$ holds (with the latter implying a larger income elasticity for goods 2 and 3 then the former) therefore seem to be relevant cases in order to explain the observed empirical pattern.

While the model focuses on the case where households consume positive quantities of all goods, we do keep observations of households that do not purchase any units of certain goods in our empirical analyses. Figure 12 depicts the household level Engel curves for spending diversity when households with zero expenditure are excluded for the case of three goods (right). Compared to the case where these expenditure are included (left), there is not much change in the shape of the Engel curves for spending diversity. Hence, including such "consumption zeros" does not appear to affect the analysis much.

# 4 The value of product variety

The insights from the last sections about the non-homothetic nature of demand heterogeneity can have important welfare consequences. In this section we discuss these by analyzing the value of product variety. This is a key

[19]Condition D, which can only be binding if $\gamma_2 < 0$ holds, is imposed to ensure that $\frac{\partial(s_{B2}(x))}{\partial x} > \frac{\partial(s_{B3}(x))}{\partial x}$ holds. If this condition is violated, $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} < 0$ holds for all values of $x$ (this is shown in the proof of Proposition 1). As this case is not in line with the empirical observations, Condition D is imposed in Proposition 1.

issue when it comes to designing optimal innovation, trade, and antitrust policies as these policies affect how large the set of goods is that households can consume. The analysis is carried out within the three-good example from Subsection 3.1.

It is assumed that initially only the “basic need” good 1 exists and that goods 2 and 3 can be introduced through innovation or can be made available through a free trade agreement. While good 1 is always sold at price $p_1 = 1$, goods 2 and 3 are now only sold at price $p_2 = p_3 = p$ when they are available, but have an infinite price when not. In order to allow to compare the welfare levels with and without goods 2 and 3, the case is considered in which $\gamma_2 = \gamma_3 < 0$ holds, i.e. in which there is no required positive subsistence consumption level for goods 2 and 3.

As above, the case in which $\beta_{i1} = 1 - \bar{\beta}$, $\beta_{B2} = \beta_{J3} > \frac{\bar{\beta}}{2}$ and $\beta_{B3} = \beta_{J2} = \bar{\beta} - \beta_{B2} \geq 0$ is considered in which household Brown prefers good 2 over good 3 and household Jones has exactly the opposite preferences and prefers good 3 over good 2. As the aggregated demand for each good does in this case not depend on the extent of preference heterogeneity (i.e. on $\beta_{B2}$) and can also be derived from the utility maximization problem of two households with average preferences ($\beta_{a2} = \beta_{a3} = \frac{\bar{\beta}}{2}$), we can analyze whether a household with heterogeneous preferences ($\beta_{i2} \neq \frac{\bar{\beta}}{2}$) values an increase in product variety in a different way than a household with average preferences does.

This is an interesting question as it allows us to evaluate whether and how ignoring the preference heterogeneity that we have identified as the driving force behind our empirical observations and instead focusing on a simpler model with hypothetical average consumers leads to biased welfare results. It should be noted that such a simpler model does not only allow to correctly derive the aggregated demand for each good, but would also allow to correctly determine the incentives to innovate in an environment with endogenous innovation by profit seeking firms (at least in a symmetric equilibrium in which the inventors of the goods 2 and 3 charge the same monopoly prices). While preference heterogeneity does not affect aggregated demand and the profits to innovate in our setting, it might, however, nevertheless affect the value that households attribute to an increase in product variety.

While it is obvious that a household benefits more from the introduction of a good that it likes a lot than from the introduction of a good that it does not like, the question considered here is whether a household benefits more or less from the joint introduction of both goods 2 and 3 when it puts a larger relative welfare weight $\beta_{ij}$ on one of them, keeping $\beta_{i2} + \beta_{i3} = \bar{\beta}$ and therefore the total quantity of the two goods that it consumes constant[20]. The extent of preference heterogeneity is then increasing in $\beta_{ij}$ when $\beta_{ij} > \frac{\bar{\beta}}{2}$ holds for a good $j \epsilon \{2; 3\}$.

To which extent a household values variety is measured by the amount $F_i$ of income $x_i$ (or by the quantity $F_i$ of good 1) that it is maximally willing to give up in order to be able to not only purchase good 1 at price 1, but to in addition purchase goods 2 and 3 at price $p$. The value that a household with average preferences ($\beta_{a2} = \beta_{a3} = \frac{\bar{\beta}}{2}$) attributes to variety is denoted by $F_a$ (so that $F_i|_{\beta_{ij}=\frac{\bar{\beta}}{2}} = F_a$ holds) and the extent to which an individual and an average household disagree about the value of product variety is measured by the term $D \equiv \frac{F_i - F_a}{x_i}$ (we divide by $x_i$ as both $F_i$ and $F_a$ depend positively on $x_i$). As before, the case is considered in which $x_i \geq \underline{x}$ holds and in which households consume positive quantities of all available goods.

**Proposition 2.** *When $\gamma_2 = \gamma_3 < 0$ and $\varepsilon \neq 1$, the following holds:*

*a) A household $i$ with heterogeneous preferences ($\beta_{ij} \neq \frac{\bar{\beta}}{2}$ for $j \epsilon \{2; 3\}$, but $\beta_{i2} + \beta_{i3} = \bar{\beta}$) values variety more than a household with average preferences ($\beta_{a2} = \beta_{a3} = \frac{\bar{\beta}}{2}$) does and the more so, the more heterogeneous these preferences are (i.e. $F_i > F_a$ holds, with $\frac{\partial F_i}{\partial \beta_{ij}} > 0$ when $\beta_{ij} > \frac{\bar{\beta}}{2}$ holds for a good $j \epsilon \{2; 3\}$).*

***Further results:***

*b) When $\gamma_2$ becomes more negative, implying a higher income elasticity for goods 2 and 3, the increase in the value of variety induced by an increase in preference heterogeneity, $\frac{\partial F_i}{\partial \beta_{ij}}$, gets larger when goods are substitutable ($\varepsilon > 0$), but smaller when goods are complementary ($\varepsilon < 1$), i.e. $sign \frac{\partial^2 F_i}{\partial \beta_{i2} \partial \gamma_2} =$*

[20] By looking at the joint introduction of two goods, one does not need to consider individual risk preferences that might play a role when instead the welfare consequences of the introduction of only one good of ex ante unknown desirability were studied.

*sign* $(1-\varepsilon)$ *holds when* $\beta_{ij} > \frac{\bar{\beta}}{2}$ *for* $j \epsilon \{2;3\}$. *Furthermore, assuming that* $\beta_{ij} > \frac{\bar{\beta}}{2}$ *holds for a good* $j \epsilon \{2;3\}$, $\lim\limits_{\gamma_2 \to -0} \frac{\partial F_i}{\partial \beta_{ij}} = 0$ *and* $\lim\limits_{\gamma_2 \to -\infty} \frac{\partial F_i}{\partial \beta_{ij}} = \infty$ *hold when* $\varepsilon > 1$, *and* $\lim\limits_{\gamma_2 \to -0} \frac{\partial F_i}{\partial \beta_{ij}} = \infty$ *and* $\lim\limits_{\gamma_2 \to -\infty} \frac{\partial F_i}{\partial \beta_{ij}} = 0$ *when* $\varepsilon < 1$. *See* ***Figure 18***.

*c) When* $\underline{x} < x_i < \gamma_1 + \frac{1}{\varepsilon}$ $(x_i > \gamma_1 + \frac{1}{\varepsilon})$ *holds, increasing preference heterogeneity leads to a larger (lower) increase in the disagreement* $D \equiv \frac{F_i - F_a}{x_i}$ *about the value of product variety when income* $x_i$ *becomes larger (when* $\beta_{ij} > \frac{\bar{\beta}}{2}$ *holds for a good* $j \epsilon \{2;3\}$, $\frac{\partial^2 D}{\partial \beta_{ij} \partial x_i} > 0$ *therefore holds when* $\underline{x} < x_i < \gamma_1 + \frac{1}{\varepsilon}$ *and* $\frac{\partial^2 D}{\partial \beta_{ij} \partial x_i} < 0$ *when* $x_i > \gamma_1 + \frac{1}{\varepsilon}$*).*

*Proof.* See Appendix A3. □

Even though aggregated consumption can be derived from the utility maximization problem of households with average preferences, such hypothetical households therefore value variety less than households with heterogeneous preferences do when $\gamma_2 < 0$ holds[21]. Under this parameter condition, newly introduced goods have a relatively high income elasticity. This is is a highly relevant case when it comes to various applications in the areas of innovation and trade. Moreover, the observation that there is in many cases a U-shaped relation between the entropy difference $\hat{E} - E_i$ and $x$ in our data can be explained by our model when $\gamma_2 < 0$ holds. Therefore, this case also seems to be relevant for the sample of goods that we look at in our empirical study.

Studying the welfare of households with average preferences without taking the empirically observed heterogeneity into account consequently leads to an underestimation of the true value that households attach to product variety, and the more so, the larger the extent of preference heterogeneity is.

This result is driven by the following mechanism: the utility that a household with a given income obtains when it consumes positive quantities of all three goods turns out to be independent of the individual values of $\beta_{i2}$ and $\beta_{i3}$ as long as $\beta_{i2} + \beta_{i3} = \bar{\beta}$ holds (as shown in Appendix A3, this even holds

[21] When all three goods are available, all households, however, suffer from the same reduction in the quantity $q^*_{i1}(x)$ of good 1 when the price $p$ of goods 2 and 3 increases (this is because $q^*_{i1}(x)$ is independent of $\beta_{B2}$, as can be seen in equation 8).

in the more general case where $\gamma_2 \neq \gamma_3$). Contrary to that, the utility of a household who only consumes good 1 falls in $\beta_{ij}$ when $\beta_{ij} > \frac{\bar{\beta}}{2}$ holds for $j\epsilon\{2;3\}$ as such an increase in preference heterogeneity reduces the utility derived from the subsistence consumption levels $\gamma_2 = \gamma_3$[22]. Consequently, the utility gain derived from being able to purchase all three goods (in optimal quantities) instead of only to good 1 increases in $\beta_{ij}$ if $\beta_{ij} > \frac{\bar{\beta}}{2}$ holds, implying a larger value of product variety.

The result that a household values the joint introduction of two goods more when its preferences regarding these goods are more heterogeneous can be generalized to settings in which, when these goods are not available, households not only have access to a single good (good 1), but to several goods of each of which they consume positive quantities (see Appendix A3).

Parts b) and c) of the proposition analyze how the size of $\gamma_2$ (determining the income elasticity of goods 2 and 3), the parameter $\varepsilon$ (that determines whether goods are substitutable or complementary to each other), and the level of income $x_i$ affect the effect of preference heterogeneity on the value of product variety. It turns out that these variables can have strong effects, implying that the effect of preference heterogeneity on the disagreement about the value of variety might be quite different for different goods and different economic environments.[23]

Interestingly, there are cases in which the disagreement between a household with heterogeneous and a household with average preferences about the value of product variety can become very large: When $\gamma_2$ is sufficiently close to zero and $\varepsilon < 1$ holds, the derivative $\frac{\partial F_i}{\partial \beta_{ij}}$ (with $j\epsilon\{2;3\}$) becomes very large as $\underset{\gamma_2 \to -0}{lim} \frac{dF_i}{d\beta_{i2}} = \infty$ holds for any value $\beta_{i2} > \frac{\bar{\beta}}{2}$, implying that even small degrees of preference heterogeneity can lead to large levels of disagreement $D = \frac{F_i - F_a}{x_i}$. The same holds true for the case where $\gamma_2$ is sufficiently neg-

[22]The utility of a household who only consumes good 1 is given by $U_i(1) = \left((1-\bar{\beta})^{\frac{1}{\varepsilon}}(x_i-\gamma_1)^{\frac{\varepsilon-1}{\varepsilon}} + \beta_{i2}^{\frac{1}{\varepsilon}}(-\gamma_2)^{\frac{\varepsilon-1}{\varepsilon}} + (\bar{\beta}-\beta_{i2})^{\frac{1}{\varepsilon}}(-\gamma_2)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$

[23]In order to properly derive the value of product variety in a particular context, one therefore needs to take all these things into account. As our data is not detailed enough to allow us to estimate all these parameters (we do not have information on relative prices), we do not try to quantify the extent of disagreement for particular goods as we fear that the results would not be very robust.

ative and where $\varepsilon > 1$ holds. The analysis therefore suggests that simple "representative household" models as advocated by Hicks might not be very useful to determine the welfare effects of product variety when heterogeneity of household consumption patterns is a prevalent feature of the data.

### 4.1 Accounting for variety demand

The above analysis focused on the case in which households are rich enough to purchase non-negative quantities of all available goods, i.e. in which $x_i \geq \underline{x}$ holds. For smaller incomes $x_i < \underline{x}$, the model can, however, also account for the empirically observed fact that richer households demand a larger variety of goods (like for example documented by Jackson (1984) and Falkinger and Zweimüller (1996)): when $\gamma_j > 0$ holds for some goods and $\gamma_j < 0$ for others, all households purchase positive quantities of the goods for which $\gamma_j > 0$ holds, while only households with sufficient income purchase positive quantities of goods for which $\gamma_j < 0$ holds (as the marginal utility of the first unit of such goods is finite while that of goods with $\gamma_j > 0$ is infinite). The variety of goods consumed therefore increases in income $x_i$ when there are several goods for which $\gamma_j < 0$ holds.

Even when the parameters $\gamma_j$ are the same for all households, households that differ with respect to the parameters $\beta_{ij}$ might then increase the variety of goods that they consume in a different order. This becomes clear by looking at the example with three goods and two households (Brown and Jones) from section 3.1, in which it is assumed that $\gamma_2 < 0$, $\beta_{i1} = 1 - \bar{\beta}$, $\beta_{B2} = \beta_{J3} > \frac{\bar{\beta}}{2}$ and $\beta_{B3} = \beta_{J2} = \bar{\beta} - \beta_{B2} \geq 0$ hold: In this case, $\underline{x} = \gamma_1 + 2p\gamma_2 - \gamma_2 \left[ \frac{\left(1-\bar{\beta}\right)p^{\varepsilon} + \bar{\beta}p}{\bar{\beta} - \beta_{B2}} \right]$ holds (see the proof of Lemma 1), implying that household Brown (Jones) stops consuming good 3 (2) when income falls below the level $\underline{x}$. Applying equation 6 to the case where households only purchase the two remaining goods, it can be shown that households stop consuming two goods and spend all their income on the basic need good 1 when there is a further fall in $x_i$ below the threshold $\dot{x} \equiv \gamma_1 - \gamma_2 \frac{1-\bar{\beta}}{\beta_{B2}} p^{\varepsilon} < \underline{x}$. When incomes increase from a level $x_i < \dot{x}$ to a level $x_i > \underline{x}$, households therefore expand the variety of goods that they consume from one to three, but in a different order: while household Brown purchases goods 1 and 2 when income lies in the range $\dot{x} < x_i < \underline{x}$, household Jones purchases goods 1 and 3 in this range as tastes are heterogeneous with respect to goods 2 and 3.

Applying these insights to a more general setting with many goods $j$, the direction in which variety demand grows can then vary across the population when households differ with respect to the parameters $\beta_{ij}$. At low income levels, households with different preferences then not only purchase different quantities of goods, but the set of goods they consume may also vary across households. This implies that the average consumption basket for the group consists of a larger variety of goods than the individual consumption baskets for each household belonging to that group. When individual goods are grouped into broader consumption categories, different households then, moreover, pick the goods which they consume in a more uneven way from these categories than households do on average. This implies that the "diversity of the variety demand" of a household with income $x_i$ is lower than the diversity of the average consumption basket of all households with income $x$.

This is indeed the case when we look at the data: Figure 13 presents the diversity of variety demand across 12 expenditure categories at the household level and representative household level using data from the year 2000. This Figure is derived in the following way: goods are grouped into 12 broader categories indexed by $h$, with the total number of goods in category $h$ given by $N_h$. Denoting the number of different goods (i.e. the varieties) that household $i$ consumes within category $h$ by $n_{ih}$, we then determine the fractions $\frac{n_{ih}}{N_h}$ for all households and categories. The entropy measure described in Section 2 is then applied to these fractions in order to estimate the diversity of household variety demand

$$D_i = \sum_{h=1}^{12} - \left( \frac{n_{ih}}{N_h} ln \left( \frac{n_{ih}}{N_h} \right) \right)$$

across the 12 expenditure categories.

For the representative households, average variety demand is calculated as

$$\hat{D} = \sum_{h=1}^{12} - \left( \frac{\hat{n}_{dh}}{N_h} ln \left( \frac{\hat{n}_{dh}}{N_h} \right) \right)$$

for the same year. In order to determine $\hat{D}$, households are grouped into deciles and the individual varieties $n_{ih}$ are replaced by the variety $\hat{n}_{dh}$ of goods of category $h$ consumed by decile $d$ (i.e. by the number of all goods of which positive quantities are consumed by at least one household falling into the decile). Figure 13 shows that the diversity of variety demand at the household level $D_i$ is lower than the diversity of variety demand $\hat{D}$ at the representative (decile) level and that both $D_i$ and $\hat{D}$ rise in income $x$. As different households grow their variety demand, the consumption baskets therefore become more diverse in terms of varieties consumed across different expenditure categories. This is highlighted in Figure 14 which presents the estimated difference between the diversity of variety demand on the household and representative level.

# 5 Conclusion

The truth about Mr Brown and Mrs Jones is that they not only possess different spending patterns, but that the differences between these patterns tend to grow in income when income is sufficiently high. In this paper we have highlighted how this 'emergent' aspect of consumption heterogeneity has important implications for the extent to which the behavior of representative consumers reflects the actual behavior and preferences of individual consumers.

While at the aggregate level the spread of household expenditure across categories, i.e. the diversity of spending, tends to always rise as income grows, this is not the case when the diversity of expenditures is examined at the household level. Rather, household spending patterns on the more disaggregated level show that rich households tend to concentrate their spending into particular expenditure categories. Because each household concentrates into different types of expenditure categories, diversity of household expenditure can nevertheless increase at the aggregate level while it declines at the individual level.

These findings, in combination with the welfare results obtained in the theoretical analysis, highlight the pitfalls of adopting representative agent models when a considerable extent of heterogeneity across households can be observed in the data. Paying attention to what Mr Brown and Mrs Jones do instead of only focusing on average behavior should therefore become a priority for future research.

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# Tables

Table 1: Categories of the UK Family Expenditure Survey,2000

| Category | Examples of spending |
|---|---|
| Food | Milk, Eggs, vegetables, meats, sweets, non-alcoholic beverages. Take away meals, food bought and consumed at work and school. |
| Fuel Light and Power | Gas, Electricity, Coal, bottled gas, paraffin, wood. |
| Alcoholic Drinks | Beer, Lager, Cider, Spirits Liqueurs. |
| Tobacco | Cigarettes, Pipe tobacco, cigars |
| Clothing and Footwear | Outerwear, Underwear, Clothing accessories, Footwear, Haberdashery and clothing materials |
| Household goods | Furniture and Furnishings, Electrical and gas appliances. Hardware, decorative goods. Toilet paper, Pet and garden expenditure. |
| Domestic and Paid services | Childcare, domestic help, laundry, postage and telephones, subscriptions and stamp duty. |
| Personal Goods and Services | Hairdressing, cosmetic requisites. Baby goods, medicines and medical goods. Personal effects and travel goods. |
| Motoring Expenditure | Accessories, parts, repairs and servicing of motor vehicles. Petrol and oil. Insurance, driving lessons and other payment. |
| Travel | Fares, other transport costs, Purchase and maintenance of non-motor vehicles. |
| Leisure Goods | TV, video and Audio equipment. Sports, camping and outdoor good and equipment. Newspapers, magazines, books and stationary. Toy, hobbies and photography. |
| Entertainment and Education Services | Cinema, spectator sports, TV rental and subscription, hotels and holiday expenses, betting stakes, educational fees and maintenance, Ad hoc school expenditure, betting stakes. |

Table 2: Average Budget for food, goods and services per decile

| **2000** | | | | **1995** | | | | **1990** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Income | Food | Goods | Services | Income | Food | Goods | Services | Income | Food | Goods | Services |
| 31.23 | 0.61 | 0.21 | 0.18 | 25.40 | 0.64 | 0.19 | 0.16 | 20.49 | 0.68 | 0.16 | 0.16 |
| 51.89 | 0.55 | 0.26 | 0.19 | 39.57 | 0.58 | 0.23 | 0.19 | 32.93 | 0.61 | 0.21 | 0.18 |
| 67.40 | 0.50 | 0.31 | 0.20 | 50.63 | 0.55 | 0.26 | 0.20 | 42.49 | 0.56 | 0.25 | 0.19 |
| 83.35 | 0.47 | 0.34 | 0.19 | 62.04 | 0.52 | 0.29 | 0.19 | 52.30 | 0.53 | 0.28 | 0.19 |
| 100.53 | 0.43 | 0.39 | 0.18 | 73.98 | 0.48 | 0.32 | 0.19 | 63.37 | 0.50 | 0.31 | 0.19 |
| 119.36 | 0.42 | 0.41 | 0.18 | 86.80 | 0.46 | 0.34 | 0.20 | 74.80 | 0.47 | 0.35 | 0.18 |
| 140.54 | 0.38 | 0.44 | 0.18 | 101.81 | 0.43 | 0.37 | 0.20 | 89.49 | 0.44 | 0.36 | 0.20 |
| 166.62 | 0.37 | 0.44 | 0.20 | 121.41 | 0.42 | 0.38 | 0.21 | 108.55 | 0.39 | 0.39 | 0.22 |
| 203.71 | 0.33 | 0.47 | 0.20 | 150.26 | 0.38 | 0.42 | 0.21 | 138.44 | 0.36 | 0.39 | 0.25 |
| 292.12 | 0.28 | 0.48 | 0.24 | 219.75 | 0.31 | 0.42 | 0.27 | 215.15 | 0.27 | 0.44 | 0.29 |

Table 3: *Notes: Each row represents a decile. Income is equal to total weekly household expenditure.*

# Figures

## The Engel curves for spending diversity

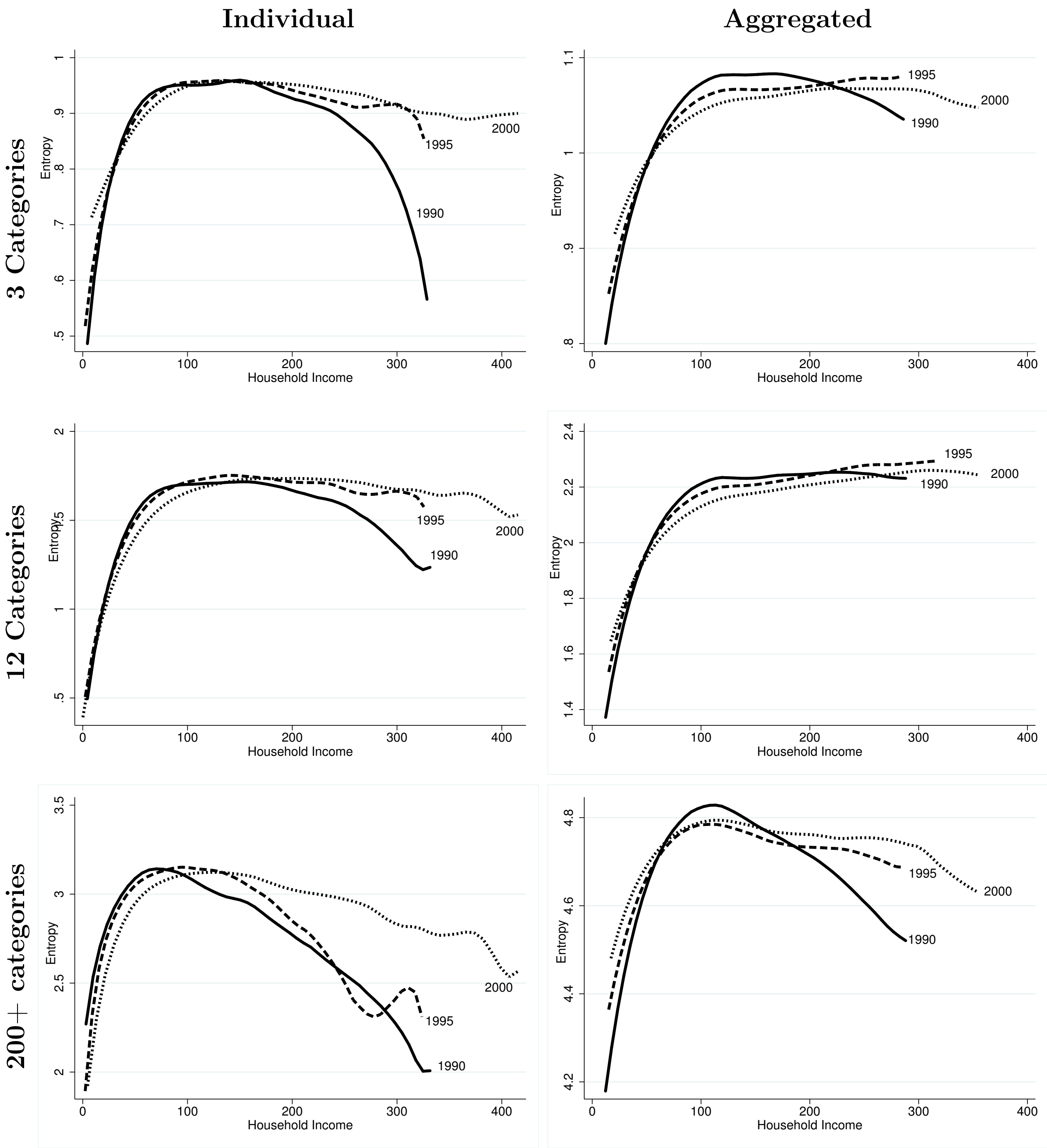


Figure 1: *Notes:* The Figures on the left show the entropies $E_i$ of consumption spending of individual households, while the Figures on the right depict the entropies $\hat{E}$ of aggregated consumption spending for groups of households with similar income levels. Households are aggregated into 50 representative groups with similar income levels. Each row represents a different level of aggregation across expenditure categories. In the first row, three broad categories are used: food, goods and services. The middle row uses the 12 expenditure categories listed in Table 1 of the Appendix, and the bottom row uses the maximum level of disaggregation of 200+ categories. The number of observations was 6,047 in 1990, 5,984 in 1995 and 5,865 in 2000.

## The case of 3 expenditure categories

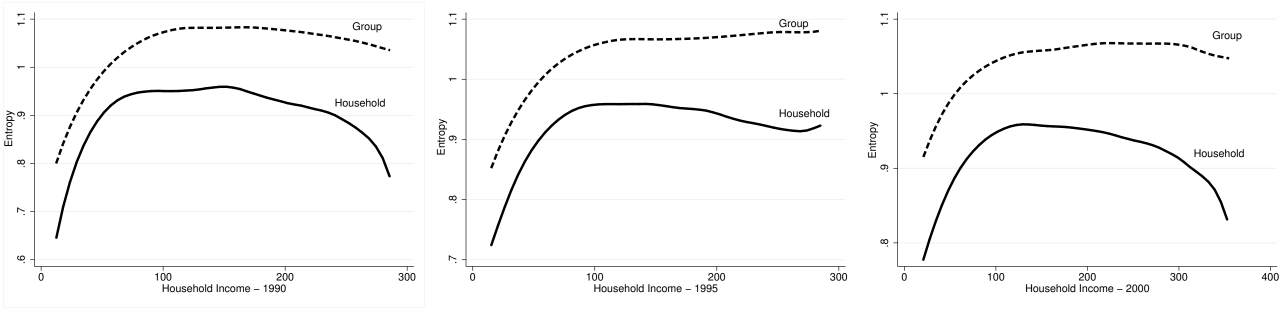


Figure 2: Spending diversity on the household and aggregated group level

*Notes:* The figures depict spending diversity on the household level (solid line, $E_i$) and on the aggregated level (dashed line, $\hat{E}$) for 1990 (left), 1995 (middle) and 1995 (right). Expenditure categories consist of 3 categories - food, goods and services. Households are aggregated into 50 representative groups with similar income levels. Note that the individual spending diversity curves are shortened to omit observations below the average income of the poorest and above the average income of the richest income group. As a result, these curves are shorter than those displayed in Figure 1

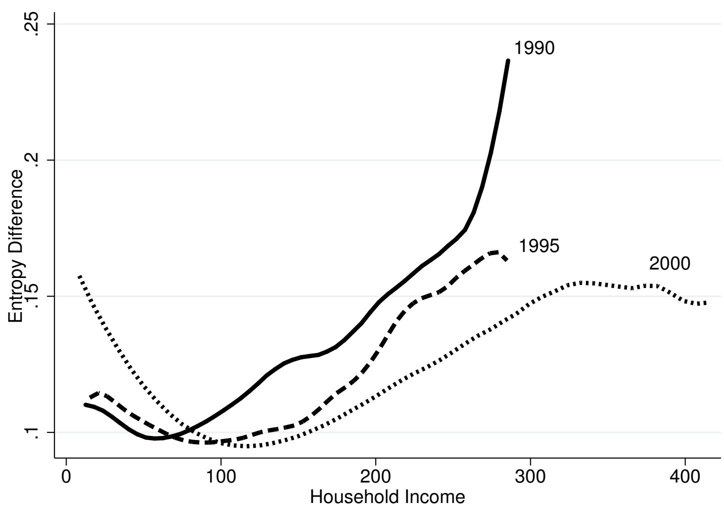


Figure 3: Difference between aggregated and household level Engel curves for spending diversity (3 categories)

Note: The curves depict the differences $\hat{E} - E_i$ between aggregate level (50 groups) and household level spending diversity. This shows that these differences tend to grow in income for large income levels.

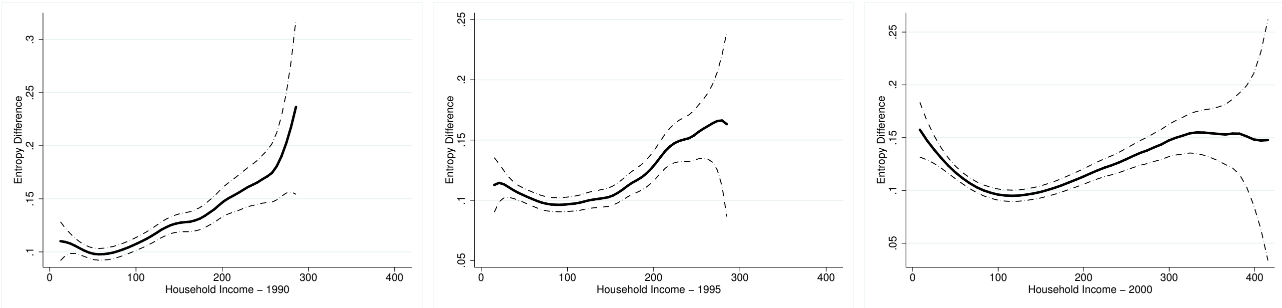


Figure 4: Estimated Difference with confidence intervals

*Notes:* The curves depict the differences $\hat{E} - E_i$ between aggregate level (50 groups) and household level spending diversity for 1990 (left), 1995 (middle) and 2000 (right). The dashed lines represent 95% confidence intervals.

## The case of 12 expenditure categories

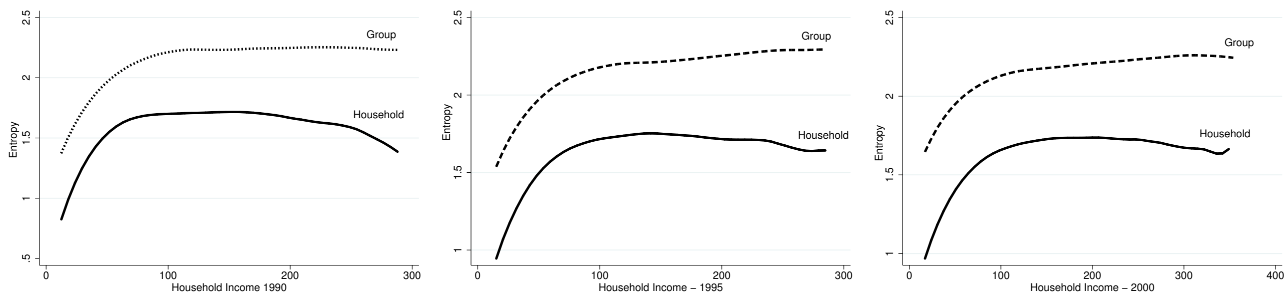


Figure 5: Household and decile entropies (12 expenditure categories)

*Notes:* The figures depict spending diversity on the household level (solid line, $E_i$) and on the decile level (dashed line, $\hat{E}$) for 1990 (left), 1995 (middle) and 1995 (right). Expenditure categories are aggregated into 12 categories - see Table 1. Households are aggregated into 50 representative groups with similar income levels. Note that the individual spending diversity curves are shortened to omit observations below the average of the poorest and above the average of the richest income group. As a result, these curves are shorter than those displayed in Figure 1

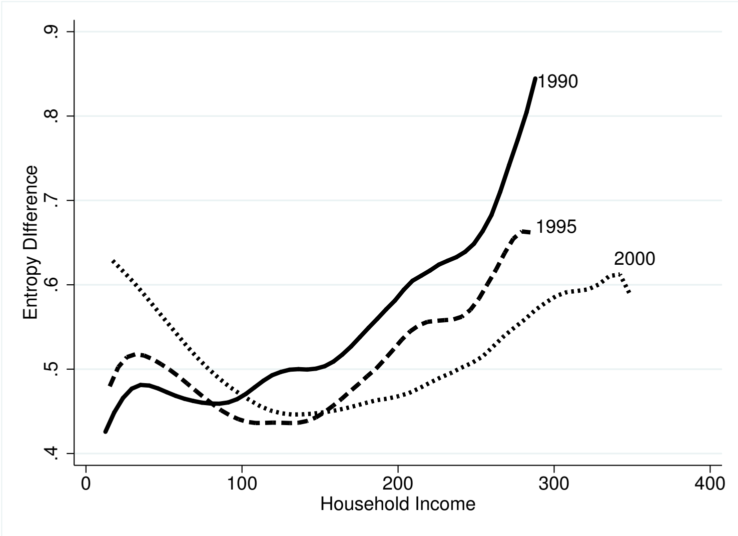


Figure 6: Difference between aggregated and household entropies (12 categories)

Note: The differences $\hat{E} - E_i$ between aggregated (group) and household level (individual) entropy of spending. This shows that these differences tend to grow in income for large income levels

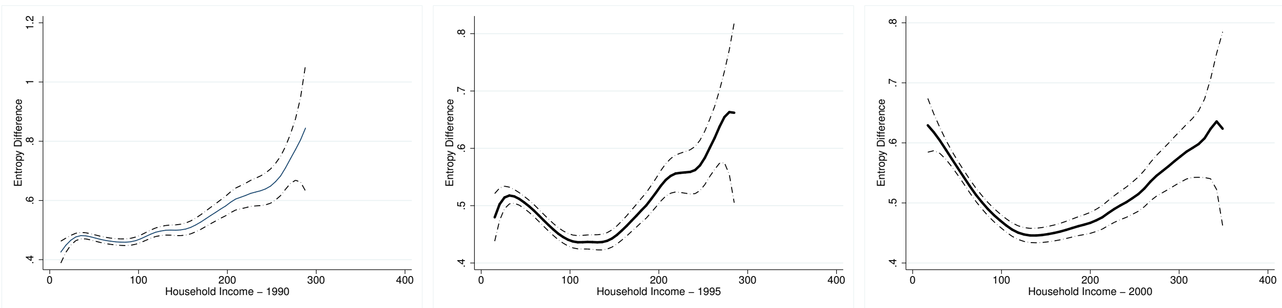


Figure 7: Estimated differences with confidence intervals

*Notes:* The curves depict the differences $\hat{E} - E_i$ between aggregate level (50 groups) and household level spending diversity for 1990 (left), 1995 (middle) and 2000 (right). The dashed lines represent 95% confidence intervals.

## The case of 200+ expenditure categories

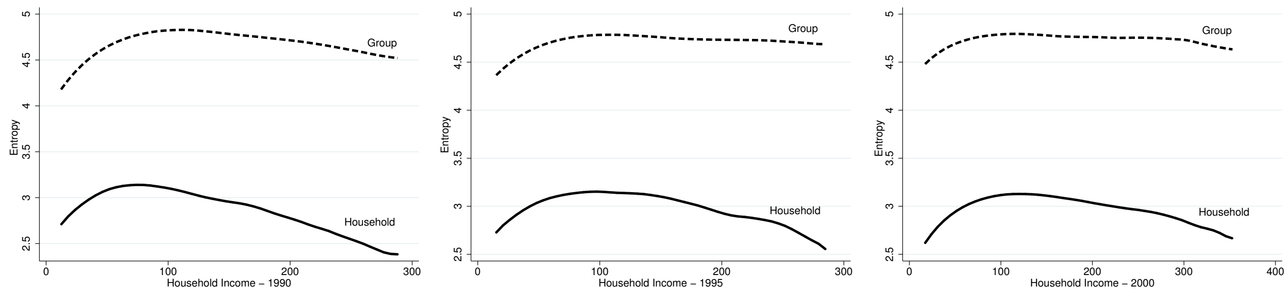


Figure 8: Household and decile entropies (200+ expenditure categories)

*Notes:* The figures depict spending diversity on the household level (solid line, $E_i$) and on the decile level (dashed line, $\hat{E}$) for 1990 (left), 1995 (middle) and 1995 (right). Expenditure categories were not aggregated. Households are aggregated into 50 representative groups with similar income levels. Note that the individual spending diversity curves are shortened to omit observations below the average of the poorest and above the average of the richest income group. As a result, these curves are shorter than those displayed in Figure 1

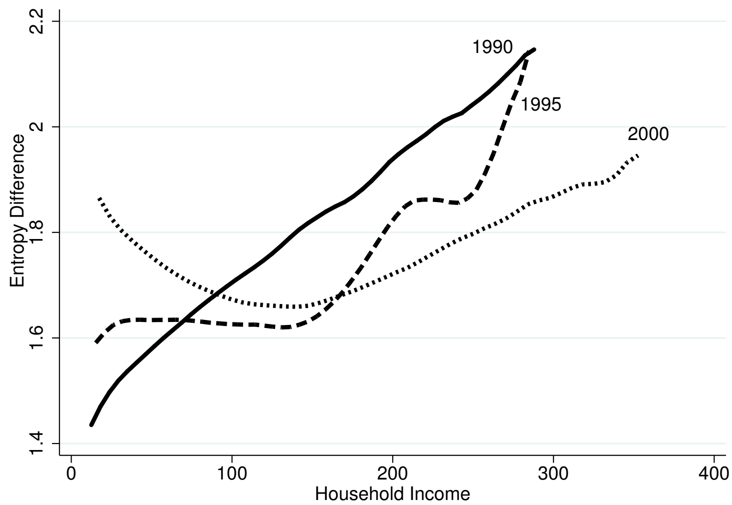


Figure 9: Difference between aggregated and household entropies (200+ Categories)

Note: The difference $\hat{E} - E_i$ between aggregated (group) level and individual (household) level entropies of spending. This shows that these differences tend to grow in income for large income levels.

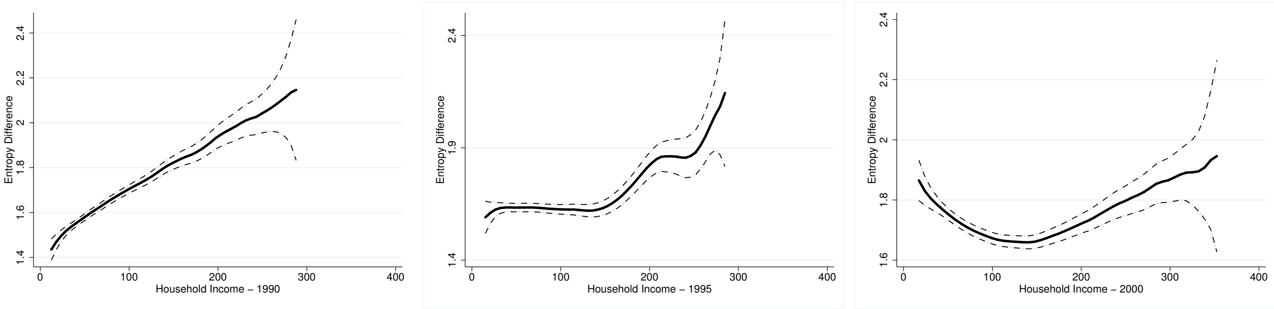


Figure 10: Estimated differences with confidence intervals

*Notes:* The curves depict the differences $\hat{E} - E_i$ between aggregate level (50 groups) and household level spending diversity for 1990 (left), 1995 (middle) and 2000 (right). The dashed lines represent 95% confidence intervals.

## Robustness check: choice of representative households

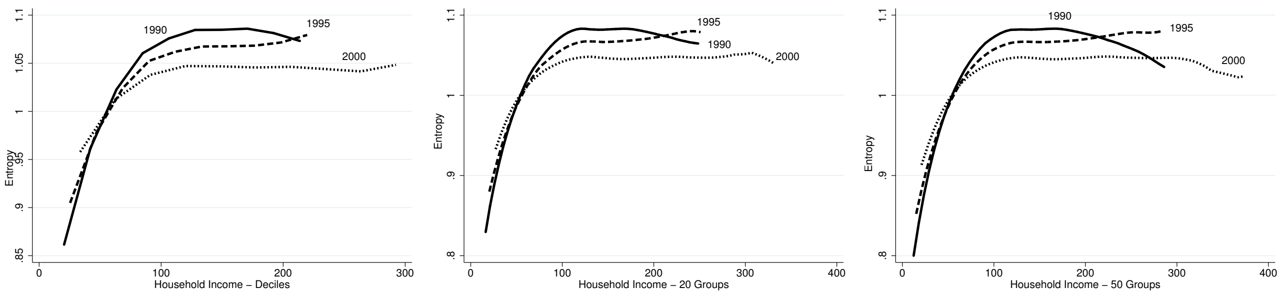


Figure 11: Aggregation of representative groups

*Notes:* This Figure depicts the the entropy $\hat{E}$ of aggregated spending across different household aggregation levels. The graph on the left depicts the case of 10 representative (decile) income groups, the one in the middle the case of 20 groups, and the one on the right the case of 50 income groups.

# Zeros removed

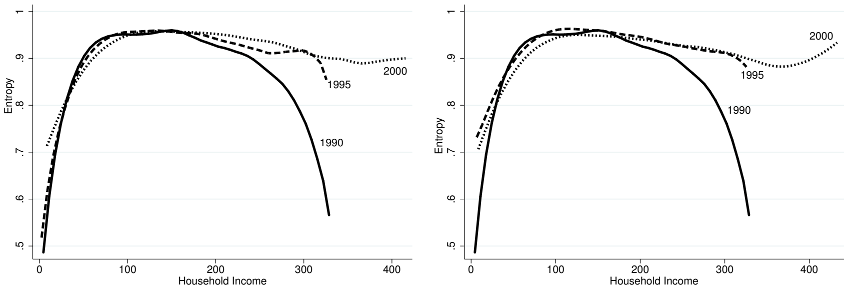


Figure 12: Zeros removed

*Notes:* This Figure compares the entropy of spending for 3 expenditure categories between the base case where households with zero expenditure in one or two of the three expenditure categories are included (left) and the case where they are removed (right). The number of observation fell by around 90 households per year as a result of excluding the zero expenditures.

## The diversity of variety demand

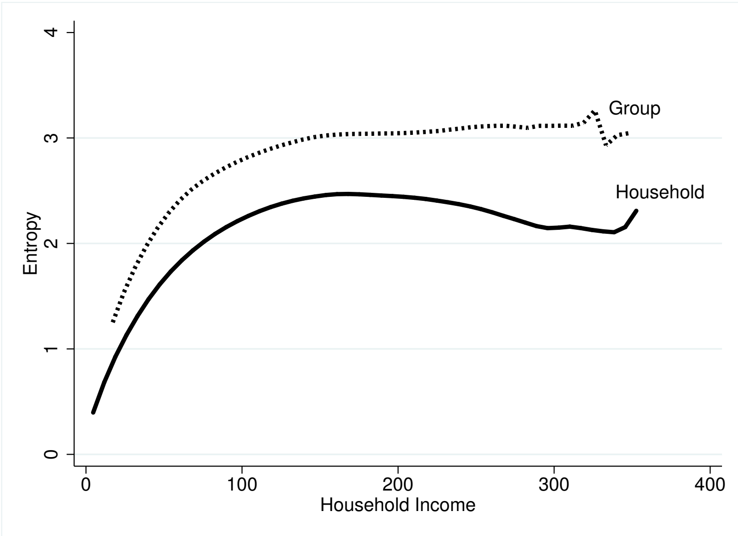


Figure 13: Diversity of variety demand

Note: This figure reports how evenly the varieties consumed by a household are distributed across the 12 categories (see Table 1) on the household level and by the representative household (50 groups) for the year 2000. This figure shows that initially this diversity increases and then flattens out.

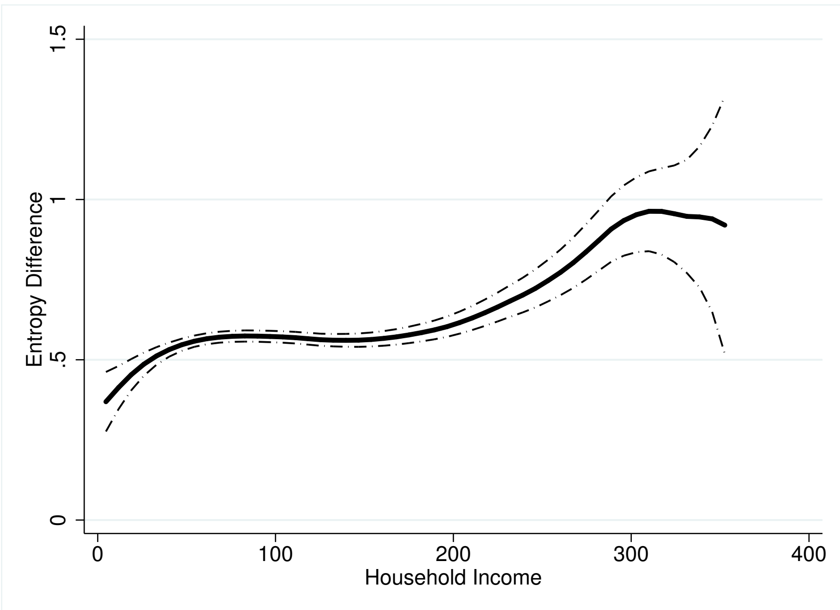


Figure 14: Estimated differences with confidence intervals

*Notes:* This figure depicts the estimated differences in the diversity of variety demand between aggregate level (50 groups) and household level spending diversity for 2000. The dashed lines represent 95% confidence intervals

## Engel curves in the case of three goods

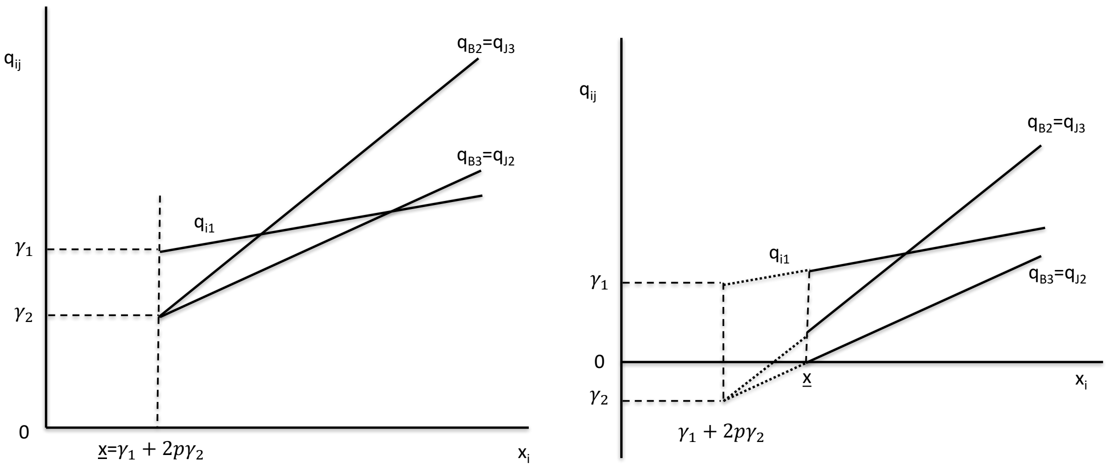


Figure 15: Engel Curves

*Notes:* The Figures show the Engel curves (i.e. the quantities $q_{ij}$ as a function of the income $x_i$) arising under the particular assumptions made in the three good example from section 3.1. The Figure on the left depicts the case where $\gamma_2 > 0$ holds and the figure on the right the case where $\gamma_2 < 0$ holds. In the latter case, the Engel curves are only drawn for income levels $x_i > \underline{x}$ for which households consume positive quantities of all goods.

## Group and individual level Engel curves for spending diversity when $\gamma_2 > 0$

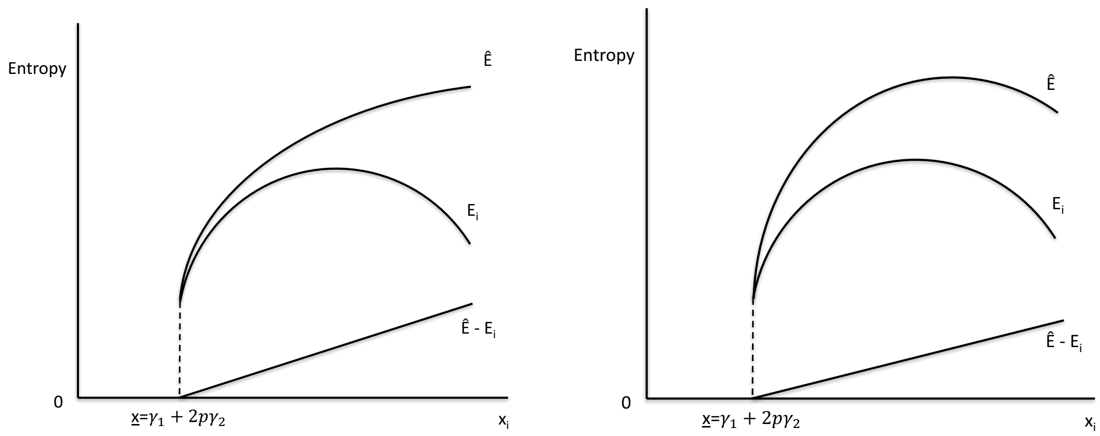


Figure 16: Entropies

*Notes:* The Figures show the shapes of the Engel curves for spending diversity that the model can generate when $\gamma_2 > 0$ holds. While the case is considered in which there is always an inverse-U relation between household consumption entropies $E_i$ and household income $x_i$, there can either be a positive relation between the entropy $\hat{E}$ of aggregated consumption spending and income $x_i$ (Figure on the left) or an inverse-U relation between $\hat{E}$ and $x_i$ (Figure on the right). The entropy difference $\hat{E} - E_i$ rises in $x_i$ in both cases.

## Group and individual level Engel curves for spending diversity when $\gamma_2 < 0$

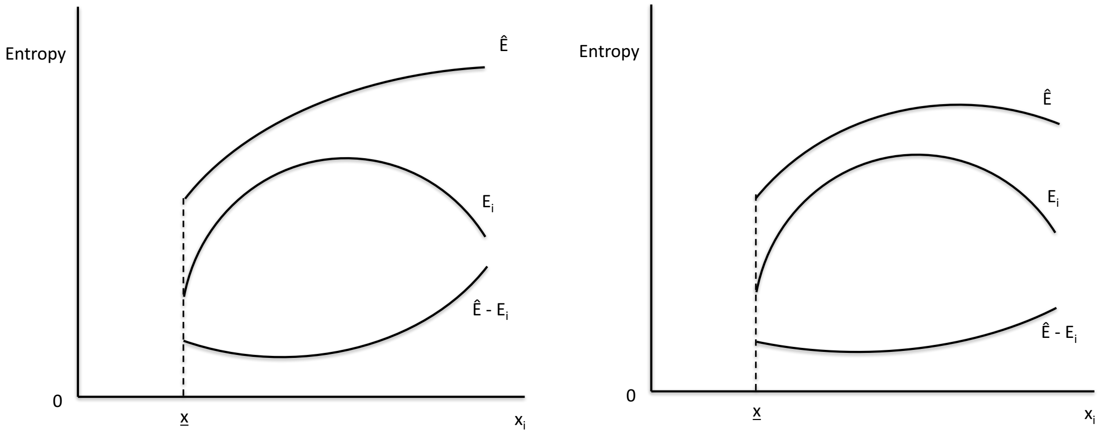


Figure 17: Entropies

*Notes:* The Figures show the shapes of the Engel curves for spending diversity that the model can generate when $\gamma_2 < 0$ holds. While the case is considered in which there is always an inverse-U relation between household consumption entropies $E_i$ and household income $x_i$, there can either be a positive relation between the entropy $\hat{E}$ of aggregated consumption spending and income $x_i$ (Figure on the left) or an inverse-U relation between $\hat{E}$ and $x_i$ (Figure on the right). The entropy difference $\hat{E} - E_i$ first falls and then rises in $x_i$ in both cases.

## The responsiveness of the value of product variety to preference heterogeneity

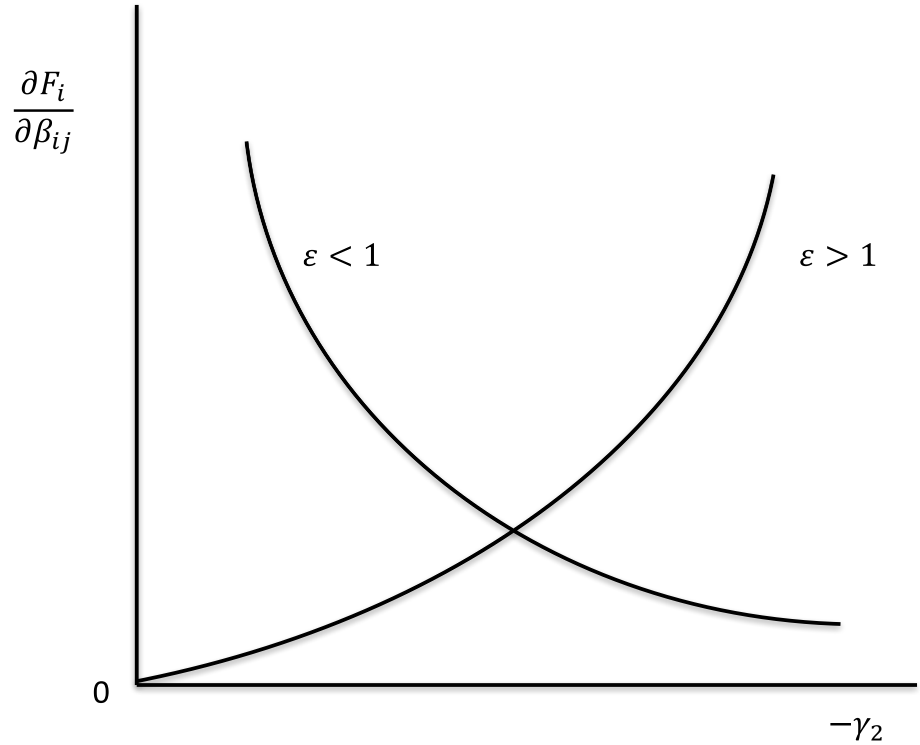


Figure 18: Value of variety and preference heterogeneity

Note: This figure plots the increase in the value of variety induced by an increase in preference heterogeneity, $\frac{\partial F_i}{\partial \beta_{ij}}$, as a function of $-\gamma_2$ (a more negative value $\gamma_2$ implies a higher income elasticity of goods 2 and 3). The slope of the curve depends on the sign of $\varepsilon$ and is positive when goods are substitutable ($\varepsilon > 0$) and negative when goods are complementary ($\varepsilon < 1$). The case is considered in which $\beta_{ij} > \frac{\bar{\beta}}{2}$ holds for $j \epsilon \{2; 3\}$.

# Appendix A: Proofs

## A1: Proof of Lemma 1

*Proof.* Differentiating equation 8, we obtain that $\frac{\partial(s_{i1}(x))}{\partial x} = \frac{2p\gamma_2\left(1-\bar{\beta}\right)-\gamma_1\bar{\beta}p^{1-\varepsilon}}{x^2\left(1-\bar{\beta}+\bar{\beta}p^{1-\varepsilon}\right)}$, so that $\frac{\partial(s_{i1}(x))}{\partial x} < 0$ holds when $\gamma_1 > \frac{2\gamma_2\left(1-\bar{\beta}\right)p^{\varepsilon}}{\bar{\beta}}$ (Condition B). Differentiating equation 12 gives

$$\frac{\partial\hat{E}}{\partial x} = -\frac{\partial\hat{s}_1}{\partial x}\left(\ln\hat{s}_1+1\right)-2\frac{\partial\hat{s}_2}{\partial x}\left(\ln\hat{s}_2+1\right) = -\frac{\partial s_{B1}}{\partial x}\left(\ln s_{B1}\right)-\left(\frac{\partial s_{B2}}{\partial x}+\frac{\partial x_{B3}}{\partial x}\right)\left(\ln\hat{s}_2\right)$$
$$= -\frac{\partial s_{B1}}{\partial x}\left(\ln s_{B1}-\ln\hat{s}_2\right) = -\frac{\partial s_{B1}}{\partial x}\left(\ln s_{B1}-ln\left(\frac{1-s_{B1}}{2}\right)\right)$$

where the conditions $\hat{s}_1 = s_{B1}$, $\hat{s}_2(x) = \frac{s_{B2}(x)+s_{B3}(x)}{2}$, $s_{B1}+s_{B2}+s_{B3} = 1$ and $\frac{\partial s_{B1}}{\partial x}+\frac{\partial s_{B2}}{\partial x}+\frac{\partial x_{B3}}{\partial x} = 0$ were used for the transformations. As $\frac{\partial s_{B1}}{\partial x} < 0$, $sign\frac{\partial\hat{E}}{\partial x} = sign\left(\ln s_{B1}-\ln\left(\frac{1-s_{B1}}{2}\right)\right) = sign\left(s_{B1}-\frac{1}{3}\right)$ holds. As $\underset{x\to\infty}{lim} s_{B1} = \frac{\left(1-\bar{\beta}\right)p^{\varepsilon}}{p\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon}}$ and as (by assumption) $s_{i1}$ continuously falls in $x$, $\frac{\partial\hat{E}}{\partial x} > 0$ therefore always holds if $\frac{\left(1-\bar{\beta}\right)p^{\varepsilon}}{p\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon}} > \frac{1}{3}$ holds, i.e. if $\bar{\beta} < \frac{2}{p^{1-\varepsilon}+2}$ holds. If $\bar{\beta} > \frac{2}{p^{1-\varepsilon}+2}$, $\frac{\partial\hat{E}}{\partial x} < 0$ holds for large values of $x$ (i.e. $x > \check{x}$), while $\frac{\partial\hat{E}}{\partial x} > 0$ still holds for lower values ($\underline{x} < x < \check{x}$) when $s_{B1}|_{x=\underline{x}} > \frac{1}{3}$ (**Condition C**) holds. The minimum income level $\underline{x}$ is given by $\underline{x} = \gamma_1+2p\gamma_2$ when $\gamma_2 = \gamma_3 \geq 0$ holds (as this income is required to purchase the positive subsistence consumption level of each good) and by $\underline{x} = \gamma_1+2p\gamma_2-\gamma_2\left[\frac{\left(1-\bar{\beta}\right)p^{\varepsilon}+\bar{\beta}p}{\bar{\beta}-\beta_{12}}\right]$ when $\gamma_2 = \gamma_3 < 0$ (in this case, $\underline{x}$ is pinned down by the condition $s_{B3}(\underline{x}) = s_{J2}(\underline{x}) = 0$). This implies that $s_{B1}|_{x=\underline{x}} = \frac{\gamma_1}{\gamma_1+2p\gamma_2}$ when $\gamma_2 \geq 0$, and $s_{B1}|_{x=\underline{x}} = \frac{\left(\bar{\beta}-\beta_{B2}\right)\gamma_1-\left(1-\bar{\beta}\right)\gamma_2}{\left(\bar{\beta}-\beta_{B2}\right)\gamma_1-\gamma_2\left(\left(1-\bar{\beta}\right)p^{\varepsilon}+2p\beta_{B2}-p\bar{\beta}\right)}$ when $\gamma_2 < 0$. Plugging these values into Condition C, we obtain that Condition C is satisfied if either $\gamma_1 > p\gamma_2 \geq 0$ (**Condition C1**) holds, or if $\gamma_1 > \frac{-\gamma_2\left(p\left(2\beta_{B2}-\bar{\beta}\right)-\left(1-\bar{\beta}\right)(3-p^{\varepsilon})\right)}{2\left(\bar{\beta}-\beta_{B2}\right)}$ and $\gamma_2 < 0$ (**Condition C2**) holds. □

## A2: Proof of Proposition 1

*Proof.* Differentiating equations 9 and 10 gives $\frac{\partial(s_{B2}(x))}{\partial x} = \frac{\gamma_1\beta_{B2}-\gamma_2\left(1-\bar{\beta}\right)p^{\varepsilon}-p\gamma_2\left(\bar{\beta}-2\beta_{B2}\right)}{x^2\left(\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon-1}\right)}$ and $\frac{\partial(s_{B3}(x))}{\partial x} = \frac{\gamma_1\left(\bar{\beta}-\beta_{B2}\right)-\gamma_2\left(1-\bar{\beta}\right)p^{\varepsilon}+p\gamma_2\left(\bar{\beta}-2\beta_{B2}\right)}{x^2\left(\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon-1}\right)}$, implying that $\frac{\partial(s_{B2}(x))}{\partial x} > \frac{\partial(s_{B3}(x))}{\partial x}$ holds when $\gamma_1 > -2\gamma_2 p$ (**Condition D**) holds. As $s_{i1}+s_{i2}+s_{i3} = 1$ and therefore $\frac{\partial(s_{i1})}{\partial x}+\frac{\partial(s_{i2})}{\partial x}+\frac{\partial(s_{i3})}{\partial x} = 0$, the conditions $\frac{\partial(s_{i1}(x))}{\partial x} < 0$ (implied by Condition B) and $\frac{\partial(s_{B2}(x))}{\partial x} > \frac{\partial(s_{B3}(x))}{\partial x}$ imply that $\frac{\partial(s_{B2}(x))}{\partial x} > 0$ needs to hold. The derivative $\frac{\partial(s_{B3}(x))}{\partial x}$ can be either positive or negative, where the latter is only possible if $\gamma_2 > 0$ holds ($\frac{\partial(s_{B3}(x))}{\partial x}$ falls in $\beta_{B2}$ and is therefore most likely negative when $\beta_{B2} = \bar{\beta}$ holds. As $sign\, \frac{\partial(s_{B3}(x))}{\partial x}\Big|_{\beta_{B2}=\bar{\beta}} = sign\,\{-\gamma_2\}$, $\frac{\partial(s_{B3}(x))}{\partial x} < 0$ can only hold if $\gamma_2 > 0$).

Subtracting equation 11 from equation 12 and differentiating with respect to $x$, we obtain that $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ holds when the following **Condition E** is satisfied:

$$\frac{\partial\left(s_{B2}(x)\right)}{\partial x}\left(\ln s_{B2} - \ln \hat{s}_2\right) > \frac{\partial\left(s_{B3}(x)\right)}{\partial x}\left(\ln \hat{s}_2 - \ln s_{B3}\right)$$

As $\hat{s}_2(x) = \frac{s_{B2}(x)+s_{B3}(x)}{2}$ and $s_{B2}(x) > s_{B3}(x)$, the terms in brackets are positive, implying that Condition E always holds when $\frac{\partial(s_{B3}(x))}{\partial x} < 0$ holds. When $\frac{\partial(s_{B3}(x))}{\partial x} < 0$, which is only possible if $\gamma_2 > 0$ (see above), $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ therefore holds. In the following, the remaining case where $\frac{\partial(s_{B3}(x))}{\partial x} > 0$ holds is considered. This is done by rewriting Condition E as follows:

$$Z \equiv \frac{\frac{\partial(s_{B2}(x))}{\partial x}}{\frac{\partial(s_{B3}(x))}{\partial x}} > Q \equiv \frac{\ln \hat{s}_2 - \ln s_{B3}}{\ln s_{B2} - \ln \hat{s}_2} \tag{13}$$

Due to the concavity of the *ln* function, $Q > 1$ holds. The proposition studies the case in which $\frac{\partial(s_{B2}(x))}{\partial x} > \frac{\partial(s_{B3}(x))}{\partial x}$, i.e. in which $Z > 1$ holds. The reason for this is that in the case where $\frac{\partial(s_{B2}(x))}{\partial x} < \frac{\partial(s_{B3}(x))}{\partial x}$, $Z < Q$ and therefore $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} < 0$ holds for all values of $x$, which would not be in line with the empirical observations. Inserting the corresponding expressions, $Z$ can be derived as:

$$Z = \frac{\gamma_1\beta_{B2} - \gamma_2\left(1-\bar{\beta}\right)p^{\varepsilon} - p\gamma_2\left(\bar{\beta} - 2\beta_{B2}\right)}{\gamma_1\left(\bar{\beta} - \beta_{B2}\right) - \gamma_2\left(1-\bar{\beta}\right)p^{\varepsilon} + p\gamma_2\left(\bar{\beta} - 2\beta_{B2}\right)} \tag{14}$$

$Z$ is therefore independent of income $x$. The proof (for the case in which $\frac{\partial(s_{B3}(x))}{\partial x} > 0$) proceeds as follows: In part $i$) it is shown that $sign\frac{\partial Q}{\partial x} = sign\gamma_2$. In part $ii$) it is shown that $Z > Q$ and therefore $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ always holds when $\gamma_2 > 0$ and the case where $\gamma_2 = 0$ is discussed. In part $iii$), the case where $\gamma_2 < 0$ is analyzed and it is shown that $Z > Q$ and therefore $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ ($Z < Q$ and therefore $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} < 0$) then holds if $\tilde{x} < x < \infty$ ($\underline{x} < x < \tilde{x}$).

$i$) Deriving $Q$ with respect to $x$ yields

$$\begin{aligned} sign\frac{\partial Q}{\partial x} = sign\Bigg\{&\frac{\partial s_{B2}}{\partial x}\left[\frac{1}{s_{B2}+s_{B3}}\left(\ln s_{B2} - \ln s_{B3}\right) - \frac{1}{s_{B2}}\left(\ln\left(\frac{s_{B2}+s_{B3}}{2}\right) - \ln s_{B2}\right)\right] \\ &+ \frac{\partial s_{B3}}{\partial x}\left[\frac{1}{s_{B2}+s_{B3}}\left(\ln s_{B2} - \ln s_{B3}\right) - \frac{1}{s_{B3}}\left(\ln s_{B2} - \ln\left(\frac{s_{B2}+s_{B3}}{2}\right)\right)\right]\Bigg\} \end{aligned}$$

Bringing all terms to a common denominator gives

$$sign\frac{\partial Q}{\partial x} = sign\left\{s_{B2}\ln s_{B2} + s_{B3}\ln s_{B3} - 2\left(\frac{s_{B2}+s_{B3}}{2}\right)\ln\left(\frac{s_{B2}+s_{B3}}{2}\right)\right\}\left[s_{B3}\frac{\partial s_{B2}}{\partial x} - s_{B2}\frac{\partial s_{B3}}{\partial x}\right]$$

As the term in curly brackets is equal to $\hat{E}(x) - E_i(x)$ and therefore positive (see above),

$$sign\frac{\partial Q}{\partial x} = sign\left[s_{B3}\frac{\partial s_{B2}}{\partial x} - s_{B2}\frac{\partial s_{B3}}{\partial x}\right]$$

Inserting $\frac{\partial s_{B2}}{\partial x} = \frac{p\beta_{B2} - s_{B2}\left(p\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon}\right)}{x\left(p\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon}\right)}$ and $\frac{\partial s_{B3}}{\partial x} = \frac{p\left(\bar{\beta}-\beta_{B2}\right) - s_{B3}\left(p\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon}\right)}{x\left(p\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon}\right)}$ (note that both derivatives are assumed to be positive here) and then $s_{B2}$ and $s_{B3}$, we get that

$$\begin{aligned} sign\frac{\partial Q}{\partial x} &= sign\left[\beta_{B2}s_{B3} - \left(\bar{\beta} - \beta_{B2}\right)s_{B2}\right] \\ &= sign\left\{\gamma_2\left[\left(1-\bar{\beta}\right)p^{\varepsilon}\left(2\beta_{B2} - \bar{\beta}\right) + p\left(\left(\beta_{B2}\right)^2 - \left(\bar{\beta}-\beta_{B2}\right)^2\right)\right]\right\} = sign\gamma_2 \end{aligned}$$

*ii*) When $\gamma_2 > 0$, $Q$ continuously rises in $x$. $Z > Q$ therefore holds for all values of $x$ if it holds for $x \to \infty$ ($Z$ does not depend on $x$). Inserting the corresponding budget shares into the expression of $Q$ and solving for the limit gives

$$\lim_{x\to\infty} Q = \frac{\ln\left(\frac{\bar{\beta}}{2}\right) - \ln\left(\bar{\beta} - \beta_{B2}\right)}{\ln\beta_{B2} - \ln\left(\frac{\bar{\beta}}{2}\right)}$$

As $Z$ continuously rises in $\gamma_2$ (using equation 14 it can be shown that $sign\frac{\partial Z}{\partial\gamma_2} = sign\left(2\beta_{B2} - \bar{\beta}\right)\left(\left(1-\bar{\beta}\right)p^{\varepsilon} + p\bar{\beta}\right) > 0$), $Z > Q$ therefore always holds if $Z|_{\gamma_2=0} = \frac{\beta_{B2}}{\bar{\beta}-\beta_{B2}} > \lim_{x\to\infty} Q$ holds. This inequality is satisfied if $\beta_{B2}\ln\beta_{B2} + \beta_{J2}\ln\beta_{J2} = E_i > 2\frac{\bar{\beta}}{2}\ln\left(\frac{\bar{\beta}}{2}\right) = \hat{E}$ holds. $E_i > \hat{E}$ holds if $\beta_{B2} > \frac{\bar{\beta}}{2}$ and $x > \underline{x}$ (see footnote 17). Consequently, $Z > Q$ and therefore $\frac{\partial\left(\hat{E}(x) - E_i(x)\right)}{\partial x} > 0$ hold when $\gamma_2 > 0$. When $\gamma_2 = 0$, $Q = \frac{\ln\left(\frac{\bar{\beta}}{2}\right) - \ln\left(\bar{\beta}-\beta_{B2}\right)}{\ln\beta_{B2} - \ln\left(\frac{\bar{\beta}}{2}\right)}$ holds independently of $x$, so that $\frac{\partial\left(\hat{E}(x) - E_i(x)\right)}{\partial x} > 0$ still holds for $x > \underline{x}$. At the point where $\gamma_2 = 0$ and $x = \underline{x}$, $s_{i2}(x) = s_{i3}(x)$ and therefore $E_i = \hat{E}$, implying that $\frac{\partial\left(\hat{E}(x) - E_i(x)\right)}{\partial x} = 0$.

*iii*) When $\gamma_2 < 0$, $Z > Q$ still holds when $x$ is sufficiently large (see part *ii*) of the proof), implying that $\frac{\partial\left(\hat{E}(x) - E_i(x)\right)}{\partial x} > 0$ still holds in this case. When $\gamma_2 < 0$ and $x$ approaches the lower bound $\underline{x}$, $s_{B3} = s_{J2}$ approaches zero (while $s_{B2} = s_{J3}$ remains positive), implying that $Q \equiv \frac{\ln\hat{s}_2 - \ln s_{B3}}{\ln s_{B2} - \ln\hat{s}_2}$ becomes infinitely large and that $Z < Q$ and therefore $\frac{\partial\left(\hat{E}(x) - E_i(x)\right)}{\partial x} < 0$ holds. As $Q$ continuously falls in $x$ when $\gamma_2 < 0$ holds (see part *i*) of the proof), $Z < Q$ and $\frac{\partial\left(\hat{E}(x) - E_i(x)\right)}{\partial x} < 0$ therefore holds in this case when $\underline{x} \le x < \tilde{x}$ and $Z > Q$ and $\frac{\partial\left(\hat{E}(x) - E_i(x)\right)}{\partial x} > 0$ when $\tilde{x} < x < \infty$, where $\tilde{x}$ ($\underline{x} < \tilde{x} < \infty$) is a positive parameter. □

## A3: Proof of Proposition 2

*Proof.* a) Let us define $x_i \equiv \tilde{x}_i + F_i$. Individual $i$ must be indifferent between only consuming good 1 and having income $x_i$ – which gives utility $U_i(1)$ –

and consuming all three goods and having income $x_i - F_i = \tilde{x}_i$ – which gives utility $U_i(3)$. Using equation 2, the condition $U_i(1) = U_i(3)$ implies that the following equation needs to hold:

$$\left(1-\bar{\beta}\right)^{\frac{1}{\varepsilon}}\left(\tilde{x}_i + F_i - \gamma_1\right)^{\frac{\varepsilon-1}{\varepsilon}} + \beta_{i2}^{\frac{1}{\varepsilon}}\left(-\gamma_2\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(\bar{\beta}-\beta_{i2}\right)^{\frac{1}{\varepsilon}}\left(-\gamma_2\right)^{\frac{\varepsilon-1}{\varepsilon}} =$$
$$\left(1-\bar{\beta}\right)^{\frac{1}{\varepsilon}}\left(q_{i1}(\tilde{x}_i) - \gamma_1\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(\beta_{i2}\right)^{\frac{1}{\varepsilon}}\left(q_{i2}(\tilde{x}_i) - \gamma_2\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(\bar{\beta}-\beta_{i2}\right)^{\frac{1}{\varepsilon}}\left(q_{i3}(\tilde{x}_i) - \gamma_2\right)^{\frac{\varepsilon-1}{\varepsilon}}$$

Subtracting the right hand side ($RHS \equiv U_i(3)^{\frac{\varepsilon-1}{\varepsilon}}$) from the left hand side ($LHS \equiv U_i(1)^{\frac{\varepsilon-1}{\varepsilon}}$) and defining $Q \equiv LHS - RHS$, we can implicitly differentiate this equation and obtain $\frac{dF_i}{d\beta_{i2}} = -\frac{\frac{\partial Q}{\partial \beta_{i2}}}{\frac{\partial Q}{\partial F_i}}$. We therefore analyze how $F_i$ depends on $\beta_{i2}$, taking $\tilde{x}_i$ as given (and $x_i$ to be variable), as this simplifies the analysis. This yields the same qualitative results as studying how $F_i = x_i - \tilde{x}_i$ depends on $\beta_{i2}$, taking $x_i$ as given.

We obtain $\frac{\partial Q}{\partial F_i} = \frac{\partial LHS}{\partial F_i} = \frac{\varepsilon-1}{\varepsilon}(1-\bar{\beta})^{\frac{1}{\epsilon}}(\tilde{x}_i + F_i - \gamma_1)^{-\frac{1}{\varepsilon}}$, so that $sign\frac{\partial Q}{\partial F_i} = sign\,(\varepsilon - 1)$ holds as the term $\tilde{x}_i + F_i - \gamma_1 = x_i - \gamma_1$ is positive when $x_i > \underline{x}$ holds.

In order to derive $\frac{\partial Q}{\partial \beta_{i2}}$, it is first shown that the right hand side (RHS) and therefore $U_i(3)$ is independent of $\beta_{i2}$:

$\frac{\partial RHS}{\partial \beta_{i2}} = \frac{1}{\varepsilon}\left[\left(\frac{q_{i2}(\tilde{x}_i)-\gamma_2}{\beta_{i2}}\right)^{\frac{\varepsilon-1}{\varepsilon}} - \left(\frac{q_{i3}(\tilde{x}_i)-\gamma_3}{\bar{\beta}-\beta_{i2}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right] +$
$\frac{\partial q_{i2}(\tilde{x}_i)}{\partial \beta_{i2}}\left(\frac{q_{i2}(\tilde{x}_i)-\gamma_2}{\beta_{i2}}\right)^{-\frac{1}{\varepsilon}} + \frac{\partial q_{i3}(\tilde{x}_i)}{\partial \beta_{i2}}\left(\frac{q_{i3}(\tilde{x}_i)-\gamma_2}{\bar{\beta}-\beta_{i2}}\right)^{-\frac{1}{\varepsilon}} = 0$

holds as the consumer's first order conditions imply that $\frac{q_{i2}(\tilde{x}_i)-\gamma_2}{\beta_{i2}} = \frac{q_{i3}(\tilde{x}_i)-\gamma_2}{\bar{\beta}-\beta_{i2}}$ holds and as $\frac{\partial q_{i2}(\tilde{x}_i)}{\partial \beta_{i2}} + \frac{\partial q_{i3}(\tilde{x}_i)}{\partial \beta_{i2}} = 0$ holds due to the assumption that $\beta_{i2} + \beta_{i3} = \bar{\beta}$ (the result that $\frac{\partial RHS}{\partial \beta_{i2}} = \frac{\partial U_i(3)}{\partial \beta_{i2}} = 0$ not only holds when $\gamma_2 = \gamma_3$, but also in the more general case where $\gamma_2 \neq \gamma_3$). Consequently, $\frac{\partial Q}{\partial \beta_{i2}} = \frac{\partial LHS}{\partial \beta_{i2}} = \frac{1}{\varepsilon}\left(-\gamma_2\right)^{\frac{\varepsilon-1}{\varepsilon}}\left[\left(\beta_{i2}\right)^{\frac{1-\varepsilon}{\varepsilon}} - \left(\bar{\beta}-\beta_{i2}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right]$ holds. When $\gamma_2 < 0$ and $\beta_{i2} > \frac{\bar{\beta}}{2}$, $\left(\beta_{i2}\right)^{\frac{1-\varepsilon}{\varepsilon}} - \left(\bar{\beta}-\beta_{i2}\right)^{\frac{1-\varepsilon}{\varepsilon}} > 0$ $(< 0)$ holds when $\varepsilon < 1$ $(\varepsilon > 1)$, implying that $sign\frac{\partial Q}{\partial \beta_{i2}} = sign\,(1-\varepsilon)$ holds. Assuming $\varepsilon \neq 1$, we therefore obtain:

$$\frac{dF_i}{d\beta_{i2}} = -\frac{\frac{\partial Q}{\partial \beta_{i2}}}{\frac{\partial Q}{\partial F_i}} = -\frac{\frac{\partial LHS}{\partial \beta_{i2}}}{\frac{\partial LHS}{\partial F_i}} = \frac{\left(-\gamma_2\right)^{\frac{\varepsilon-1}{\varepsilon}}\left[\left(\beta_{i2}\right)^{\frac{1-\varepsilon}{\varepsilon}} - \left(\bar{\beta}-\beta_{i2}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right]}{\left(1-\varepsilon\right)\left(1-\bar{\beta}\right)^{\frac{1}{\epsilon}}\left(\tilde{x}_i + F_i - \gamma_1\right)^{-\frac{1}{\varepsilon}}} > 0$$

Due to symmetry, also $sign\frac{dF_i}{d\beta_{i3}} > 0$ holds if $\beta_{i3} > \frac{\bar{\beta}}{2}$ (if $\beta_{ij} = \frac{\bar{\beta}}{2}$, $\frac{\partial F_i}{\partial \beta_{ij}} = 0$ holds). $F_i$ therefore increases in $\beta_{ij}$ when $\gamma_2 < 0$ and when $\beta_{ij} > \frac{\bar{\beta}}{2}$ holds.

This result can be generalized to the case where, absent innovation or trade, households not only have access to good 1, but to several goods of each of which they always consume positive quantities and where there is only preference heterogeneity with respect to the two newly introduced goods. The value of increasing variety by two goods is then measured by the income $F_i$ that households $i$ is willing to give up in order to have access to them. In such an extended setting, there are additional additive terms on the RHS that are independent of $\beta_{i2}$ and of $F_i$ and additional terms on the LHS that depend on $F_i$, but not on $\beta_{i2}$. Then, $\frac{\partial Q}{\partial \beta_{i2}} = \frac{\partial LHS}{\partial \beta_{i2}}$ stays the same as in the analysis above and $sign\frac{\partial Q}{\partial F_i} = sign\,(\varepsilon - 1)$ still holds, implying that $\frac{dF_i}{d\beta_{i2}} > 0$ still holds.

b) In order to determine the sign of $\frac{\partial^2 F_i}{\partial \beta_{i2}\partial \gamma_2}$, we again take $\tilde{x}_i$ as given (and $x_i$ to be variable), as this simplifies the analysis compared to the case where $x_i$ as given and leads to the same qualitative results. Taking into account that $\frac{dF_i}{d\beta_{i2}} = -\frac{\frac{\partial Q}{\partial \beta_{i2}}}{\frac{\partial Q}{\partial F_i}}$, that $\frac{\partial Q}{\partial F_i} = \frac{\varepsilon-1}{\varepsilon}(1-\bar{\beta})^{\frac{1}{\epsilon}}(\tilde{x}_i + F_i - \gamma_1)^{-\frac{1}{\varepsilon}}$ is independent of $\gamma_2$, and that $\frac{\partial^2 Q}{\partial \beta_{i2}\partial \gamma_2} = -\frac{(\varepsilon-1)}{\varepsilon^2}(-\gamma_2)^{-\frac{1}{\varepsilon}}\left[(\beta_{i2})^{\frac{1-\varepsilon}{\varepsilon}} - \left(\bar{\beta} - \beta_{i2}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right]$, we obtain (assuming $\varepsilon \neq 1$):

$$\frac{\partial^2 F_i}{\partial \beta_{i2}\partial \gamma_2} = -\frac{\frac{\partial^2 Q}{\partial \beta_{i2}\partial \gamma_2}}{\frac{\partial Q}{\partial F_i}} = \frac{(-\gamma_2)^{-\frac{1}{\varepsilon}}\left[(\beta_{i2})^{\frac{1-\varepsilon}{\varepsilon}} - \left(\bar{\beta} - \beta_{i2}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right]}{\varepsilon(1-\bar{\beta})^{\frac{1}{\epsilon}}(\tilde{x}_i + F_i - \gamma_1)^{-\frac{1}{\varepsilon}}}$$

As $sign\left[(\beta_{i2})^{\frac{1-\varepsilon}{\varepsilon}} - \left(\bar{\beta} - \beta_{i2}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right] = sign\,(1-\varepsilon)$ when $\beta_{i2} > \frac{\bar{\beta}}{2}$ holds and as $\tilde{x}_i + F_i - \gamma_1 = x_i - \gamma_1$ is positive when $x_i > \underline{x}$ holds, $sign\frac{\partial^2 F_i}{\partial \beta_{i2}\partial \gamma_2} = sign\,(1-\varepsilon)$ holds when $\gamma_2 < 0$ (and $sign\frac{\partial^2 F_i}{\partial \beta_{i2}\partial \gamma_2} = 0$ when $\beta_{i2} = \frac{\bar{\beta}}{2}$). As $\frac{\partial F_i}{\partial \beta_{i2}} = \frac{(-\gamma_2)^{\frac{\varepsilon-1}{\varepsilon}}\left[(\beta_{i2})^{\frac{1-\varepsilon}{\varepsilon}} - \left(\bar{\beta} - \beta_{i2}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right]}{(1-\varepsilon)(1-\bar{\beta})^{\frac{1}{\epsilon}}(\tilde{x}_i + F_i - \gamma_1)^{-\frac{1}{\varepsilon}}}$, $\lim_{\gamma_2 \to -0}\frac{\partial F_i}{\partial \beta_{i2}} = 0$ and $\lim_{\gamma_2 \to -\infty}\frac{\partial F_i}{\partial \beta_{i2}} = \infty$ hold when $\varepsilon > 1$ holds, while $\lim_{\gamma_2 \to -0}\frac{dF_i}{d\beta_{i2}} = \infty$ and $\lim_{\gamma_2 \to -\infty}\frac{\partial F_i}{\partial \beta_{i2}} = 0$ hold when $\varepsilon < 1$ holds. Due to symmetry, the same results apply when $\beta_{i2}$ is replaced by $\beta_{i3}$ and when $\beta_{i3} > \frac{\bar{\beta}}{2}$ holds.

c) Instead of directly calculating the sign of $\frac{\partial^2 D}{\partial \beta_{i2}\partial x_i}$, we simplify the analysis by making use of the fact that analyzing $\frac{F_i - F_a}{\tilde{x}_i}$, taking $\tilde{x}_i$ as given (and $x_i = \tilde{x}_i + F_i$ as endogenous) leads to the same qualitative results as analyzing $\frac{F_i - F_a}{x_i}$, taking $x_i$ as given (and $\tilde{x}_i = x_i - F_i$ as endogenous), i.e. that $sign\frac{\partial^2 D}{\partial x_i \partial \beta_{i2}} = sign\frac{\partial^2 \tilde{D}}{\partial \tilde{x}_i \partial \beta_{i2}}$, where $\tilde{D} \equiv \frac{F_i - F_a}{\tilde{x}_i}$. Taking into account that $F_a$ does not depend on $\beta_{i2}$, that $\frac{dF_i}{d\beta_{i2}} = -\frac{\frac{\partial Q}{\partial \beta_{i2}}}{\frac{\partial Q}{\partial F_i}}$, $\frac{\partial Q}{\partial F_i} = \frac{\varepsilon-1}{\varepsilon}(1-\bar{\beta})^{\frac{1}{\epsilon}}(\tilde{x}_i + F_i - \gamma_1)^{-\frac{1}{\varepsilon}}$, and that $\frac{\partial^2 Q}{\partial \beta_{i2}\partial \tilde{x}_i} = 0$, we obtain

$$\frac{\partial^2\left(\frac{F_i - F_a}{\tilde{x}_i}\right)}{\partial \beta_{i2}\partial \tilde{x}_i} = \frac{\frac{\partial^2 F_i}{\partial \beta_{i2}\partial \tilde{x}_i}\tilde{x}_i - \frac{\partial F_i}{\partial \beta_{i2}}}{\tilde{x}_i^2} = \frac{1}{\tilde{x}_i}\frac{\frac{\partial Q}{\partial \beta_{i2}}}{\frac{\partial Q}{\partial F_i}}\left[1 + \frac{\frac{\partial^2 Q}{\partial F_i \partial \tilde{x}_i}}{\frac{\partial Q}{\partial F_i}}\right] = \frac{1}{\tilde{x}_i}\frac{\frac{\partial Q}{\partial \beta_{i2}}}{\frac{\partial Q}{\partial F_i}}\left[1 - \frac{1}{\varepsilon\,(\tilde{x}_i + F_i - \gamma_1)}\right]$$

The term $\tilde{x}_i + F_i - \gamma_1 = x_i - \gamma_1$ is positive when $x_i > \underline{x}$ holds. As $sign\frac{\partial Q}{\partial \beta_{i2}} = sign\,(1-\varepsilon)$ holds if $\beta_{i2} > \frac{\bar{\beta}}{2}$ and if $\gamma_2 < 0$ hold, and as $sign\frac{\partial Q}{\partial F_i} = sign\,(\varepsilon - 1)$, we therefore obtain that

$$sign\frac{\partial^2\left(\frac{F_i - F_a}{\tilde{x}_i}\right)}{\partial \beta_{i2}\partial \tilde{x}_i} = sign\left[\frac{1}{\varepsilon} - (\tilde{x}_i + F_i - \gamma_1)\right] = \begin{cases} > 0 & if \;\; x_i < \gamma_1 + \frac{1}{\varepsilon} \\ < 0 & if \;\; x_i > \gamma_1 + \frac{1}{\varepsilon} \end{cases}$$

Using the fact that $sign\frac{\partial^2 D}{\partial x_i \partial \beta_{i2}} = sign\frac{\partial^2 \tilde{D}}{\partial \tilde{x}_i \partial \beta_{i2}}$ then gives the result that $\frac{\partial^2 D}{\partial x_i \partial \beta_{i2}} > 0$ when $\underline{x} \leq x_i < \gamma_1 + \frac{1}{\varepsilon}$ holds (Case 1) and that $\frac{\partial^2 D}{\partial x_i \partial \beta_{i2}} < 0$ holds if $x_i > \gamma_1 + \frac{1}{\varepsilon}$

(Case 2). Due to symmetry, the same results apply when $\beta_{i2}$ is replaced by $\beta_{i3}$ and when $\beta_{i3} > \frac{\bar{\beta}}{2}$ holds.

As $\underline{x} = \gamma_1 + 2p\gamma_2 - \gamma_2 \left[\frac{(1-\bar{\beta})p^{\varepsilon} + \bar{\beta}p}{\bar{\beta} - \beta_{12}}\right]$ when $\gamma_2 = \gamma_3 < 0$ and $\beta_{12} = \beta_{23} > \frac{\bar{\beta}}{2}$ (see the proof of Lemma 1), there is a non-empty parameter range $\underline{x} \leq x_i < \gamma_1 + \frac{1}{\varepsilon}$ for which Case 1 results if $\gamma_2 > -\frac{(\bar{\beta} - \beta_{12})}{\varepsilon\left((1-\bar{\beta})p^{\varepsilon} + 2p\left(\beta_{12} - \frac{\bar{\beta}}{2}\right)\right)}$ holds. $\square$