Quantitative Models of Commercial Policy*

Ralph Ossa†

University of Chicago and NBER

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Abstract

What tariffs would countries impose if they did not have to fear any retaliation? What would occur if there was a complete breakdown of trade policy cooperation? What would be the outcome if countries engaged in fully efficient trade negotiations? And what would happen to trade policy cooperation if the world trading system had a different institutional design? While such questions feature prominently in the theoretical trade policy literature, they have proven difficult to address empirically, because they refer to what-if scenarios for which direct empirical counterparts are hard to find. In this chapter, I introduce research which suggests overcoming this difficulty by applying quantitative models of commercial policy.

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†University of Chicago, Booth School of Business, 5807 South Woodlawn Avenue, Chicago, IL 60637, United States; ralph.ossa@chicagobooth.edu.

1 Introduction

At least since Johnson's (1953-54) pioneering analysis of optimal tariffs and retaliation, what-if questions regarding potential policy scenarios have dominated the theoretical trade policy literature: What tariffs would countries impose if they did not have to fear any retaliation? What would occur if there was a complete breakdown of trade policy cooperation? What would be the outcome if countries engaged in fully efficient trade negotiations? And what would happen to trade policy cooperation if the world trading system had a different institutional design?

In this chapter, I introduce research which takes this theoretical literature to the data using quantitative models of commercial policy. Quantitative models are a natural tool for empirical work in this area because they are designed for counterfactual analyses. As a result, they can shed light on the what-if scenarios emphasized in the theoretical literature without necessarily requiring historical precedents. This is especially useful for key benchmark scenarios such as fully escalated trade wars or fully efficient trade talks for which direct empirical counterparts are hard to find.

My particular focus is on quantitative research which assumes that trade policy choices are made by optimizing governments. While this is a standard assumption in the theoretical literature, it has long been avoided in quantitative applications because of the unique methodological challenges it brings about. As a result, little was known about the magnitudes of optimal tariffs, the potential welfare costs of a breakdown of trade policy cooperation, or the potential welfare gains which can be achieved in future trade negotiations. Instead, quantitative analyses mainly focused on comparative statics exercises such as predicting the effects of particular trade agreements.

While this is still very much an emerging literature, I believe a separate introduction is valuable to have since the required tools go beyond what is commonly used in the quantitative trade literature. Moreover, excellent accounts of the broader quantitative trade literature are already easy to obtain. In particular, a thorough introduction to traditional Quantitative General Equilibrium (CGE) models is provided in the Handbook of Computable General Equilibrium Modeling edited by Dixon and Jorgenson (2013). Moreover, a comprehensive review of the more recent quantitative gravity literature building on the work of Eaton and Kortum (2002) is available from the Handbook of International Economics chapter by Costinot and Rodriguez-Clare (2014).[1]

My goal is to equip readers who are interested in this area with the knowledge required to expand its frontier. Assuming no background in quantitative modeling, I provide an in-depth discussion of the key tools, the key findings, and the key limitations of the literature so far. An integral part of this chapter is a programming toolkit which is available from the accompanying website. It contains a set of fully documented MATLAB programs which can be used to efficiently compute counterfactuals, optimal tariffs, Nash tariffs, and cooperative tariffs. While they are tailored to the workhorse model used in this chapter, they can be easily modified to apply to other environments.

I structure my explanations around my analysis in Ossa (2014) which is the most comprehensive one available to date. However, I go beyond it by elaborating more extensively on the underlying methods, including the "exact hat algebra" technique of Dekle et al (2007), the elasticity estimations of Feenstra (1994), Broda and Weinstein (2006), and Caliendo and Parro (2015), and the mathematical programming with equilibrium constraints (MPEC) algorithm of Su and Judd (2012). The central theme throughout this chapter is numerical optimization which differentiates it from the abovementioned contribution of Costinot and Rodriguez-Clare (2014). As will become clear shortly, this is a critical theme for this literature as challenges associated with it have forced earlier studies to confine attention to low dimensional setups with only a few countries and industries.

To the best of my knowledge, there are only five papers other than Ossa (2014) which have seriously attempted to quantify trade policy equilibria featuring optimizing governments. Hamilton and Whalley (1983) were the first to attempt a serious calibration of optimal trade policy. Employing simple CES specifications on the demand and the supply side of the economy, they explore optimal tariffs with and without retaliation in a range of simple two region, one import good models. They conclude that optimal tariffs are far away from the tariffs observed in the data and that the margin for tariff retaliation in a worldwide tariff war

[1] See also Spearot (forthcoming) and Caliendo et al (2015) for two more recent quantitative analyses of trade policy counterfactuals.

is potentially large.

Markusen and Wigle (1989) explore Nash equilibrium tariffs and their welfare effects in a much richer numerical general equilibrium framework featuring scale economies and capital mobility. While their framework allows for eight regions and six industries, they only consider a tariff war between the US and Canada further constraining the tariffs to vary proportionately across all industries. They find that the Nash equilibrium tariffs are much lower than the ones computed by Hamilton and Whalley (1983) and conjecture that this is due to scale economies and capital mobility.

Perroni and Whalley (2000), Ossa (2011), and Ossa (2012) calculate optimal tariffs with and without retaliation in a quantitative Armington model, a quantitative Krugman (1980) model with free entry, and a quantitative Krugman (1980) model without free entry, respectively. They now allow for seven instead of two regions but still assume that each country imposes a single tariff against all imports from a given trading partner. While Perroni and Whalley (2000) are particularly interested in the potential effect of regional trade agreements on Nash tariffs, Ossa's (2011, 2012) calculations are part of a broader attempt to explore optimal trade policy in "new trade" environments.[2]

The quantitative approach discussed in this chapter is more closely related to the quantitative gravity literature pioneered by Eaton and Kortum (2002) than the traditional Computable General Equilibrium (CGE) literature. As argued by Costinot and Rodriguez-Clare (2014), this newer literature distinguishes itself by having more appealing micro-theoretical foundations, offering a tighter connection between theory and data, and prioritizing transparency over realism.

While this is true, it is also important to recognize that the differences between newer and older quantitative trade models are often overemphasized. For example, Eaton and Kortum (2002) indeed develop a full-fledged Ricardian model which has more appealing micro-

[2] Rudimentary quantitative analyses can already be found in the early formal trade policy literature. Johnson (1953-54) numerically calculated demand elasticity combinations for which a country gains or loses in a tariff war. His analysis was subsequently extended by Gorman (1958) to a broader class of models, by Kuga (1973) to many countries and industries, and by Kennan and Riezman (1990) to allow for customs unions. While numerical calculations feature prominently in all of these papers, they were clearly meant to be numerical illustrations rather than serious calibration exercises. For completeness, let me also mention that Baldwin and Clarke (1987) calculate various equilibria of a simple two-country tariff game which is meant to capture some of the salient features of the Tokyo Round and that Alvarez and Lucas (2007) provide a short discussion of optimal tariffs in small open economies.

theoretical foundations than the ad-hoc Armington (1969) model the CGE literature typically relies on. Yet, we now know from the work of Arkolakis et al (2012) that these models are actually isomorphic in terms of their quantitative predictions so that these micro-theoretical differences matter much less than it originally seemed.

Also, some of the new techniques of connecting theory to data have close counterparts in the earlier CGE literature. For example, the "exact hat algebra" approach of Dekle et al (2007) closely resembles a standard method in the CGE literature of expressing equilibrium conditions in "calibrated share form". In light of this, the main difference between newer and older quantitative trade models seems to be that important model parameters such as trade elasticities are now usually estimated using the same model relationships that are then also used for counterfactual analyses instead of just taken from existing studies in the literature.

In that sense, the newer quantitative trade literature comes somewhat closer to full-fledged structural estimation than the earlier CGE literature did even though it is still best described as "theory with numbers" in my view. This means that its quantitative findings so far should not be taken at face value but rather as offering a sense of the magnitudes. Of course, this is not only interesting in its own right but can also provide valuable insights into the plausibility of the underlying theory. For example, we will see that the trade war equilibrium predicted by the benchmark model of this chapter seems broadly consistent with the observed trade war following the Great Depression which is an encouraging result.

The remainder of this chapter is divided into three sections. The first section introduces the main methods, including the theoretical framework of Ossa (2014), the "exact hat algebra" technique of Dekle et al (2007), the elasticity estimations of Feenstra (1994), Broda and Weinstein (2006), and Caliendo and Parro (2015), ways to deal with aggregate trade imbalances, and the MPEC algorithm of Su and Judd (2012). The second section illustrates these methods in an application to 10 countries and 33 industries calculating optimal tariffs, Nash tariffs, and efficient tariffs. The last section considers a number of extensions which have been analyzed in the literature and discusses ideas for future work.

2 Methods

2.1 Theory

In this section, I introduce the model of Ossa (2014) which I use as a workhorse model throughout. As will become clear shortly, it nests many of the forces emphasized in the theoretical literature, which makes it a natural starting point for quantitative trade policy work. Having said this, none of the methods discussed in this chapter are specific to this model and could be applied readily to any of the other gravity models surveyed in Costinot and Rodriguez-Clare (2015). I will further elaborate on this in the following and also point to some specific alternative models which seem particularly interesting to me.

2.1.1 Setup

There are N countries indexed mainly by i or j and S industries indexed mainly by s. Households have access to a continuum of differentiated varieties and make their consumption decisions according to the following nested Cobb-Douglas-CES utility functions:

$$C_j = \prod_{s=1}^{S} \left(\sum_{i=1}^{N} \int_0^{M_{is}} c_{ijs} \left(\omega_{is}\right)^{\frac{\sigma_s - 1}{\sigma_s}} d\omega_{is} \right)^{\frac{\sigma_s}{\sigma_s - 1}\mu_{js}} \tag{1}$$

where c_{ijs} denotes consumption of an industry s variety from country i in country j, M_{is} is the mass of industry s varieties produced in country i, $\sigma_s > 1$ is the elasticity of substitution between industry s varieties, and μ_{js} is the share of expenditure country j households spend on industry s varieties. Households collect all labor income, profits, and tariff revenue generated in the economy and there are a total of L_i workers residing in country i.

Each consumption variety is produced by a single monopolistic firm. Firms hire labor only, produce output using constant returns to scale technologies, and incur iceberg shipping costs. Their technologies are summarized by the following inverse production functions:

$$l_{is} = \sum_{j=1}^{N} \frac{\theta_{ijs} c_{ijs}}{\varphi_{is}} \tag{2}$$

where l_{is} is the amount of labor hired by an industry s firm in country i, φ_{is} is the productivity of industry s firms in country i, and $\theta_{ijs} > 1$ is an iceberg trade barrier in the sense that θ_{ijs} units of an industry s variety have to be shipped out of country i for one unit to arrive in country j. There are no fixed costs of production and the mass of firms is exogenous everywhere.

Governments impose import tariffs but do not have access to other policy instruments. I denote the ad valorem tariff imposed by country j against industry s imports from country i by t_{ijs}, where $t_{ijs} \geq 0$ for all $i \neq j$ and $t_{ijs} = 0$ for all $i = j$, and define the shorthand $\tau_{ijs} \equiv 1 + t_{ijs}$ for future use. Government preferences are given by:

$$G_j = \sum\nolimits_{s=1}^{S} \lambda_{js} W_{js} \tag{3}$$

where W_{js} is the welfare of industry s in country j and λ_{js} is a political economy weight. I will elaborate on the details of this specification in the application section and for now only consider the special case of welfare maximizing governments which arises if $\lambda_{js} = 1$ for all j and s which then implies $G_j = C_j$.[3]

Notice that this model is a hybrid between a multi-sector Krugman (1980) model and a multi-sector Armington (1969) model. Unlike a standard Krugman (1980) model, it abstracts from fixed costs of production and does not allow for free entry. Unlike a standard Armington (1969) model, it features imperfect competition and products which are differentiated at the level of firms. These modifications ensure that there are no corner solutions and that there is more to trade policy than just terms-of-trade effects. While this is useful in practice, it is in no way critical for the applicability of the methods discussed in this chapter which can be used for any gravity model as indicated before.[4]

[3] As will become clear in the application section, industry welfare is simply defined as nominal industry income deflated by the ideal aggregate price index which then sums to total real income or total welfare if $\lambda_{js} = 1$ for all j and s.

[4] Readers familiar with the work of Arkolakis et al (2012) will know that these models anyway all have the same predictions in the special case of one industry. However, as soon as there is more than one industry, differences between these models emerge in the sense that "new trade" production relocation or profit shifting effects appear in addition to traditional terms-of-trade effects in imperfectly competitive environments.

2.1.2 Equilibrium in levels

Utility maximization implies that firms in industry s of country i face demands:

$$c_{ijs} = \frac{(p_{is}\theta_{ijs}\tau_{ijs})^{-\sigma_s}}{P_{js}^{1-\sigma_s}}\mu_{js}E_j \tag{4}$$

where p_{is} is the ex-factory price set by industry s firms in country i, P_{js} is the ideal price index of industry s varieties in country j, and E_j is the total expenditure of consumers in country j.

Profit maximization requires that firms charge a constant markup over marginal costs:

$$p_{is} = \frac{\sigma_s}{\sigma_s - 1}\frac{w_i}{\varphi_{is}} \tag{5}$$

where w_i is the wage rate in country i. This also implies that profits account for a fraction $\frac{1}{\sigma_s}$ of industry s revenues, as is easy to verify by substituting equations (2), (4), and (5) into the definition of industry profits $\pi_{is} = M_{is}\left(\sum_{j=1}^{N} p_{is}\theta_{ijs}c_{ijs} - w_i l_{is}\right)$.

The ideal industry price indices are given by $P_{js} = \left(\sum_{i=1}^{N}(p_{is}\theta_{ijs}\tau_{ijs})^{1-\sigma_s}\right)^{\frac{1}{1-\sigma_s}}$ so that:

$$P_{js} = \left(\sum_{i=1}^{N} M_{is}\left(\frac{\sigma_s}{\sigma_s - 1}\frac{w_i}{\varphi_{is}}\theta_{ijs}\tau_{ijs}\right)^{1-\sigma_s}\right)^{\frac{1}{1-\sigma_s}} \tag{6}$$

together with equation (5). They combine to the ideal aggregate price indices in a Cobb-Douglas fashion implying $P_j = \prod_{s=1}^{S}\left(\frac{P_{js}}{\mu_{js}}\right)^{\mu_{js}}$. For future reference, recall that the ideal aggregate price indices are unit expenditure functions so that $C_j = \frac{E_j}{P_j}$.

Defining $X_{ijs} = M_{is}p_{is}\theta_{ijs}c_{ijs}$ as the value of trade flowing from country i to country j in industry s evaluated at ex-factory prices, equation (4) and (5) imply:

$$X_{ijs} = M_{is}(\tau_{ijs})^{-\sigma_s}\left(\frac{\sigma_s}{\sigma_s - 1}\frac{w_i}{\varphi_{is}}\theta_{ijs}\right)^{1-\sigma_s}(P_{js})^{\sigma_s - 1}\mu_{js}E_j \tag{7}$$

This is, of course, just a standard gravity equation decomposing bilateral trade flows into bilateral trade costs as well as origin and destination effects. Notice that the elasticity of trade with respect to tariffs is different from the elasticity of trade with respect to iceberg trade costs. This is simply because the considered trade flows are evaluated at ex-factory prices which are net of tariffs but gross of iceberg trade costs.

Recall that all labor incomes, profits, and tariff revenues ultimately accrue to households which can be captured by the budget constraint:

$$E_i = \sum_{n=1}^{N}\sum_{s=1}^{S} X_{ins} + \sum_{m=1}^{N}\sum_{s=1}^{S} t_{mis} X_{mis} - \Omega_i \tag{8}$$

Notice that $\sum_{n=1}^{N}\sum_{s=1}^{S} X_{ins}$ is the sum of labor incomes and profits and $\sum_{m=1}^{N}\sum_{s=1}^{S} t_{mis} X_{mis}$ are tariff revenues. Ω_i are exogenous international transfers satisfying $\sum_{i=1}^{N}\Omega_i = 0$ which will prove useful later on.

Since a fraction $\frac{1}{\sigma_s}$ of revenues is distributed as profits, the remaining fraction $1-\frac{1}{\sigma_s}$ is distributed as labor income. As a result:

$$w_i L_i = \sum_{n=1}^{N}\sum_{s=1}^{S}\left(1-\frac{1}{\sigma_s}\right) X_{ins} \tag{9}$$

This can also be interpreted as a labor market clearing condition since it reduces to $L_i = \sum_{s=1}^{S} M_{is} l_{is}$ after substituting equations (2), (4), (5), and (7).

For given tariffs, conditions (7) - (9) jointly determine the equilibrium of the model. For future reference, it is useful to summarize this in Definition 1:

Definition 1 *For given tariffs, an equilibrium can be defined as a set of* $\{E_i, w_i\}$ *such that*

$$E_i = \sum_{n=1}^{N}\sum_{s=1}^{S} X_{ins} + \sum_{m=1}^{N}\sum_{s=1}^{S} t_{mis} X_{mis} - \Omega_i$$

$$w_i L_i = \sum_{n=1}^{N}\sum_{s=1}^{S}\left(1-\frac{1}{\sigma_s}\right) X_{ins}$$

where

$$X_{ijs} = M_{is}\left(\tau_{ijs}\right)^{-\sigma_s}\left(\frac{\sigma_s}{\sigma_s-1}\frac{w_i}{\varphi_{is}}\theta_{ijs}\right)^{1-\sigma_s}\left(P_{js}\right)^{\sigma_s-1}\mu_{js}E_j$$

$$P_{is} = \left(\sum_{m=1}^{N} M_{ms}\left(\frac{\sigma_s}{\sigma_s-1}\frac{w_m}{\varphi_{ms}}\theta_{mis}\tau_{mis}\right)^{1-\sigma_s}\right)^{\frac{1}{1-\sigma_s}}$$

Notice that the equilibrium for given tariffs could be summarized in a very similar fashion in other gravity models. For example, a multi-sector Armington (1969) model would feature the same conditions with the exceptions that there would be no markups and profits so that the respective terms would be dropped from the second, third, and fourth equation of Definition 1. Similar arguments can be made for other well-known frameworks including Eaton and Kortum (2002), Krugman (1980) with free entry, and Melitz (2003) with Pareto distributed productivity draws.

2.1.3 Equilibrium in changes

In principle, it would be possible to take the equilibrium as summarized in Definition 1 to the data by calibrating the structural parameters $\{L_i, M_{is}, \varphi_{is}, \theta_{ijs}\}$ as well as the elasticities σ_s. However, these structural parameters are hard to recover in practice which makes Definition 1 a challenging starting point for quantitative work. Notice that calibrating μ_{js} would not be a problem since it is just an expenditure share: $\mu_{js} = \frac{\sum_{i=1}^{N} \tau_{ijs} X_{ijs}}{\sum_{m=1}^{N} \sum_{t=1}^{S} \tau_{mjt} X_{mjt}}$. As will become clear later, Ω_i would also be easy to recover since it corresponds to aggregate trade surpluses here: $\Omega_i = \sum_{j=1}^{N} \sum_{s=1}^{S} (X_{ijs} - X_{jis})$.

A technique which has come to be known as "exact hat algebra" in the literature circumvents this identification problem in an elegant way. It is usually attributed to Dekle et al (2007) but a version of it is also frequently applied in the traditional computable general equilibrium literature where researchers refer to it as expressing equations in "calibrated share form".[5] I will illustrate this technique step-by-step in the following using the equilibrium conditions from Definition 1.

The basic idea is to perform a quantitative comparative statics exercise taking some observed equilibrium of the world economy as a starting point. In the application, I will later focus on a case with 10 regions and 33 industries in the year 2007 but any equilibrium for which sufficient data is available will do. The comparative statics exercise can, in principle, be conducted with respect to any exogenous variable but I will focus on changes in tariffs and international side payments and assume that all other exogenous variables from Definition 1 remain unchanged.

[5] See, for example, the note prepared by Rutherford (1995) which can be viewed under http://www.gams.com/solvers/mpsge/cesfun.htm.

Denote the factual (i.e. observed) values of tariffs and side payments by $\{t_{ijs}, \tau_{ijs}, \Omega_i\}$ and consider the effects of changing them to some counterfactual (i.e. different from observed) values $\left\{t'_{ijs}, \tau'_{ijs}, \Omega'_i\right\}$ holding all other exogenous variables from Definition 1 unchanged. Clearly, changing tariffs and side payments also changes all endogenous variables from Definition 1 to some counterfactual values $\left\{E'_i, w'_i, P'_{is}, X'_{ijs}\right\}$ but all equilibrium conditions from Definition 1 must continue to hold.

Applying this logic to the gravity equation means that it comes in a factual and counterfactual version. Denoting proportional changes of variables with a "hat", $\hat{x} = \frac{x'}{x}$, the trick is now to simply divide one by the other yielding:

$$\begin{aligned} X_{ijs} &= M_{is} \left(\tau_{ijs}\right)^{-\sigma_s} \left(\frac{\sigma_s}{\sigma_s - 1} \frac{w_i}{\varphi_{is}} \theta_{ijs}\right)^{1-\sigma_s} \left(P_{js}\right)^{\sigma_s - 1} \mu_{js} E_j \\ X'_{ijs} &= M_{is} \left(\tau'_{ijs}\right)^{-\sigma_s} \left(\frac{\sigma_s}{\sigma_s - 1} \frac{w'_i}{\varphi_{is}} \theta_{ijs}\right)^{1-\sigma_s} \left(P'_{js}\right)^{\sigma_s - 1} \mu_{js} E'_j \end{aligned}$$

$$\Rightarrow \hat{X}_{ijs} = \left(\hat{\tau}_{ijs}\right)^{-\sigma_s} \left(\hat{w}_i\right)^{1-\sigma_s} \left(\hat{P}_{js}\right)^{\sigma_s - 1} \hat{E}_j \tag{10}$$

Notice that all parameters that enter multiplicatively simply cancel because they take on the same values before and after the change. This eliminates much of the original complication because it is then no longer necessary to estimate them.

A slightly extended version of this approach needs to be taken when transforming the other equations because they also include additive terms. The basic idea is to express these terms as a weighted sum of proportional changes, where the weights have some factual empirical counterpart. For example, if the factual and counterfactual equations were $x = y + z$ and $x' = y' + z'$, one would divide one by the other generating $\hat{x} = \frac{y}{x}\hat{y} + \frac{z}{x}\hat{z}$ and transform the weights $\frac{y}{x}$ and $\frac{z}{x}$ until they can be measured somehow.

This basic idea can be applied directly to the budget constraint from Definition 1. Defining $\beta_{ijs} = \frac{X_{ijs}}{E_i}$ as the sales of country i to country j in industry s as a share of the total expenditure of country i and $\gamma_{ijs} = \frac{X_{ijs}}{E_j}$ as the sales of country i to country j in industry s as a share of the total expenditure of country j, it should be straightforward to verify that it can be written in changes as follows:

$$\begin{aligned} E_i &= \sum_{n=1}^{N}\sum_{s=1}^{S} X_{ins} + \sum_{m=1}^{N}\sum_{s=1}^{S} t_{mis} X_{mis} - \Omega_i \\ E'_i &= \sum_{n=1}^{N}\sum_{s=1}^{S} X'_{ins} + \sum_{m=1}^{N}\sum_{s=1}^{S} t'_{mis} X'_{mis} - \Omega'_i \end{aligned}$$

$$\Rightarrow \hat{E}_i = \sum_{n=1}^{N}\sum_{s=1}^{S} \beta_{ins}\hat{X}_{ins} + \sum_{m=1}^{N}\sum_{s=1}^{S} t'_{mis}\gamma_{mis}\hat{X}_{mis} - \frac{\Omega'_i}{E_i} \tag{11}$$

$E_j = \sum_i \tau_{ijs} X_{ijs}$ by definition so that β_{ijs} and γ_{ijs} can be easily computed with factual data on bilateral tariffs and trade. I have avoided expressing t_{ijs} and Ω_i in changes to allow for special cases featuring zero factual tariffs or transfers.

An equally simple transformation can be applied to the labor income equation from Definition 1. Defining $\delta_{ijs} = \frac{\left(1-\frac{1}{\sigma_s}\right)X_{ijs}}{w_i L_i}$, one obtains:

$$\begin{aligned} w_i L_i &= \sum_{n=1}^{N}\sum_{s=1}^{S}\left(1-\frac{1}{\sigma_s}\right) X_{ins} \\ w'_i L_i &= \sum_{n=1}^{N}\sum_{s=1}^{S}\left(1-\frac{1}{\sigma_s}\right) X'_{ins} \end{aligned}$$

$$\Rightarrow \hat{w}_i = \sum_{n=1}^{N}\sum_{s=1}^{S} \delta_{ins}\hat{X}_{ins} \tag{12}$$

δ_{ijs} is also straightforward to recover from trade data given some elasticity estimates since a share $1-\frac{1}{\sigma_s}$ of revenues accrue to workers so that $w_i L_i = \sum_n \sum_s \left(1-\frac{1}{\sigma_s}\right) X_{ins}$.

Defining $\alpha_{ijs} = \frac{\tau_{ijs} X_{ijs}}{\sum_m \tau_{mjs} X_{mjs}}$ as the expenditure of country j consumers on industry s varieties from country i as a share of the expenditure of country j consumers on industry s varieties overall, the price index equation from Definition 1 can be manipulated as follows:

$$\begin{aligned} P_{is} &= \left(\sum_{m=1}^{N} M_{ms}\left(\frac{\sigma_s}{\sigma_s-1}\frac{w_m}{\varphi_{ms}}\theta_{mis}\tau_{mis}\right)^{1-\sigma_s}\right)^{\frac{1}{1-\sigma_s}} \\ P'_{is} &= \left(\sum_{m=1}^{N} M_{ms}\left(\frac{\sigma_s}{\sigma_s-1}\frac{w'_m}{\varphi_{ms}}\theta_{mis}\tau'_{mis}\right)^{1-\sigma_s}\right)^{\frac{1}{1-\sigma_s}} \end{aligned}$$

$$\Rightarrow \quad \frac{P'_{is}}{P_{is}} = \left(\sum_{m=1}^{N} \frac{M_{ms}\left(\frac{\sigma_s}{\sigma_s-1}\frac{w_m}{\varphi_{ms}}\theta_{mis}\tau_{mis}\right)^{1-\sigma_s}}{\sum_{k=1}^{N} M_{ks}\left(\frac{\sigma_s}{\sigma_s-1}\frac{w_k}{\varphi_{ks}}\theta_{kis}\tau_{kis}\right)^{1-\sigma_s}} \frac{M_{ms}\left(\frac{\sigma_s}{\sigma_s-1}\frac{w'_m}{\varphi_{ms}}\theta_{mis}\tau'_{mis}\right)^{1-\sigma_s}}{M_{ms}\left(\frac{\sigma_s}{\sigma_s-1}\frac{w_m}{\varphi_{ms}}\theta_{mis}\tau_{mis}\right)^{1-\sigma_s}} \right)^{\frac{1}{1-\sigma_s}} \tag{13}$$

$$\Leftrightarrow \quad \hat{P}_{is} = \left(\sum_{m=1}^{N} \alpha_{mis} \left(\hat{w}_m \hat{\tau}_{mis}\right)^{1-\sigma_s} \right)^{\frac{1}{1-\sigma_s}}$$

The last step follows from substituting the gravity equation into the above definition of α_{ijs}, as should be straightforward to verify. Intuitively, this says that price index changes are expenditure share weighted averages of changes in prices, which in turn are driven by changes in wages and tariffs in this environment.

Just like equations (6) - (9) can be used to solve for the equilibrium in levels, equations (10) - (13) can be used to solve for the equilibrium in changes, which is useful to summarize in Definition 2:

Definition 2 *For given tariff changes, an equilibrium is a set of* $\left\{\hat{E}_i, \hat{w}_i\right\}$ *such that*

$$\hat{E}_i = \sum_{n=1}^{N}\sum_{s=1}^{S} \beta_{ins}\hat{X}_{ins} + \sum_{m=1}^{N}\sum_{s=1}^{S} \gamma_{mis} t'_{mis}\hat{X}_{mis} - \frac{\Omega'_i}{E_i}$$

$$\hat{w}_i = \sum_{n=1}^{N}\sum_{s=1}^{S} \delta_{ins}\hat{X}_{ins}$$

where

$$\hat{X}_{ijs} = \left(\hat{\tau}_{ijs}\right)^{-\sigma_s} \left(\hat{w}_i\right)^{1-\sigma_s} \left(\hat{P}_{js}\right)^{\sigma_s-1} \hat{E}_j$$

$$\hat{P}_{is} = \left(\sum_{m=1}^{N} \alpha_{mis} \left(\hat{w}_m \hat{\tau}_{mis}\right)^{1-\sigma_s} \right)^{\frac{1}{1-\sigma_s}}$$

and

$$\begin{aligned}
\alpha_{ijs} &= \frac{\tau_{ijs}X_{ijs}}{\sum_{m=1}^{N}\tau_{mjs}X_{mjs}} \\
\beta_{ijs} &= \frac{X_{ijs}}{E_i} \\
\gamma_{ijs} &= \frac{X_{ijs}}{E_j} \\
\delta_{ijs} &= \frac{\left(1-\frac{1}{\sigma_s}\right)X_{ijs}}{w_i L_i}
\end{aligned}$$

While the equilibrium formulation in Definition 2 therefore circumvents the need to explicitly estimate $\{L_i, M_{is}, \varphi_{is}, \theta_{ijs}\}$, it also ensures that the counterfactual effects of changes in tariffs and international transfers can be computed from a reference point which perfectly matches industry-level trade flows and tariffs. Essentially, it imposes a restriction on the set of unknown parameters $\{L_i, M_{is}, \varphi_{is}, \theta_{ijs}\}$ such that the predicted X_{ijs} perfectly match the observed X_{ijs} for given τ_{ijs} and σ_s. Recall that $\left\{\alpha_{ijs}, \beta_{ijs}, \gamma_{ijs}, \delta_{ijs}\right\}$ can all be expressed as simple functions of X_{ijs}, τ_{ijs}, and σ_s.

Notice that this procedure does not deliver any estimates of $\{L_i, M_{is}, \varphi_{is}, \theta_{ijs}\}$ simply because there are too many degrees of freedom. For example, the iceberg trade costs alone could be adjusted to perfectly match any pattern of industry-level trade. Of course, this also means that many different gravity models could be perfectly matched to the same trade data using "exact hat algebra" techniques. As a result, there is a real issue of how to discriminate between different gravity models, which I discuss more extensively below.[6]

As should now almost go without saying, the equilibrium for given tariff changes could be summarized in very similar ways in other gravity models. For example, removing the term $\left(1 - \frac{1}{\sigma_s}\right)$ from the definition of δ_{ijs} is all it takes to instead implement a multi-sector Armington (1969) model since the markups cancel from the third and fourth equations of Definition 1 as a result of applying the "exact hat algebra" technique. Of course, small differences in the equations can cause large differences in the outcomes so that this does not mean that those differences have to be economically irrelevant.

2.1.4 First-order conditions

The "exact hat algebra" approach is not only a useful tool to calculate counterfactual tariff changes but can also be used to go one step further and characterize which of those tariff changes is chosen by optimizing governments. I now illustrate this point using optimal tariffs as an example but the analysis can be readily extended to Nash tariffs and cooperative tariffs. The idea is to express optimal tariffs as functions of endogenous elasticities and then use the structure of the model to solve for them. A special case of this is Gros' (1987) well-known

[6] Costinot and Rodriguez-Clare (2015) discuss in more detail for which trade models this "exact hat algebra" approach works.

version of the classic optimal tariff formula that a country's optimal tariff equals the inverse of its trading partner's export supply elasticity.

As mentioned above, I abstract from political economy considerations for now and simply assume that tariffs are set by welfare maximizing governments. As a result, country l chooses its tariffs against its trading partners $k \neq l$ in all industries t, τ_{klt}, to maximize its real income $G_l = \frac{E_l}{\prod_s \left(\frac{P_{ls}}{\mu_{ls}}\right)^{\mu_{ls}}}$. To be able to leverage the "exact hat algebra" approach, it is useful to think of the equivalent formulation in changes, where country l chooses its tariff changes $\hat{\tau}_{klt}$ to maximize its real income change $\hat{G}_l = \frac{\hat{E}_l}{\prod_s \left(\hat{P}_{ls}\right)^{\mu_{ls}}}$, all relative to the factual equilibrium. Using this formulation, it should be easy to verify that the associated first-order conditions can be written as:

$$\frac{\partial \hat{E}_l / \hat{E}_l}{\partial \hat{\tau}_{klt}} = \sum_{s=1}^{S} \mu_{ls} \frac{\partial \hat{P}_{ls} / \hat{P}_{ls}}{\partial \hat{\tau}_{klt}} \qquad (14)$$

The semi-elasticities $\frac{\partial \hat{E}_l / \hat{E}_l}{\partial \hat{\tau}_{klt}}$ and $\frac{\partial \hat{P}_{ls} / \hat{P}_{ls}}{\partial \hat{\tau}_{klt}}$ are equilibrium objects which can be characterized by differentiating the equilibrium conditions in changes from Definition 2. However, these equilibrium conditions first have to be evaluated at the optimal tariffs because this is where the first-order conditions have to hold. As a result, optimal tariff changes can be defined as a set of $\hat{\tau}_{klt}$ such that the equilibrium conditions in changes from Definition 2 predict a set of counterfactual parameters $\left\{\alpha'_{ijs}, \beta'_{ijs}, \gamma'_{ijs}, \delta'_{ijs}\right\}$ for which the derivatives of the equilibrium conditions in changes from Definition 2 are consistent with the first-order conditions (14).[7] Differentiating the equilibrium conditions in changes from Definition 2, this logic is summarized as Definition 3:

Definition 3 *Country l's optimal tariff changes are characterized by a set of* $\left\{\frac{\partial \hat{E}_i / \hat{E}_i}{\partial \hat{\tau}_{klt}}, \frac{\partial \hat{w}_i / \hat{w}_i}{\partial \hat{\tau}_{klt}}, \hat{\tau}_{klt}\right\}$ *such that*

$$\frac{\partial \hat{E}_l / \hat{E}_l}{\partial \hat{\tau}_{klt}} = \sum_{s=1}^{S} \mu_{ls} \frac{\partial \hat{P}_{ls} / \hat{P}_{ls}}{\partial \hat{\tau}_{klt}}$$

$$\frac{\partial \hat{E}_i / \hat{E}_i}{\partial \hat{\tau}_{klt}} = \sum_{n=1}^{N} \sum_{s=1}^{S} \beta'_{ins} \frac{\partial \hat{X}_{ins} / \hat{X}_{ins}}{\partial \hat{\tau}_{klt}} + \sum_{m=1}^{N} \sum_{s=1}^{S} \gamma'_{mis} \tau'_{mis} \left(\frac{\partial \hat{\tau}_{mis} / \hat{\tau}_{mis}}{\partial \hat{\tau}_{klt}} + \frac{t'_{mis}}{\tau'_{mis}} \frac{\partial \hat{X}_{mis} / \hat{X}_{mis}}{\partial \hat{\tau}_{klt}} \right)$$

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[7] To be clear, $\alpha'_{ijs} = \frac{\tau'_{ijs} X'_{ijs}}{\sum_{m=1}^{N} \tau'_{mjs} X'_{mjs}}$, $\beta'_{ijs} = \frac{X'_{ijs}}{E'_i}$, $\gamma'_{ijs} = \frac{X'_{ijs}}{E'_j}$, and $\delta'_{ijs} = \frac{\left(1 - \frac{1}{\sigma_s}\right) X'_{ijs}}{w'_i L_i}$, where τ'_{ijs} are the optimal tariffs in levels and $\left\{E'_i, w'_i, X'_{ijs}\right\}$ can be calculated from the equations in Definition 2. Notice that counterfactual levels and changes can always be linked using the identity $x' = x\hat{x}$.

$$\frac{\partial \hat{w}_i/\hat{w}_i}{\partial \hat{\tau}_{klt}} = \sum_{n=1}^{N}\sum_{s=1}^{S}\delta'_{ins}\frac{\partial \hat{X}_{ins}/\hat{X}_{ins}}{\partial \hat{\tau}_{klt}}$$

where

$$\frac{\partial \hat{X}_{ijs}/\hat{X}_{ijs}}{\partial \hat{\tau}_{klt}} = -\sigma_s\frac{\partial \hat{\tau}_{ijs}/\hat{\tau}_{ijs}}{\partial \hat{\tau}_{klt}} - (\sigma_s - 1)\frac{\partial \hat{w}_i/\hat{w}_i}{\partial \hat{\tau}_{klt}} + (\sigma_s - 1)\frac{\partial \hat{P}_{js}/\hat{P}_{js}}{\partial \hat{\tau}_{klt}} + \frac{\partial \hat{E}_j/\hat{E}_j}{\partial \hat{\tau}_{klt}}$$

$$\frac{\partial \hat{P}_{is}/\hat{P}_{is}}{\partial \hat{\tau}_{klt}} = \sum_{m=1}^{N}\alpha'_{mis}\left(\frac{\partial \hat{w}_m/\hat{w}_m}{\partial \hat{\tau}_{klt}} + \frac{\partial \hat{\tau}_{mis}/\hat{\tau}_{mis}}{\partial \hat{\tau}_{klt}}\right)$$

and $\left\{\alpha'_{ijs}, \beta'_{ijs}, \gamma'_{ijs}, \delta'_{ijs}\right\}$ *are calculated using the equilibrium conditions from Definition 2.*

Optimal tariffs can therefore be calculated as the solution to a system of linear and non-linear equations. While I explain below that this approach is not the most efficient to actually calculate optimal tariffs, I suspect that it could prove useful to study the properties of optimal tariffs, Nash tariffs, and cooperative tariffs theoretically. For example, it might be possible to formally establish conditions for existence and uniqueness in the spirit of Allen and Arkolakis (2014) by leveraging existing knowledge about the properties of systems of equations with these particular functional forms.

Moreover, this formulation might help shed light on the qualitative and quantitative determinants of optimal trade policy. An encouraging start is that it can be reduced to Gros' (1987) well-known version of the classic optimal tariff formula in the special case $N = 2$, $S = 1$, and $\Omega_1 = \Omega_2 = 0$:[8] $t'_{21} = \frac{1}{\alpha'_{22}(\sigma-1)}$. This is a version of the classic optimal tariff formula because $\alpha'_{22}(\sigma - 1)$ can be shown to correspond to country 2's export supply elasticity. It depends on the trade elasticity $\sigma - 1$ and the own trade share α'_{22}, where the apostrophe indicates that it is evaluated at the optimal tariff.

2.2 Calibration

2.2.1 Elasticity estimation

I now discuss two complementary approaches that are widely used to estimate the elasticities σ_s. I begin with the traditional approach due to Feenstra (1994) which requires panel data

[8] A detailed derivation of this is available from me upon request. It is straightforward but tedious so I will not reproduce it here.

on values and quantities of trade flows. I then turn to the alternative approach suggested by Caliendo and Parro (2015) which can be implemented in principle using cross-sectional data on tariffs and values of trade flows alone. While I illustrate these approaches in the context a model in which the σ_s correspond to substitution elasticities, the methods really focus on estimating trade elasticities, that is the partial elasticities of trade flows with respect to trade costs. As is well-known, trade elasticities are associated with different structural parameters in different gravity models which should be kept in mind when exploring variations of the workhorse model emphasized here.

Feenstra (1994) The approach of Feenstra (1994) is based on an earlier insight of Leamer (1981) which I now briefly summarize: Suppose you have time-series data on prices and quantities and ask which supply and demand elasticities maximize the likelihood of this data given that the supply and demand curves have constant elasticity forms. While it is impossible to uniquely identify these elasticities for standard endogeneity reasons, Leamer (1981) shows that one can still narrow them down to combinations described by a hyperbolic curve whose precise shape depends on the variances and covariances of the supply and demand shocks generating the data.

Feenstra's (1994) basic idea is to exploit cross-country variation in the variances and covariances of these supply and demand shocks to obtain unique estimates of the supply and demand elasticities. Loosely speaking, a different Leamer hyperbola can be constructed for each country and the estimation approach simply determines which elasticity combination is the best fit for all. The key identifying assumptions are that the supply and demand elasticities are constant and do not vary across countries and that the supply and demand shocks are all drawn independently.

This idea is surprisingly easy to implement using a panel of import values and quantities. The first step is to use the import data to construct a panel of unit values p_{ist} and expenditure shares α_{ijst} for a particular importer j and a particular industry s in which there is variation across exporters i and time t. The definitions of p_{ist} and α_{ijst} are the same as the ones used before with the exception that the subscript t is now added to indicate the time dimension which was absent before. Feenstra (1994) shows that a consistent estimate of σ_s can then be obtained by applying the following simple procedure assuming that the above identifying assumptions hold:[9]

1. Define $Y_{ist} \equiv (\Delta \ln p_{ist} - \Delta \ln p_{kst})^2$, $Z_{1ijst} \equiv (\Delta \ln \alpha_{ijst} - \Delta \ln \alpha_{kjst})^2$, and $Z_{2ijst} \equiv (\Delta \ln p_{ist} - \Delta \ln p_{kst}) (\Delta \ln \alpha_{ijst} - \Delta \ln \alpha_{kjst})$, where Δ denotes time differences and k is an arbitrary reference country, and use ordinary least-squares to estimate the following linear regression:[10]
$$Y_{ist} = \theta_{1s} Z_{1ijst} + \theta_{2s} Z_{2ijst} + u_{ijst} \tag{15}$$

2. Take the estimated coefficients $\tilde{\theta}_{1s}$ and $\tilde{\theta}_{2s}$ from the above regression and back out the estimated supply elasticities $\tilde{\rho}_s$ using the formulas:
$$\begin{aligned} \tilde{\rho}_s &= 0.5 + \left(0.25 - \left(4 + \tilde{\theta}_{2s}^2 / \tilde{\theta}_{1s}\right)^{-1}\right)^{\frac{1}{2}}, \text{ if } \tilde{\theta}_{2s} > 0 \\ \tilde{\rho}_s &= 0.5 - \left(0.25 - \left(4 + \tilde{\theta}_{2s}^2 / \tilde{\theta}_{1s}\right)^{-1}\right)^{\frac{1}{2}}, \text{ if } \tilde{\theta}_{2s} < 0 \end{aligned} \tag{16}$$

3. Use the estimated supply elasticities $\tilde{\rho}_s$ from the above formulas and calculate the estimated demand elasticities $\tilde{\sigma}_s$ from the relationship:
$$\tilde{\sigma}_s = 1 + \left(\frac{2\tilde{\rho}_s - 1}{1 - \tilde{\rho}_s}\right) \frac{1}{\tilde{\theta}_{2s}} \tag{17}$$

Notice that this procedure cannot be applied if $0.25 < \left(4 + \tilde{\theta}_{2s}^2 / \tilde{\theta}_{1s}\right)^{-1}$ because then the elasticity formulas would have imaginary values as results. However, Broda and Weinstein (2006) suggest a grid search approach for this case with which one can still recover estimates of ρ_s and σ_s. The idea is to simply find the values of ρ_s and σ_s which minimize the residual sum of squares of regression (15) subject to the constraints that $0 \leq \rho_s < 1$ and $\sigma_s > 1$. It makes use of the theoretical restrictions derived in Feenstra (1994) that $\theta_{1s} = \frac{\rho_s}{(\sigma_s - 1)^2 (1 - \rho_s)}$ and $\theta_{2s} = \frac{2\rho_s - 1}{(\sigma_s - 1)(1 - \rho_s)}$.

[9] This is based on the explanations in Feenstra (2010), in which the Feenstra (1994) procedure is particularly clearly explained.

[10] To obtain efficient estimates of the elasticities, this regression needs to be run a second time using weighted least squares, where the weights are computed from the inverse of the standard deviation of the residuals from the unweighted regression. This is done in the STATA code I use to estimate the elasticities for this chapter which follows the STATA code from Feenstra (2010).

As is the case with any estimation procedure, the Feenstra (1994) method is not without flaws. One major drawback is that the assumption of independent supply and demand shocks is likely to be violated in practice leading to inconsistent estimates. For example, one might think that productivity and spending simultaneously fall in recessions in which case both the supply and the demand curves shift in. Another issue is that Feenstra's (1994) assumption of constant export supply elasticities does not apply in standard gravity models such as the one discussed in this chapter. Strictly speaking, the Feenstra (1994) method is therefore not correctly specified to estimate demand elasticities in such settings but it is frequently applied to them anyway.

Despite these caveats, the Feenstra (1994) method can deliver plausible estimates of σ_s. In particular, Table 1 reports elasticity estimates calculated by applying the STATA code provided in Feenstra (2010) to Comtrade data for the years 1994-2008.[11] In anticipation of the below application section, they are provided for the 33 industries with which I work later on. My only departure from the standard procedure is that I pool the data over a number of major importers.[12] This is consistent with my earlier assumption that σ_s does not vary across countries and allows me to use a larger dataset leading to more precise estimates. As can be seen, homogeneous goods such as wheat are estimated to have the highest elasticties. Also, the mean elasticity is found to be 3.44 which is within the range of estimates from the literature.

While the Feenstra (1994) method is mostly applied to estimate import demand elasticities, Broda et al (2008) also make use of the associated export supply elasticities in an interesting

[11] The Comtrade data is originally at the SITC-Rev2 4-digit level and I convert it, first, to the SITC-Rev3 4-digit level using a concordance from the Center for International Data at UC Davis and, second, to the GTAP sector level using a concordance which I manually constructed with the help of various concordances available from the GTAP website. This involved combining the original GTAP sectors "raw milk" and "dairy products" into a new GTAP sector "raw and processed dairy", the original GTAP sectors "paddy rice" and "processed rice" into a new GTAP sector "raw and processed rice", and the original GTAP sectors "raw and processed sugar" and "sugar cane, sugar beet" into a new GTAP sector "sugar". This is exactly the same procedure I follow in Ossa (2014).

[12] In particular, I pool over the importers with which I work later on, namely Canada, China, India, Japan, Korea, Russia, and the US as well as the EU-25 countries Austria, Belgium, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, The Netherlands, Poland, Portugal, Slovakia, Slovenia, Sweden, Spain, and the United Kingdom and the Mercosur countries Argentina, Brazil, Paraguay, and Uruguay. Soderbery (2015) has recently suggested an interesting extension to the Feenstra (1994) method which deals with a small sample bias found to be present in the original methodology. While I have not yet applied this methodology myself, I know that my elasticity estimates look much less plausible if I do not pool across importers which I suspect is exactly due to the fact that the sample size used for each elasticity estimation is then much reduced.

trade policy application. Focusing on a number of non-WTO member countries, they show that tariffs are increasing in the export supply elasticities faced by importing countries just as the classic optimal tariff formula predicts. The focus on non-WTO member countries is necessary because the export supply elasticities themselves depend on tariffs under all but the most restrictive assumptions. As a result, one has to assume that factual tariffs are equal to optimal tariffs for the exercise to make sense, which is more plausible for non-WTO member countries.

Caliendo and Parro (2015) Caliendo and Parro (2015) suggest an alternative method which identifies σ_s from the effect tariff changes have on trade flows. It exploits the particular structure of gravity equations and can be applied directly to equation (7). In particular, consider industry s trade flows from country i to country j, from country j to country k, and from country k to country i and multiply them together as $X_{ijs}X_{jks}X_{kis}$. Now divide this by the same term just with trade flows of the reverse direction and substitute the gravity equation (7) to obtain:

$$\frac{X_{ijs}X_{jks}X_{kis}}{X_{jis}X_{kjs}X_{iks}} = \left(\frac{\tau_{ijs}\tau_{jks}\tau_{kis}}{\tau_{jis}\tau_{kjs}\tau_{iks}}\right)^{-\sigma_s} \left(\frac{\theta_{ijs}\theta_{jks}\theta_{kis}}{\theta_{jis}\theta_{kjs}\theta_{iks}}\right)^{1-\sigma_s} \tag{18}$$

Notice that all terms which are specific to a particular origin or destination have been cancelled by taking this ratio so that only pair-specific tariffs and iceberg trade costs remain. The next step is to assume that iceberg trade costs can be decomposed into an origin-specific, a destination-specific, and a pair-specific component, where the pair-specific component has a deterministic and a stochastic part: $\theta_{ijs} = \vartheta_{ijs}\vartheta_{is}\vartheta_{js}\varepsilon_{ijs}$. Under the plausible restriction that the pair-specific component is symmetric in the sense that $\vartheta_{ijs} = \vartheta_{jis}$, the above equation simplifies to:

$$\frac{X_{ijs}X_{jks}X_{kis}}{X_{jis}X_{kjs}X_{iks}} = \left(\frac{\tau_{ijs}\tau_{jks}\tau_{kis}}{\tau_{jis}\tau_{kjs}\tau_{iks}}\right)^{-\sigma_s} \left(\frac{\varepsilon_{ijs}\varepsilon_{jks}\varepsilon_{kis}}{\varepsilon_{jis}\varepsilon_{kjs}\varepsilon_{iks}}\right)^{1-\sigma_s} \tag{19}$$

Taking logs and defining the error term $\nu_{ijks} \equiv \frac{\varepsilon_{ijs}\varepsilon_{jks}\varepsilon_{kis}}{\varepsilon_{jis}\varepsilon_{kjs}\varepsilon_{iks}}$ to simplify the notation, this yields Caliendo and Parro's (2015) estimating equation which can be written in levels or changes, where a "hat" denotes a proportional change just as above:

$$\ln\left(\frac{X_{ijs}X_{jks}X_{kis}}{X_{jis}X_{kjs}X_{iks}}\right) = -\sigma_s \ln\left(\frac{\tau_{ijs}\tau_{jks}\tau_{kis}}{\tau_{jis}\tau_{kjs}\tau_{iks}}\right) + \nu_{ijks} \quad (20)$$

$$\ln\left(\frac{\hat{X}_{ijs}\hat{X}_{jks}\hat{X}_{kis}}{\hat{X}_{jis}\hat{X}_{kjs}\hat{X}_{iks}}\right) = -\sigma_s \ln\left(\frac{\hat{\tau}_{ijs}\hat{\tau}_{jks}\hat{\tau}_{kis}}{\hat{\tau}_{jis}\hat{\tau}_{kjs}\hat{\tau}_{iks}}\right) + \hat{\nu}_{ijks}$$

Just like the Feenstra (1994) method, the Caliendo and Parro (2015) approach is based on a strong identifying assumption. In particular, $\frac{\tau_{ijs}\tau_{jks}\tau_{kis}}{\tau_{jis}\tau_{kjs}\tau_{iks}}$ has to be independent of ν_{ijks} (or its equivalent in changes) for regression (20) to yield consistent estimates. This is violated, for example, if pair-specific tariff and non-tariff barriers are correlated which is likely to be the case. An additional problem is that all identification comes from discriminatory tariff barriers because all MFN tariff barriers cancel out. While this does not invalidate the method in any manner, it is still likely to limit its power in many applications because it eliminates much of the variation the tariff data contain.[13]

While Caliendo and Parro (2015) have shown that their methodology can be successfully applied using trade and tariff data from the North American Free Trade Agreement (NAFTA), I was unable to obtain meaningful estimates using a cross-section of trade and tariff data from the Global Trade Analysis Project (GTAP) for the year 2007. This is the data I use in the below application section and it features the tariffs and trade flows of 10 regions in 33 agricultural and manufacturing industries. Less than half of the estimates were significant and some even had negative signs. Presumably, this is because the included regions comprise mainly WTO member countries so that there is not enough variation in discriminatory tariff barriers.

Discussion In many ways, the elasticity estimation is the Achilles' heel of quantitative trade policy analyses. Not only is it plagued by serious identification problems, but also do most results critically depend on the elasticity estimates. This will become clear in the below

[13] A subtle point that is developed more fully in Costinot and Rodriguez-Clare (2014) is that it matters whether tariffs are assumed to be imposed before markups or after markups (thereby acting as cost shifters or demand shifters) in gravity models featuring monopolistic competition and selection effects such as Melitz (2003). This implies that care must be taken when interpreting the elasticity of trade flows with respect to tariffs obtained from Caliendo and Parro (2015) type estimations in such environments. See also Felbermayr et al (2013) and Felbermayr et al (2015).

application section where the elasticity estimates are shown to be important drivers of optimal tariffs, Nash tariffs, and cooperative tariffs as well as their associated welfare effects. Unfortunately, it seems difficult to overcome the identification problem since convincing instruments are hard to come by for many countries and industries. The natural alternative is to present all results for a range of elasticity estimates and then interpret them with the level of caution they need.

Moreover, the elasticity estimation is often the only time when the model is seriously confronted with the data because all gravity models can be trivially made to match bilateral trade flows in levels by choosing appropriate iceberg trade costs. The analysis of Caliendo and Parro (2015) represents a commendable exception to this rule because the authors actually try to match the trade growth following NAFTA given the tariff cuts NAFTA implied. In my view, much more work is needed along these lines because different gravity models have different predictions even though they can all match the levels perfectly fine. For example, multi-sector Krugman (1980) models feature home-market effects whereas multi-sector Eaton and Kortum (2002) models don't and the quantitative literature so far has little to say about which one works best.

A more general concern in the same spirit is that most gravity models so far impose very strong assumptions on the nature of demand elasticities. In particular, demand elasticities are usually assumed not to vary across countries which amounts to saying that preferences are the same everywhere. Also, demand elasticities are assumed to be constant which seems more plausible as a local approximation than as a global property holding along the entire demand curve. While the first concern seems to apply to all quantitative trade analyses, the second one could be particularly important for optimal tariff calculations such as the ones performed here. As will become clear in the below application section, the estimated optimal tariffs are rather high in constant elasticity models so that the extrapolation is taken quite far.[14]

[14] While the methods of Feenstra (1994) and Caliendo and Parro (2014) are commonly used to estimate trade elasticities, they are by no means the only approaches offered in the literature. For example, Eaton and Kortum (2002) develop an alternative procedure based on price data which has recently been extended by Simonovska and Waugh (2014).

2.2.2 Trade deficits

Static models like the one used in this chapter really have no compelling way of rationalizing aggregate trade deficits. As a result, they are usually accounted for in an ad hoc manner by introducing international transfers such as the ones labelled Ω_i above.[15] Effectively, the assumption is that countries running trade surpluses finance the trade deficits of the other ones. The mechanics of this can be seen by combining the budget constraint (8) with the requirement that $E_i = \sum_{m=1}^{N} \sum_{s=1}^{S} \tau_{mis} X_{mis}$ by definition. Recalling the shorthand $\tau_{ijs} = 1 + t_{ijs}$ from above, this yields

$$\Omega_i = \sum_{n=1}^{N} \sum_{s=1}^{S} X_{ins} - \sum_{m=1}^{N} \sum_{s=1}^{S} X_{mis} \tag{21}$$

A common assumption is now to leave the trade deficits unchanged when performing counterfactuals by imposing $\Omega_i' = \Omega_i$ for all i. However, this assumption is problematic for two reasons. First, it implies extreme general equilibrium adjustments for high tariffs as the model then tries to reconcile falling trade volumes with constant aggregate trade deficits and cannot hold at all in the limit as tariffs approach infinity. Second, it requires a decision in which units the aggregate trade deficits are to be measured which often seems to be made unconsciously in the literature by choosing a particular numeraire. To see this, notice that the budget constraint (8) implies that real income includes a term $\frac{\Omega_i}{P_i}$ so that it matters in what units Ω_i is held fixed.

In Ossa (2014), I suggest one possible solution to this problem which is to simply eliminate all aggregate trade deficits before performing any trade policy counterfactuals. In particular, the idea is to set $\Omega_i' = 0$ for all i and use the equations summarized in Definition 2 above to calculate a counterfactual matrix of bilateral trade flows X_{ijs}' which is free of trade deficits. This is essentially a replication of the exercise performed by Dekle et al (2007) which popularized the "exact hat algebra" approach introduced above. Notice that the abovementioned measurement problem does not arise in this particular application because all transfers are set equal to zero anyway.

[15] Instead of modeling Ω_i as an exogenous transfer of country i, one could also model it as the endogenous portion of country i's income accruing to foreign shareholders who own part of country i's economy. This idea is developed more fully by Caliendo et al (2014) in an economic geography environment.

Table 2 summarizes the effects of this procedure using the 10 region, 33 industry, GTAP data used in the below application section. The first column lists the aggregate trade deficits in the raw data calculated as aggregate exports minus aggregate imports as a share of aggregate export plus aggregate imports. The second and third columns summarize the percentage changes in the values of exports and imports the model predicts as a result of setting $\Omega_i' = 0$ for all i. As one would expect, aggregate trade imbalances are large in the raw data so that exports and imports have to change substantially to eliminate them.

While I work with this purged dataset in the below application section, it seems clear to me that a better solution to the aggregate trade deficit problem needs to be found. The ideal, of course, would be to set up a dynamic model which can rationalize aggregate trade deficits by appealing to intertemporal trade. Such a model would also allow for the possibility to study how trade policy and aggregate trade imbalances interact. So far, the literature largely treats aggregate trade imbalances as orthogonal to trade policy and investigating this further seems like a great opportunity for future work.

2.2.3 Optimization

While solving for optimal tariffs, Nash tariffs, or cooperative tariffs is not a hard problem in principle, making it feasible in practice has been the main challenge faced by this literature. To see this, consider the problem of calculating optimal tariffs in a model with 10 regions and 33 industries such as the one considered in the below application section. Since each country can choose a different tariff against each trading partner, solving for one country's optimal tariffs already involves solving an optimization problem with 9*33=297 arguments. This becomes even more complex when cooperative tariffs are considered because countries then have to jointly set 10*9*33=2,970 tariffs efficiently.

In light of this, it is important to carefully choose the numerical approach. In my experience, three strategies have proven particularly effective. First, formulating the problem in such a way that the number of variables which have to be solved for numerically is minimized. Second, using an algorithm in the mathematical programming with equilibrium constraints (MPEC) tradition such as the one recently suggested by Su and Judd (2012). And third, providing the solver with analytical first-derivatives of the objective functions and the constraints so that they do not have to be repeatedly approximated numerically.

While the second and third point require a more extensive elaboration, the first point is much more easily explained. As will become clear shortly, calculating optimal tariffs, Nash tariffs, and cooperative tariffs is best done using the equations summarized in Definition 2. In this context, minimizing the number of numerically solved variables just means writing everything as a system of nonlinear equations in the $2N$ unknowns $\left\{\hat{E}_i, \hat{w}_i\right\}$. While this is already made explicit in the statement of Definition 2, the problem could have been formulated alternatively as one which includes $\hat{P}_{is}$ or $\hat{X}_{ijs}$ in the list of unknowns which would have drastically increased the number of numerically solved variables.

Optimal tariffs To understand the idea behind the MPEC approach, it is useful to begin by considering a more naive way of formulating the optimal tariff problem using the equations summarized in Definition 2:

Problem 1 *Solve*

$$\min_{\{\hat{\tau}_{klt}\}} - \hat{C}_l\left(\hat{\tau}_{klt}\right)$$

where $\hat{C}_l\left(\hat{\tau}_{klt}\right)$ *is calculated as*

$$\hat{C}_l = \frac{\hat{E}_l}{\prod_{s=1}^{S}\left(\hat{P}_{ls}\right)^{\mu_{ls}}}$$

after solving for $\hat{E}_i$ *and* $\hat{w}_i$ *from*

$$\begin{aligned} 0 &= \hat{E}_i - \sum_{n=1}^{N}\sum_{s=1}^{S}\beta_{ins}\hat{X}_{ins} - \sum_{m=1}^{N}\sum_{s=1}^{S}\gamma_{mis}t'_{mis}\hat{X}_{mis} \\ 0 &= \hat{w}_i - \sum_{n=1}^{N}\sum_{s=1}^{S}\delta_{ins}\hat{X}_{ins} \end{aligned}$$

where $\hat{X}_{ijs}$ *and* $\hat{P}_{is}$ *are given by*

$$\begin{aligned} \hat{X}_{ijs} &= \left(\hat{\tau}_{ijs}\right)^{-\sigma_s}\left(\hat{w}_i\right)^{1-\sigma_s}\left(\hat{P}_{js}\right)^{\sigma_s-1}\hat{E}_j \\ \hat{P}_{is} &= \left(\sum_{m=1}^{N}\alpha_{mis}\left(\hat{w}_m\hat{\tau}_{mis}\right)^{1-\sigma_s}\right)^{\frac{1}{1-\sigma_s}} \end{aligned}$$

and $\{\alpha_{ijs}, \beta_{ijs}, \gamma_{ijs}, \delta_{ijs}\}$ *are defined as above.*

This formulation would be implemented in MATLAB following a two-stage approach. First, one would define a function which computes welfare changes as a function of tariff changes by solving a system of nonlinear equations, for example using "fsolve". Second, one would apply an optimization routine to this function calculating the tariff changes which maximize the welfare change for the country in question, for example "fminunc". While this would work fine in applications with few countries and industries, it would quickly become inefficient for larger scale problems simply because the "fsolve" algorithm would solve the function with high accuracy for each iteration of the "fminunc" routine.

The MPEC approach circumvents this problem by treating the equilibrium conditions as constraints. It has recently received much attention in the context of structural estimation following the work of Su and Judd (2012). Using their logic, it is useful to restate Problem 1 as follows:

Problem 2 *Solve*

$$\min_{\{\hat{C}_i, \hat{w}_i, \hat{E}_i, \hat{\tau}_{ils}\}} -\hat{C}_l$$

subject to

$$\begin{aligned} 0 &= \hat{E}_i - \sum_{n=1}^{N}\sum_{s=1}^{S} \beta_{ins}\hat{X}_{ins} - \sum_{m=1}^{N}\sum_{s=1}^{S} \gamma_{mis} t'_{mis} \hat{X}_{mis} \\ 0 &= \hat{w}_i - \sum_{n=1}^{N}\sum_{s=1}^{S} \delta_{ins}\hat{X}_{ins} \\ 0 &= \hat{C}_i - \frac{\hat{E}_i}{\prod_{s=1}^{S}\left(\hat{P}_{is}\right)^{\mu_{is}}} \end{aligned}$$

where $\hat{X}_{ijs}$ *and* $\hat{P}_{is}$ *are given by*

$$\begin{aligned} \hat{X}_{ijs} &= \left(\hat{\tau}_{ijs}\right)^{-\sigma_s}\left(\hat{w}_i\right)^{1-\sigma_s}\left(\hat{P}_{js}\right)^{\sigma_s - 1}\hat{E}_j \\ \hat{P}_{is} &= \left(\sum_{m=1}^{N} \alpha_{mis}\left(\hat{w}_m \hat{\tau}_{mis}\right)^{1-\sigma_s}\right)^{\frac{1}{1-\sigma_s}} \end{aligned}$$

and $\left\{\alpha_{ijs}, \beta_{ijs}, \gamma_{ijs}, \delta_{ijs}\right\}$ *are defined as above.*[16]

This formulation can be implemented in MATLAB using a constrained optimization solver such as "fmincon". While Problem 1 and Problem 2 are formally identical, Problem 2 can be solved much more quickly numerically. This is simply because most solvers do not enforce constraints to be satisfied until the final iteration which eliminates much of the abovementioned redundant accuracy.

Most solvers allow the user to manually supply the first-derivatives of the objective function and the constraints improving speed and accuracy. While this is a relatively tedious endeavour, it is well worth the effort in my experience because it improves the algorithm's performance in large-scale problems significantly. The derivative of the objective function from Problem 2 is simply a $3N+(N-1)S$-by-1 vector with a -1 as its lth element and zeros everywhere else. Denoting the constraints from Problem 2 by the functions $F_i(.)$, $G_i(.)$, and $H_i(.)$, the derivative of the constraints is a $3N+(N-1)S$-by-$3N$ matrix:

$$D = \begin{bmatrix} \frac{\partial F_1}{\partial \hat{C}_1} & \cdots & \frac{\partial F_N}{\partial \hat{C}_1} & \frac{\partial G_1}{\partial \hat{C}_1} & \cdots & \frac{\partial G_N}{\partial \hat{C}_1} & \frac{\partial H_1}{\partial \hat{C}_1} & \cdots & \frac{\partial H_N}{\partial \hat{C}_1} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial F_1}{\partial \hat{C}_N} & \cdots & \frac{\partial F_N}{\partial \hat{C}_N} & \frac{\partial G_1}{\partial \hat{C}_N} & \cdots & \frac{\partial G_N}{\partial \hat{C}_N} & \frac{\partial H_1}{\partial \hat{C}_N} & \cdots & \frac{\partial H_N}{\partial \hat{C}_N} \\ \frac{\partial F_1}{\partial \hat{E}_1} & \cdots & \frac{\partial F_N}{\partial \hat{E}_1} & \frac{\partial G_1}{\partial \hat{E}_1} & \cdots & \frac{\partial G_N}{\partial \hat{E}_1} & \frac{\partial H_1}{\partial \hat{E}_1} & \cdots & \frac{\partial H_N}{\partial \hat{E}_1} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial F_1}{\partial \hat{E}_N} & \cdots & \frac{\partial F_N}{\partial \hat{E}_N} & \frac{\partial G_1}{\partial \hat{E}_N} & \cdots & \frac{\partial G_N}{\partial \hat{E}_N} & \frac{\partial H_1}{\partial \hat{E}_N} & \cdots & \frac{\partial H_N}{\partial \hat{E}_N} \\ \frac{\partial F_1}{\partial \hat{w}_1} & \cdots & \frac{\partial F_N}{\partial \hat{w}_1} & \frac{\partial G_1}{\partial \hat{w}_1} & \cdots & \frac{\partial G_N}{\partial \hat{w}_1} & \frac{\partial H_1}{\partial \hat{w}_1} & \cdots & \frac{\partial H_N}{\partial \hat{w}_1} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial F_1}{\partial \hat{w}_N} & \cdots & \frac{\partial F_N}{\partial \hat{w}_N} & \frac{\partial G_1}{\partial \hat{w}_N} & \cdots & \frac{\partial G_N}{\partial \hat{w}_N} & \frac{\partial H_1}{\partial \hat{w}_N} & \cdots & \frac{\partial H_N}{\partial \hat{w}_N} \\ \frac{\partial F_1}{\partial \hat{\tau}_{beg}} & \cdots & \frac{\partial F_N}{\partial \hat{\tau}_{beg}} & \frac{\partial G_1}{\partial \hat{\tau}_{beg}} & \cdots & \frac{\partial G_N}{\partial \hat{\tau}_{beg}} & \frac{\partial H_1}{\partial \hat{\tau}_{beg}} & \cdots & \frac{\partial H_N}{\partial \hat{\tau}_{beg}} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial F_1}{\partial \hat{\tau}_{end}} & \cdots & \frac{\partial F_N}{\partial \hat{\tau}_{end}} & \frac{\partial G_1}{\partial \hat{\tau}_{end}} & \cdots & \frac{\partial G_N}{\partial \hat{\tau}_{end}} & \frac{\partial H_1}{\partial \hat{\tau}_{end}} & \cdots & \frac{\partial H_N}{\partial \hat{\tau}_{end}} \end{bmatrix} \tag{22}$$

[16] Notice that it is actually not necessary to include all $\hat{C}_i$ in the list of arguments as well as all $0 = \hat{C}_i - \frac{\hat{E}_i}{\prod_{s=1}^{S}\left(\hat{P}_{is}\right)^{\mu_{is}}}$ in the list of constraints (instead only the one for $i = l$). However, this is how I implement it in the MATLAB code available on the website accompanying this chapter to also calculate the welfare effects on all other countries right away.

where $\{\hat{\tau}_{beg},...,\hat{\tau}_{end}\}$ abbreviates the $(N-1)S$ tariffs imposed by country l, the partial derivatives of the first set of constraints from Problem 2 are:

$$\begin{aligned}
\frac{\partial F_i}{\partial \hat{C}_k} &= 0 \\
\frac{\partial F_i}{\partial \hat{E}_k} &= \frac{\partial \hat{E}_i}{\partial \hat{E}_k} - \sum_{n=1}^{N}\sum_{s=1}^{S} \beta_{ins} \frac{\partial \hat{X}_{ins}}{\partial \hat{E}_k} - \sum_{m=1}^{N}\sum_{s=1}^{S} \gamma_{mis} t'_{mis} \frac{\partial \hat{X}_{mis}}{\partial \hat{E}_k} \\
\frac{\partial F_i}{\partial \hat{w}_k} &= -\sum_{n=1}^{N}\sum_{s=1}^{S} \beta_{ins} \frac{\partial \hat{X}_{ins}}{\partial \hat{w}_k} - \sum_{m=1}^{N}\sum_{s=1}^{S} \gamma_{mis} t'_{mis} \frac{\partial \hat{X}_{mis}}{\partial \hat{w}_k} \\
\frac{\partial F_i}{\partial \hat{\tau}_{klt}} &= -\sum_{n=1}^{N}\sum_{s=1}^{S} \beta_{ins} \frac{\partial \hat{X}_{ins}}{\partial \hat{\tau}_{klt}} - \sum_{m=1}^{N}\sum_{s=1}^{S} \gamma_{mis} t'_{mis} \hat{X}_{mis} \left(\frac{1}{\hat{X}_{mis}} \frac{\partial \hat{X}_{mis}}{\partial \hat{\tau}_{klt}} + \frac{\tau_{mis}}{t'_{mis}} \frac{\partial \hat{\tau}_{mis}}{\partial \hat{\tau}_{klt}} \right)
\end{aligned} \tag{23}$$

the partial derivatives of the second set of constraints from Problem 2 are:

$$\begin{aligned}
\frac{\partial G_i}{\partial \hat{C}_k} &= 0 \\
\frac{\partial G_i}{\partial \hat{E}_k} &= -\sum_{n=1}^{N}\sum_{s=1}^{S} \delta_{ins} \frac{\partial \hat{X}_{ins}}{\partial \hat{E}_k} \\
\frac{\partial G_i}{\partial \hat{w}_k} &= \frac{\partial \hat{w}_i}{\partial \hat{w}_k} - \sum_{n=1}^{N}\sum_{s=1}^{S} \delta_{ins} \frac{\partial \hat{X}_{ins}}{\partial \hat{w}_k} \\
\frac{\partial G_i}{\partial \hat{\tau}_{klt}} &= -\sum_{n=1}^{N}\sum_{s=1}^{S} \delta_{ins} \frac{\partial \hat{X}_{ins}}{\partial \hat{\tau}_{klt}}
\end{aligned} \tag{24}$$

the partial derivatives of the third set of constraints from Problem 2 are:

$$\begin{aligned}
\frac{\partial H_i}{\partial \hat{C}_k} &= \frac{\partial \hat{C}_i}{\partial \hat{C}_k} \\
\frac{\partial H_i}{\partial \hat{E}_k} &= -\frac{\hat{E}_i}{\prod_{s=1}^{S} \left(\hat{P}_{is}\right)^{\mu_{is}}} \frac{1}{\hat{E}_i} \frac{\partial \hat{E}_i}{\partial \hat{E}_k} \\
\frac{\partial H_i}{\partial \hat{w}_k} &= \frac{\hat{E}_i}{\prod_{s=1}^{S} \left(\hat{P}_{is}\right)^{\mu_{is}}} \sum_{s=1}^{S} \mu_{is} \alpha'_{kis} \frac{1}{\hat{w}_k} \\
\frac{\partial H_i}{\partial \hat{\tau}_{klt}} &= \frac{\hat{E}_i}{\prod_{s=1}^{S} \left(\hat{P}_{is}\right)^{\mu_{is}}} \mu_{it} \alpha'_{kit} \frac{1}{\hat{\tau}_{kit}} \frac{\partial \hat{\tau}_{kit}}{\partial \hat{\tau}_{klt}}
\end{aligned} \tag{25}$$

the partial derivatives of the equations describing $\hat{X}_{ijs}$ in Problem 2 are:

$$\begin{aligned} \frac{\partial \hat{X}_{ijs}}{\partial \hat{C}_k} &= 0 \\ \frac{\partial \hat{X}_{ijs}}{\partial \hat{E}_k} &= \frac{\hat{X}_{ijs}}{\hat{E}_j} \frac{\partial \hat{E}_j}{\partial \hat{E}_k} \\ \frac{\partial \hat{X}_{ijs}}{\partial \hat{w}_k} &= (1-\sigma_s)\, \hat{X}_{ijs} \left(\frac{1}{\hat{w}_i} \frac{\partial \hat{w}_i}{\partial \hat{w}_k} - \alpha'_{kjs} \frac{1}{\hat{w}_k} \right) \\ \frac{\partial \hat{X}_{ijs}}{\partial \hat{\tau}_{klt}} &= \hat{X}_{ijs} \left(-\sigma_s \frac{1}{\hat{\tau}_{ijs}} \frac{\partial \hat{\tau}_{ijs}}{\partial \hat{\tau}_{klt}} + (\sigma_s - 1)\, \alpha'_{kjs} \frac{1}{\hat{\tau}_{kjs}} \frac{\partial \hat{\tau}_{kjs}}{\partial \hat{\tau}_{klt}} \right) \end{aligned} \tag{26}$$

and the partial derivatives of the equations describing $\hat{P}_{is}$ in Problem 2 are:

$$\begin{aligned} \frac{\partial \hat{P}_{is}}{\partial \hat{C}_k} &= 0 \\ \frac{\partial \hat{P}_{is}}{\partial \hat{E}_k} &= 0 \\ \frac{\partial \hat{P}_{is}}{\partial \hat{w}_k} &= \alpha'_{kis} \frac{\hat{P}_{is}}{\hat{w}_k} \\ \frac{\partial \hat{P}_{is}}{\partial \hat{\tau}_{klt}} &= \alpha'_{kis} \frac{\hat{P}_{is}}{\hat{\tau}_{kis}} \frac{\partial \hat{\tau}_{kis}}{\partial \hat{\tau}_{klt}} \end{aligned} \tag{27}$$

To be clear, $\frac{\partial \hat{C}_i}{\partial \hat{C}_k}$ represents an N-by-1 vector which has a value of 1 at position $i = k$ and zeros everywhere else with a similar logic applying to similar expressions above. Also, α'_{ijs} refers to the value of α_{ijs} at the counterfactual tariffs with a similar logic applying to similar expressions above. All expressions are simply partial derivatives and should be relatively easy to derive. Of course, it is easy to make mistakes when implementing this in practice especially since all derivatives also have to be stacked in exactly the right way. However, most solvers can check user-supplied derivatives numerically so that those mistake are usually relatively easy to find.

Figures 1a and 1b display the results of testing the above procedure for calculating optimal tariffs in a simple two-country one-industry example. The example uses the same data I use in the below application section but aggregates it to two countries and one industry with the two countries being Canada and the US. As will become clear later, countries set tariffs purely for terms-of-trade motives in the one industry special case of this model so that the classic optimal tariff argument applies. In particular, the terms-of-trade gain outweighs the distortion loss for small enough tariffs so that the optimal tariff is positive.

Figure 1a plots the welfare effects of unilateral changes in the US tariff against Canada using the equations summarized in Definition 2. It also shows the optimal US tariff calculated using the above MPEC procedure as a grey vertical line. As can be seen, the welfare maximizing US tariff indeed coincides with the shown optimal US tariff so that the applied optimization procedure seems to work. Figure 1b repeats this for the optimal Canadian tariff yielding the same basic results. Besides providing simple verification checks, Figures 1a and 1b also reveal interesting economic points which I discuss in more detail later on. In particular, notice that the US optimal tariff is larger than the Canadian optimal tariff and that the welfare effects on Canada are always larger than the welfare effects on the US.

Nash tariffs Given an efficient algorithm to calculate optimal tariffs, Nash tariffs can be computed relatively straightforwardly. Recall that Nash tariffs are optimal tariffs with retaliation, that is the best-response tariffs one would expect to prevail in a full-blown tariff war. Perroni and Whalley (2000) already report that iteration over optimal tariffs typically yields fast convergence to a seemingly unique result. The procedure is to simply impose optimal tariffs given factual tariffs and then reoptimize repeatedly until the best-response equilibrium is found. Figure 2 illustrates why this works in the simple Canada-US example by plotting the best-response functions of both countries. As can be seen, the best response functions are relatively flat and have a unique intersection which explains why an iterative algorithm quickly converges.

While it is obvious that there is a unique Nash equilibrium in Figure 2, this of course does not necessarily have to extend to cases with many countries and industries. However, I have been unable to find multiple Nash equilibria even in more complicated applications when exploring the implications of choosing different starting points. A trivial exception to this is autarky which is a Nash equilibrium in all tariff games simply because any country's unilateral trade liberalization would not trigger any trade in general equilibrium if all other countries keep their tariffs at infinity. Trying to formally prove the conjecture that there is a unique interior Nash equilibrium would be an interesting objective for future work. One possible approach would be to explore the formal properties of the first-order conditions summarized in Definition 3.

Cooperative tariffs Calculating cooperative tariffs is far more challenging than calculating optimal tariffs and Nash tariffs because all countries' tariffs have to be chosen at the same time. It is also best done by using an MPEC algorithm and supplying analytical first-derivatives. As I discuss more extensively in the below application section, cooperative tariffs are not just zero for three reasons. First, there is an entire efficiency frontier even in perfectly competitive models of which free trade is only one point. Second, governments might use cooperative tariffs as a second-best policy to correct existing distortions in imperfectly competitive environments. And third, political economy pressures might imply that free trade is not efficient from the point of view of governments even though it might be efficient from the point of view of the population as a whole.

Cooperative tariffs can be calculated by applying a bargaining protocol in the spirit of symmetric Nash bargaining assuming that countries equally split all efficiency gains. This can be implemented by extending the MPEC formulation of the optimal tariff problem summarized in Problem 2 above. In particular, allowing the optimization to be over all tariff changes and not just the tariff changes of country l, assuming that the objective is to maximize the welfare change of country 1 and not country l, and imposing a fourth set of constraints ensuring that all countries' welfare changes are the same, the bargaining problem can be formulated as follows:[17]

Problem 3 *Solve*

$$\min_{\left\{\hat{C}_i,\hat{w}_i,\hat{E}_i,\hat{\tau}_{ijs}\right\}} -\hat{C}_1$$

[17] As will become clear shortly in the discussion surrounding Figure 3, there is no unique efficient tariff vector but instead an entire efficiency frontier. While the suggested algorithm identifies the point on the efficiency frontier which ensures that all countries' welfare changes are the same, one could alternatively try to identify the point on the efficiency frontier which maximizes the sum of all countries' welfares as is often done in the theoretical literature. In practice, this would involve maximizing the sum of all countries' welfares in changes, i.e. $\sum_i \frac{C_i}{\sum_n C_n}\hat{C}_i$, which would require data on $\frac{C_i}{\sum_n C_n}$.

subject to

$$
\begin{aligned}
0 &= \hat{E}_i - \sum_{n=1}^{N}\sum_{s=1}^{S}\beta_{ins}\hat{X}_{ins} - \sum_{m=1}^{N}\sum_{s=1}^{S}\gamma_{mis}t'_{mis}\hat{X}_{mis} \\
0 &= \hat{w}_i - \sum_{n=1}^{N}\sum_{s=1}^{S}\delta_{ins}\hat{X}_{ins} \\
0 &= \hat{C}_i - \frac{\hat{E}_i}{\prod_{s=1}^{S}\left(\hat{P}_{is}\right)^{\mu_{is}}} \\
0 &= \hat{C}_i - \hat{C}_1
\end{aligned}
$$

where $\hat{X}_{ijs}$ and $\hat{P}_{is}$ are given by

$$
\begin{aligned}
\hat{X}_{ijs} &= \left(\hat{\tau}_{ijs}\right)^{-\sigma_s}\left(\hat{w}_i\right)^{1-\sigma_s}\left(\hat{P}_{js}\right)^{\sigma_s-1}\hat{E}_j \\
\hat{P}_{is} &= \left(\sum_{m=1}^{N}\alpha_{mis}\left(\hat{w}_m\hat{\tau}_{mis}\right)^{1-\sigma_s}\right)^{\frac{1}{1-\sigma_s}}
\end{aligned}
$$

and $\left\{\alpha_{ijs}, \beta_{ijs}, \gamma_{ijs}, \delta_{ijs}\right\}$ are defined as above.

Given that Problem 3 is just an extension of Problem 2, the derivatives associated with Problem 3 also build on the derivatives associated with Problem 2. The derivative of the objective function from Problem 3 is now a $3N+N(N-1)S$-by-1 vector with a -1 as its first element and zeros everywhere else. Denoting the constraints from Problem 3 by the functions $F_i(.)$, $G_i(.)$, $H_i(.)$, and $I(.)$ the derivative of the constraints is a $3N+N(N-1)S$-by-$4N$

matrix:

$$
D = \begin{bmatrix}
\frac{\partial F_1}{\partial \hat{C}_1} & \cdots & \frac{\partial F_N}{\partial \hat{C}_1} & \frac{\partial G_1}{\partial \hat{C}_1} & \cdots & \frac{\partial G_N}{\partial \hat{C}_1} & \frac{\partial H_1}{\partial \hat{C}_1} & \cdots & \frac{\partial H_N}{\partial \hat{C}_1} & \frac{\partial I_1}{\partial \hat{C}_1} & \cdots & \frac{\partial I_N}{\partial \hat{C}_1} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\frac{\partial F_1}{\partial \hat{C}_N} & \cdots & \frac{\partial F_N}{\partial \hat{C}_N} & \frac{\partial G_1}{\partial \hat{C}_N} & \cdots & \frac{\partial G_N}{\partial \hat{C}_N} & \frac{\partial H_1}{\partial \hat{C}_N} & \cdots & \frac{\partial H_N}{\partial \hat{C}_N} & \frac{\partial I_1}{\partial \hat{C}_N} & \cdots & \frac{\partial I_N}{\partial \hat{C}_N} \\
\frac{\partial F_1}{\partial \hat{E}_1} & \cdots & \frac{\partial F_N}{\partial \hat{E}_1} & \frac{\partial G_1}{\partial \hat{E}_1} & \cdots & \frac{\partial G_N}{\partial \hat{E}_1} & \frac{\partial H_1}{\partial \hat{E}_1} & \cdots & \frac{\partial H_N}{\partial \hat{E}_1} & \frac{\partial I_1}{\partial \hat{E}_1} & \cdots & \frac{\partial I_N}{\partial \hat{E}_1} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\frac{\partial F_1}{\partial \hat{E}_N} & \cdots & \frac{\partial F_N}{\partial \hat{E}_N} & \frac{\partial G_1}{\partial \hat{E}_N} & \cdots & \frac{\partial G_N}{\partial \hat{E}_N} & \frac{\partial H_1}{\partial \hat{E}_N} & \cdots & \frac{\partial H_N}{\partial \hat{E}_N} & \frac{\partial I_1}{\partial \hat{E}_N} & \cdots & \frac{\partial I_N}{\partial \hat{E}_N} \\
\frac{\partial F_1}{\partial \hat{w}_1} & \cdots & \frac{\partial F_N}{\partial \hat{w}_1} & \frac{\partial G_1}{\partial \hat{w}_1} & \cdots & \frac{\partial G_N}{\partial \hat{w}_1} & \frac{\partial H_1}{\partial \hat{w}_1} & \cdots & \frac{\partial H_N}{\partial \hat{w}_1} & \frac{\partial I_1}{\partial \hat{w}_1} & \cdots & \frac{\partial I_N}{\partial \hat{w}_1} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\frac{\partial F_1}{\partial \hat{w}_N} & \cdots & \frac{\partial F_N}{\partial \hat{w}_N} & \frac{\partial G_1}{\partial \hat{w}_N} & \cdots & \frac{\partial G_N}{\partial \hat{w}_N} & \frac{\partial H_1}{\partial \hat{w}_N} & \cdots & \frac{\partial H_N}{\partial \hat{w}_N} & \frac{\partial I_1}{\partial \hat{w}_N} & \cdots & \frac{\partial I_N}{\partial \hat{w}_N} \\
\frac{\partial F_1}{\partial \hat{\tau}_{beg}} & \cdots & \frac{\partial F_N}{\partial \hat{\tau}_{beg}} & \frac{\partial G_1}{\partial \hat{\tau}_{beg}} & \cdots & \frac{\partial G_N}{\partial \hat{\tau}_{beg}} & \frac{\partial H_1}{\partial \hat{\tau}_{beg}} & \cdots & \frac{\partial H_N}{\partial \hat{\tau}_{beg}} & \frac{\partial I_1}{\partial \hat{\tau}_{beg}} & \cdots & \frac{\partial I_N}{\partial \hat{\tau}_{beg}} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\frac{\partial F_1}{\partial \hat{\tau}_{end}} & \cdots & \frac{\partial F_N}{\partial \hat{\tau}_{end}} & \frac{\partial G_1}{\partial \hat{\tau}_{end}} & \cdots & \frac{\partial G_N}{\partial \hat{\tau}_{end}} & \frac{\partial H_1}{\partial \hat{\tau}_{end}} & \cdots & \frac{\partial H_N}{\partial \hat{\tau}_{end}} & \frac{\partial I_1}{\partial \hat{\tau}_{end}} & \cdots & \frac{\partial I_N}{\partial \hat{\tau}_{end}}
\end{bmatrix} \tag{28}
$$

where $\{\hat{\tau}_{beg},...,\hat{\tau}_{end}\}$ abbreviates the $N(N-1)S$ tariffs jointly imposed by all countries. The partial derivatives of $F_i(.)$, $G_i(.)$, $H_i(.)$, $\hat{X}_{ijs}$, and $\hat{P}_{is}$ are exactly the same as the ones shown above for Problem 2 with the important exception that the derivatives with respect to tariffs now have to be taken with respect to all $l = \{1, ..., N\}$ and not just one particular l. Other than $\frac{\partial I_i}{\partial \hat{C}_k} = \frac{\partial \hat{C}_1}{\partial \hat{C}_k} - \frac{\partial \hat{C}_i}{\partial \hat{C}_k}$, the partial derivatives of $I(.)$ are all zero, namely $\frac{\partial I_i}{\partial \hat{E}_k} = 0$, $\frac{\partial I_i}{\partial \hat{w}_k} = 0$, and $\frac{\partial I_i}{\partial \hat{\tau}_{klt}} = 0$.

Figure 3 illustrates the cooperative tariffs using the simple US-Canada example from above and compares them to the Nash tariffs calculated previously. While cooperative tariffs are calculated using the MPEC approach summarized in Problem 3, I have allowed for asymmetric bargaining weights just for this example to be able to trace out a whole segment of the efficiency frontier. Readers familiar with the theoretical trade policy literature will recognize Figure 3 as a quantitative example of a familiar illustration of cooperative and noncooperative tariffs which can be found, for example, in Bagwell and Staiger (2002).

The curve labelled E is simply the efficiency frontier tracing out combinations of Pareto efficient tariffs obtained by solving a version of Problem 3 with varying bargaining weights. As can be seen, free trade is on the efficiency frontier in the one sector special case of our model which I discuss in more detail later on. The other curves are iso-welfare loci of the US and Canada which are solid and dashed, respectively. I have drawn one small pair passing through free trade and another larger pair passing through the Nash equilibrium tariffs which I have labelled N. As one would expect, the smaller ones are tangent to one another at zero tariffs whereas the larger ones intersect perpendicularly at Nash tariffs.

3 Application

I now put the above tools to work in a more serious application, calculating the optimal tariffs, Nash tariffs, and cooperative tariffs for the 10 largest trading blocks of the world. This is essentially an extension of my analysis in Ossa (2014) from 7 to 10 regions and is supposed to highlight some of the main results which have been obtained so far. In the interest of minimizing replication, I limit my discussion to some key points in this section and refer the reader to Ossa (2014) for a more comprehensive analysis.

3.1 Data

Obtaining the necessary data on trade flows and tariffs is not as easy as one might think. The main complication regarding the trade data is to obtain the diagonal elements of the trade matrix capturing within-country flows especially at the industry level. The main difficulty regarding the tariff data is to get accurate ad valorem equivalents of specific tariffs which are hard to compute because converting per-unit tariffs into per-value equivalents requires price data. In my experience, a particularly convenient source is the Global Trade Analysis Project (GTAP) database which is available from the Department of Agricultural Economics at Purdue University. While commonly used for a quantitative general equilibrium model called the GTAP model, the database can also be downloaded for alternative uses which is what I did for my application here.

In particular, I work with a 10 region and 33 industry aggregation of the GTAP 8 database for the year 2007. I include the world's 9 largest trading blocks and a residual Rest of the World as well as all available agricultural and manufacturing industries. The GTAP 8 database is itself based on a number of data sources which are carefully cleaned by a large team of researchers to ensure global consistency. Specifically, the international trade data is mainly drawn from the UN's Comtrade database, its domestic trade data is mainly constructed from national input-output accounts, and the tariff data is mainly taken from the International Trade Centre's Market Access Map database. The database is documented in Narayanan et al (2012) which can be accessed directly from the GTAP website under https://www.gtap.agecon.purdue.edu.[18]

While the most recent editions of the GTAP data have to be purchased, earlier versions are available free of charge. To use the data outside of the GTAP model, one has to follow a number of steps. First, one has to aggregate the data as needed using a GTAP-supplied software called "GTAPAgg" which saves the aggregated data as ".har" files. Second, these ".har" files have to be converted into ".csv" files using a GEMPACK program called "har2csv" which can be executed with a simple batch file. These ".csv" files can then be imported into MATLAB and used normally. The original GTAP data spans all sectors of the economy. I simply drop all industries which do not belong to the agricultural or manufacturing sector and further aggregate a few agricultural sectors to match the Comtrade data used in the elasticity estimation above.

3.2 Welfare effects

Before considering optimal tariffs, Nash tariffs, and efficient tariffs, it is useful to illustrate the welfare effects of tariff changes in this environment. This can be done most clearly by totally differentiating the budget constraint and the price index from Definition 1. Assuming $d\Omega_i = 0$ for simplicity and recalling that industry revenues are split between industry labor income and industry profits, $\sum_{n=1}^{N} X_{ins} = w_i L_{is} + \pi_{is}$, this yields:

$$\frac{dE_i}{E_i} = \frac{w_i L_i}{E_i}\frac{dw_i}{w_i} + \sum_{s=1}^{S}\frac{\pi_{is}}{E_i}\frac{d\pi_{is}}{\pi_{is}} + \sum_{m=1}^{N}\sum_{s=1}^{S}\frac{t_{mis}X_{mis}}{E_i}\left(\frac{dX_{mis}}{X_{mis}} + \frac{dt_{mis}}{t_{mis}}\right) \tag{29}$$

[18] While the GTAP data is particularly useful in my experience, there are a number of other excellent data sources such as CEPII databases which can be accessed under http://www.cepii.fr/cepii/en/bdd_modele/bdd.asp and the World Input Output Database which can be accessed under http://www.wiod.org/new_site/home.htm. Many researchers also make use of the World Bank's Word Integrated Trade Solution available under http://wits.worldbank.org/ and the NBER-UN data available under http://cid.econ.ucdavis.edu/data/undata/undata.html.

$$\frac{dP_{is}}{P_{is}} = \sum_{m=1}^{N} \frac{\tau_{mis} X_{mis}}{\mu_{is} E_i} \left(\frac{dw_m}{w_m} + \frac{d\tau_{mis}}{\tau_{mis}} \right) \tag{30}$$

These two derivatives can now be combined to an informative decomposition of the welfare change $\frac{dC_i}{C_i} = \frac{dE_i}{E_i} - \frac{dP_i}{P_i}$. In particular, recalling that $P_i = \prod_{s=1}^{S} \left(\frac{P_{is}}{\mu_{is}} \right)^{\mu_{is}}$ so that $\frac{dP_i}{P_i} = \sum_{s=1}^{S} \mu_{is} \frac{dP_{is}}{P_{is}}$, that $\tau_{ijs} = 1 + t_{ijs}$ so that $d\tau_{ijs} = dt_{ijs}$, that $p_{is} = \frac{\sigma_s}{\sigma_s - 1} \frac{w_i}{\varphi_{is}}$ so that $\frac{dp_{is}}{p_{is}} = \frac{dw_i}{w_i}$, and that $\sum_{n=1}^{N} \sum_{s=1}^{S} X_{ins} = E_i - \sum_{m=1}^{N} \sum_{s=1}^{S} t_{mis} X_{mis}$, it should be easy to verify that around zero tariffs:[19]

$$\frac{dC_i}{C_i} = \underbrace{\sum_{m=1}^{N} \sum_{s=1}^{S} \frac{X_{mis}}{E_i} \left(\frac{dp_{is}}{p_{is}} - \frac{dp_{ms}}{p_{ms}} \right)}_{\text{terms-of-trade effect}} + \underbrace{\sum_{s=1}^{S} \frac{\pi_{is}}{E_i} \left(\frac{d\pi_{is}}{\pi_{is}} - \frac{dp_{is}}{p_{is}} \right)}_{\text{profit shifting effect}} \tag{31}$$

The first term of this decomposition is a traditional terms-of-trade effect. It captures that a country benefits if the value of its export bundle increases relative to the value of its import bundle. Here, the terms-of-trade effect can also be interpreted as a relative wage effect because a country's export bundle only becomes more expensive relative to its import bundle if its wage goes up relative to the wages of its trading partners. Notice that this close link between relative prices and relative wages implies that tariffs always change the terms-of-trade in all industries at the same time. It would be interesting to explore setups in which this would no longer be true for example by allowing for variable markups, changing marginal costs, multiple mobile factors of production, or input-ouput linkages.

The second term of this decomposition is a "new trade" profit shifting effect. It captures that a country benefits if its profits increase on aggregate because of a reallocation of resources towards more profitable industries. To see this, notice that $\frac{d\pi_{is}}{\pi_{is}} - \frac{dp_{is}}{p_{is}} = \frac{dL_{is}}{L_{is}}$ so that the profit shifting effect can be rewritten as $\sum_{s=1}^{S} \frac{\pi_{is}}{E_i} \frac{dL_{is}}{L_{is}}$, which follows straightforwardly from the fact that industry profits equal a share $\frac{1}{\sigma_s}$ of industry revenues, $\pi_{is} = \frac{1}{\sigma_s} \sum_{j=1}^{N} M_{is} p_{is} \theta_{ijs} c_{ijs}$, technology (2), and the identity $L_{is} = M_{is} l_{is}$. It can be shown that the profit shifting effect is equal to zero if $\sigma_s = \sigma$ for all s which makes sense since there is no variation in profitability across industries if markups are the same everywhere.

[19] Without setting tariffs to zero, the decomposition includes a third term $+\sum_{m=1}^{N} \sum_{s=1}^{S} \frac{t_{mis} X_{mis}}{E_i} \left(\frac{dX_{mis}}{X_{mis}} - \frac{dp_{is}}{p_{is}} \right)$, which captures that generally tariff revenues change if trade volumes change.

To illustrate this, the upper panel of Table 3 shows the general equilibrium adjustments following a unilateral increase in the tariffs protecting the US chemicals or apparel industry by 50 percentage points. As can be seen, this intervention increases the US wage relative to other countries and allows the US to expand its protected industry at the expense of its other industries. Intuitively, a unilateral increase in protection makes imports more expensive for domestic consumers so that they switch expenditure towards the protected domestic industry. This then allows the protected US industry to expand which bids up wages and forces the other industries in the US to contract. These general equilibrium effects are computed using the equilibrium conditions in changes summarized in Definition 2.

The lower panel of Table 3 then turns to the associated welfare effects. As can be seen, the US benefits from the unilateral intervention in the chemicals industry but loses from the unilateral intervention in the apparel industry. While the terms-of-trade effects are positive in both cases as a result of the increase in the US relative wage, the profit shifting effect is positive if the US protects the chemicals industry but negative if the US protect the apparel industry. The explanation for this is simply that the chemicals industry is a relatively high profitability industry because its products have a relatively low elasticity of substitution whereas the apparel industry is a relatively low profitability industry because its products have a relatively low elasticity of substitution, as can be seen from the elasticity estimates in Table 1.

Notice that the profit shifting effect disappears from decomposition (31) in the special case of only one industry because then $\pi_i = \frac{1}{\sigma-1} w_i L_i$ so that $\frac{d\pi_i}{\pi_i} - \frac{dp_i}{p_i} = 0$. In this case, the model is actually isomorphic to an Armington (1969) model (and indeed many other gravity models) which is just an example of the point made by Arkolakis et al (2012). However, different versions of decomposition (31) apply in different multi-sector gravity models because the strict isomorphism then breaks down. For example, it can be shown that only the terms-of-trade effect remains in a multi-sector Armington (1969) model. Also, it can be shown that the profit shifting effect is replaced by the production relocation effect $\sum_{m=1}^{N} \sum_{s=1}^{S} \frac{1}{\sigma_s - 1} \frac{X_{mis}}{E_i} \frac{dM_{ms}}{M_{ms}}$ in a multi-sector Krugman (1980) model with free entry.[20]

[20] In this case, countries would still benefit from a reallocation of resources towards low σ_s industries but now because this decreases the aggregate price index. In particular, consumers prefer local varieties to imported varieties because they are delivered without trade costs. Moreover, this preference is more pronounced for more differentiated varieties so that consumers would rather have low σ_s varieties produced nearby.

3.3 Optimal tariffs

Figure 4 illustrates the optimal tariffs of all countries calculated using the methods introduced above. Recall that optimal tariffs are defined as welfare-maximizing tariffs without retaliation so that this figure contains 10 separate policy experiments. In particular, each panel focuses on one country and shows its optimal tariffs for all trading partners and all industries assuming that all other countries continue to impose their factual tariffs. Each dot represents the optimal tariff in one industry against one trading partner and industries are ranked along the horizontal axis in increasing order of their elasticities. I was able to calculate these optimal tariffs in around 15 minutes using a high-end desktop which suggests that the analysis could be easily extended beyond 10 regions and 33 industries.

As one would expect from the above examples, the optimal tariffs are positive for all countries, industries, and trading partners and tend to be lower in higher elasticity industries. By imposing positive tariffs, countries improve their relative wages generating positive terms-of-trade effects. By tilting tariffs towards low elasticity industries, they further shift resources towards more profitable industries generating positive profit shifting effects. While optimal tariffs tend to be lower in higher elasticity industries, the figure also makes clear that this is not always the case. This simply reflects the fact that optimal tariffs also depend on other factors such as the industry's trade exposure as the simple two-country one-industry optimal tariff formula discussed earlier already made clear.

Table 4a turns to the welfare effects of these optimal tariffs always listing the effects on the tariff imposing country as well as the averages of the effects on all other countries. As can be seen, real income increases by an average 2.4 percent for the tariff imposing country and decreases by an average -0.6 percent elsewhere. Countries can benefit at the expense of other countries because the terms-of-trade and profit shifting effects have a beggar-thy-neighbor character. As can be seen, wages go up in the tariff imposing country relative to all other countries and profits go up in the tariff imposing country at the other countries' expense. Table 4a also lists the median optimal tariffs which reveal that the optimal tariffs tend to be higher for economically larger countries as one would expect.

Table 4b illustrates that the optimal tariffs as well as their welfare effects are strongly decreasing in the average elasticity. In particular, I take the original elasticity estimates from Table 1 and scale them proportionately to have a mean of 3.5, 5.0, or 6.5. I then redo all calculations required to construct Table 4a for these scaled values and report the last row of Table 4a in Table 4b. As can be seen, the average optimal tariffs more than halves when using elasticity estimates with mean 6.5 instead of mean 3.5 which simply reflects the fact that countries then have less monopoly power to exploit in world markets. Recall that the elasticity estimates average to 3.44 in the original calculations which is why the last row of Table 4a differs slightly from the first row of Table 4b.

In Ossa (2014), I also explore the case in which governments are politically motivated so that the welfare weights in the government preferences (3) deviate from 1. W_{js} is then defined as the welfare of industry s in country j which is just the nominal income accruing to the associated workers and firms deflated by the ideal aggregate price index. The welfare weights λ_{js} are normalized to satisfy $\frac{1}{S}\sum_{s=1}^{S}\lambda_{js}=1$ so that one dollar of income accruing to industry s in country j matters λ_{js} as much to the government as one dollar of income accruing to an industry which receives average political support. This is meant to capture political economy motives such as the ones emphasized by Grossman and Helpman (1994) in a reduced form way.

I show that the political economy weights can be calibrated such that the distribution of optimal tariffs matches the distribution of noncooperative tariffs from the data. These noncooperative tariffs include tariffs such as the so-called column-two tariffs of the US which are applied to countries with which the US does not have normal trade relations. With the exception of China, the predicted optimal tariffs are substantially higher than the measured noncooperative tariffs given the baseline elasticity estimates. However, the levels can also be brought in line much more closely if the higher elasticities from the sensitivity checks are used. The bottom line is that the average optimal tariffs and their average welfare effects are quite similar with and without political economy pressures. This is because political economy pressures are more about the intranational rather than the international redistribution of rents.

While the calibrated political economy weights appear highly plausible with the most favored sectors being wearing apparel, dairy, textiles, beverages and tobacco products, and wheat, they could also just capture other determinants of trade policy that the underlying model fails to account for. In order to investigate this possibility further, it would be interesting to relate the calibrated political economy weights to observables such as campaign contributions which the earlier empirical literature has emphasized (see, for example, Gawande and Krishna (2003) for an excellent overview).

3.4 Trade wars

Figure 5 illustrates the Nash tariffs of all countries using the same template as Figure 4. Recall that Nash tariffs capture the best-response tariffs that would prevail in a full-blown tariff war so that Figure 5 now summarizes a single policy experiment. As can be seen from comparing Figure 4 and Figure 5, the Nash tariffs are very similar to the optimal tariffs suggesting that the best response functions are again relatively flat just like in the simple US-Canada example discussed earlier. Using the optimal tariffs as a starting point, the iterative algorithm calculating Nash tariffs discussed above converges in around 20 minutes on my desktop so that the analysis could again easily be extended beyond 10 regions and 33 industries.

Table 5a summarizes the welfare effects of moving from factual tariffs to Nash tariffs. The real-world analog to this is a breakdown of trade policy cooperation escalating in a full-blown tariff war. As can be seen, all countries lose from the tariff war with the average welfare loss equalling -3.5 percent. The losses are most severe for Canada which is explained by the fact that Canada is the most open economy in the sample followed by the Rest of the World, Korea, and Russia who also lose a lot. The losses are least severe for Japan which is due to Japan's inefficient factual trade policy. In particular, Japan imposes extreme tariffs on agricultural products such as a 237 percent tariff on rice so that a move to Nash tariffs actually reduces its self-inflicted distortions significantly.

Table 5b again reports sensitivity checks for proportionately scaled versions of the elasticities. Just like optimal tariffs, Nash tariffs and their welfare effects are also strongly decreasing in the elasticities, as one would expect from the similarity between the two. I consider mean elasticities between 3.5 and 6.5 because this corresponds to the range of aggregate trade elasticities suggested by Simonovska and Waugh (2014). This range also captures what most empirical trade economists would regard as reasonable today so that the average Nash tariffs consistent with the model are somewhere between 25 percent and 57 percent. This is broadly consistent with the tariffs imposed during the trade war following the Great Depression which are typically reported to average around 50 percent.

3.5 Trade talks

Figures 6a, 6b, and 6c illustrate the outcomes of efficient trade negotiations starting at Nash tariffs, factual tariffs, and free trade, respectively. They are computed by implementing a bargaining protocol in the spirit of symmetric Nash bargaining just as discussed above. In particular, I first simulate the equilibria given Nash tariffs, factual tariffs, and free trade and then solve for the tariff change which maximizes country 1's welfare subject to the condition that all countries gain the same. This is supposed to simulate the outcome of perfect trade negotiations, that is the best-case scenario of what can be achieved under the WTO. Calculating these cooperative tariffs is very demanding computationally and takes approximately one full day per case on my desktop computer.

As can be seen from these figures, cooperative tariffs have very similar cross-industry distributions across all three cases but vary with respect to the average levels with which they apply. The cross-industry distributions reflect countries' attempts to correct a distortion originating from the fact that prices are too high in low elasticity industries. The cross-country distribution reflects countries' attempt to make implicit side payments ensuring that the bargaining protocol is satisfied and all countries gain the same. For that reason, the tariff levels vary across the three cases because the three different benchmarks require three different sets of side payments for all efficiency gains to be equally spread.

For example, we have seen above that Canada loses most as a result of the tariff war which is why all countries impose high tariffs against Canada in the case of trade negotiations starting at Nash tariffs summarized in Figure 6a. Essentially, Canada's terms-of-trade have to be sufficiently bad in equilibrium to ensure that Canada does not gain more than anyone else. Similarly, Japan faces the highest tariffs following the trade negotiations starting at factual tariffs summarized in Figure 6b because Japan would otherwise gain too much from dismantling its inefficient factual tariff regime. Of course, this would look different if one explicitly allowed for side payments, which might be more realistic since import subsidies are rarely seen.

Table 6a lists the welfare benefits associated with these trade negotiations. As can be seen, moving from Nash tariffs to cooperative tariffs improves each country's welfare by 3.5 percent so that this number can be seen as the maximum possible value of the WTO. In contrast, moving from factual tariffs to efficient tariffs only improves welfare by 0.4 percent suggesting that almost 90 percent of all possible welfare gains have already been realized in past trade negotiations. The finding that trade negotiations starting at free trade would only increase welfare by 0.1 percent confirms that tariffs are a poor instrument to address distortions. This should have been expected from the targeting principle which implies that distortions are best addressed with the appropriate direct policy instruments.

In Ossa (2014), I also explore the implications of the most-favored nation (MFN) clause of the WTO which generally prohibits countries from applying discriminatory tariffs against other countries. I start by recalculating optimal tariffs and Nash tariffs under the restriction of MFN to see if this clause by itself has any bite. I find that the welfare results are almost identical to the unrestricted case which also makes sense given the small amount of discrimination shown in Figures 4 and 5. I then consider if trade liberalizations of a group of insider countries affect nonparticipating outsider countries if the insider countries extend their concessions to the outsider countries in an MFN fashion. I find that these MFN tariffs cuts would actually overcompensate the outsiders for the trade diversion they experience which qualifies earlier results from Bagwell and Staiger (2005).

3.6 Discussion

If one is willing to take the model as a maintained hypothesis, the above results can be viewed as providing answers to questions of immediate policy relevance. For example, they illustrate what would happen if there was a complete breakdown of trade policy cooperation and quantify how much there is to gain from future tariff negotiations. If one instead takes a more cautious approach, they can be interpreted as a plausibility check on some of the leading models of trade policy making. For example, they show that the Nash tariffs are of the same order of magnitude as the tariffs observed during the trade war following the Great Depression and highlight that there is enough to gain from trade policy cooperation to plausibly justify the ordeal of real-world trade negotiations organized by the WTO.

While I believe that the model captures many important forces, I lean towards taking a cautious interpretation for now, simply because much more can be done to explore the model's validity. One obvious task seems to be to assess how sensitive the results are to alternative assumptions about the economic environment. For example, it would be interesting to explore what happens if one introduces intermediate goods and nontraded goods into the analysis which clearly feature prominently in the real world. Also, it would be fascinating to carefully confront the quantitative predictions of the model with the trade war following the Great Depression to see if the highlighted forces are consistent with what has been observed.

With that in mind, it is still insightful to elaborate on what the numbers suggest so far. In particular, if one combines the welfare effects from Tables 5a and 6a with data on manufacturing and agricultural value added from 2007, it is straightforward to calculate that a complete breakdown of trade policy cooperation would cost the world approximately \$300 billion per year while further tariff negotiations could bring about gains of approximately \$40 billion per year. So, to the extent that one credits the WTO for preventing the outbreak of trade wars and blames the WTO for failing to promote further trade talks, one has to conclude that the WTO's track record is quite impressive so far.[21]

4 Extensions

In this section, I discuss a number of promising directions in which this literature could be taken in future work. Overall, the trade policy literature so far is long on theory and short on quantification so that there is an abundance of opportunities for interesting research. I organize my discussion into three main categories, namely alternative models, other trade policy applications, and applications in other fields. As will become clear shortly, the applications in other fields I envision also have a trade policy character but answer questions which are normally associated with other literatures.

[21] These calculations refer to 2005 US dollars and are analogous to the ones I performed in a VOX column which can be accessed under http://www.voxeu.org/article/wto-success-no-trade-agreement-no-trade-war.

4.1 Alternative models

Perhaps the most obvious direction for future work in this area is to analyze optimal tariffs, Nash tariffs, and cooperative tariffs in a range of alternative gravity models applying the techniques introduced above. The Handbook of International Economics chapter of Costinot and Rodriguez-Clare (2015) is a great resource for researchers interested in this area because it provides a comprehensive overview of how the many different gravity models relate. In fact, their chapter already includes a short section on the effects of tariffs in various gravity models which can serve as an excellent starting point.

An important contribution of such analyses would be to assess the robustness of the results provided so far. As mentioned earlier, introducing intermediate goods should be a high priority in my opinion given how central they are to international trade. In the process of writing this chapter, I have actually experimented with this already but ultimately concluded that it requires a more thorough treatment than I can offer here. One complication is that intermediate goods introduce a factor market distortion into the Ossa (2014) model giving rise to a new margin for optimal tariffs to manipulate. In particular, firms then buy labor on perfectly competitive markets but intermediate goods on imperfectly competitive markets so that relative intermediate goods prices are too high. While this could turn out to be an interesting mechanism, it might make more sense to first characterize optimal tariffs with intermediate goods in a perfectly competitive model such as Caliendo and Parro's (2015) extension of Eaton and Kortum (2002).

Another interesting extension would be to relax the exclusive focus on tariffs and allow for alternative policy instruments. In my assessment, it would be particularly useful to disentangle the questions of optimal allocation and implementation by following the primal approach of the public finance literature. An interesting example of this is the recent analysis by Costinot et al (2015) who solve for optimal trade policy in a Ricardian trade model. They first assume that governments can directly control the resource allocation as social planners and then ask how their preferred allocations can be implemented with a particular set of policy instruments. While their paper is predominantly theoretical, they also provide some quantifications which would be interesting to expand upon.

4.2 Other trade policy applications

While a quantification of optimal tariffs, Nash tariffs, and cooperative tariffs is probably a natural first application of the above techniques, they could really be used to connect most branches of the theoretical trade policy literature to the data. As an illustration, let me focus on the part of the theoretical literature which emphasizes the world trading system's institutional design. In a nutshell, WTO members are supposed to concentrate all protective measures into tariffs, apply these tariffs on a nondiscriminatory basis, and change these tariffs in a reciprocal fashion. Influenced by Bagwell and Staiger (1999), an extensive literature has studied the implications of these and other WTO principles and their exceptions for the efficiency of trade negotiations.

Consider first the principle of nondiscrimination which has the important exception that WTO members are allowed to enter into preferential trade agreements under some conditions. This exception is controversial in the theoretical literature for three main reasons. First, there is a concern that joining preferential trade agreements causes trade diversion which can even make the parties to the preferential trade agreement worse off (Viner, 1950). Second, there is a discussion whether preferential trade agreements encourage or discourage further liberalizations and are therefore building blocks or stumbling blocks on the way to free trade (Bhagwati, 1991). And third, there is an argument that MFN tariffs protect outsiders to reciprocal trade liberalizations from any externalities and work in conjunction with the principle of reciprocity to guide countries to the efficiency frontier (Bagwell and Staiger, 1999).

A quantitative analysis could shed some new light on this debate. First of all, the equilibrium conditions in changes summarized in Definition 2 could be used to quantify the extent of trade diversion real-world trade agreements bring about. My conjecture is that these effects are negligible when focusing on tariff changes alone but that they might become more important when changes in non-tariff barriers are also taken into account. Nowadays, many preferential trade agreements also reduce non-tariff barriers as part of some deep integration process and measuring such reductions convincingly would be the key challenge to overcome.

Moreover, a modified version of the optimization procedure summarized in Problem 2 could be used to assess which preferential trade agreements countries would optimally sign. Besides speaking to the building blocks versus stumbling blocks debate, this could readdress the influential question asked by Baier and Bergstrand (2004) whether the observed proliferation of preferential trade agreements is consistent with welfare maximization. In this context, it could also be explored how members of preferential trade agreements would optimally adjust their external tariffs thereby assessing the abovementioned Bagwell and Staiger (1999) argument.

Consider now the principle of reciprocity which is only a norm during phases of trade liberalization but binds more strictly when retaliatory actions are concerned. In particular, WTO member countries are authorized to withdraw substantially equivalent concessions if one of their trading partners reneges on previously made tariff commitments. In practice, this institutionalized threat of retaliation tends to prevent trading partners from reneging in the first place and can thus be interpreted as an explicit tit-for-tat rule aimed at ensuring that trade agreements are self-enforcing.

A quantitative analysis could help determine how much bite this interpretation has. In particular, the welfare effects associated with optimal tariffs summarized in Table 4a quantify the value of unilateral deviations from the status quo. Also, the welfare effects associated with Nash tariffs summarized in Table 4b quantify the costs of a breakdown of trade policy cooperation which a unilateral deviation would likely entail. Given a discount factor, one could now calculate the most cooperative tariffs which can be sustained in a repeated game in the spirit of Bagwell and Staiger (1990). In doing so, one could also distinguish between bilateral and multilateral enforcement mechanisms along the lines of Maggi (1999).

Moreover, it would be interesting to quantify how important it is for the success of trade negotiations that the principle of reciprocity applies multilaterally instead of bilaterally. As is discussed more extensively in Bagwell et al (2015) as well as Chapter 8 of this Handbook, one of the key innovations of the General Agreement on Tariffs and Trade (GATT) relative to the earlier US Reciprocal Trade Agreements Act was to relax the bilateral reciprocity constraint and replace it with a multilateral one. The basic idea is that it is easier to find an overall balance of concessions if it does not have to apply individually for each country pair.

4.3 Applications in other fields

While these examples should suffice to illustrate that there are many interesting opportunities for quantitative work within the traditional trade policy literature, it is also easy to think of closely related applications in other fields. For example, I use similar tools in Ossa (2015) to study subsidy competition among US states in a quantitative economic geography model. US states spend substantial resources to subsidize the relocation of firms from other states. I show that this is consistent with welfare maximization because firm relocations allow states to benefit at the expense of other states. I also show that observed subsidies are much closer to cooperative than to noncooperative levels but that the potential costs of an escalation of subsidy competition are large.

It would be interesting to extend this analysis beyond the domestic US economy and analyze subsidies offered to attract foreign firms. This could then be tied in again with the traditional trade policy literature because WTO rules also limit the foreign direct investment (FDI) policies governments can apply. For example, the Trade Related Investment Measures (TRIMS) agreement limits the local content requirements which can be imposed on foreign-owned firms. Also, FDI flows are increasingly subject to bilateral investment treaties which would be fascinating to analyze.

Another promising area is climate policy. For example, Nordhaus (forthcoming) argued in his 2015 Presidential Address to the American Economic Association that small tariff penalties could be a powerful tool to entice countries to join climate clubs which impose stricter policies fighting climate change. His argument is based on the so-called Coalition Dynamic Integrated Model of Climate and the Economy (C-DICE) which is a quantitative model developed by Nordhaus and his team. It uses the results from Ossa (2014) to construct a reduced-form trade benefit function which is needed to quantify the effectiveness of tariff penalties.

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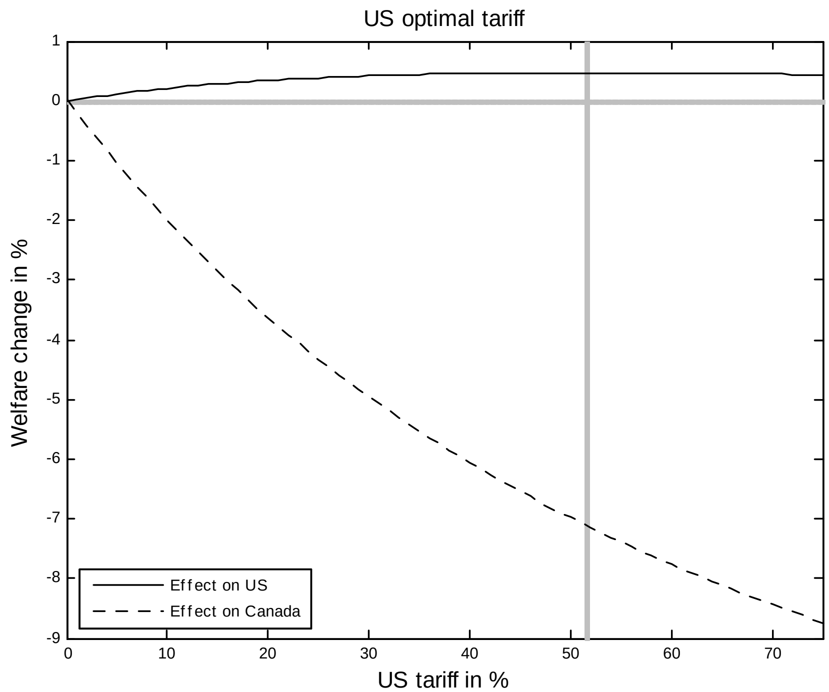

Figure 1a: US optimal tariff in simple example

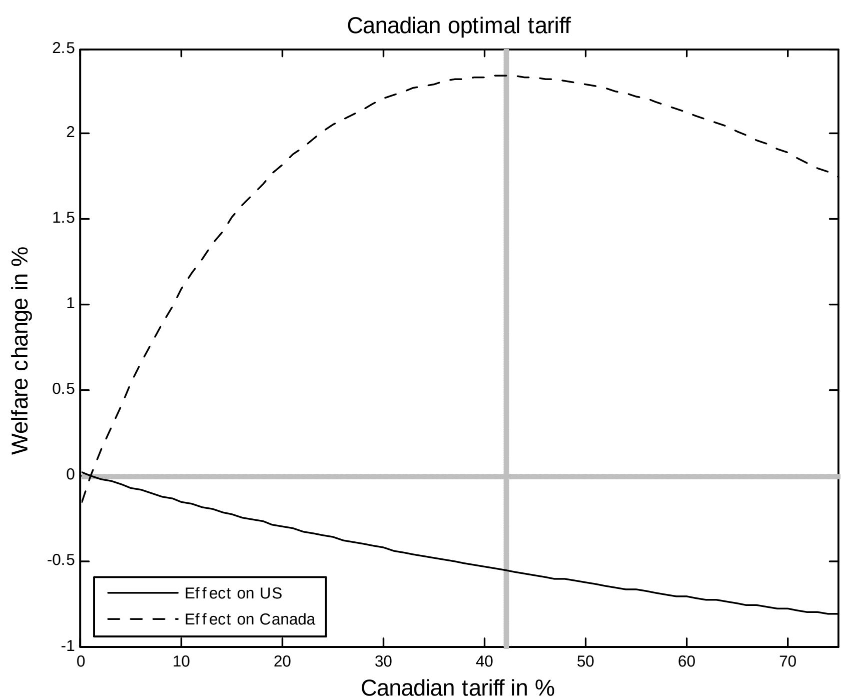

Figure 1b: Canadian optimal tariff in simple example

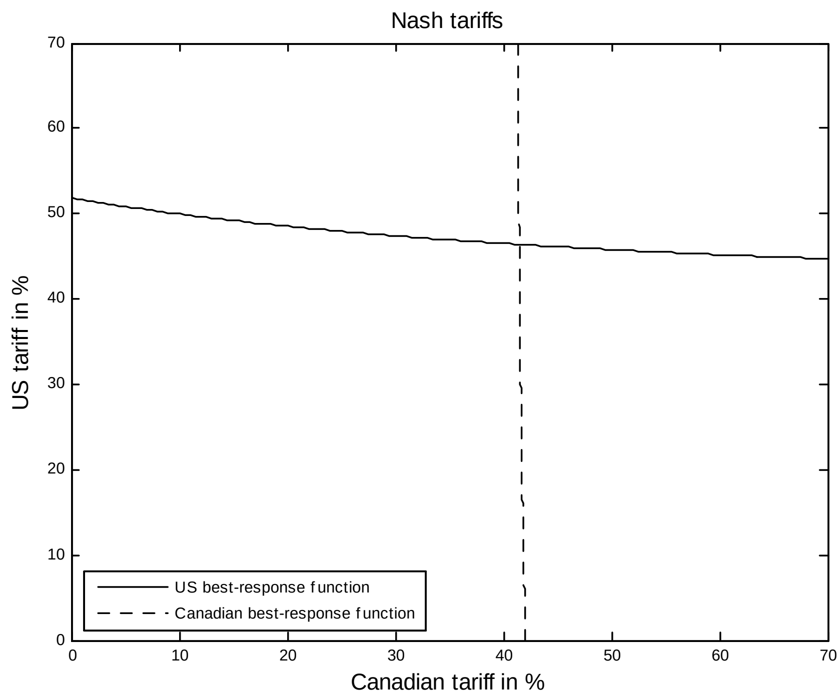

Figure 2: Reaction functions in simple example

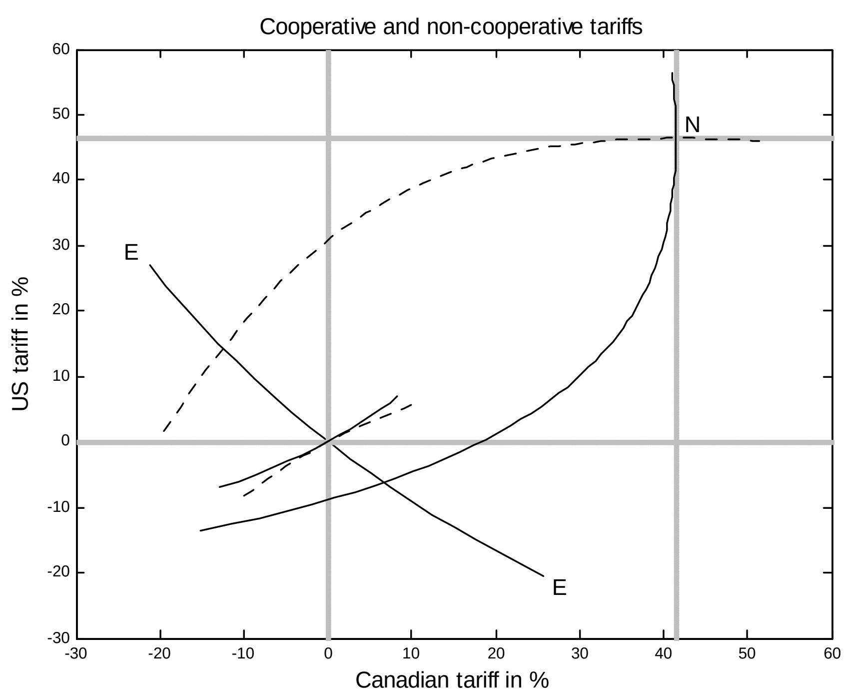

Figure 3: Trade wars and trade talks in simple example

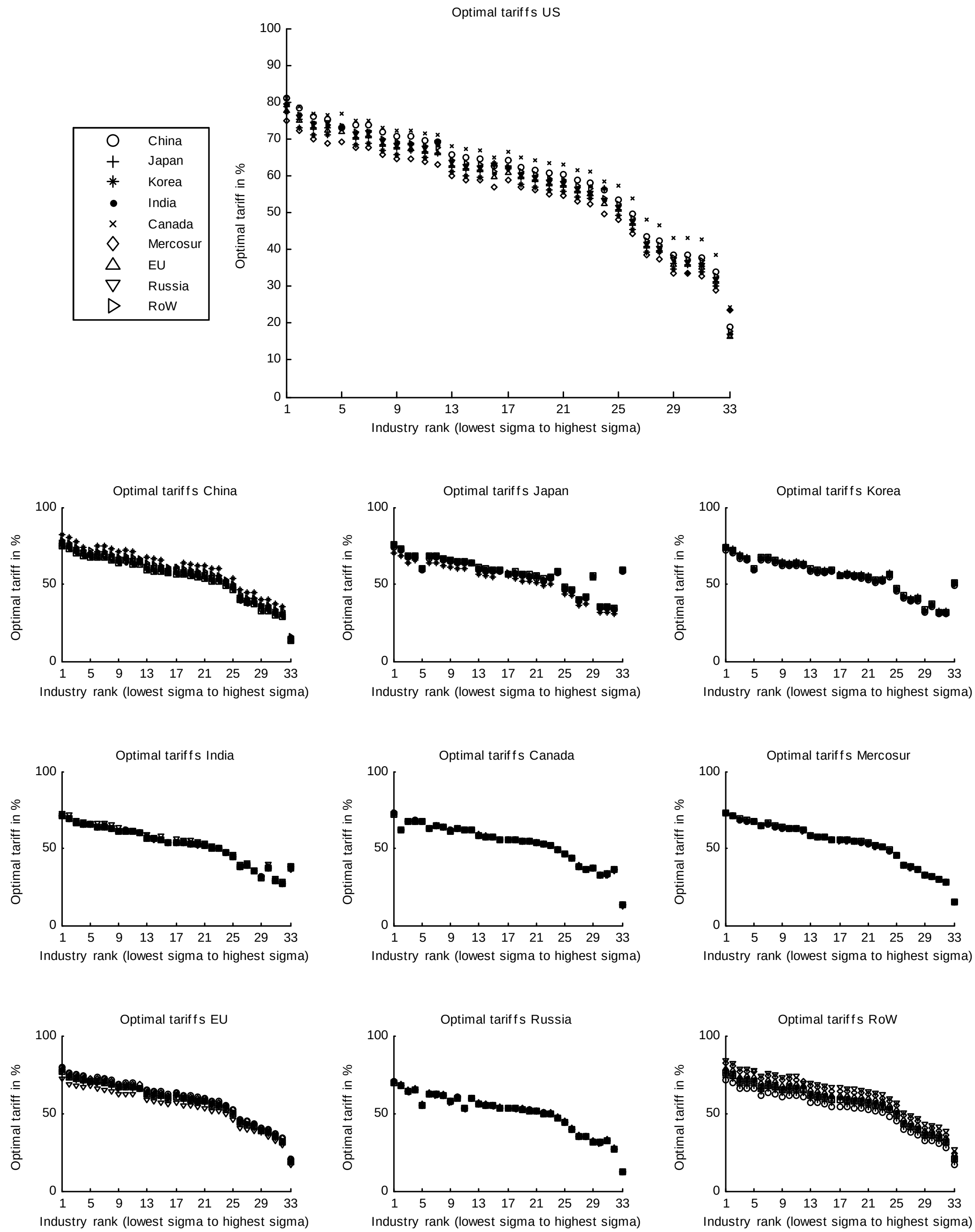

Figure 4: Optimal tariffs

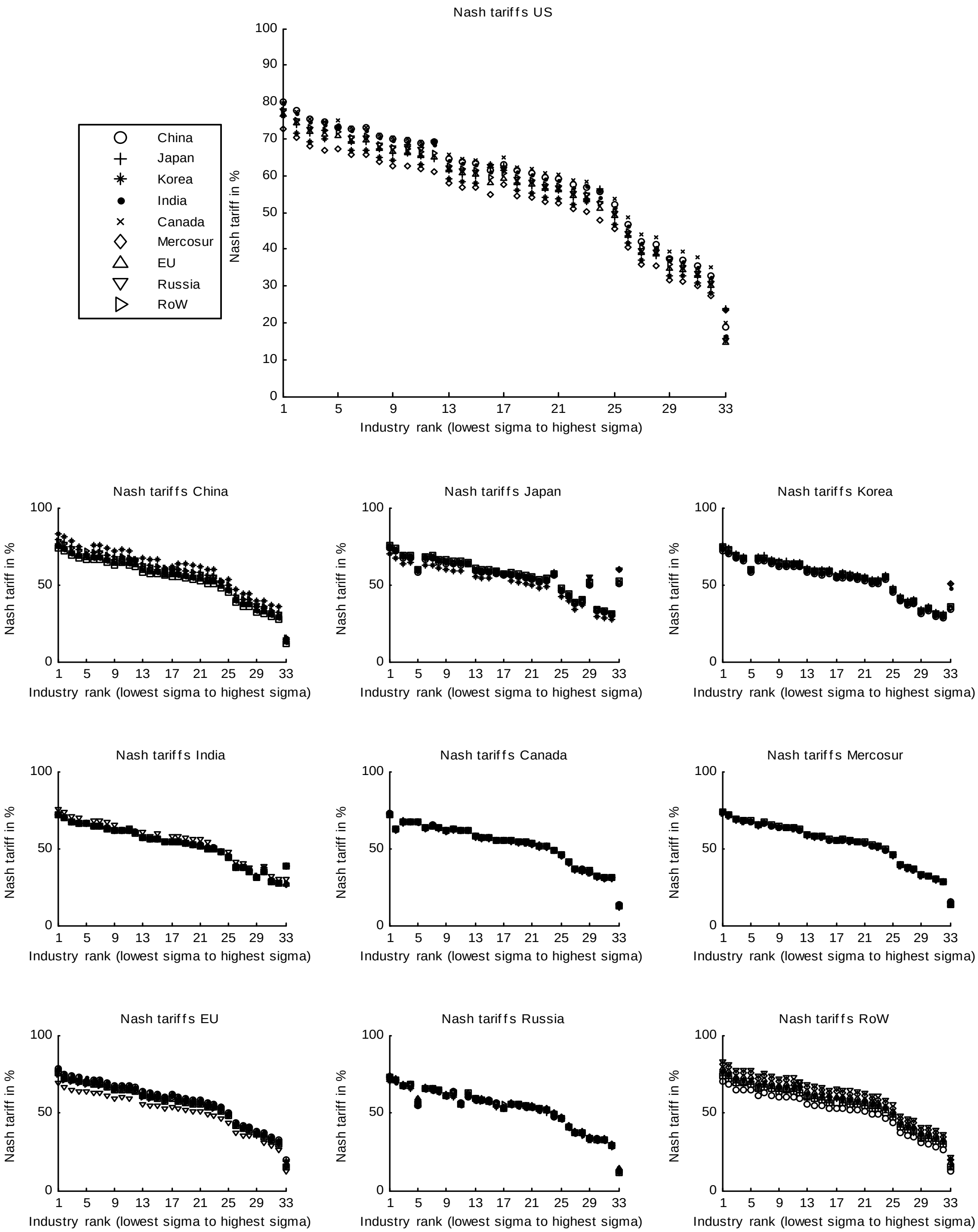

Figure 5: Nash tariffs

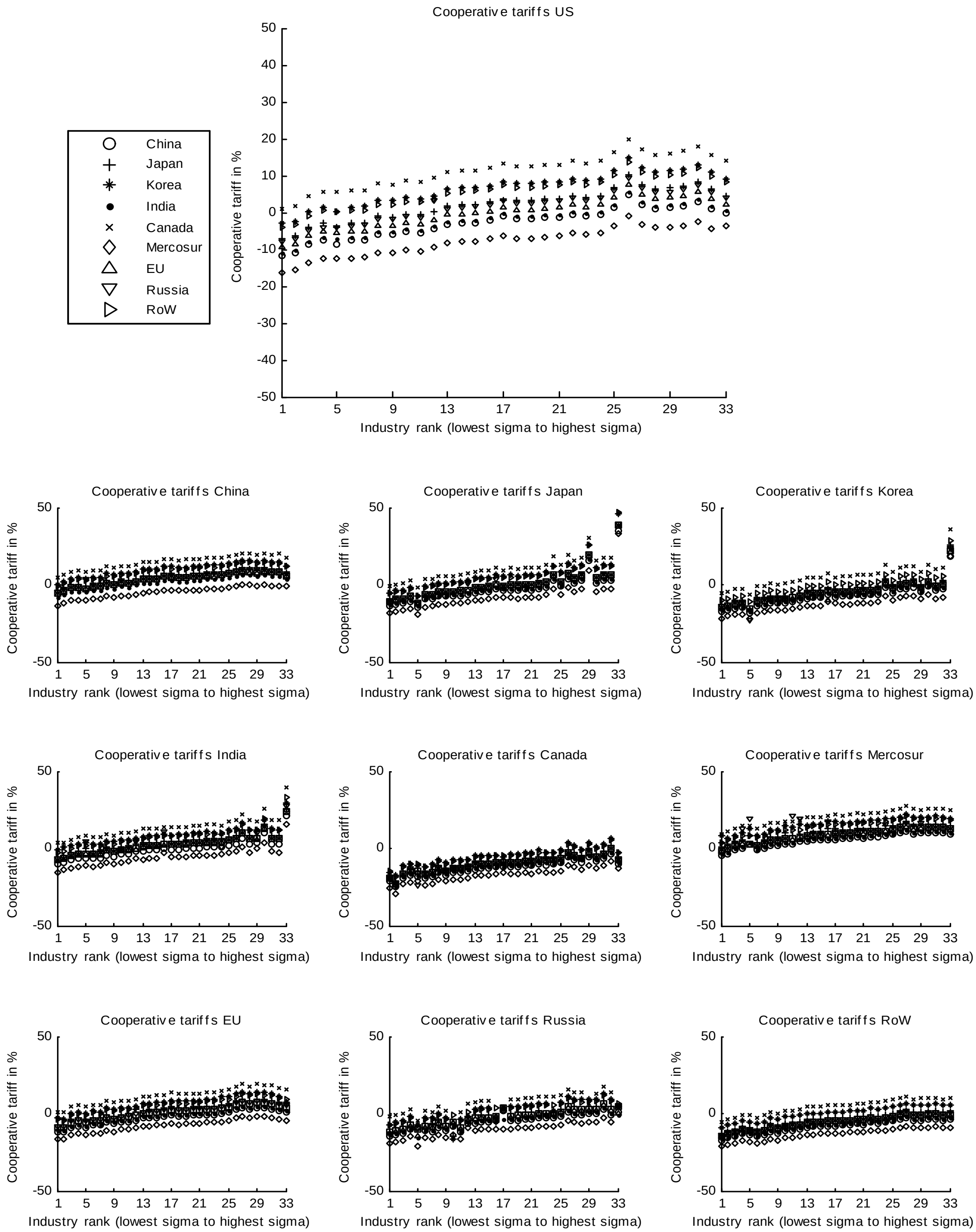

Figure 6a: Trade negotiations starting at Nash tariffs

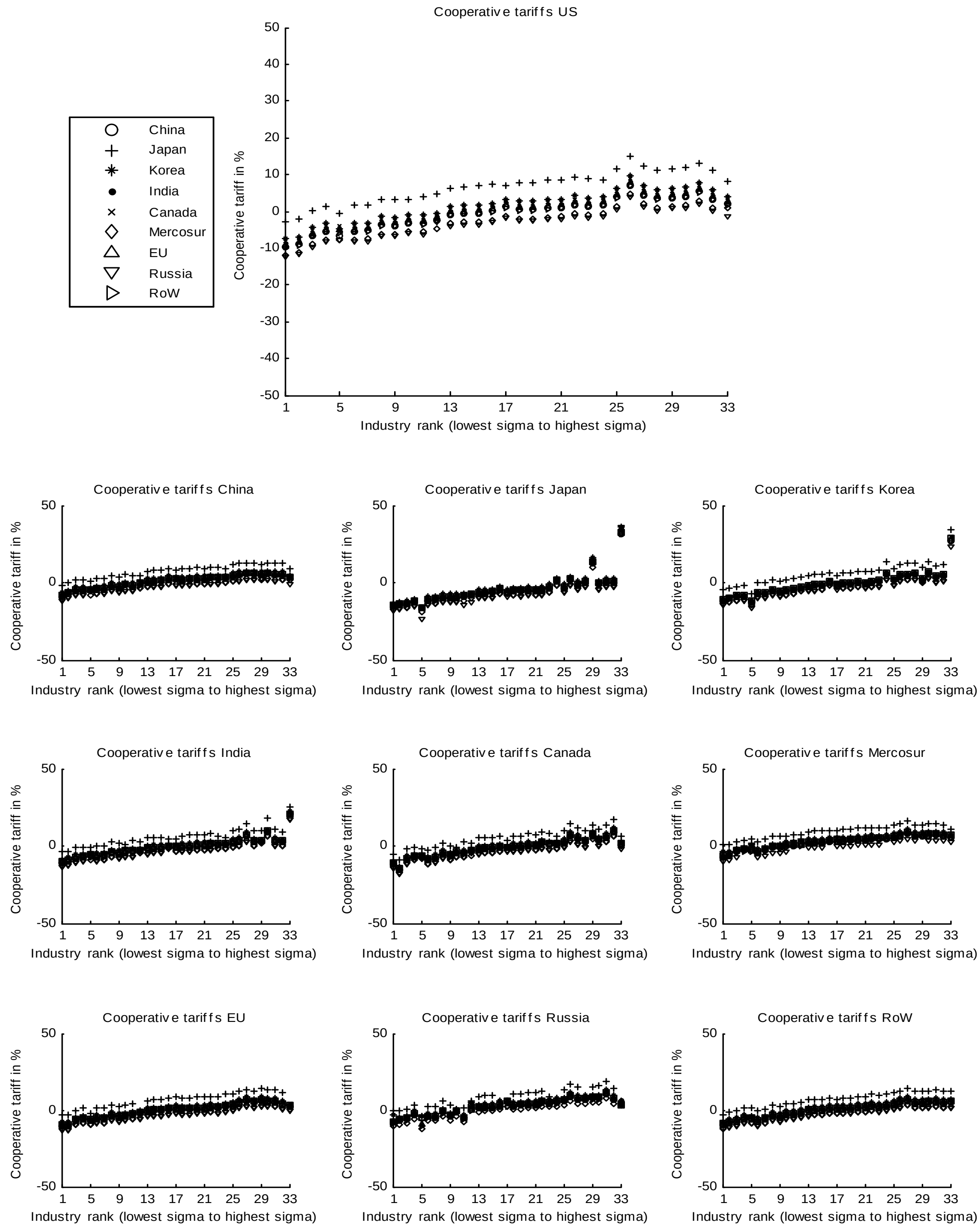

Figure 6b: Trade negotiations starting at factual tariffs

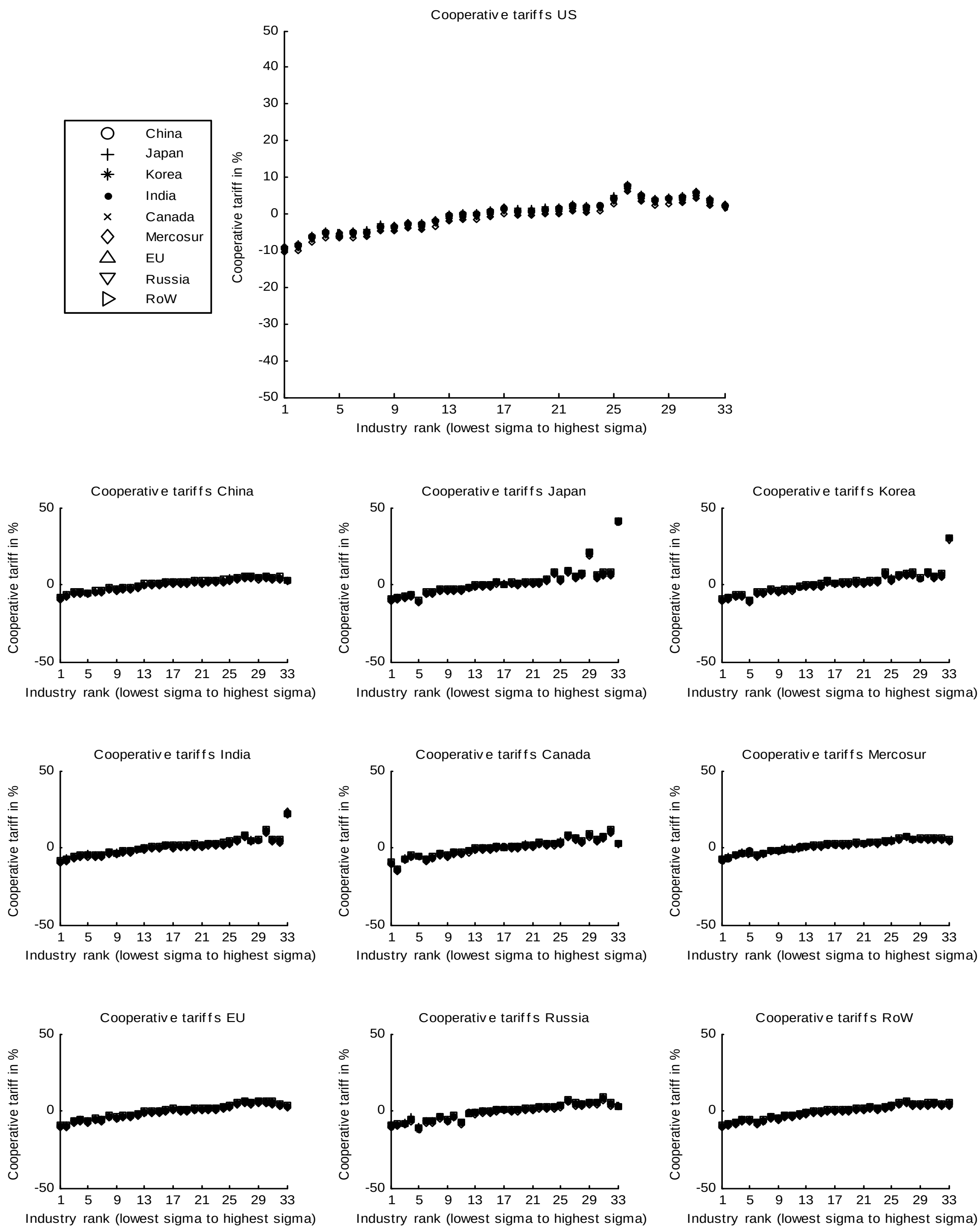

Figure 6c: Trade negotiations starting at free trade

TABLE 1: Elasticity estimates

Wheat	12.37	Oil seeds	2.89
Dairy	5.60	Metal products	2.79
Wearing apparel	5.31	Other food products	2.78
Vegetable oils, etc	4.98	Paper products, etc.	2.73
Rice	4.87	Bovine cattle, etc.	2.58
Bovine meat products	4.39	Other crops	2.54
Other metals	4.38	Sugar	2.52
Leather products	4.11	Electronic equipment	2.49
Other manufactures	3.52	Other mineral products	2.47
Other cereal grains	3.29	Chemical products, etc.	2.37
Other meat products	3.14	Other machinery, etc.	2.37
Motor vehicles, etc	3.13	Plant-based fibers	2.33
Ferrous metals	3.01	Forestry	2.33
Other transport equipment	2.99	Wood products	2.29
Beverages, etc.	2.93	Vegetables, etc.	2.19
Textiles	2.90	Other animal products	2.12
Wool, etc	2.89	Mean	3.44

Notes: These are the elasticities of substitution estimated following the Feenstra (1994) method for the 33 GTAP industries included in the analysis.

TABLE 2: Effects of eliminating aggregate trade deficits

	net exports (in %)	Δ exports (in %)	Δ imports (in %)
Canada	-2	-7	-11
China	21	-17	28
EU	8	-10	5
India	-4	1	-8
Japan	28	-17	46
Korea	20	-11	34
Mercosur	18	-17	21
RoW	-11	9	-13
Russia	-29	24	-32
US	-22	16	-26

Notes: "net exports" refers to (exports-imports)/(exports+imports) in the raw data, "Δ exports" refers to the change in exports resulting from setting aggregate trade deficits equal to zero, and "Δ imports" refers to the change in imports resulting from setting aggregate trade deficits equal to zero.

TABLE 3: Effect of a 50 percentage point increase in US tariff

	Δ US wage (in %)	Δ US production in protected (in %)	Δ US production in other (in %)
Chemicals	1.52	5.85	-1.41
Apparel	0.69	33.23	-0.99
	Δ US welfare (in %)	**Terms-of-trade effect (in %)**	**Profit shifting effect (in %)**
Chemicals	0.17	0.35	0.12
Apparel	-0.13	0.16	-0.14

Notes: The entries in the upper panel are the wage change of the US normalized such that the average wage change across all countries equals zero, the change in the quantity of output in the US chemicals or apparel industry, and the average change in the quantity of output in all other US industries. The entries in the lower panel list the associated welfare effects decomposed into terms-of-trade and profit shifting effects according to the formula in the main text. The terms-of-trade and profit shifting effects do not add up to the overall welfare effects because they are computed using a linear approximation.

TABLE 4a: Optimal tariffs

	Δ welfare (in %)		Δ wage (in %)		Δ profits (in %)		tariff (in %)
	own	other	own	other	own	other	median
China	1.6	-0.9	23.3	-2.6	0.7	-0.2	58.8
Japan	3.4	-0.4	19.4	-2.2	1.1	-0.1	58.3
Korea	2.5	-0.2	19.8	-2.2	0.0	0.0	56.5
India	2.0	-0.1	8.9	-1.0	3.2	-0.2	54.0
Canada	4.5	-0.1	14.8	-1.6	3.8	-0.1	55.6
US	2.4	-1.6	26.0	-2.9	0.6	-0.3	61.0
Mercosur	1.1	-0.1	17.6	-2.0	1.3	-0.1	55.2
EU	1.9	-1.3	24.0	-2.7	0.2	-0.3	60.6
Russia	1.9	-0.1	13.9	-1.5	2.6	0.0	52.9
RoW	2.7	-1.2	19.6	-2.2	1.1	-0.5	60.3
Mean	2.4	-0.6	18.7	-2.1	1.5	-0.2	57.3

TABLE 4b: Sensitivity of optimal tariffs wrt. σ

σ	Δ welfare (in %)		Δ wage (in %)		Δ profits (in %)		tariff (in %)
mean	own	other	own	other	own	other	median
3.5	2.4	-0.6	18.2	-2.0	1.5	-0.2	55.9
5.0	1.7	-0.3	9.7	-1.1	1.5	-0.2	33.8
6.5	1.5	-0.2	6.1	-0.7	1.5	-0.2	24.2

Notes: The entries under "welfare" are the changes in real income, the entries under "wage" are the changes in wages normalized such that the average wage change across all countries equals zero, the entries under "profits" are the changes in profits due to changes in industry output, and the entries under "tariff" are the optimal tariffs. The columns labelled "own" refer to effects on the tariff imposing country while the changes labelled "other" refer to the average of the effects on all other countries. The last row of Table 4a reports averages. Table 4b reports only such averages.

TABLE5a: Nash tariffs

	Δ welfare (in %)	Δ wage (in %)	Δ profits (in %)	tariff (in %)
China	-2.1	4.4	0.1	58.2
Japan	-1.5	0.0	0.1	57.5
Korea	-4.6	-0.1	-1.2	56.2
India	-1.9	-9.4	2.4	54.1
Canada	-7.8	-5.1	0.5	54.9
US	-2.4	6.1	-0.5	60.1
Mercosur	-1.9	1.6	1.0	55.3
EU	-2.5	4.2	-1.0	59.0
Russia	-4.7	-1.2	-0.2	55.2
RoW	-5.5	-0.4	-0.9	59.6
Mean	-3.5	0.0	0.0	57.0

TABLE5b: Sensitivity of Nash tariffs wrt. σ

σ	Δ welfare (in %)	Δ wage (in %)	Δ profits (in %)	tariff (in %)
3.5	-3.4	0.0	0.0	55.6
5.0	-1.8	0.0	0.2	33.9
6.5	-1.1	0.0	0.2	24.6

Notes: The entries under "welfare" are the changes in real income, the entries under "wage" are the changes in wages normalized such that the average wage change across all countries equals zero, the entries under "profits" are the changes in profits due to changes in industry output, and the entries under "tariff" are the Nash tariffs. The last row of Table 5a reports averages. Table 5b reports only such averages.

TABLE 6a: Cooperative tariffs

	Δ welfare (in %)			Δ wage (in %)			Δ profits (in %)		
	Nash	Fact.	Free	Nash	Fact.	Free	Nash	Fact.	Free
China	3.5	0.4	0.1	1.8	2.1	-0.1	-0.2	-0.2	0.3
Japan	3.5	0.4	0.1	-0.6	-6.7	0.7	1.4	1.4	0.0
Korea	3.5	0.4	0.1	-4.9	-0.6	0.4	1.6	0.3	0.0
India	3.5	0.4	0.1	7.1	-6.8	0.1	-0.8	1.4	0.3
Canada	3.5	0.4	0.1	-5.4	-0.9	-0.4	0.7	1.6	1.1
US	3.5	0.4	0.1	-2.8	2.1	-0.1	0.5	0.0	0.3
Mercosur	3.5	0.4	0.1	10.0	3.9	0.0	-1.5	-0.6	0.3
EU	3.5	0.4	0.1	-0.7	2.0	0.2	1.2	0.2	0.1
Russia	3.5	0.4	0.1	-0.2	3.8	-0.7	-2.0	-2.0	1.1
RoW	3.5	0.4	0.1	-4.5	1.1	-0.2	1.4	0.5	0.7
Mean	3.5	0.4	0.1	0.0	0.0	0.0	0.2	0.3	0.4

TABLE 6b: Sensitivity of cooperative tariffs wrt. σ

σ	Δ welfare (in %)			Δ wage (in %)			Δ profits (in %)		
mean	Nash	Fact.	Free	Nash	Fact.	Free	Nash	Fact.	Free
3.5	3.4	0.4	0.1	0.0	0.0	0.0	0.2	0.3	0.4
5.0	2.1	0.4	0.0	0.0	0.0	0.0	-0.2	0.0	0.2
6.5	1.6	0.5	0.0	0.0	0.0	0.0	-0.3	-0.2	0.2

Notes: The entries under "welfare" are the changes in real income, the entries under "wage" are the changes in wages normalized such that the average wage change across all countries equals zero, the entries under "profits" are the changes in profits due to changes in industry output, and the entries under "tariff" are the cooperative tariffs. The columns labelled "Nash", "Fact.", and "Free" refer to trade negotiations stating at Nash tariffs, factual tariffs, and zero tariffs, respectively. The last row of Table 6a reports averages. Table 6b reports only such averages.