Inequality, Market Structure, and Innovation

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Contents

Abstract

This thesis analyzes the impact of research productivity, inequality and patent breadth on deterministic horizontal innovations (i.e. product invention) and stochastic vertical innovations (i.e. cost reduction) in a one-period closed-economy framework with non-homothetic consumer preferences. Inequality tends to reduce vertical innovations that thrive on the purchasing power of the poor and fosters horizontal innovations that target the fancies of the rich. However, the most intriguing result is that increasing incentives for vertical innovation, either by a higher productivity of the researchers or a weaker leading breadth, benefit horizontal innovation contrary to the literature on sequential innovations.

Chapter 1

Introduction

During the history of mankind, inventions have always played a crucial role for development and progress. Be it the discovery of tools and the invention of the wheel in early pre-historic civilizations, the improvement in warfare that lead to the vast realm of the roman empire, the printing press or the advances in shipbuilding and navigation during the age of discovery.
Although the impact of these inventions is felt unil today, they were mostly driven by curiosity or served the means of a single monarch. In the course of the industrial revolution a strengthened patent law lead to an increasing interest by enterpreneurs in inventions and their commercial application as innovations. Prominent examples are James Watt's steam engine, Samuel Crompton's spinning mule or James Beaumont Nielson's hot blast. The commercial application of these inventions enabled a rapid increase in productivity and as such fostered economic growth.
With the ignition of innovations set in the industrial revolution they have eversince played a crucial role in economies and for growth especially. This includes 20^{th} century pioneers in transportation like the automotive industry and aviation who facilitated trade and led to a more connected world, or innovations in information and communication technologies like the (mobile)phone, the television or the computer and the internet which are crucial drivers of growth in our modern, service-oriented economices. With an ever growing importance of innvovations throughout history it almost feels as if today only a constantly innovating firm can succeed in our ephemeral modern world. In 2016, Volkswagen, the leading private R&D firm in terms of expenditures, invested a staggering 13.2 billion dollars in research and development while the top ten firms' expenditures amounted to 103.3 billion[1], roughly the GDP of Morocco in 2016 or Ecuador in 2014[2]. Another colourful statistic that emphasizes the prominence of innovations are the 2.9 million patent applications and 1.2 million patent grants recorded worldwide

[1]Source: The Global Innovation 1000
[2]Source: World Bank Database

in 2015 by the WIPO[3].
This thesis tries to shed some light on the mechanisms that drive innovation. It does so by assuming a framework with non-homothetic consumer preferences, a closed economy[4] and one sole time period with a preceding two-stage innovation process. In the deterministic first stage, firms can freely enter a horizontal R&D market that is concerned with the invention of a new consumer good, a new variety. The stochastic second stage features a vertical R&D market with limited entry where a successful innovator discovers a more efficient production technique for an already existing variety invented in the first stage.
In the first part, we refrain from inequality and patent breadth and simply assume a completely egalitarian society and patents that solely protect the discovered production technique of a variety. In such a setting, an increasing productivity of researchers in the second stage not only raises the incentives for vertical innovation but also extends the range of goods invented in the first stage. This contrasts the existing literature featuring sequential innovations, where each entrant challenges the incumbent's market. As such, innovations that build upon already existing products destroy the old market structures and create new ones embeding the idea of creative destruction mentioned by Schumpeter [1942]. It thus seems only natural that stronger vertical innovation incentives reduce the incentives for horizontal innovation, especially when lacking any protection in form of a blocking patent. Yet, the opposite is the case and horizontal innovations flourish with an increasing presence of process innovations. As we will see, this seemingly peculiar phenomenon stems from the resource-efficient production technique of the process innovators and the redistribution of firm profits to the consumers. On the one hand, a bigger vertical innovation rate reduces the average labor intensity in production releasing the necessary resources for the development of new varieties on the other hand it increases a representative consumer's profit income that is needed to purchase these additional varieties.
In the second part, I investigate the impact of inequality by introducing a two-class society of rich and poor. If we were to assume homothetic preferences, poorer individuals would purchase the same set of goods as the rich but in a smaller quantity. Therefore, a mean-preserving spread of the income distribution would not affect the aggregate demand of a firm and thus, changes in inequality would have no impact on the innovation activity. Therefore, following the approaches of Falkinger [1994] and Zweimüller [2000], I assume non-homothetic preferences. More precisely, the focus is set on the extensive margin (i.e. whether or not to consume) rather than the intensive margin (i.e. how much to consume) of consumption. That is,

[3] Source: WIPO statistics database

[4] Open economy endogenous growth models featuring a product life cycle à la Vernon [1966] include Flam and Helpman [1987], Segerstrom et al. [1990], Grossman and Helpman [1991b]

with an increasing income, a consumer turns to new varieties that weren't part of his consumption basket before rather than purchasing more units of the same set of goods in which he is already satiated. Thus, a wealthier individual will indeed buy a broader consumption basket opposed to a poorer one such that inequality matters for the demand of an innovator and hence for their innovation incentives.

Following the terminology of Foellmi et al. [2014], I analyze the impact of an increasing income gap and an increasing income concentration. The former reduces the income share of the poor and increases the one of the rich. As such, an income concentration always fosters horizontal innovation and might have a detrimental effect on the vertical innovation rate. The latter increases the fraction of poor individuals but leaves their income unchanged such that the residual income is concentrated among fewer rich individuals resulting in higher incomes for them. If the fraction of poor consumers is relatively high, an increasing income concentration raises the amount of produt inventions and leaves the vertical innovation incentives unchanged. If however, the fraction is small enough such an increase in inequality lowers the vertical innovation rate and has an ambiguous impact on horizontal innovation.

In the third an last part, I analyze the impact of leading patent breadth on both innovation types within an egalitarian society. The introduction of a leading breadth implies that any superior version of a product can be blocked by the current incumbent, such that the second stage process innovator can only commercially produce the variety after a license payment to the product inventor. I only consider ex-ante licensing such that the fee, that defines the fraction of profits acruing to the horizontal innovator in case the new process is actually discovered, is determined before the vertical innovator engages in his R&D activity.[5] As expected, a stronger patent protection of the incumbent by means of an increase in the license payment reduces the vertical innovation rate, however, it also reduces the number of product inventions. Therefore, patent breadth has a detrimental impact on the type of innovations it is desigend to protect. This counter-intuitive result is yet again driven by the redistribution of firm profits. The free entry into the horizontal R&D market and the limited entry into the vertical R&D market imply that the sole profits of this economy stem from process innovation. As such, a lowered vertical innovation rate by means of a higher license fee reduces total profits and thus a representative consumer's income restricting him to purchase a narrower range of goods reducing the number of horizontal innovations.

[5]If we were to consider ex-post licensing, the vertical innovator would have to form beliefs about the fraction of revenue he has to pay to the incumbent in order to employ the optimal amount of researchers. As we want to abstract from beliefs we refrain from such a scenario.

Innovations are one of the most important drivers of growth, be it by introducing more productive technologies, by allowing for a more efficient utilization of scarce or depletable resources or simply by intesifying the competitive pressure for incumbent firms which in turn are forced to be innovative as well. As such, they have also found their way into economic theory and especially into growth theory. While the neoclassical growth models, influenced by the work of Solow [1956] and Ramsey [1928], focus on pure capital accumulation and thus feature a more investment-driven growth process, the endogenous growth theory puts its emphasis on innovation-driven growth. The main distinction within these models lies within the type of innovation that is either horizontal or vertical.
The former models, influenced by the paper of Romer [1990] ,Grossman and Helpman [1990] and Grossman and Helpman [1991b], feature innovations that introduce new, previously inexistent products such that in the process of growth the variety of goods is ever expanding.
The latter models feature innovations that improve already existing goods. This improvement either takes the form of an increased product quality or a better production process and the concomitant lower production costs. While I focus on the latter, the most important contributions to the endogenous growth theory made by Aghion and Howitt [1992], Grossman and Helpman [1991a] and Segerstrom et al. [1990] focus on the former, featuring a sequential upgrading of the product quality for a fixed range of goods.
Besides innovations, another influence on growth has recieved growing interest in recent years, namely inequality. One of the earliest contributions to the growth-inequality literature comes from Kuznets [1955] who noted an inverse u-shaped relationship such that in the course of economic development inequality first increases and later on decreases. The two other main bodys of work focus either on capital market imperfections and the concomitant restrictions for human capital investment (Galor and Zeira [1993], Matsuyama [2011]) or a politico-economic explanation, where an increasing incequality reduces the income of the median voter who in turn demands a higher redistributive taxation of the accumulating factor capital, thus reducing growth (Alesina and Rodrik [1994],Persson and Tabellini [1994]).
Another possibility of inequality to affect the long-run growth rate is via its impact on innovations. This is exactly what is done in the models featuring demand-induced innovations where inequality affects the innovation incentives by means of the innovators' demand schedules. As previously mentioned, mean-preserving spreads of the income distribution would have no effect on the demand of an innovator in case of homothetic preferences. As such, a prerequisite for these models is the introduction of non-homothetic preferences.
The earliest contribution to this body of work comes from Falkinger [1994] who assumes a hierarchic consumer demand and horizontal innovations only. An increasing inequality extends the excess income of the rich over the poor

leading to the creation of new luxuries for the rich and thus the extension of the range of goods. If technical change is driven by the stock of knowledge as measured by the number of innovations, a frequently made assumption in the endogenous growth theory, inequality increases growth.

Zweimüller [2000] builds upon the assumption of hierarchic preferences and assumes that innovations reduce the production costs opposed to a freely accesible backstop technology. More basic needs are thus satisfied by innovative firms and more fancy needs by the traditional firms. Focusing on the time path of demand, he argues that an increasing inequality harms growth since it takes the now poorer individuals a longer time to purchase the most recent innovators product while the even richer turn their excess demand to the traditional sector.

The results in the previous papers rely on fixed prices and thus focus on the demand effect of inequality while completely abstracting from any price effects. Foellmi and Zweimüller [2006] assume that both effects are present and note that an increasing inequality has a positive price effect and a negative demand effect on the innovation incentives. The former comes about an increase in the rich's willingness to pay while the latter is either due to a reduced initial market size or an extended time until the poor can buy the good. The price effect always dominates the demand effect such that inequality benefits innovation and promotes growth.

Foellmi and Zweimüller [2017] reintroduce a competitive fringe in the fashion of Zweimüller [2000] that absorbs the residual demand to study the impact of inequality on innovation incentives. A redistribution of the poor to the rich has a positive price effect, by increasing the willingness to pay of the latter, but also has a negative market size effect, by diverting demand away from the innovative goods towards the non-innovative ones. Since the presence of a competitive fringe limits the price setting scope of the innovators, the relative size of their productivity advantage will determine whether the price effect dominates or the market size effect.

In my models, price and demand effects are also present. Although they impact the number of horizontal innovations, they will never affect the incentives of horizontal innovators as their expected one time profits will always amount to the fixed R&D costs, such that a positive (negative) demand effect is counterbalanced by an adequate negative (positive) price effect. On the other hand, changes in inequality do affect the vertical innovation incentives via price and demand effects that both have an unambiguously negative influence.

Foellmi et al. [2014] also study the influence of inequality on innovation-driven growth via price and market size effects, however, assume a twofold innovation process. Horizontal innovators introduce new high-quality luxuries for the rich while vertical innovators target the poor by developing mass production techniques that lower the production costs and product quality, but the former more than the latter. In their base framework, they assume that both innovation activities are undertaken by the same firm and that the rich consume a broad range of high-quality goods while the poor consume a narrow range of low-quality goods. In this scenario the impact of inequality on growth depends on the relative importance of vertical innovations in driving the stock of knowledge and hence productivity.

Although this closely resembles the setup of my model there are some key differences. I assume a static world rather than a dynamic one, model the second R&D process stochastically rather than deterministically and assume that different firms engage in the two innovation activities. Yet, an increasing inequality also tends to benefit horizontal innovation and harm vertical innovation. This directly follows from the intent of the respective innovation type. Horizontal innovations create luxuries for the rich while vertical innovations make use of mass production techniques that open up the market for the poor. Strengthening the relative cosumer group by changes in inequality thus benefits the one or the other type of innovation.

It should have become unequivocally clear that the long-run growth rate in endogenous growth models critically depends on the innovation incentives. In case of sequential innovations, where future innovations derive from past innovations, it might be the case the the most recent innovator might not get enough profits to cover his research costs in which case neither his nor any subsequent innovations occur. As mentioned in Green and Scotchmer [1995] or Scotchmer [2006], this might either stem from the competition imposed by entrants, reducing the profits accruing to the incumbent, or more subtle, by neglecting that the social value of the first innovation includes the enabling of every subsequent innovtion as well. This is especially crucial if the first innovator provides basic research that by itself has no commercial application and only applied future research creates profits. In either case, a redistribution of profits from future innovators to the incumbent is necessary to guarantee any innovation activity, which can be achieved by changes in the patent policy.

Generally, a patent contract extends in four directions. The first is patent length, that defines the duration of the patent. The second is a patentability requirement, that defines the prerequisite distinction from the most recent innovation another one has to fulfill to recieve a patent. The third and forth are lagging and leading patent breadth. The former defines the range of inferior innovations that can be blocked from commercial production, preventing imitation. The latter defines the range of superior versions that can be blocked, limiting subsequent innovations. A strengthening of each category makes the patent more valuable, thus promoting current innovation. However, it might also lower the incentives for future innovations which may be as crucial to growth and social welfare as the initial innovation.

O'Donoghue and Zweimüller [2004] study the role of a patentability requirement and leading patent breadth on growth in a quality-ladder model. Their findings suggest that both instruments can stimulate the R&D activity, which in turn increases the growth rate. However, as is the case in my model, a collusion prohibition implies that leading patent breadth curbs the innovation incentives.

Chu et al. [2012] extend this analysis by allowing for horizontal (i.e product creation) and vertical innovations (i.e. quality improvements). Their findings suggest, that a more backloaded (frontloaded) profit division rule increases the incentives for horizontal (vertical) innovation. As such, a stronger protection of incumbents against future entrants, by increasing the share of profits that the latter has to pay to the former, directs the innovation incentives away from vertical innovation, thus lowering the growth rate, towards horizontal innovation, which increases social welfare.

Although increasing license payments of the entrant to the incumbent reduce the vertical innovation incentives in my model as well, they also reduce the horizontal innovation activity which they are designed to foster. Yet again, this peculiar phenomenon occurs due to the lower total profits induced by the dampened vertical innovation rate which reduces the profit income of the consumers and as such the range of goods they can purchase.

The upcoming chapter first introduces to the notational convention by restricting the attention to an egalitarian economy without patent breadth and vertical innovation. The latter is introduced in the second section constituting the benchmark framework. The role of inequality in this framework is analyzed in section 3, the one of different patent breadth policies in section 4. The fifth section concludes.

Chapter 2

Models

2.1 Introductory Model

To get acquainted with the notation[1] that will be used throughout the upcoming sections we first restrict our attention to the simplest of worlds that features a completely egalitarian society and horizontal innovations only. Any equilibrium requires optimal consumer behavior, optimal firm behavior in R&D and production and labor market clearing.

Consumers

Our economy features a discrete, positive but finite number N of consumers, indexed by i, which have identical non-homothetic preferences defined over a possibly infinite continuum of indivisible consumer goods, indexed by j. More specifically, following Falkinger [1994], Zweimüller [2000], Foellmi and Zweimüller [2006] among others we restrict the consumption decision to the extensive margin, however we shall refrain from a hierarchic structure such that each variety yields the same marginal utility. Formally, a consumer's utility is defined as

$$u_i = \int_0^\infty d_i(j)dj$$

where $d_i(j) \in \{0, 1\}$ reflects the consumption decision of individual i for good j and takes on 1 if he decides to consume the good or 0 otherwise. A direct implication of this specification is that the consumer goods are perfect substitutes such that a consumer will always buy the lower priced goods first. Another noteworthy trait is that although there is complete satiation after consumption of one unit within a specific category of demand there is no satiation with respect to variety.

[1] A notational glossary is provided in the appendix

The range of goods a consumer buys, which withal constitutes his (lifetime) utility, is thus rationed by his finite means which can be written as

$$y_i = \lambda_i + \rho_i$$

where the means y_i comprise labor income λ_i, i.e. the market clearing wage w multiplied with the inelastically supplied labor endowment l_i, and profit income ρ_i that is determined by the shares in firm profits. There is neither capital nor inhereted wealth positions in this economy. Futhermore, since only relative prices matter we henceforth set the wage rate as our numeraire, i.e. $w = 1$.
The consumer problem is thus given by choosing the optimal range of goods $\{d_i(j)\}_{j=0}^{\infty}$ that maximizes his utility subject to the budget constraint which can be written as

$$e_i \leq y_i$$

where $e_i = \int_0^{\infty} d_i(j)p(j)dj$ are the expenditures of the individual. Because there are no further periods and due to the lacking satiation with respect to variety, a consumer will always fully expend his means. Seeking the broadest range of goods, the optimal consumer behavior can thus be summarized by choosing one unit of the cheapest goods until his means are fully spent.

Firms

Labor is the only productive factor and there is free entry into the horizontal R&D market. Before a consumer good j can be produced and sold it has to be invented first[2]. Employing F labor units guarantees the discovery of a new variety which can then be produced with marginal labor requirement a. Since the wage rate is set as numeraire, F also reflects the fixed research cost of a product invention while a denotes the marginal production costs of a horizontal innovator. Each innovation is patent protected, prohibiting any other firm from commercially producing this variety. Implied by the lacking indexation, this innovation process is completely identical across all consumer goods. Production features Bertrand competition, such that depending on the prices set, the profits of such an investment amount to

$$\pi(j) = D(j)[p(j) - a]$$

where $D(j) = \sum_{i=1}^{N} d_i(j)$ measures the aggregate demand which depends on the own prices, the prices of the competitors and the means of the consumers. Free entry into horizontal R&D requires that in equilibrium these profits will

[2]Opposed to Murphy et al. [1989], Zweimüller [2000], Foellmi and Zweimüller [2017] there is no freely accesible backstop technology.

exactly amount to the innovation costs, i.e. $\pi(j) = F$, such that the zero profit prices are given by

$$p(j) = a + \frac{F}{D(j)}$$

Labor market

The labor market has to clear at the numeraire wage rate w. The supply side is determined by the aggreagtion over each indiviudals labor endowment which is inelastically supplied. This implies that the labor supply L^s is given by

$$L^s = \sum_{i=1}^{N} l_i$$

The labor demand comes from two sources: production and research. In equilibrium, there will be a positive but finite number G of horizontal innovators. Each of them uses F labor units to invent the variety and a units of labor for each of the $D(j)$ demanded units. The aggregate labor demand can thus be written as

$$L^d = \int_0^G [D(j)a + F]dj = aD + GF$$

where $D = \int_0^G D(j)dj$ is the aggregate consumer demand. Labor market clearing is given by $L^s = L^d$.

Equilibrium

We are interested in the equilibrium range of goods G. Since each variety has to be invented first, we refer to G as the equilibrium amount of horizontal innovation. Due to perfect substitutability of the consumer goods and completely homogeneous consumers, the unique equilibrium will be perfectly symmetric and features identical horizontal innovators that serve the entire market, i.e. $D(j) = N = D_h$, at the same zero profit prices, $p(j) = a + \frac{F}{N} = p_h$[3], that cover the marginal production cost and the average fixed innovation cost per sold unit.[4]

[3] The subscript h will be used throughout the models to signal values corresponding to horizontal innovators.

[4] Assume an equilibrium within which the consumers would only buy a subset of the produced goods. This would imply a smaller market for at least some innovators which in turn set a higher zero profit price. Due to free entry into R&D, another innovator could enter the market, charge incrementally lower prices and get demand from the whole population since the goods are perfectly substitutable. Since the price range is continuous, the new price would be strictly above the zero profit price for serving the whole population and the entrant would have strictly positive profits, implying that the initial constelation cannot constitute an equilibrium.

Thus, the consumers have no profit income, i.e. $\rho_i = 0$. Furthermore, the assumption of a completely egalitarian society requires each individual to have the same labor endowment l. Together with the numeraire wage rate, this implies that the labor income of an individual amounts to $\lambda_i = l$ which also constitutes his total means y_i. Furthermore, equal firm prices imply that the expenditures of the representative consumer are given by $e_i = \int_0^G p(j)dj = Gp_h$. The full expenditure constraint requires $y_i = e_i$ which, using the derived values, gives the equilibrium amount of horizontal innovation as

$$G_0^* = \frac{Nl}{F + Na} \tag{2.1}$$

An alternative derivation of this value follows via the labor market clearing condition. Due to an egalitarian labor endowment, the labor supply is given by $L^s = Nl$. The labor demand is given by $L^d = G[F + Na]$. The clearing condition yields the result.
Before we analyze their impact on the equilibrium amount of horizontal innovation G_0^*, we shall define the reasonable range within wich the exogenous parameters lie.

Assumption 0

$$0 < \varphi < \infty \quad , \quad \varphi \in \{F, a, l, N\}$$

This assumption will hold throughout the upcoming models. Furthermore it also guarantess that $0 < G_0^* < \infty$.
As in all endogenous growth models, increasing R&D cost F or production costs a have a detrimental impact on the innovation activity. The free entry condition requires an adequate increase in innovator prices p_h that together with the fixed individual incomes $y_i = l$, evoked by the fixed productive factor, restrict a representative consumer to purchase a narrower range of goods[5].
An increasing labor endowment l[6] benefits horizontal innovation. It increases the labor income (labor supply) while the prices (labor demand) remain unchanged, such that more goods will be consumed (produced).
Last but not least, an increasing population size N also benefits the degree of horizontal innovation. By distributing the fixed setup costs F on a broader customer base, the prices p_h charged by each innovator fall while the

[5]Arguing over the labor market, a fixed labor supply and an increased labor demand per innovator reduces the range of innovative goods allowed.

[6]A frequently made distinction of the early endogenous growth models is between raw labor and human capital or specialized labor. The former is used in the production process while the latter is used primarily in R&D. This model refrains from such a distinction, such that labor benefits production and R&D in the same productive manner.

income of our representative consumer $y_i = l$ remains unchanged, allowing the consumption of a broader set of goods. In this sense, the model features scale effects with resepct to the innovation activity. This crucially depends on the fact that an increasing population size ceteris paribus raises the aggregate amount of labor, i.e. $L^s = Nl$, the sole productive factor which is equally applicable to production and research. In fact, if we assume that the aggregate labor supply is fixed, i.e. $L^s = L$, an increasing population size would decrease the number of product inventions. The prices charged by each innovator would still sink, however, so would the income of our representative consumer, $y_i = \frac{L}{N}$. The latter effect dominates the former, such that a consumer can only afford a smaller range of goods. This leads us to the following proposition

Proposition 0

*In an egalitarian society without the possibility of vertical innovations and where Assumption **0** holds, an increasing population size N*

(a) raises the number of horizontal innovations $\boldsymbol{G_0^*}$

(i) if the individual labor endowment l_i is fixed

(b) lowers the number of horizontal innovations $\boldsymbol{G_0^*}$

(ii) if the aggregate labor endowment $\sum_{i=1}^{N} l_i$ is fixed

An analogous proof of this proposition follows over the labor market but the intuition should be clear. In fact, throughout the upcoming models the argumentation focuses on one of the two explanatory approaches, via impact on consumer behavior or the resource based approach over the labor market, whichever seems more appropriate especially with respect to drawing comparisons to the existing literature[7].

Having established familiarity with the notation at hand we can now turn to the benchmark model by introducing vertical innovation.

[7] This is attributable to the trinity of consumer goods, R&D and labor market. By walras's law it follows, that if two of these markets clear at given prices so does the third.

2.2 Benchmark Model

We are still in a static world featuring a completely egalitarian society. However, preceding the sole production period where firms compete in prices, there is a two-stage innovation process. The first stage is concerned with the creation of new consumer goods (horizontal innovation) while the second stage tries to find a more cost-efficient production technique (vertical innovation) for the previously invented goods[8]. To understand the impact of the second stage on the first stage we first concider the optimal behavior of the vertical innovators.

<u>Vertical Innovators</u>

Opposed to the horizontal R&D market, the entry is restricted to one firm per product j and prohibited for the inventor of the variety. Furthermore, the process is stochastic rather than deterministic[9]. Technically, by employing $l_v(j)$ labor units[10] a firm targeting good j succeeds in its R&D efforts with probability $x(l_v(j))$, allowing the production of the good with marginal labor cost $b < a$[11]. The success probability has the following functional form[12]

$$x(l_v(j)) = 1 - e^{-q l_v(j)}$$

where $0 < q < \infty$ is a measure of labor productivity or effectivness in the development of the new production technique. The two most important traits of this exponential density are that it features a decreasing marginal impact of the employed labor units and that a finite labor input prevents a success certainty. Figure 3.1 illustrates this probability function for different levels

[8] This is similar to the Foellmi et al. [2014] paper, where vertical innovations introduce mass production techniques. However, in this model the marginal production cost reduction does not come at the expense of quality and horizontal and vertical innovations are achieved by different firms.

[9] If the vertical innovator would certainly succeed, the inventor of the product would be driven out of the market with certainty and fail to cover his research costs, such that he would not invent the product in the first place and neither vertical nor horizontal innovations would ever exist. This, however, crucially depends on the fact that only new firms are allowed in vertical innovation.

[10] The subscript v will be used throughout the models to signal values corresponding to vertical innovators.

[11] The cost reduction is exogenously given. Thus a vertical innovator cannot choose the innovation size, as in the extension of Aghion and Howitt [1992], nor is it a random draw from a distribution as in Minniti et al. [2013].

[12] In the quality-ladder models of Aghion and Howitt [1992] and Grossman and Helpman [1991a], the innovator chooses the amount of labor employed, impacting the poisson arrival rate of the innovation, such that the time between to successful innovations is exponentially distributed. Since we consider a static framework we have to abstract from arrival rates and model the instantaneous success probability. The uncertainty is thus not with respect to *when* the innovation occurs but rather *if* it occurs.

of researcher's productivity q. The innovation is patent protected such that neither the inventor of the variety nor any other firm has access to this production technique of the consumer good j. Although a vertical innovation is only applicable to the targeted product the process itself is completely symmetric across all varieties. Broadly speaking, the cost reduction from a to b and the probability function $x(l_v(j))$ are independent of the variety j. If the firm succeeds in innovating the production process it engages in Bertrand competition with all other producers, including the inventor of this variety. This is due to the special patent structure that prohibits the commercial production of a variety *with a given technology*, not the production in general. The production profits of a vertical innovator are given by[13]

$$\pi_v(j) = D_v(j)[p_v(j) - b]$$

where $D_v(j)$ is the aggregate demand of the vertical innovator providing variety j which depends on his charged prices $p_v(j)$. Thus, there are two choice variables a vertical innovator faces: the optimal amount of labor in R&D $l_v(j)$ and the subsequent prices in production $p_v(j)$.
The latter is found by analyzing the strategic interaction between entrant (vertical innovator) and incumbent (horizontal innovator) of a specific variety. Since they provide one and the same good the consumers will buy from whoever charges the lower prices. Since the incumbent has already incurred the invention cost F in the preliminary first stage, he will underbid any price above his marginal production cost a, as this would help to cover the sunk cost at least partially. Therefore, the vertical innovators will charge $p_v(j) = a$ which is independent of j due to the symmetry of the R&D process across all varieties[14].
The former choice variable is set, such that the expected profits will be maximized, which are given by

$$x(l_v(j))\pi_v(j) - l_v(j)$$

Since the production profits $\pi_v(j)$ are independent of $l_v(j)$, the solution to this maximization problem is given by[15]

$$l_v^*(j) = \frac{ln(q\pi_v(j))}{q}$$

Straightaway, one notes that the optimal amount of employed labor is strictly increasing in the innovator's production profits $\pi_v(j)$[16]. If there lurk more

[13] Remember the numeraire wage rate.

[14] The tie-breaker assumption assures, that with equal prices, the consumers prefer the less labor intense production process.

[15] See the calculation section in the appendix for this and all upcoming derivations.

[16] Consult the calculation section in the appendix for the comparative statics of this and all upcoming models.

profits from production, the innovation is more worthwile, justifying the employment of more researchers to increase the likelihood of a discovery. The impact of the labor productivity q is somewhat more diversified. It first raises, then lowers the optimal labor demand, eventually driving it down to zero as the productivity tends to infinity. An illustration of this relationship for different profit levels is provided in Figure 3.2. The intuition behind this hump-shaped relationship can best be explained via the impact of the employed labor units on the expected net profits. On the one hand, additional researchers increase the likelihood of discovering the new production technique $x(l_v(j))$. On the other hand, this requires larger wage payments. For low levels of q, an increasing efficiency of the employed labor units attracts further demand, justifying the additional costs. Eventually, the researchers' productivity will be high enough, such that a relatively small number of units employed already leads to a relatively high likelihood of discovering the production technique and an even further increase in efficiency will lower the demand for researchers in order to save wage payments.
Plugging this optimizer back into the probability function, the optimal success rate of the vertical innovator targeting variety j is given by

$$x^*(j) = 1 - \frac{1}{q\pi_v(j)}$$

As in the previous model, each consumer demands the whole range of goods and each innovator will serve the entire population.[17] This implies that the production profits are identical for all vertical innovators and given by $\pi_v(j) = N(a-b)$. It directly follows that the optimal labor demand $l_v(j)$ and therefore the success probability $x^*(j)$ are independent of the variety as well. To ensure a positive labor demand and to interpret x^* as a proper probability, we make the following assumption

Assumption 1a

$$qN(a-b) > 1$$

Assumption 0 and Assumption 1a guarantee that $0 < l_v^* < \infty$ and $0 < x^* < 1$. In words, it guarantees that the vertical innovators demand a strictly positive and finite amount of labor and that there will be vertical innovation. Fixing the production profits $\pi_v = N(a-b)$, vertical innovations will only occur if the labor productivity is big enough, i.e. $q > \bar{q} \equiv \frac{1}{N(a-b)}$, otherwise the uncertainty of the research process will erase any firm activity. If the production profits increase, either via a bigger cost reduction $a-b$ or a broader market N, the minimal labor efficiency threshold $\bar{q}$ falls and

[17] This is again follows from perfect substitutability, egality in means and the free entry into horizontal R&D. This implies that even the incumbents that remain market leaders will serve the entire population. Furthermore, the vertical innovators will charge lower prices and attract consumer demand even before the horizontal innovators.

vertical innovations are more likely to occur[18]. Furthermore, as there cannot be a negative labor demand, a violation of Assumption 1a restricts $l_v = 0$ and we are in the economy of the introductory model where only horizontal innovations occur.

In equilibrium, there will be exactly one firm engaged in vertical innovation for each previously developed variety $j \in [0, G]$. As the range of consumer goods is continuous and since every vertical innovator demands the same amount of research labor, x not only measures the probability with which a single firm invents a new production process but also implies the fraction of goods where an innovation actually occurs. Thus, we will refer to this value as the vertical innovation rate which in this case is given by

$$x_1^* = 1 - \frac{1}{qN(a-b)} \tag{2.2}$$

As we have seen, increasing production profits π_v, be it by means of a larger profit margin $a - b$ or a larger market N, increase the optimal labor demand l_v which in turn raises the vertical innovation rate. While the labor productivity in vertical R&D q either increases or decreases the optimal demand for researchers, depending on the efficiency/cost tradeoff, it unambiguously increases the vertical innovation rate.

Having understood the second-stage behavior we can turn to the first stage.

Horizontal Innovators

The horizontal R&D market is unchanged. There is still free entry and a one-time investment of F labor units gives access to a production technology with marginal labor requirement a which will be patent protected. The only thing that changes is the introduction of the second-stage and its add-on innovations. Due to complete symmetry, we again consider the market for a representative consumer variety j. The second stage innovator succeeds with probability x_1^*, in which case the incumbent product inventor will be driven out of the market due to price competition and higher marginal production costs. Thus, a horizontal innovator will only make profits if the vertical innovator fails in his research, which naturally occurs with probability $1 - x_1^*$. In this case he will make the following production profits

$$\pi_h = D_h[p_h - a]$$

[18] Another interpretation would state that for a given labor productivity q and population size N, the cost reduction must be sufficiently large to guarantee the existence of vertical innovations. Generally speaking, there is a tradeoff between research productivity and the concomitant likelihood of discovery and subsequent production profits. A fall in the former can be counterbalanced by an increase the latter and leave innovation incentives unchanged and vice versa.

As there is only one completely homogeneous consumer type which demands exactly one unit of each available variety, $D_h = N$ will hold in equilibrium. Free entry into the horizontal R&D market requires that the *expected* profits will amount to the innovation cost. The free entry condition states

$$(1 - x_1^*)\pi_h = F$$

Using the previously derived values for x_1^* and π_h we get that the zero expected profit prices of each horizontal innovator are given by

$$p_h = a + Fq(a - b)$$

Comparing these prices to the ones in the introductory model, one notes that if Assumption 1a holds, the horizontal innovators charge higher prices. Although the cost structure is unchanged, a positive vertical innovation rate (Assumption 1a) reduces the expected demand $(1 - x_1^*)N$, raising the *expected* average costs.

The range of goods G that is feasible in equilibrium again depends on the fixed amount of the only productive factor labor and can either be found via the full expenditure constraint or the labor market.

Consumers

There is still no inequality and consumer preferences are unchanged such that the optimal behavior implies that they consume exactly one unit of the cheapest varieties until their means are fully spent.[19]. Due to the steady innovation rate, there will be $G_v = x_1^*G$ vertical innovators providing varieties at prices $p_v = a$ and $G_h = (1 - x_1^*)G$ horizontal innovators that remain market leaders providing their varieties at prices $p_h = a + Fq(a - b)$. The representative consumer's expenditures are thus given by

$$e_i = G_h p_h + G_v p_v$$

The means y_i still comprise labor income λ_i and profit income ρ_i. Due to the egalitarian society, each consumer $i \in \{1, ..., N\}$ still has the same labor endowment and labor income

$$\lambda_i = l$$

Due to the free entry into the horizontal R&D market, there occur no profits from product inventions. However, the restricted access to vertical R&D

[19] A utility backing up this behavior would be $u_i = \int_0^\infty max\{d_{i,h}(j), d_{i,v}(j)\}dj$ where $d_{i,x}(j) \in \{0, 1\}$ measures whether they consume variety j provided by innovator $x \in \{h, v\}$ or not. This basically implies that they maximally consume one unit of a variety, depending on the prices charged by the vertical and horizontal innovators. Henceforth, we shall tacitly assume such a utility while focusing solely on the expenditure constraint.

implies some market power for the process innovators such that their expected profits are strictly positive[20]. Due to the continuous product range the expected profits of a single firm correspond to the average profits of all firms. These profits are equally redistributed to the consumers[21] such that their profit incomes amount to[22]

$$\rho_i = \frac{G}{N}[x_1^* \pi_v - l_v^*]$$

The full expenditure constraint $y_i = e_i$ and the previously derived equilibrium values imply, that the equilibrium amount of horizontal innovation is given by

$$G_1^* = \frac{Nl}{F + Nb + \frac{1}{q}[1 + ln(qN(a-b))]} \tag{2.3}$$

Assumption 0 and Assumption 1a guarantee that $0 < G_1^* < \infty$. Furthermore, comparing Equation (3.1) and (3.3) we get that $G_0^* < G_1^*$ as long as Assumption 1a holds. This implies, that in an economy where vertical innovations are present, the equilibrium range of goods increases as opposed to a world where they are absent. The explanation behind this result is twofold. On the one hand, a consumer faces higher prices charged by the horizontal innovators while facing the likelihood of lower vertical innovator prices alike. These two effects cancel each other out, such that the average price faced by a consumer, i.e. $p_\phi = x_1^* p_v + (1 - x_1^*)p_h = a + \frac{F}{N}$, remains unchanged. On the other hand, each consumer recieves a bigger income stream due to positive profits from vertical innovators. Therefore, the consumers can afford a broader set of goods and vertical innovation increases the amount of horizontal innovation. Before we compare this rather odd effect to the existing literature, we shall alternatively show the derivation of G_1^* over the labor market.

Labor Market

The aggregate labor supply remains unchanged

$$L^s = \sum_{i=1}^{N} l_i = Nl$$

Broadly speaking, the labor demand stems from production and R&D. For each of the G varieties, there is exactly one horizontal innovator, requiring F

[20]Provided that Assumption 1a holds and vertical innovations actually occur.

[21]One can assume equal endowments in firm shares explaining such a distribution.

[22]Remember that there is exactly one firm engaged in vertical R&D per previously developed variety, such that there is a total number of G vertical research firms. However, only a fraction x_1^* of them will successfully innovate the production technique.

labor units for the invention of the consumer good, and one vertical innovator, employing l_v labor units trying to develop a new production technique. Only a fraction x_1^* of the vertical innovators will succeed in their research and engage in production, producing N goods with unit labor requirement b. On the other hand, only a fraction $1 - x_1^*$ of the horizontal innovators won't be driven out of production and serve the entire population N with unit labor requirement a. Gathering these insights, the aggregate labor demand amounts to

$$L^d = G[N(x_1^* b + (1 - x_1^*)a) + F + l_v^*]$$

Using the labor market clearing $L^s = L^d$ and the equilibrium vertical innovation rate from equation (3.2), we arrive at the same expression for G_1^* as in equation (3.3).
Although vertical innovation increases the aggregate labor demand in R&D, it also reduces the aggregate labor demand in production due to the labor saving production technique. As long as Assumption 1a holds, such that there is a positive vertical innovation rate, the latter dominates the former such that vertical innovations release labor that is used for the invention of new consumer goods[23].

Equilibrium

The impact of a second-stage R&D market can most vividly be described by analyzing the impact of q, the researchers' productivity in process R&D, on the vertical innovation rate x_1^* and the amount of horizontal innovations G_1^* in equilibrium. Provided that process innovations occur (Assumption 1a), an increasing labor productivity in vertical R&D first increases the optimal demand for researchers l_v^* per vertical innovator and then decreases this demand later on due to efficiency/cost considerations. However, it unambiguously raises the optimal success probability and thus the vertical innovation rate x_1^*. While this comes as no surprise, the impact on the amount of horizontal innovations does. An increase in q unambiguously raises G_1^*. This can best be understood via the full expenditure constraint. While the average prices remain unaffected, the income of a consumer increases due to a larger profit income, such that they can expand the range of goods bought. This allows us to formulate the following proposition

[23]The average labor savings have to exceed the average innovation costs to make vertical innovations worthwile. This is specifically attributable to the fact that the profit margin exactly equals the cost reduction.

Proposition 1

*If Assumption **0** and Assumption **1a** hold:*

(i) there is a positive and finite amount of vertical innovations, $0 < \boldsymbol{x_1^*} < 1$*, and horizontal innovations,* $0 < \boldsymbol{G_1^*} < \infty$

(ii) the presence of vertical innovation, $\boldsymbol{x_1^*} > 0$*, increases the amount of horizontal innovations* $\boldsymbol{G_1^*}$

(iii) an increasing labor productivity $\boldsymbol{q}$ *in vertical R&D increases the vertical innovation rate* $\boldsymbol{x_1^*}$ *as well as the number of horizontal innovations* $\boldsymbol{G_1^*}$

Especially the two latter points seem counter-intuitive. As in every endogenous growth model featuring vertical innovation, there is creative destruction, where the latest innovation renders the previous ones obsolete since they build upon the same good. As such, one could expect that the presence of further innovations and moreover a strengthened incentive to engage in these activities would be detrimental for the incumbent innovator. Therefore, the introduction of a second-stage add-on innovation and an increased incentive to engage in them by an increased productivity of the researchers[24], should lower the incentives for the horizontal innovators as they are more likely to be driven out of the market.

However, the amount of product inventions increases. By means of the free entry condition, an increasing likelihood of being driven out of the market, i.e. a reduction in the expected demand, is counterbalanced by an adjustment of the prices charged by the horizontal innovators, such that their expected period profits remain unchanged, amounting to the research costs F. This implies, that the incentives for a single product inventor remain unaffected and the increasing profitability of vertical innovations extends the range of goods due to an increased demand for new varieties triggered by the bigger (profit) incomes of the consumers.

In the endogenous growth models the period profits would remain unchanged and increasing incentives for future innovations shorten the time for which they occur. Therefore, the introduction as well as stronger incentives for future innovators would lower the incentives for current innovation requiring a lower discount that in turn reduces the growth rate.

For the remainder of this work we normalize $q = 1$ and center the attention on the impact of inequality and patent breadth on the innovation activities begining with the former.

[24]The same accounts for lower production costs b.

2.3 Inequality

Thus far we have assumed an egalitarian society where everyone consumed the whole range of goods available. Building on the structure of the benchmark model, we now introduce inequality and analyze its impact on vertical and horizontal innovation.
Inequality extends in two directions: the income of a consumer group and the size of this group.[25] We restrict our attention to a two class society of rich and poor individuals.
Of the N individuals, a fraction β belongs to the latter, the rest to the former. Thus, there are $N_r = (1-\beta)N$ rich individuals and $N_p = \beta N$ poor individuals[26].
These two groups have the same non-homothetic preferences and only differ with respect to their means that still comprise labor income λ_i and profit income ρ_i. Income inequality can thus emerge by means of different endowments in labor and firm shares. To keep things as tractable as possible, we assume that the poor have the same percentage θ of the average labor endowments and average firm shares and thus of the average means. By definition, the rich's percentage of average endowments and means is given by $\frac{1-\beta\theta}{1-\beta}$[27]. This allows us to write the income of the consumer types as follows

$$y_i = \begin{cases} \theta y_\phi & , \quad i \in N_p \\ \frac{1-\beta\theta}{1-\beta} y_\phi & , \quad i \in N_r \end{cases}$$

where $y_\phi = \lambda_\phi + \rho_\phi$ is the average income of the consumers.
To guarantee that inequality exists and that the rich actually have a bigger income we need the following assumption

Assumption 2

$$0 < \beta < 1 \quad ; \quad 0 < \theta < 1$$

If Assumption 2 holds $\theta < 1 < \frac{1-\beta\theta}{1-\beta}$. Thus, as long as this assumption is fulfilled, there are poor and rich individuals present and the former actually have a lower income opposed to the latter.
Comparing the relative income of the rich to the poor

$$\omega = \frac{y_r}{y_p} = \frac{1-\beta\theta}{(1-\beta)\theta}$$

[25] Among others, this approach follows Zweimüller [2000], Foellmi and Zweimüller [2006]

[26] The subscript r and p will be used throughout this section to index values corresponding to rich and poor, respectively.

[27] $y_\phi = \beta y_p + (1-\beta) y_r$

we note that an increase in β and a decrease in θ both raise ω and thus foster inequality.[28] Following Foellmi et al. [2014], we term the former as an increasing *income concentration*[29] and the latter an increasing *income gap*.[30] An illustration of these effects is shown by means of the Lorenz curve in Figure 3.3 in the graphic section of the appendix.
Although the consumers differ in their income, they are completely homogeneous in any other respect. Therefore, their optimal behavior still implies that they fully expend their means on the available goods[31]. Due to the higher income of the rich, they necessarily have higher expenditures as well. These excess expenditures over the poor can only be accomplished in two ways: price discrimination or excess demand. Assuming that the former is either prohibited by law or impossible[32] we restrict our attention to the latter scenario. Hence, the rich will consume a broader range of goods as opposed to the poor whereas their excess demand is satisfied by innovative firms.[33]
Depending on the purchasing power of the poor, there are three different equilibrium scenarios that emerge which are illustrated in Figure 3.5 in the appendix. If they are relatively wealthy, they consume but a small fraction of the goods provided by horizontal innovators (Scenario 1). A reduction in their purchasing power will increase this fraction until they exclusively consume the whole range of goods provided by vertical innovators (Scenario 2). If their incomes fall even further they will only purchase a subset of these goods (Scenario 3).

[28] Comparative statics are again to be found in the appendix.

[29] Leaving the income of the poor unaffected, an increase in β concentrates the remaining income among a smaller number of rich individuals.

[30] A fall in θ lowers the fraction of income acruing to the poor and raises the one of the rich.

[31] Remember the tacitly assumed utility $u_i = \int_0^\infty max\{d_{i,h}(j), d_{i,v}(j)\}dj$ that holds for every individual.

[32] If price discrimination were possible and allowed, both consumer types would demand the same number of consumer goods. While the price schedule of a vertical innovator would remain unchanged, a horizontal innovator would simply charge higher prices for the rich and lower ones for the poor individuals leaving the average price charged unchanged, such that we would again be in the economy described in the benchmark model. This, however, only holds as long as the purchasing power of the poor allows them to buy at least the whole range of vertical innovator goods at prices a.

[33] Alternatively, the excess demand could be met by a non-innovative sector employing a freely accessible backstop technology as it is the case in Zweimüller [2000], Foellmi and Zweimüller [2017].

Scenario 1

Vertical Innovation

Since we are still in the framework of the benchmark model the R&D process is unchanged. Thus, a vertical innovator targeting variety j will still choose the optimal number of researchers that maximizes his expected profits, the solution to which is still given by[34]

$$l_v^*(j) = ln(\pi_v(j))$$

and the production profits can yet again be written as

$$\pi_v(j) = D_v(j)[p_v(j) - b]$$

where $D_v(j)$[35] is the aggregate demand for variety j provided by the vertical innovator which depends on his price $p_v(j)$ charged. Due to price competition with the incumbent product inventor, this price can never exceed the marginal production costs $a \equiv \overline{p_v}$ of the horizontal innovator, constituting the natrual price limit of a vertical innovator. Therefore, $p_v(j) \leq \overline{p_v}$ will hold in every equilibrium irrespective of the variety. This directly implies that the prices charged by successful vertical innovators will be strictly lower than the ones charged by horizontal innovators that maintain their market, as they will also cover their fixed setup costs F. Due to perfect substitutability of the varieties, the consumers will always target the vertical innovators' goods before turning their attention to the horizontal innovators' goods. As long as the purchasing power of the poor is high enough, such that they will consume part of the horizontal innovators' goods, they can also afford to buy the whole range of vertical innovators' goods at prices $p_v(j) = a$, which will consequently be charged by each successful process innovator. Furthermore, they will serve the entire population, i.e. $D_v(j) = N$, leaving their production profits unchanged at $\pi_v(j) = N(a-b)$ such that the optimal labor demand in the research process is given by

$$l_v^*(j) = ln(N(a-b)) \quad , \quad j \in [0, G]$$

To guarantee that the firms will demand a strictly positive number of researchers and thus to guarantee the existence of vertical innovations we need a slightly modified version of Assumption 1a

Assumption 1b

$$N(a-b) > 1$$

[34]Remember the normalization of $q = 1$.

[35]With the tacitly assumed utility, the aggregate demand is given $D_v(j) = \sum_{i=1}^{N} d_{i,v}(j)$.

This simply states that due to uncertainty, the profits awaiting in case of a successful process innovation have to be big enough to counterbalance the costs you have to incur to get them in order to make vertical R&D worthwile. This can either be achieved via a big enough cost reduction $a - b$ and thus a big enough profit margin or a big enough market N.
Since every vertical innovator expects the same production profits and thus employs the same number of researchers, the steady vertical innovation rate is given by

$$x_{2a}^{*} = 1 - \frac{1}{N(a-b)} \tag{2.4}$$

which unsurprisingly equals the vertical innovation rate of the benchmark model (Equation (3.2)) with the productivity normalization $q = 1$.

<u>Horizontal Innovation</u>

Free entry into horizontal R&D requires that the expected profits amount to the innovation cost such that the equilibrium is characterized by

$$(1 - x_{2a}^{*})\pi_h(j) = F$$

where x_{2a}^{*} is the success probability of the vertical innovator, F are the fixed setup costs and the production profits in case of a failure in process innovations $\pi_h(j)$ is given by

$$\pi_h(j) = D_h(j)[p_h(j) - a]$$

$D_h(j)$[36] is the aggregate demand for the variety j provided by the product inventor and $p_h(j)$ his price charged. Using the free entry condition the zero expected profit prices can be written as

$$p_h(j) = a + \frac{F}{(1 - x_{2a}^{*})D_h(j)}$$

The equilibrium features two types of product inventors. A fraction m sells to everyone and is labeled as mass producers, such that their demand is given by $D_h^m = N$[37]. The other fraction $1 - m$ sells exclusively to the rich, satisfying their excess demand, and is labeled as exclusive producers, such that their demand is given by $D_h^e = (1-\beta)N$[38]. Plugging these values in the zero profit prices we get

$$p_h(j) = \begin{cases} a + F(a-b) & , \quad j \in G_h^m \\ a + \frac{F(a-b)}{(1-\beta)} & , \quad j \in G_h^e \end{cases}$$

[36] With the tacitly assumed utility, this is given by $D_h(j) = \sum_{i=1}^{N} d_{i,h}(j)$.
[37] The superscript m will be used to signal mass producers that sell to everyone.
[38] The superscript e will be used to signal exclusive producers that sell solely to the rich.

where $G_h^m = m(1-x_{2a}^*)G$ is the number of goods provided by mass producing horizontal innovators and $G_h^e = (1-m)(1-x_{2a}^*)G$ is the number of goods provided by exclusive producing horizontal innovators. The prices of the former are henceforth labeled as p_h^m and the ones of the latter as p_h^e. The fraction of mass producers m is endogenous and depends on the purchasing power of the poor. Therefore, we turn to the demand side of the economy.

<u>Consumers</u>

There are $N_p = \beta N$ poor individuals with a fraction θ in average endowments and $N_r = (1-\beta)N$ rich individuals with a share $\frac{1-\beta\theta}{1-\beta}$ in average endowments which are given by

$$y_\phi = \lambda_\phi + \rho_\phi$$

The average labor endowments are unchanged such that the average labor income $\lambda_\phi = l$. Profits only accrue to vertical innovators and the aggregate profits amount to the expected profits of all G firms engaged in the second stage R&D markets. The expected profits of a single vertical innovator are still given by $x_{2a}^*\pi_v - l_v^*$. Multiplying with G and dividing by N gives the average profit per individual ρ_ϕ. This allows us to write the average income as

$$y_\phi = l + \frac{G}{N}(x_{2a}^*\pi_v - l_v^*) = l + \frac{G}{N}\big(N(a-b) - 1 - ln(N(a-b))\big)$$

where the last step follows using the previously derived equilibrium values. As was already mentioned, the rich consume all G goods available in equilibrium and the poor purchase solely goods from vertical innovators $G_v = x_{2a}^*G$ and mass producing horizontal innovators G_h^m. Therefore, we can write the expenditures as

$$e_i = \begin{cases} G_v p_v + G_h^m p_h^m & , \quad i \in N_p \\ G_v p_v + G_h^m p_h^m + G_h^e p_h^e & , \quad i \in N_r \end{cases}$$

The equilibrium features full income expenditure, i.e. $y_i = e_i$, such that we can summarize the consumer side of the economy by the following expression

$$\omega = \frac{y_r}{y_p} = \frac{e_r}{e_p} = \varphi$$

where ω measures relative incomes of rich to poor and φ measures the relative expenditures. After some rearrangements and usage of the equilibrium values we arrive at an expression for the equilibrium fraction of horizontal innovators' goods purchased by the poor[39]

$$m_1^*(\theta) = 1 - \frac{(F+Na)(a-b)(1-\theta)}{a(1-\beta\theta) + F(a-b)} \tag{2.5}$$

[39] The derivation can be found in the appendix.

By Assumption 0, Assumption 1b and Assumption 2, that guarantee the existence of vertical innovations and inequality, it follows that $m_1^* < 1$. Therefore, the the poor will actually consume only a subset of all goods. However, to ensure that the fraction of goods bought from horizontal innovators is strictly positive, we need the following assumption

Assumption 3a

$$\theta > \frac{[N(a-b)-1]a}{[F+Na](a-b)-a\beta} \equiv \overline{\theta}$$

Assumption 0, 1b and 2 imply that $0 < \overline{\theta} < 1$. Assumption 3a then implies that if $\overline{\theta} < \theta < 1$, $0 < m_1^* < 1$ holds as well. Otherwise put, the income share of the poor θ and thus their purchasing power must be big enough to guarantee Scenario 1. The following proposition summarizes this insight

Proposition 2a

If Assumption ***0****,* ***1b****,* ***2*** *and* ***3a*** *hold:*

An equilibrium exists where vertical and horizontal innovations occur and inequality is present. Furthermore, the rich consume all $\boldsymbol{G}$ *goods available and the poor consume but a fraction* $\boldsymbol{m}$ *of the* $\boldsymbol{G_h}$ *goods provided by the horizontal innovators.*

Broadly speaking, Proposition 2a states that the equilibrium according to Scenario 1 exists. An illustration of this economy is provided in Figure 3.5 (b).
To close the model, we need to find the equilibrium value for the endogenous range of goods G, the number of horizontal innovations. To this end we look at the labor market.

Labor Market

The aggregate labor supply is unchanged since the average labor endowment λ_ϕ, which is still inelastically supplied, is unchanged. Therefore,

$$L^s = \sum_{i=1}^{N} l_i = N_p l_p + N_r l_r = Nl$$

where the last equality follows from the labor endowments of the poor, $l_p = \theta l$, and the rich, $l_r = \frac{1-\beta\theta}{1-\beta} l$, respectively.
The aggregate labor demand stems from production and R&D. Every product inventor demands F labor units to invent a new variety and every vertical innovator demands l_v^* labor units in research. Of the G firms engaged in hor-

izontal R&D, only a fraction $1 - x^*_{2a}$ will actually produce with marginal labor requirement a. A fraction m^*_1 of them sells to everyone while the rest produces exclusively for the rich.[40] On the other hand, only a fraction x^*_{2a} of all G firms engaged in vertical R&D will actually develop the new production technique and serve the entire population with marginal labor requirement b. The aggregate labor demand is given by

$$L^d = G\Big(N\Big(x^*_{2a}b + (1 - x^*_{2a})a\big(m^*_1 + (1 - m^*_1)(1 - \beta)\big)\Big) + F + l^*_v\Big)$$

The labor market clears, i.e. $L^s = L^d$, at the unit wage rate $w = 1$. Rearranging and using the previously derived equilibrium values gives the equilibrium amount of horizontal innovation as

$$G^*_{2a} = \frac{Nl}{F + Nb + 1 + ln(N(a-b)) - (F + Na)\frac{a\beta(1-\theta)}{a(1-\beta\theta)+F(a-b)}} \tag{2.6}$$

Assumption 0, 1b, 2 and 3a guarantee that $0 < G^*_{2a} < \infty$[41].

<u>Discussion</u>

To understand the impact of inequality on the innovation activities, we compare our equilibrium values to the ones of the benchmark model, with the normalization $q = 1$.

Comparing equation (3.4) and (3.2) we see that $x^*_1 = x^*_{2a}$. That is, the introduction of inequality doesn't affect the vertical innovation rate as long as we are in an equilibrium according to Proposition 3a. Furthermore, in such an economy neither an increasing income concentration, $\beta \uparrow$, nor an increasing income gap, $\theta \downarrow$, affects the vertical innovation rate. This straightforward follows from the fact that in this equilibrium scenario, the poor are always wealthy enough to purchase the whole range of vertical innovators' goods at prices $\overline{p_v}$, such that the production profits of a process innovator remain unchanged.

However, inequality affects the fraction of horizontal innovators' goods purchased by the poor, i.e. m^*_1, and the number of horizontal innovations G^*_{2a}. Comparig equations (3.6) and (3.3) we see that $G^*_1 < G^*_{2a}$. Moreover, an increasing income concentration and an increasing income gap both raise the equilibrium number of product inventions G^*_{2a}. As such, not only the mere introduction of inequality but also an increasing degree of inequality benefit horizontal innovation.

A lower income share of the poor, θ, leads to a concomitant increasing income share of the rich, $\frac{1-\beta\theta}{1-\beta}$, thus raising the relative income and the relative expenditures. With unchanged prices, this implies a smaller fraction

[40]Since each consumer only demands one unit of a variety, the customer base corresponds to the number of produced units.

[41]By ensuring that m^*_1 (3a), x^*_{2a} (1b), β & θ (2) $\in (0, 1)$

of goods consumed by the poor and a fall in m_1^*. The exact opposite is the case for the rich. The additional income is spent on new goods due to the non-homothetic preferences. As these new goods have to be invented first[42], an increasing relative income fosters horizontal innovations. This is exactly the same effect as described in Falkinger [1994] with the slight difference that he assumes hierarchic consumer preferences and the increasing demand has to be accompanied by an increasing (labor) productivity via the stock of knowledge to make this extension feasible.
The adjustments within an economy according to Proposition 2a in case of an increasing income gap are illustrated in Figure 3.6 (b).
On the other hand, an increasing fraction of poor individuals, β, leaves their income unchanged but increases the one of the rich by concentration among a smaller number of individuals. Opposed to an increasing income gap, there is also an impact on prices charged. The exclusive horizontal producers have a smaller customer base which is counterbalanced by increasing prices to guarantee zero expected profits. However, the increasing income dominates the increasing prices, such that the rich extend the range of goods consumed, increasing the number of horizontal innovations. The income of the poor is unaffected as are the prices they face, $\overline{p_v}$ and p_h^m, and the fraction of horizontal and vertical innovators. To counterbalance the increasing number of goods, the poor thus consume a smaller fraction of horizontal innovators' goods such that m_1^* decreases in β.
The adjustments within an economy according to Proposition 2a in case of an increasing income concentration are illustrated in Figure 3.6 (c).
The following proposition summarizes these insights

Proposition 3a

*In an economy according to Proposition **2a**, an increasing inequality in form of a increasing income gap, $\boldsymbol{\theta} \downarrow$, or an increasing income concentration, $\boldsymbol{\beta} \uparrow$*

(i) leaves the vertical innovation rate $\boldsymbol{x^*_{2a}}$ <u>unchanged</u>

(ii) <u>increases</u> the number of horizontal innovations $\boldsymbol{G^*_{2a}}$

According to Assumption 3a this scenario requires $\theta > \overline{\theta}$, that is that the poor are wealthy enough. The next scenario considers the possibility that the poor still consume the whole range of vertical innovators' goods within a certain price range but are no longer able to buy any horizontal innovators' goods.

[42]Again, a competitive fringe as in Zweimüller [2000], Foellmi and Zweimüller [2017] could also absorb this excess demand opposed to an innovative sector.

Scenario 2

Vertical Innovation

If $\theta < \overline{\theta}$ the poor won't be able to afford the whole range of vertical innovators' goods at prices $\overline{p_v} = a$. A vertical innovator targeting variety j thus faces a choice between two options. Adjust the prices, such that the poor can afford the good as well, or charge $p_v(j) = \overline{p_v}$ and face demand from the rich only.[43] The latter, exclusive producer would thus make the following production profits

$$\pi_v^e(j) = (1-\beta)N(a-b)$$

whereas the former's profits are given by

$$\pi_v^m(j) = N(p_v(j) - b)$$

Comparing these two production proftis, a successful vertical innovator would always choose to be a mass producer as long as the poor can afford this variety at a price $p_v(j) > b + (1-\beta)(a-b) \equiv \underline{p_v}$.
Within this second equilibrium scenario, all vertical innovators will be mass producers charging the same price $p_v \in [\underline{p_v}, \overline{p_v}]$, depending on the purchasing power of the poor. Since the production profits are independent of the targeted variety, each firm engaged in vertical R&D demands the same optimal amount of labor given by

$$l_v^*(j) = ln(N(p_v - b)) \quad , \quad j \in [0, G]$$

To ensure that this equilibrium will feature a strictly positive labor demand in vertical research, and thus a positive vertical innovation rate, we need a slightly modified version of Assumption 1b

Assumption 1c

$$N(p_v - b) > 1 \quad , \quad p_v \in [\underline{p_v}, \overline{p_v}]$$

where $\underline{p_v} < \overline{p_v}$ is guaranteed by Assumption 2.
With a similar reasoning to before, this assumption states that the profits that await a successful vertical innovator have to be big enough to counterbalance the costs of getting them in order to make it worthwile to engage in vertical R&D.
With every vertical innovator demanding the same number of researchers, we get the steady vertical innovation rate as

$$x_{2b}^* = 1 - \frac{1}{N(p_v - b)} \quad , \quad p_v \in [\underline{p_v}, \overline{p_v}] \tag{2.7}$$

[43]Demand from the rich is always guaranteed at $\overline{p_v}$ since this price is strictly lower than the ones charged by horizontal innovators.

By Assumption 1c, $0 < x^*_{2b} < 1$ is indeed fulfilled, such that in equilibrium, we have a positive fraction of goods provided by horizontal and vertical innovators alike.
Straightforward, a smaller price p_v charged, lowers the profit margin of a vertical innovator and reduces the optimal number of researchers employed, which in turn lowers the success probability of discovering the new production technique and hence the vertical innovation rate. The opposite is the case for the new marginal labor requirement b.

Horizontal Innovation

The free entry into the horizontal R&D market again requires that the expected profits amount to the fixed setup costs such that

$$(1 - x^*(j))\pi_h(j) = F$$

holds for the inventor of variety j. With probability $1 - x^*(j)$ the second stage innovator will fail to discover the new production technique and the inventor will serve the market making profits

$$\pi_h(j) = D_h(j)(p_h(j) - a)$$

where $D_h(j)$ is the demand for this variety provided by the product inventor depending on his charged prices $p_h(j)$. While the poor still purchase the whole range of vertical innovators' goods, they can no longer afford any horizontal innovators' goods, such that each incumbent horizontal innovator will be an exclusive producer, i.e. $D_h(j) = (1-\beta)N$. Furthermore, they face the same probability of being driven out of the market, that is $x^*(j) = x^*_{2b}$, such that the zero expected profit prices are given by

$$p_h(j) = a + \frac{F(p_v - b)}{1 - \beta} \quad , \quad j \in G_h$$

where $G_h = (1 - x^*_{2b})G$ is the mass of horizontal innovators serving the market. A bigger probability of being driven out by a successful vertical innovator, spurred by an increasing profit margin of the process innovator $p_v - b$, or a smaller market size of the exclusive producing horizontal innovators, i.e. a reduction in the fraction $1 - \beta$ of rich individuals, increase the prices charged by the producing product inventors to counterbalance the smaller expected profit stream, such that they can still cover their innovation costs F.
To find the determinants of the endogenous vertical innovator prices p_v and the range of income shares θ that ensure the existence of such an equilibrium scenario, we turn to the optimal consumer behavior.

Consumers

Once again, the optimal consumer behavior can be summarized by equating the relative income to the relative expenditures. As in the separating equilibrium of Foellmi et al. [2014], the poor consume all $G_v = x^*_{2b}G$ vertical innovators' goods at prices p_v while the rich additionally buy the G_h horizontal innovators' goods at the above zero expected profit prices p_h. This allows us to write

$$\omega = \frac{1-\beta\theta}{(1-\beta)\theta} = \frac{x^*_{2b}p_v + (1-x^*_{2b})p_h}{x^*_{2b}p_v} = \varphi$$

where ω again measures the relative means of rich to poor and φ measures their relative expenditures.
Using the previously derived values for x^*_{2b} and p_h we get an expression of the endogenous vertical innovator prices p_v depending on the exogenous parameters β, θ, a, b, F and N. We are especially interested in the relationship between the poor's income share θ and the prices p_v set by the vertical innovators. After rearrangement of the above equation we get

$$\theta(p_v) = \frac{(N(p_v-b)-1)p_v}{(N(p_v-b)-1)p_v + a(1-\beta) + F(p_v-b)}$$

which gives an expression of θ as a function of p_v whose range we know. By Assumption 0, 1c and 2 it follows that for any $p_v \in [\underline{p_v}, \overline{p_v}]$, $0 < \theta(p_v) < 1$ holds as well. Pluggin in the upper bound of this range we get

$$\theta(\overline{p_v}) = \frac{[N(a-b)-1]a}{(F+Na)(a-b)-a\beta} \equiv \overline{\theta}$$

which unsurprisingly corresponds to the critical value we derived in the first equilibrium scenario. When the poor's income share is exactly given by $\overline{\theta}$, they can purchase exactly the whole range of vertical innovators' goods at prices $\overline{p_v}$.
Plugging in the lower bound of the price range we get

$$\theta(\underline{p_v}) = \frac{((1-\beta)N(a-b)-1)(a-\beta(a-b))}{(F+N(a-\beta(a-b)))(1-\beta)(a-b)-b\beta} \equiv \underline{\theta}$$

There are two things worth noting. By Assumption 0, 1c and 2 it holds that $\underline{\theta} < \overline{\theta}$, as expected. The second thing is that, taking the partial derrivative of $\theta(p_v)$ with respect to p_v, the same assumptions guarantee a strictly postive relationship. These insights are summarized in the following assumption

Assumption 3b

$$\underline{\theta} < \theta < \overline{\theta}$$

This assumption states within which range the poor's income share must lie to guarantee an equilibrium where only mass prodcuing vertical innovators and exclusive producing horizontal innovators are present. As expected, with an income share $\overline{\theta}$, the vertical innovators can charge the highest prices $\overline{p_v} = a$ and subsequently lower their prices as this share falls, until it reaches $\underline{\theta}$ in which case they still sell to everyone at prices $\underline{p_v} = b + (1-\beta)(a-b)$. This leads us to the following proposition

Proposition 2b

*If Assumption **0**, **1c**, **2** and **3b** hold:*

*An equilibrium exists where vertical and horizontal innovations occur and inequality is present. Furthermore, the rich consume all **G** goods available and the poor consume all $\boldsymbol{G_v}$ goods provided by the vertical innovators, solely.*

Following Proposition 2a, this proposition states that an equilibrium according to Scenario 2 exists. An illustration of this economy is provided in Figure 3.5 (c).
Yet again, we need to derrive the number of horizontal innovations G feasible in equilibrium. To this end we center on the labor market.

Labor Market

The labor demand stems from the customary sources production and R&D. In the former market, G_v firms produce N goods with unit labor requirements b and G_h firms produce $(1-\beta)N$ goods with marginal labor requirement a. In R&D, there are G firms present in both stages demanding F labor units in the first stage and l_v^* labor units in the second stage. All together the labor demand can be written as

$$L^d = G\Big(N\Big(x_{2b}^* b + (1 - x_{2b}^*)a(1-\beta)\Big) + F + l_v^*\Big)$$

The labor supply is unchanged and the labor market clears at unit wage rate. After rearrangments and usage of the equilibrium values, we can write the equilibrium range of goods as

$$G_{2b}^* = \frac{Nl}{F + Nb + 1 + ln(N(p_v - b)) - \frac{p_v - a(1-\beta)}{p_v - b}} \tag{2.8}$$

Assumption 0, 1c and 2 ensure that $0 < G_{2b}^* < \infty$.

Discussion

How do the innovation activities adjust to an increasing income gap, a reduction of the poor's income share θ, in an equilibrium according to Proposition 2b?
We have already noted that there is a strict positive relationship between this parameter and p_v. As such, a worsened income position of the poor reduces the prices the mass producing vertical innovators can charge. This straightaway follows from the fact that as long as we are in this second equilibrium scenario and the poor are still relatively wealthy, i.e. $\theta \geq \underline{\theta}$, it is always worthwile for any vertical innovator to lower the prices and remain a mass producer rather than to sell exclusively to the rich at the limit price $\overline{p_v}$.[44]
Falling prices lower the profit margin of a vertical innovator and as such his production profits $\pi_v = N(p_v - b)$. Since the optimal number of researchers employed, i.e. l_v^*, solely depends on these profits, each vertical innovator employs less labor units in the R&D process lowering his success probability. Since this applies to all G firms engaged in vertical R&D, the vertical innovation rate x_{2b}^* is falling in case of an increasing income gap.
Contrasting that, an increasing income gap strictly increases the number of horizontal innovations G_{2b}^*.
Lower vertical innovator prices p_v impact the labor demand in vertical R&D and production. As before, the lower the prices a vertical innovator can charge, the lower his labor demand l_v^*, such that the former sector will employ less labor. In the latter sector, the falling prices lower the fraction x_{2b}^* of goods produced by vertical innovators and increase the fraction $1 - x_{2b}^*$ of goods produced by horizontal innovators. While the vertical innovators have a less labor intense production technology they also serve a broader market. If $\beta > 1 - \frac{b}{a}$, that is if there are relatively few rich individuals, the market size effect is dominant such that the increasing fraction of horizontal innovators will result in labor savings in production. If $\beta < 1 - \frac{b}{a}$ the production technique dominates and the lower fraction of vertical innovators will result in an augmented labor demand in production. Irrespective of the impact on the labor demand in production, the overall result of a falling price p_v will always be a damped labor demand in the prevalent markets which releases resources for the invention of new varieties.
The adjustments within an economy according to Proposition 2b in case of an increasing income gap are illustrated in Figure 3.7 (b).
What about an increasing income concentration, an increasing fraction β of poor individuals?

[44]Strictly speaking, in the case of $\theta = \underline{\theta}$ the innovators would be indifferent between mass and exclusive production such that it wouldn't be worthwile to remain a producer of the former format. However, in the case of indifference they might choose either option and an equilibrium according to Proposition 2b is surely supported.

It leaves the income of a poor agent unaffected. Therefore, the vertical innovators can still charge the same prices and have an unchanged profit margin as well as an unchanged market size and thus unchanged profits. This results in the same unchanged number of reaserchers l_v^* employed per firm engaged in process R&D and thus an unchanged vertical innovation rate x_{2b}^*.
It, however, impacts the number of horizontal innovations. The labor demand from both R&D sectors remains unchanged but the production sector is affected by a decreasing fraction of rich agents. There is a smaller number of individuals consuming the whole range of goods, leading to a decreasing overall number of goods produced. A less labor absorbing production sector releases resources for the extension of the range of varieties. Therefore, an increasing income concentration benefits horizontal innovation.[45]
The adjustments within an economy according to Proposition 2b in case of an increasing income concentration are illustrated in Figure 3.7 (c).

The previous insights are gathered in the following proposition

Proposition 3b

*In an economy according to Proposition **2b***

- *(a) an increasing income gap,* $\boldsymbol{\theta} \downarrow$
  - *(i) lowers the vertical innovation rate* $\boldsymbol{x_{2b}^*}$
  - *(i) increases the number of horizontal innovations* $\boldsymbol{G_{2b}^*}$
- *(b) an increasing income concentration,* $\boldsymbol{\beta} \uparrow$
  - *(ii) leaves the vertical innovation rate* $\boldsymbol{x_{2b}^*}$ *unchanged*
  - *(ii) increases the number of horizontal innovations* $\boldsymbol{G_{2b}^*}$

The last equilibrium scenario occurs if $\theta < \underline{\theta}$. In this case, the poor cannot afford the whole range of vertical innovators' goods at prices $p_v \geq \underline{p_v}$, such that a fraction of the process innovators will become exclusive producers selling only to the rich while the other fraction remains in mass production.

[45]It is crucial to note, that the described effects of an income concentration are no *ceteris paribus* effects. The two endogenous variables of this scenario are p_v and G. As such, β would also affect p_v, impacting the vertical innovation rate. An increasing range of goods will also increase the number goods provided by vertical innovators. With an unchanged income share θ and unchanged vertical innovator prices p_v, the poor would thus have increasing expenditures that cannot be covered by their income. The endogenous prices p_v would thus fall to balance the poor's expenditures, leading to a damped vertical innovation rate x_{2b}^*. As argued above, falling prices p_v will increase G_{2b}^* such that an increasing income concentration will surely benefit horizontal innovation. We shall abstract from this *ceteris paribus* effect and tacitly assume that a higher β is accompanied by a slightly higher θ that counterbalances the poor's expenditures such that we can stick to the above interpretation.

Scenario 3

Vertical Innovation

If the poor's income share is small enough, i.e. $\theta < \underline{\theta}$, they could only afford the whole range of vertical innovators' goods at prices below $\underline{p_v} \equiv b + (1-\beta)(a-b)$. However, no vertical innovator will ever consent to these prices as it is more lucrative to charge $\overline{p_v} \equiv a$ and sell exclusively to the rich. Therefore, in such an economy, the poor will solely buy a subset of these goods at the lower bound price.

Depending on the actual income share θ a poor agent possesses, there will be an endogenous fraction m of mass producing vertical innovators that sell their goods at prices $p_v^m = \underline{p_v}$, satisfying the $D_v^m = N$ demanded units. On the other hand, a fraction $1-m$ of exclusive producing vertical innovators charges prices $p_v^e = \overline{p_v}$ while facing an aggregate demand $D_v^e = (1-\beta)N$.

A vertical innovator is indifferent between these two strategies as they yield the same production profits

$$\pi_v = (1-\beta)N(a-b)$$

As the production profits a firm engaged in vertical R&D expects to recieve in case of a successful process innovation are independent of the variety, they also demand the same number of researchers

$$l_v^*(j) = ln((1-\beta)N(a-b)) \quad , \quad j \in [0, G]$$

To guarantee that vertical innovations occur, we make the following assumption

Assumption 1d

$$(1-\beta)N(a-b) > 1$$

that once again states that the production profits π_v need to be big enough to make an innovation effort worthwile.

A reformulated version of this assumption states that $\beta < 1 - \frac{1}{N(a-b)} \equiv \overline{\beta}$. Otherwise, the customer base of the exclusively producing vertical innovators, $D_v^e = N_r$, or the prices charged by the mass producing vertical innovators, $p_v^m = a(1-\beta) + b\beta$, would be too small.

A uniform labor demand implies a uniform success probability and therefore a steady vertical innovation rate, such that the equilibrium fraction of goods where a process innovation actually occurs is given by

$$x_{2c}^* = 1 - \frac{1}{(1-\beta)N(a-b)} \tag{2.9}$$

Assumption 1d ensures that $0 < x_{2c}^* < 1$ holds. That is, vertical innovations occur but not for each variety.

Horizontal Innovation

Horizontal R&D is still characterized by free entry such that the expected profits have to amount to the innovation costs. That is

$$(1 - x^*(j))\pi_h(j) = F$$

holds for each variety j.
Remembering that the marginal production costs are a and therefore the production profits of a product inventor are given by $\pi_h(j) = D_h(j)[p_h(j) - a]$ we can write the zero expected profit prices as

$$p_h(j) = a + \frac{F}{(1 - x^*(j))D_h(j)}$$

where $x^*(j)$ is the probability that the vertical innovator targeting variety j succeeds in discovering the new production technique and $D_h(j)$ is the aggregate demand the horizontal innovator faces. Since these prices cover the setup costs F, they exceed the prices charged by the exclusive producing vertical innovators, such that a horizontal innovator will only face demand from the rich. In the symmetric equilibrium, the demand schedule, $D_h(j) = (1 - \beta)N$, and the probability of being driven out, $x^*(j) = x^*_{2c}$, are identical across all varieties, such that a product inventor that remains the only producer of the respective consumer good will charge

$$p_h = a + F(a - b)$$

which corresponds to the prices set by mass producing horizontal innovators in Scenario 1. Although they serve a smaller market, which would increase the prices, there is a lower probability of being driven out by a successful process innovator, offsetting the former effect, such that the prices remain unchanged.
Before we determine the equilibrium number of horizontal innovations G, we turn our attention to the consumers to find the driving mechanisms behind the fraction of mass producing vertical innovators m.

Consumers

As before, the optimal consumer behavior can be summarized by equating the relative incomes ω to the relative expenditures φ. The poor only consume the goods produced by the $G_v^m = mx^*_{2c}G$ mass producing vertical innovators. The rich additionally purchase the $G_v^e = (1 - m)x^*_{2c}G$ products provided by the exclusive producing vertical innovators and the ones provided by the $G_h = (1 - x^*_{2c})G$ horizontal innovators. Thus we can write

$$\frac{1 - \beta\theta}{(1 - \beta)\theta} = \frac{x^*_{2c}(mp_v^m + (1 - m)p_v^e) + (1 - x^*_{2c})p_h}{x^*_{2c}mp_v^m}$$

Using the values for x^*_{2c}, p_h, $p^e_v = \overline{p_v}$ and $p^m_v = \underline{p_v}$, we can express the endogenous fraction of mass producers m in terms of the exogenous parameters θ, β, a, b, F and N.

$$m^*_2 = \frac{[F + (1-\beta)Na](a-b)(1-\beta)\theta}{[(1-\beta)N(a-b)-1][a(1-\beta)+b\beta(1-\theta)]} \tag{2.10}$$

By Assumption 0, Assumption 1d and Assumption 2 it holds that $m^*_2 > 0$. The last assumption guarantees that $\theta > 0$, that is the poor always recieve a strictly positive income such that a strictly positive number of goods will be provided by mass producers. As we are in an equlibrium within which the poor only consume a subset of all vertical innovators' goods we need $m^*_2 < 1$ which is given whenever the following assumption holds

Assumption 3c

$$\theta < \underline{\theta}$$

Obviously, the income share of the poor has to be small enough as they would otherwise be in a position to purchase the whole range of vertical innovators' goods at prices at or above $\underline{p_v}$ and we would be in either of the previous equilibrium scenarios. Furthermore, $0 < \underline{\theta}$ is ensured by Assumption 0, 1d and 2.
This allows us to summarize these facts in the following proposition

Proposition 2c

*If Assumption **0**, **1d**, **2** and **3c** hold:*

An equilibrium exists where vertical and horizontal innovations occur and inequality is present. Furthermore, the rich consume all $\boldsymbol{G}$ goods available and the poor consume only a subset $\boldsymbol{m}$ of the $\boldsymbol{G_v}$ goods provided by the vertical innovators.

This proposition states that an economy according to the third equilibrium scenario exists given the prerequisite parameter ranges defined in the respective assumptions. An illustration of this economy is shown in Figure 3.5 (d).
Yet again, we derrive the equilibrium range of goods via the labor market.

Labor Market

The labor market clears at the unit wage rate and the aggregate inelastic labor supply is unchanged. Labor demand again stems from vertical and horizontal R&D and the production sector and is given by the self-explaining equation

$$L^d = G\Big(N\Big(x^*_{2c}b(m^*_2 + (1-m^*_2)(1-\beta)) + (1-x^*_{2c})a(1-\beta)\Big) + F + l^*_v\Big)$$

After some rearrangements and usage of the above equilibrium values for x^*_{2c}, l^*_v and m^*_2 the labor market clearing condition gives the following equilibrium number of product inventions

$$G^*_{2c} = \frac{Nl}{F + (1-\beta)Nb + 1 + ln((1-\beta)N(a-b)) + \frac{[F+((1-\beta)Na)]b\beta\theta}{a(1-\beta)+b\beta(1-\theta)}} \quad (2.11)$$

Assumption 0, 1d and 2 ensure that $0 < G^*_{2c} < \infty$.

Discussion

Having derived the equilibrium values for the vertical innovation rate x^*_{2c} and the amount of horizontal innovations G^*_{2c}, we can yet again ask ourselves how they react to changes in inequality.

Beginning with an increasing income gap, we note that a smaller income share of the poor θ doesn't afffect the production profits of a vertical innovator, irrespective of whether he is a mass or an exclusive producer since they still set the same prices and satisfy the same demand. As the production profits directly determine the vertical innovation incentives, the optimal labor demand l^*_v and therefore the vertical innovation rate x^*_{2c} remain unaffected by an increasing income gap.

What changes though is the fraction of mass producing vertical innovators m^*_2. With fixed prices, it straightaway follows that a smaller income of the poor reduces the range of goods they purchase and as such the fraction of mass producing vertical innovators. Due to a smaller share of goods consumed by everyone, the production process becomes less labor absorbing. Together with an unchanged labor demand in both R&D markets, a smaller income share of the poor releases labor that is used for the invention of new varieties, such that an increasing income gap implies more horizontal innovations.

The adjustments within an economy according to Proposition 2c in case of an increasing income gap are illustrated in Figure 3.8 (b).

The more diversified impact comes from an income concentration. Opposed to the previous two scenarios, a larger fraction of poor individuals affects the vertical innovation rate. Either by lowering the customer base of the exlusive producing vertical innovators or by reducing the prices set by the mass producing process innovators, a higher β leads to smaller production profits and therefore, via a lower optimal demand for researchers in the second stage R&D market, to a lower vertical innovation rate.

Contrasting this clear cut relationship is the impact of an increasing income concentration on the number of horizontal innovations.

First off, it lowers the optimal labor demand of each vertical innovator l_v^*. Secondly, it changes the labor demand in production in a threefold way.
As in the previous scenarios, there will be less rich individuals present in the economy which consume the whole range of goods and more poor individuals that only consume a subset of them. While this would clearly reduce the amount of labor absorbed in production such that, together with the smaller labor demand in vertical R&D markets, the range of goods invented could be extended, there are other forces present working in the opposite direction.
The smaller vertical innovation rate x_{2c}^* implies that the average production process becomes more labor intense as there will be more firms present serving the market with the old production technique. Whereas this is an entirely new effect, the one on the fraction of goods purchased by the poor is not.
The relative income ω rises with an increasing income concentration. In the previous scenarios, the relative expenditures φ were affected by changes in the prices set by the exclusively producing horizontal innovators since they had to spread the fixed setup costs F on a smaller customer base. In this last scenario, the prices set by the horizontal innovators remain unaffected but rather the ones of the mass producing vertical innovators fall. Furthermore, the decreasing vertical innovation rate lowers the expenditures of the poor even further while increasing the ones of the rich since they purchase a larger fraction of high priced goods provided by horizontal innovators. Opposed to the previous scenarios, these two effects that increase the relative expenditures might be strong enough, such that the equilibrium amount of mass producers m_2^* might increase to counterbalance them. If this were to be the case, a broader set of goods would be sold to everyone such that the aggregate labor demand in production would increase.
Therefore, we can say that a higher β lowers l_v^* and N_r, which makes a borader set of varieties invented feasible due to a lower labor demand in vertical R&D and production, respectively , lowers x_{2c}^*, which makes production less labor efficient, and has an ambiguous effect on m_2^*. Depending on the relative strength of these forces, the number of horizontal innovations might increase or decrease. Although the actual direction depends on parameter constelations, it is more likely for an increasing income concentration to benefit horizontal innovation whenever the degree of inequality is already relatively high, either by means of a large β or a small θ.
The adjustments within an economy according to Proposition 2c in case of an increasing income concentration are illustrated in Figure 3.8 (c).
Altogether, we can summarize these results in the following proposition

Proposition 3c

In an economy according to Proposition $\boldsymbol{2c}$

(a) an increasing *income gap*, $\boldsymbol{\theta} \downarrow$

  (i) leaves the vertical innovation rate $\boldsymbol{x^*_{2c}}$ *unchanged*

  (i) *increases* *the number of horizontal innovations* $\boldsymbol{G^*_{2c}}$

(b) an increasing *income concentration*, $\boldsymbol{\beta} \uparrow$

  (ii) *lowers* *the vertical innovation rate* $\boldsymbol{x^*_{2c}}$

  (ii) *ambiguously* *impacts the number of horizontal innovations* $\boldsymbol{G^*_{2c}}$

Now, before we turn to the next section, we shall conclude this one by compactly recapitulating the most important findings and drawing comparisons to the existing literature.

Summary

The overall impact of inequality on the R&D activities is summarized in the following proposition

Proposition 3

*If Assumption **0**, **1d**[46] and **2** hold:*

An economy exists where horizontal and vertical innovations occur and inequality is present. Furthermore, an increasing inequality by means of

(a) an increasing income gap, $\boldsymbol{\theta} \downarrow$

(i) never fosters vertical innovations $\boldsymbol{x}^*$

(i) always promotes horizontal innovations $\boldsymbol{G}^*$

(b) an increasing income concentration, $\boldsymbol{\beta} \uparrow$

(ii) never fosters vertical innovations $\boldsymbol{x}^*$

(ii) promotes horizontal innovations $\boldsymbol{G}^*$ *if* $\theta > \underline{\theta}$

(ii) ambiguously impacts horizontal innovations $\boldsymbol{G}^*$ *if* $\theta < \underline{\theta}$

In the most basic form, it could be argued that horizontal innovations are targeted at the rich by creating new, previously inexisting varieties while vertical innovations are targeted at the poor by introducing new production techniques that allow the charging of lower prices, making these goods available for a broader customer base. An increasing inequality tends to weaken the position of the poor while strengthening the one of the rich, thus stimulating horizontal R&D and retarding vertical R&D.
As in all studies focusing on the impact of inequality on innovation-driven growth, the assumption of non-homothetic preferences implies that the rich consume a broader range of goods. Falkinger [1994] abstracts from any price effects by assuming that firms charge constant prices, such that an increase in inequality extends the excess demand of the rich which is met by the invention of new luxuries by the innovative sector.
Zweimüller [2000] also assumes constant prices, however, the excess demand of the rich is met by a non-innovative sector operating with a freely accesible backstop technology. The innovative firms introduce cost-efficient production technologies and provide more basic goods. As such, an increasing inequality lowers the innovation incentives by affecting the time path of demand of the most recent innovator due to a postponed profit flow from the now even poorer consumers.

[46] Assumption 1d is a sufficient condition to guarantee this proposition. Although it is not a necessary one to guarantee an equilibrium according to Scenario 1 or 2 it is required for Scenario 3.

This study builds upon the assumption of the former paper, that the excess demand of the rich is met by an innovative sector. Yet, an increasing inequality also affects the prices charged by the innovators which partially absorbs the excess income of the rich. However, except for an increasing income concentration in case $\theta < \underline{\theta}$ inequality unambiguously fosters product inventions. On the other hand, inequality tends to retard the vertical innovation incentives by lowering the production profits. This is either due to a negative price effect, i.e. lower prices charged by vertical innovators in Scenario 2 (p_v) and Scenario 3 (p_v^e), or a negative demand effect, i.e. a smaller customer base of exclusive vertical innovators in Scenario 3 (N_r). Therefore, an increasing inequality tends to stifle cost-reducing innovtions that thrive on the purchasing power of the poor, as in Zweimüller [2000].
Foellmi and Zweimüller [2006] allow for price effects and note that an increasing inequality raises the incentives to innovate. On the one hand, it increases the willingness to pay of the rich and as such the prices charged by the most recent innovator, constituting the positive price effect. On the other hand, a negative demand effect is present lowering the innovation incentives, which either takes the form of a smaller initial customer base or a longer transition time to mass markets, depending on whether the increasing inequality is due to an increasing income concentration or an increasing income gap, respectively. However, the price effect always dominates the demand effect, such that the innovation incentives rise in case inequality increases.
Foellmi and Zweimüller [2017] also allow for price effects, however, assume a market structure with a competitive fringe à la Zweimüller [2000]. An increasing inequality impacts the innovation incentives by means of a positive price effect and a negative market size effect. The former is due to an increasing willingness to pay of the rich, the latter due to the redirection of demand from the innovative to the non-innovative sector. Which effect dominates depends on the relative cost advantage of the innovators over the competitive fringe and as such on the fomer's price setting scope.
While the former studies focus on one innovation type only, the model of Foellmi et al. [2014] closely resembles the setup of the present study by allowing for a twofold innovation process with horizontal innovations that introduce new varieties and vertical innovations that introduce mass production techniques which, contrasting the present study, come at the expense of product quality.
In their base model, where both innovations are undertaken by the same firm, an increasing *income gap* raises the rich's willingness to pay for quality while lowering the one of the poor, directing the R&D incentives away from process innovations towards product inventions. If mass production techniques are not crucial contributors to the knowledge stock and thus minor drivers of the productivity level, this raises the growth rate. In the current study, an increasing income gap also benefits horizontal innovation

and tends to retard vertical innovations, however not due to changes in the willingness to pay but rather because of a positive demand effect, i.e a strengthened excess demand for new varieties triggered by the higher excess income of the rich, and a negative price effect, i.e. a smaller price charged by vertical innovators, respectively.

In their study, an increasing *income concentration* raises the willingness to pay of the rich as well but reduces the customer base of the most recent horizontal innovator. If the mass production techniques are irrelevent for technical change, this increase in inequality raises the growth rate, otherwise it has an ambiguous impact. This ambiguity of an increasing income concentration is also featured in the present study with respect to the horizontal innovation activity in case inequality is relatively pronounced, i.e. if $\theta < \underline{\theta}$. In this scenario, inequality also reduces the vertical innovation incentives by means of negative price effects for mass producing process innovators or negative demand effects of exclusive producing process innovators. Otherwise, i.e. if $\theta > \underline{\theta}$, it unambiguously benefits horizontal innovations while leaving the vertical innovation incentives unaffected.

Now that we have extensively analyzed the implications of inequality within a two-class society on the two innovation activities we return to our benchmark model by normalizing $\theta = 1$. In this one-class society we shall analyze the impact a protection of first stage against second-stage innovators has on both R&D activities by means of a leading patent breadth.

2.4 Patent Breadth

As mentioned in the introduction, one major issue with vertical innovations lies within their schumpeterian nature. In case sequential innovations are rival to one another, future entrants erode the profit stream of the incumbent. This profit erosion dampens today's innovation incentives, leading to an underinvestment in R&D and subsequently a suboptimally low growth rate. As is the case in the present study, this future threat is especially pronounced if every superior innovation renders its predecessor obsolete such that the profit stream of the current innovator stops with the next innovation. If this profit erosion becomes severe enough, such that today's innovators can no longer cover their research costs, no innovations occur today and subsequently none in the future. To counteract this phenomenon, there needs to be an adjustment in the patent structure. A particularly suitable instrument to this end is leading patent breadth that defines the range of superior versions that can be blocked from commercial production by the patent holder. However, while an increasing leading patent breadth undoubtably extends the profit stream of the most recent innovator, and as such not only possesses the power to guarantee any innovation activity at all but also promotes today's innovation incentives, it also lowers the incentives for future innovators. As such, a too strong patent protection of current innovations might stifle economic growth by retarding the incentives for future innovations.

Up to now, there was no patent breadth at all and a successful innovator was solely granted a patent that covered his developed production technique for the respective variety. There was neither a lagging breadth, that restricted the commercial production with the inferior production techniques of the horizontal innovators, nor was there a leading breadth, that restricted the commercial production with the superior techniques of the vertical innovators. While we still abstain from the former patent breadth, we analyze the implications of the latter on both R&D activities. In particular, we assume that the leading breadth comes in form of a blocking patent that prevents a successful process innovator from commercial production without a license from the incumbent product inventor. Furthermore, we consider ex-ante licensing, i.e. the license bargaining is done before the second R&D stage, and analyze different strength of the blocking patent as measured by the fraction $\alpha \in [0, 1]$ a vertical innovator must pay from his production profits as license fee in case he succeeds in his R&D activity. The ex-ante licensing is crucial for the outcome of the model since it affects the process innovator's expected profits and as such his optimal demand for researchers. If we were to consider ex-post licensing, the vertical innovator would have to form beliefs about the height of the license payment. Since we want to abstract form belief theory, we thus have to consider ex-ante licenses. Furthermore, the patent structure, i.e. that patents comprise a leading breadth and that the license bargaining is done prior to the second-stage, is common knowledge to all market participants before the two-stage R&D process.

Vertical Innovation

As we are still in the benchmark model framework, the access to each variety in the second-stage R&D market is restricted to one firm and prohibited for the product inventor. As such, a firm targeting the consumer good j yet again faces the problem of employing the optimal number of reaserachers that maximizes his expected profits. The only thing that changes is the fact that he only recieves a fraction $1 - \alpha$ of his production profits while paying the rest as license fee to the product inventor. Furthermore, the ex-ante bargaining implies that the profits a second-stage innovator expects to recieve if he engages in the vertical R&D market can be written as

$$(1 - e^{-l_v(j)})(1 - \alpha)\pi_v(j) - l_v(j)$$

where $l_v(j)$ is the number of researchers employed, α the share of profits payed to the product inventor as part of the licensing agreement and $\pi_v(j)$ the production profits which have the familiar form

$$\pi_v(j) = D_v(j)[p_v(j) - b]$$

where $D_v(j)$ and $p_v(j)$ are the well known aggregate demand faced by the vertical innovator producing variety j and his charged prices, respectively.
Without patent breadth, the first stage innovator always had an incentive to underbid any vertical innovator's price above his own marginal production costs a to cover his sunk innovation costs F. However, if he is compensated with a fraction α of the profits from the second-stage innovator in case the cost-reducing production technique is actually discovered, he might have an incentive to collude with them in order to keep prices high and recieve higher license payments. In fact, O'Donoghue and Zweimüller [2004] point out that it is the facilitated collusion enabeled by a leading patent breadth that leads to a stimulation of R&D, such that an efficient cooperation between the patent and antitrust authorities is inevitable for promoting innovations.
To remain in the simplest of worlds, we assume that the antitrust authorities prohibit any form of collusion. Similar to before, the Bertrand competition implies a marginal cost pricing such that $p_v(j) = a$. Absent inequality, the completely homogeneous consumers will demand the whole range of goods in equilibrium such that $D_v(j) = N$.
Since these values are independent of the variety, so are the production profits $\pi_v(j)$. With the same leading patent breadth structure across the consumer goods, the optimal labor demand of a representative second stage innovator is given by

$$l_v^*(j) = ln((1 - \alpha)N(a - b)) \quad , \quad j \in [0, G]$$

As we are accustomed to, we need a certain assumption that guarantees that vertical innovations actually occur

Assumption 1e

$$(1-\alpha)N(a-b) > 1$$

Exchanging α with β, we would have Assumption 1d. Similarly, this assumption states that the profits accruing to a successful vertical innovator need to be big enough such that it is worthwile to engage in vertical R&D. A restated version implies that $\alpha < 1 - \frac{1}{N(a-b)} \equiv \overline{\alpha}$. With a finite cost reduction $a-b$ and a finite population size N, the threshold value $\overline{\alpha} < 1$. Therefore, as long as $\overline{\alpha} < \alpha \leq 1$, that is as long as the fraction of profits paid as licensing fee is big enough, no firm would engage in vertical R&D and we find ourselves in an economy according to the introductory model. Henceforth, we thus assume that Assumption 1e holds such that we are indeed in an economy with both types of innovations and a leading patent breadth.

The uniform labor demand across all firms in the second stage implies a uniform success probability and, due to the continuous range of goods, a steady vertical innovation rate that is given by

$$x_3^* = 1 - \frac{1}{(1-\alpha)N(a-b)} \tag{2.12}$$

By Assumption 0 and 1e, $0 < x_3^* < 1$ such that vertical innovations occur but not for each variety.

To find the equilibrium range of goods, we need to know which prices the representative consumer faces and thus turn our attention to the first stage R&D market.

Horizontal Innovation

There is still free entry into the first stage R&D market, such that in equilibrium the expected profits amount to the innovation cost. Yet again, the patent structure is common knowledge before the two-stage innovation process takes place, such that the expected profits of a horizontal innovator are twofold. If the product inventor remains the sole producer, which happens with probability $1-x_3^*$, he reaps production profits $\pi_h(j)$, whereas in case of a successful process innovation, that happens with probability x_3^*, he will be compensated for his lost monopoly position by the licensing fee, a fraction α of the production profits π_v of a vertical innovator. The free entry condition thus states

$$(1-x_3^*)\pi_h(j) + x_3^*\alpha\pi_v = F$$

Using the knowledge that the production profits of a horizontal innovator are given by $\pi_h(j) = D_h(j)[p_h(j) - a]$, that the equilibrium will feature $D_h(j) = N$ due to the homogeneous consumer group and using the previously derived value for x_3^*, the zero profit price a horizontal innovator charges in case he remains the sole producer of the respective variety is given by

$$p_h(j) = a + F(a-b) - \alpha(a-b)[F + (1-\alpha)N(a-b) - 1] \quad , \quad j \in G_h$$

Not only does the symmetric market structure imply that these prices are equal across all producing horizontal innovators $G_h = (1 - x_3^*)G$, but they are also smaller than the ones charged once patent breadth is absent. On one hand, the introduction of a license payment reduces the vertical innovation rate and thus increases the probability that the horizontal innovator remains in the market, lowering the need to cover the fixed setup costs F by high prices. Furthermore, even in case he is driven out of the market, he is compensated via the license fee paid by the vertical innovator, reducing the zero expected profit prices even further.
We are now in a position to determine which range of goods the homogeneous consumers are able to afford in equilibrium.

Consumers

There is no inequality and the consumers share the same non-homothetic preferences where they are fully satiated after consumption of one unit of a variety. Together with the lack of satiation with respect to an increasing range of goods and the lacking further time periods, their optimal behavior still implies full income expenditure. The means of our representative consumer are given by

$$y_i = \lambda_i + \rho_i$$

where $\lambda_i = l$ is the unchanged labor endowment and labor income, respectively, and $\rho_i = \frac{G}{N}(x_3^*(1-\alpha)\pi_v - l_v^*)$ is the profit income which comes about the restricted access to the second-stage R&D market.[47]
On the other hand the expenditures are given by

$$e_i = G_v p_v + G_h p_h$$

where $G_v = x_3^* G$ measures the range of goods provided by vertical innovators at prices $p_v = a$ and $G_h = (1 - x_3^*)G$ measures the range of goods provided by

[47]Remember that, due to the continuous range of goods, the expected values of a single firm correspond to the averages of this type of firm. As such, the expected profits of a horizontal innovator will correspond to the average profits of all horizontal innovators and vice versa for the vertical innovators.

horizontal innovators at prices p_h. Using the previously derived equilibrium values, the average price faced by a consumer is given by

$$p_\phi = x_3^* p_v + (1 - x_3^*) p_h = a + \frac{F}{N} - ((1-\alpha)N(a-b) - 1)\frac{\alpha}{(1-\alpha)N}$$

An interesting feature of these average prices is, that once we set α either to zero or to $\overline{\alpha}$ we arrive at the old average prices from the introductory and the benchmark model.
By using the full expenditure constraint $y_i = e_i$ and the equilibrium values we arrive at the expression for the equilibrium amount of product inventions

$$G_3^* = \frac{Nl}{F + Nb + 1 + ln[(1-\alpha)N(a-b)] + \frac{\alpha}{1-\alpha}} \qquad (2.13)$$

Assumption 0 and 1e yet again guarantee that $0 < G_3^* < \infty$.

Discussion

Since we are in an egalitarian society, we compare our equilibrium vertical innovation rate and number of product inventions to the benchmark model.
Consulting equation (2.2) with the adequate normalization of the productivity parameter $q = 1$, we see that $x_1^* \geq x_3^*$ as long as $0 \leq \alpha \leq \overline{\alpha}$.
That is, if there is no leading patent breadth, i.e. $\alpha = 0$, we obviously find ourselves in the old benchmark scenario where the incentives to engage in process R&D are unchanged.
However, if leading patent breadth exists such that a strictly positive license fee is required, i.e. $\alpha > 0$, the vertical innovation rate sinks. Furthermore, an increase in α, and thus a stronger protection for the inventor of the variety, reduces the profits left for the vertical innovator and as such his optimal demand for researchers l_v^* that in turn reduces the vertical innovation rate x_3^*.
The vertical innovation rate continues to fall in α up to the crticial threshold value $\overline{\alpha} < 1$, above which no new production technique will be discovered and we find ourselves once again in the world of the introductory model.
Since a leading patent breadth is designed to reduce the innovation incentives for superior versions of a variety, it comes as no surprise that the vertical innovation rate x_3^* is falling in α. However, it should also increase the incentives for the incumbent innovator since he recieves a stronger protection of his invention, such that we should expect a positive impact of α on the amount of horizontal innovations G_3^*.
Consulting equation (2.3), with the adequate normalization, we note that $G_1^* \geq G_3^*$ as long as $0 \leq \alpha \leq \overline{\alpha}$.[48]

[48] $ln(1-\alpha) + \frac{\alpha}{1-\alpha}$ is a strictly positive function for $0 < \alpha \leq 1$.

If there is no leading patent breadth, i.e. $\alpha = 0$, we obviously land in the benchmark economy such that $G_1^* = G_3^*$. Once we introduce patent breadth and allow for a positive license fee, i.e. $\alpha > 0$, we get that $G_1^* > G_3^*$.
Furthermore, G_3^* falls in α until it reaches the threshold value $\overline{\alpha}$, above which we have no vertical innovations, i.e. $x_3^* = 0$, such that we find ourself in a world according to the introductory model, i.e. $G_0^* = G_3^*$.
This is exactly the opposite of what we expected. Not only does the introduction of a leading patent breadth reduce the number of product inventions but a strengthening of the blocking patent by means of a higher license payment harms horizontal innovations even further. The driving force behind this result is the dependence of a representative consumer's income on process innovation profits. An increase in α lowers the expected profits of a vertical innovator and thus the profit income ρ_i of our representative consumer. Although the average prices p_ϕ might rise or fall in α[49], the reduced income always dominates such that a higher license fee reduces the range of goods a consumer can purchase.
The following proposition summarizes these insights.

Proposition 4

*In an egalitarian economy where collusion is forbidden and Assumption **0** and **1e** hold, there occur horizontal as well as vertical innovations. Furthermore, strengthening the blocking patent of the leading breadth by means of a higher license fee* $\boldsymbol{\alpha}$

(a) reduces the vertical innovation rate $\boldsymbol{x_3^*}$

(b) reduces the number of horizontal innovations $\boldsymbol{G_3^*}$

Interestingly enough, O'Donoghue and Zweimüller [2004] also detect that within a classical quality-ladder model, leading patent breadth stifles the follow-on innovation activity in case collusion is forbidden. In the present study, collusion is prohibited as well and leading patent breadth also has a detrimental impact on the R&D activity, however not only on sequential vertical innovations but for horizontal innovations as well.
Chu et al. [2012] allow for two types of innovations and find that strengthening the effect of blocking patents by means of bigger license payments stimulates horizontal innovation and stifles vertical innovation. Horizontal innovations create new varieties and are thus the first innovations on a quality ladder while every subsequent vertical innovation builds upon them by increasing the product quality. Either increasing the range of superior goods covered by the leading patent breadth or increasing the license payments to

[49] p_ϕ is a convex function of α and falls in the range of $0 < \alpha < 1 - \frac{1}{\sqrt{N(a-b)}} \equiv \underline{\alpha}$ and increases in the range $\underline{\alpha} < \alpha < \overline{\alpha}$.

every innovator whose patent is infringed upon makes the profit stream more backloaded and raises the horizontal innovation incentives while dampening the ones of the vertical innovators.
While the latter effect is also present in this model, increasing license payments also stifle the horizontal innovation activity. Yet again, this counter-intuitive peculiarity is driven by the redistribution of firm profits to the consumers. Although the expected profits of a single horizontal innovator remain unaffected by any changes in the patent scheme, the expected profits of each vertical innovator fall with a stronger leading breadth. As such, a stronger patent protection reduces total profits and therefore a representative consumer's income restricting him to purchase a narrower range of goods which implies a smaller number of horizontal innovations. Thus, leading patent breadth has a detrimental impact on the type of innovations it is designed to foster.

2.5 Conclusion

This study has shed some light on the driving forces behind innovations in a one-period, closed-economy model with a preceding two-stage R&D process and non-homothetic consumer preferences.

The first driving force is the *researchers' productivity* in process R&D, q. An increase in this parameter raises the vertical innovation incentives by making it increasingly likely to discover the new production technology. Furthermore, it also raises the number of horizontal innovations. Although the incumbent horizontal innovator is more likely to be driven out of the market, an adequate increase in his prices leaves the one-period profits unchanged and as such his innovation incentives. Yet, the fostering of the cost-efficient new production technology releases resources that are used for the development of new varieties.

The second driving force is the strength of the *patent breadth*, α. An increase in this parameter strictly lowers the vertical innovation rate by reducing the fraction of profits that remain for a successful innovator. Although it should protect the incumbent horizontal innovator, it reduces the number of product inventions. Again, the free entry into the horizontal R&D market requires that the expected profits of a product inventor amount to his research costs, such that a stronger patent breadth leaves the incentives for a single horizontal innovator unchanged. However, it reduces the fraction of firms employing a cost-efficient production technology, absorbing more resource in the production process, and lowers the labor demand in vertical R&D, which releases resources. The former effect always dominates the latter, explaining the controversial impact of patent breadth on horizontal innovation.

The last driving force is *inequality*. This either takes the form of an increasing income gap, i.e. a lower θ, or an increasing income concentration, i.e. an increasing β. If there is only a mild degree of inequality and both parameters are relatively high, they do not impact the vertical innovation rate. However, in case of a pronounced inequality, they lower the vertical innovation incentives by means of price or demand effects. On the other hand, an increasing inequality tends to raise the number of product inventions. An increasing *income gap* always benefits horizontal innovation, mainly by reducing the fraction of goods consumed by the poor and thus the number of mass markets. An increasing *income concentration* also raises the number of product inventions as long as $\theta > \underline{\theta}$, i.e. if the poor are relatively wealthy. In this case, the lower number of rich individuals, that consume the whole range of goods, releases resources. If however $\theta \leq \underline{\theta}$, the additional reduction in the vertical innovation rate, which dominates the lower labor demand in vertical R&D, counteracts this effect, such that the overall impact is ambiguous.

The most intriguing and controversial result of this study is, that stronger incentives for sequential vertical innovation increase the horizontal innovation activity. Albeit this clearly contradicts the existing literature on sequential innovations, it can easily be traced back to the difference in how the respective R&D markets are modeled. The horizontal innovation market is characterized by a free access and a deterministic research process while the vertical innovation market has a limited access and a stochastic research process. This implies that any changes in the aforementioned parameters do not affect the incentives of a single product inventor however they do impact the incentives of a process innovator. Furthermore, horizontal innovators generate zero profits while vertical innovators make positive profits such that all profits occur from second stage innovation. Therefore, any parameter changes that lower the vertical innovation incentives also imply a reduction in total profits and as such in the income of the consumers. Although the average prices might be affected as well the reduction in the consumers' purchasing power always dominates such that they are restricted to purchase a narrower range of goods implying a fall in the number of product inventions.

To add some much needed realism to this model, the most obvious extension would be to introduce further time periods. Not only would this allow the verification of the robustness of the results in a dynamic setting, but it would also enalbe us to draw inferences from the various parameters about the growth rate.

Chapter 3

Appendix

3.1 Glossary

Consumers

i	Indexation for individuals/consumers
N	Number of individuals/consumers
β	Fraction of poor individuals
y_i	Means of individual i
λ_i	Labor income of individual i
l_i	Labor endowment of individual i
ρ_i	Profit income of individual i
θ	Poor's income share
ω	Relative income of rich to poor
e_i	Expenditures of individual i
m	Fraction of mass producers/goods bought by the poor
φ	Relative expenditures of rich to poor

Firms

j	Indexation for consumer goods
G	Equilibrium range of consumer goods
h	Indexation for horizontal innovators

v	Indexation for vertical innovators
x	Success probability in vertical R&D / vertical innovation rate
$\mathbf{l}_v$	Labor demand in vertical R&D
e	Indexation for exclusive producers
m	Indexation for mass producers
F	Horizontal R&D cost/labor requirement
a	Marginal production cost of horizontal innovators
b	Marginal production cost of vertical innovators
q	Labor productivity in vertical R&D
α	Patent breadth / license fee

<u>Labor market</u>

w	Market clearing wage

3.2 Calculations

3.2.1 Derivations

Introductory Model

$$\boldsymbol{G_0^*}$$

$$y_i = e_i$$
$$l = Gp_h$$

$$\Rightarrow \qquad G = \frac{l}{p_h}$$

Using Equilibrium values

$$p_h = a + \frac{F}{N}$$

$$\Rightarrow \qquad G_0^* = \frac{Nl}{F + Na}$$

Benchmark Model

Optimal $\boldsymbol{l_v}$

$$\max_{l_v} \quad x(l_v)\pi_v - l_v$$
$$s.t. \quad x(l_v) = 1 - e^{-ql_v}$$

$$\Rightarrow \qquad \mathcal{L} = (1 - e^{-ql_v})\pi_v - l_v$$

FOC

$$\frac{\partial \mathcal{L}}{\partial l_v} = qe^{-ql_v}\pi_v - 1 \stackrel{!}{=} 0$$

$$\Rightarrow \quad l_v^* = \frac{ln(q\pi_v)}{q}$$

SOC

$$\frac{\partial^2 \mathcal{L}}{\partial l_v^2} = -q^2 e^{-ql_v}\pi_v \stackrel{A0}{<} 0$$

$\boldsymbol{G_1^*}$

$$y_i = e_i$$
$$l + \frac{G}{N}(x\pi_v - l_v) = G[xp_v + (1-x)p_h]$$

$$\Rightarrow \qquad G = \frac{Nl}{N\Big(xp_v + (1-x)p_h\Big) - x\pi_v + l_v}$$

Using Equilibrium values

$$\pi_v = N(a-b)$$
$$l_v = \frac{ln(qN(a-b))}{q}$$
$$x = 1 - \frac{1}{qN(a-b)}$$
$$p_v = a$$
$$p_h = a + Fq(a-b)$$

$$\Rightarrow \qquad G_1^* = \frac{Nl}{F + Nb + \frac{1}{q}[1 + ln(qN(a-b))]}$$

Inequality

Scenario 1

$$\mathbf{m_1^*}$$

$$\omega = \varphi$$

$$\frac{1-\beta\theta}{1-\beta} = \frac{xp_v + (1-x)[mp_h^m + (1-m)p_h^e]}{xp_v + (1-x)p_h^m}$$

$$\Rightarrow m = \frac{(1-x)p_h^e - x\kappa p_v}{(1-x)[p_h^e + \kappa p_h^m]}$$

Using Equilibrium values

$$\begin{aligned} x &= x_{2a}^* = 1 - \frac{1}{N(a-b)} \\ \kappa &= \omega - 1 = \frac{1-\theta}{(1-\beta)\theta} \\ p_v &= a \\ p_h^m &= a + F(a-b) \\ p_h^e &= a + \frac{F(a-b)}{1-\beta} \end{aligned}$$

$$\Rightarrow \qquad m_1^* = 1 - \frac{(F+Na)(a-b)(1-\theta)}{a(1-\beta\theta) + F(a-b)}$$

$$m_1^* \overset{A0,A1b,A2}{<} 1$$

$$m_1^* \quad > \quad 0 \qquad \Leftrightarrow \qquad \theta > \frac{(N(a-b)-1)a}{(F+Na)(a-b)-a\beta} \equiv \overline{\theta}$$

$$\boldsymbol{G^*_{2a}}$$

$$L^s = L^d$$

$$Nl = G\Big[N\Big(xb + (1-x)a[m + (1-m)(1-\beta)]\Big) + F + l_v\Big]$$

$$\Rightarrow \qquad G = \frac{Nl}{N(xb + (1-x)a[1-\beta(1-m)]) + F + l_v}$$

Using equilibirum values

$$\begin{aligned} l_v^* &= ln(N(a-b)) \\ x_{2a}^* &= 1 - \frac{1}{N(a-b)} \\ m_1^* &= 1 - \frac{(F+Na)(a-b)(1-\theta)}{a(1-\beta\theta) + F(a-b)} \end{aligned}$$

$$\Rightarrow G_{2a}^* = \frac{Nl}{F + Nb + 1 + ln[N(a-b)] - [F+Na]\frac{a\beta(1-\theta)}{a(1-\beta\theta)+F(a-b)}}$$

Scenario 2

$$\theta(\mathbf{p_v})$$

$$\omega = \varphi$$

$$\frac{1-\beta\theta}{(1-\beta)\theta} = \frac{xp_v + (1-x)p_h}{xp_v}$$

$$\Rightarrow \quad \theta = \frac{xp_v}{xp_v + (1-x)(1-\beta)p_h}$$

Using Equilibrium values

$$x = x^*_{2b} = 1 - \frac{1}{N(p_v - b)}$$
$$p_h = a + \frac{F(p_v - b)}{1-\beta}$$

$$\Rightarrow \qquad \theta(p_v) = \frac{[N(p_v - b) - 1]p_v}{[N(p_v - b) - 1]p_v + a(1-\beta) + F(p_v - b)}$$

$$\frac{\partial\theta(p_v)}{\partial p_v} = \frac{(1-\beta)a[N(2p_v - b) - 1] + F[N(p_v - b)^2 + b]}{\Big[[N(p_v - b) - 1]p_v + a(1-\beta) + F(p_v - b)\Big]^2} \overset{A0,A1c,A2}{>} 0$$

$$\mathbf{G^*_{2b}}$$

$$L^s = L^d$$

$$Nl = G\Big(N\Big(xa + (1-x)b(1-\beta)\Big) + F + l_v^*\Big)$$

$$\Rightarrow \qquad G = \frac{Nl}{N\Big(xa + (1-x)b(1-\beta)\Big) + F + l_v^*}$$

Using Equilibrium values

$$x = x^*_{2b} = 1\frac{1}{N(p_v - b)}$$
$$l_v^* = ln(N(p_v - b))$$

$$\Rightarrow \qquad G^*_{2b} = \frac{Nl}{F + Nb + 1 + ln(N(p_v - b)) - \frac{p_v - (1-\beta)a}{p_v - b}}$$

Scenario 3

$$\mathbf{m_2^*}$$

$$\omega = \varphi$$

$$\frac{1-\beta\theta}{1-\beta} = \frac{x[mp_v^m + (1-m)p_v^e] + (1-x)p_h]}{xmp_v^e}$$

$$\Rightarrow \qquad m = \frac{xp_v^e + (1-x)p_h}{x[p_v^e + \kappa p_v^m]}$$

Using Equilibrium values

$$\begin{aligned} x = x_{2c}^* &= 1 - \frac{1}{(1-\beta)N(a-b)} \\ \kappa = \omega - 1 &= \frac{1-\theta}{(1-\beta)\theta} \\ p_v^m &= (1-\beta)a + \beta b \\ p_v^e &= a \\ p_h &= a + F(a-b) \end{aligned}$$

$$\Rightarrow \qquad m_2^* = \frac{[F + (1-\beta)Na](a-b)(1-\beta)\theta}{[(1-\beta)N(a-b) - 1][a(1-\beta) + b\beta(1-\theta)]}$$

$$m_2^* \overset{A0,A1d,A2}{>} 0$$

$$m_2^* \quad < \quad 1 \qquad \Leftrightarrow \qquad \theta < \frac{((1-\beta)N(a-b)-1)(a-\beta(a-b))}{(F+N(a-\beta(a-b)))(1-\beta)(a-b)-b\beta} \equiv \underline{\theta}$$

$$\mathbf{G^*_{2c}}$$

$$L^s = L^d$$

$$Nl = G\Big[N\Big(xb(m + (1-m)(1-\beta)) + (1-x)a(1-\beta)\Big) + F + l_v\Big]$$

$$\Rightarrow \qquad G = \frac{Nl}{N(\beta xmb + (1-\beta)(xb + (1-x)a)) + F + l_v}$$

Using equilibirum values

$$\begin{aligned} l^*_v &= ln((1-\beta)N(a-b)) \\ x^*_{2c} &= 1 - \frac{1}{(1-\beta)N(a-b)} \\ m^*_2 &= \frac{[F + (1-\beta)Na](a-b)(1-\beta)\theta}{[(1-\beta)N(a-b)-1][a(1-\beta)+b\beta(1-\theta)]} \end{aligned}$$

$$\Rightarrow G^*_{2c} = \frac{Nl}{F + (1-\beta)Nb + 1 + ln[(1-\beta)N(a-b)] + [F + (1-\beta)Na]\frac{b\beta\theta}{a(1-\beta)+b\beta(1-\theta)}}$$

Patent Breadth

G_3^*

$$y_i = e_i$$

$$l + \frac{G}{N}(x(1-\alpha)\pi_v - l_v) = G[xp_v + (1-x)p_h]$$

$$\Rightarrow \qquad G = \frac{Nl}{N\Big(xp_v + (1-x)p_h\Big) - x(1-\alpha)\pi_v + l_v}$$

Using Equilibrium values

$$\begin{aligned}
\pi_v &= N(a-b) \\
l_v &= ln((1-\alpha)N(a-b)) \\
x &= 1 - \frac{1}{(1-\alpha)N(a-b)} \\
p_v &= a \\
p_h &= a + F(a-b) - \alpha(a-b)[F + (1-\alpha)N(a-b) - 1]
\end{aligned}$$

$$\Rightarrow \qquad G_3^* = \frac{Nl}{F + Nb + 1 + ln(1-\alpha)N(a-b) + \frac{\alpha}{1-\alpha}}$$

3.2.2 Comparative Statics

Introductory Model

$$\frac{\partial G_0^*}{\partial F} = \frac{-Nl}{[F+Na]^2} \overset{A0}{<} 0$$

$$\frac{\partial G_0^*}{\partial a} = \frac{-N^2 l}{[F+Na]^2} \overset{A0}{<} 0$$

$$\frac{\partial G_0^*}{\partial l} = \frac{N}{F+Na} \overset{A0}{>} 0$$

$$\frac{\partial G_0^*}{\partial N} = \frac{Fl}{[F+Na]^2} \overset{A0}{>} 0$$

Benchmark Model

$$\mathbf{l_v^*}$$

$$\frac{\partial l_v^*}{\partial a} = \frac{1}{q(a-b)} \overset{A0,A1a}{>} 0$$
$$\frac{\partial l_v^*}{\partial b} = \frac{-1}{q(a-b)} \overset{A0,A1a}{<} 0$$
$$\frac{\partial l_v^*}{\partial N} = \frac{1}{qN} \overset{A0}{>} 0$$
$$\frac{\partial l_v^*}{\partial q} = \frac{1}{q^2}(1 - ln(qN(a-b)) \lesseqgtr 0 \Leftrightarrow q \gtreqless \frac{e}{N(a-b)}$$

$$\mathbf{x_1^*}$$

$$\frac{\partial x_1^*}{\partial a} = \frac{1}{qN(a-b)^2} \overset{A1a}{>} 0$$
$$\frac{\partial x_1^*}{\partial b} = \frac{-1}{qN(a-b)^2} \overset{A1a}{<} 0$$
$$\frac{\partial x_1^*}{\partial N} = \frac{1}{qN^2(a-b)} \overset{A1a}{>} 0$$
$$\frac{\partial x_1^*}{\partial b} = \frac{1}{q^2N(a-b)} \overset{A1a}{>} 0$$

$$\mathbf{G_1^*}$$

$$\frac{\partial G_1^*}{\partial b} = \frac{-Nl[N - \frac{1}{q(a-b)}]}{[F + Nb + \frac{1}{q}(1 + ln(qN(a-b))]^2} \overset{A0,A1a}{<} 0$$
$$\frac{\partial G_1^*}{\partial q} = \frac{Nl\frac{ln(qN(a-b))}{q^2}}{[F + Nb + \frac{1}{q}(1 + ln(qN(a-b))]^2} \overset{A0,A1a}{>} 0$$

Inequality

$$y_i = \begin{cases} \theta y_\phi & , i \in N_p \\ \frac{1-\beta\theta}{1-\beta} y_\phi & , i \in N_r \end{cases}$$

$$\omega = \frac{y_r}{y_p} = \frac{1-\beta\theta}{(1-\beta)\theta}$$

Income concentration

$$\frac{\partial y_p}{\partial \beta} = 0$$
$$\frac{\partial y_r}{\partial \beta} = \frac{1-\theta}{(1-\beta)^2} y_\phi > 0$$
$$\frac{\partial \omega}{\partial \beta} = \frac{1-\theta}{(1-\beta)^2\theta} > 0$$

Income gap

$$\frac{\partial y_p}{\partial \theta} = y_\phi > 0$$
$$\frac{\partial y_r}{\partial \theta} = \frac{-\beta}{1-\beta} y_\phi < 0$$
$$\frac{\partial \omega}{\partial \theta} = \frac{-1}{(1-\beta)\theta^2} < 0$$

Scenario 1

$\mathbf{m_1^*}$

$$\frac{\partial m_1^*}{\partial \theta} = \big(F + Na\big)\big(a - b\big)\frac{a(1-\beta) + F(a-b)}{[a(1-\beta\theta) + F(a-b)]^2} \overset{A0,A1b,A2}{>} 0$$

$$\frac{\partial m_1^*}{\partial \beta} = -\big(F + Na\big)\big(a - b\big)\frac{a\theta(1-\theta)}{[a(1-\beta\theta) + F(a-b)]^2} \overset{A0,A1b,A2}{<} 0$$

$\mathbf{G_{2a}^*}$

$$\frac{\partial G_{2a}^*}{\partial \theta} = \frac{-Nl\big(F + Na\big)a\beta\frac{a(1-\beta)+F(a-b)}{[a(1-\beta\theta)+F(a-b)]^2}}{\big[F + Nb + 1 + ln[N(a-b)] - [F + Na]\frac{a\beta(1-\theta)}{a(1-\beta\theta)+F(a-b)}\big]^2} \overset{A0,A1b,A2}{<} 0$$

$$\frac{\partial G_{2a}^*}{\partial \beta} = \frac{Nl\big(F + Na\big)a(1-\theta)\frac{a+F(a-b)}{[a(1-\beta\theta)+F(a-b)]^2}}{\big[F + Nb + 1 + ln[N(a-b)] - [F + Na]\frac{a\beta(1-\theta)}{a(1-\beta\theta)+F(a-b)}\big]^2} \overset{A0,A1b,A2}{>} 0$$

Scenario 2

$$\boldsymbol{x^*_{2b}}$$

$$\frac{\partial x^*_{2b}}{\partial p_v} = \frac{1}{N(p_v - b)^2} \overset{A1c}{>} 0$$

$$\mathbf{G^*_{2b}}$$

$$\frac{\partial G^*_{2b}}{\partial p_v} = \frac{-Nl[\frac{p_v - a(1-\beta)}{(p_v-b)^2}]]}{(F + Nb + ln[N(p_v - b)] + \frac{a(1-\beta)-b}{p_v-b})^2} \overset{\substack{A0,A1c,A2 \\ p_v \geq (1-\beta)a+\beta b}}{<} 0$$

$$\frac{\partial G^*_{2b}}{\partial \beta} = \frac{Nl[\frac{a}{p_v-b}]}{(F + Nb + ln[N(p_v - b)] + \frac{a(1-\beta)-b}{p_v-b})^2} \overset{A0,A1c,A2}{>} 0$$

<u>Scenario 3</u>

$$\boldsymbol{m_2^*}$$

$$\frac{\partial m_2^*}{\partial \theta} = \frac{[F + (1-\beta)Na](a-b)(1-\beta)(a(1-\beta)+b\beta)}{((1-\beta)N(a-b)-1)\big(a(1-\beta)+b\beta(1-\theta)\big)^2} \overset{A0,A1d,A2}{>} 0$$

$$\frac{\partial m_2^*}{\partial \beta} = \frac{(a-b)\theta\Big[\Big((1-\beta)N\Big(a+F(a-b)\Big)\Big(a(1-\beta)+b\beta(1-\theta)\Big)\Big)-\Big(\Big((1-\beta)N(a-b)-1\Big)\Big((1-\beta)Na+F\Big)b(1-\theta)\Big)\Big]}{\Big(\Big((1-\beta)N(a-b)-1\Big)\Big(a(1-\beta)+b\beta(1-\theta)\Big)\Big)^2} \overset{???}{\lesseqgtr} 0$$

$$\boldsymbol{G_{2c}^*}$$

$$\frac{\partial G_{2c}^*}{\partial \theta} = \frac{-Nl\big[[F+(1-\beta)Na]\frac{b\beta(a(1-\beta)+b\beta)}{[a(1-\beta)+b\beta(1-\theta)]^2}\big]}{\big(F+(1-\beta)Nb+1+ln[(1-\beta)N(a-b)]+[F+(1-\beta)Na]\frac{b\beta\theta}{a(1-\beta)+b\beta(1-\theta)}\big)^2} \overset{A0,A1d,A2}{<} 0$$

$$\frac{\partial G_{2c}^*}{\partial \beta} = \frac{-Nl\big(-Nb-\frac{1}{1-\beta}+\frac{ab\theta(N[a(1-\beta)^2-b\beta^2(1-\theta)]+F)}{(a(1-\beta)+b\beta(1-\theta))^2}\big)}{[F+(1-\beta)Nb+1+ln[(1-\beta)N(a-b)]+((1-\beta)Na+F)\frac{b\beta\theta}{a(1-\beta)+b\beta(1-\theta)}]^2} \overset{???}{\lesseqgtr} 0$$

Patent Breadth

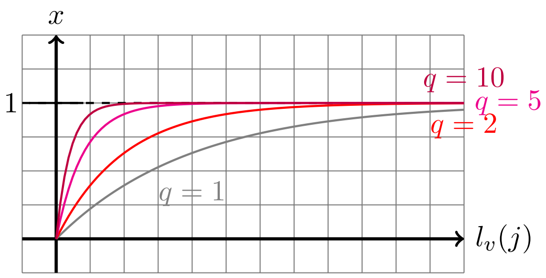
$$\boldsymbol{x_3^*}$$

$$\frac{\partial x_3^*}{\partial \alpha} = -\frac{1}{(1-\alpha)^2 N(a-b)} \overset{A0,1e}{<} 0$$

$$\boldsymbol{G_3^*}$$

$$\frac{\partial G_3^*}{\partial \alpha} = -\frac{Nl\frac{\alpha}{(1-\alpha)^2}}{[F + Nb + 1 + ln(1-\alpha)N(a-b) + \frac{\alpha}{1-\alpha}]^2} \overset{A0,1e}{<} 0$$

3.3 Graphs



(a) x(ql$_v(j)$) $= 1 - e^{-ql_v(j)}$

Figure 3.1: Success Probability in Vertical R&D

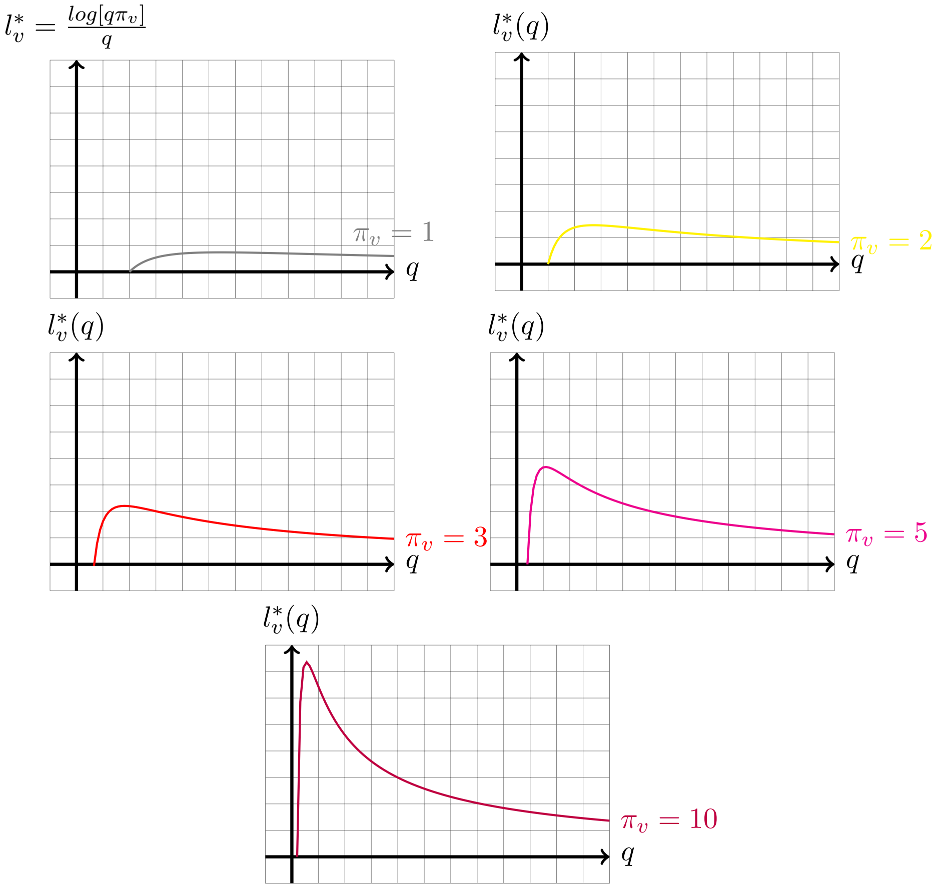
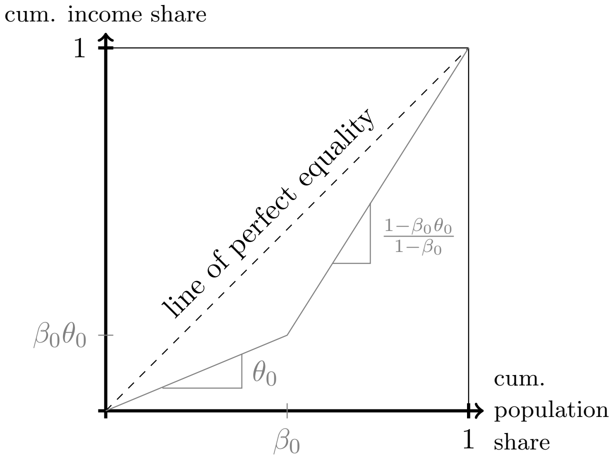
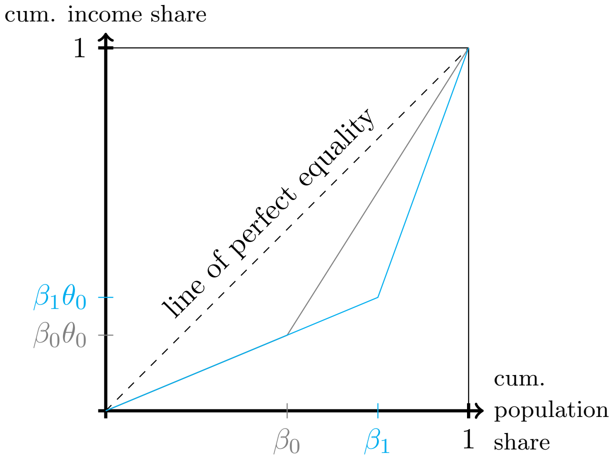


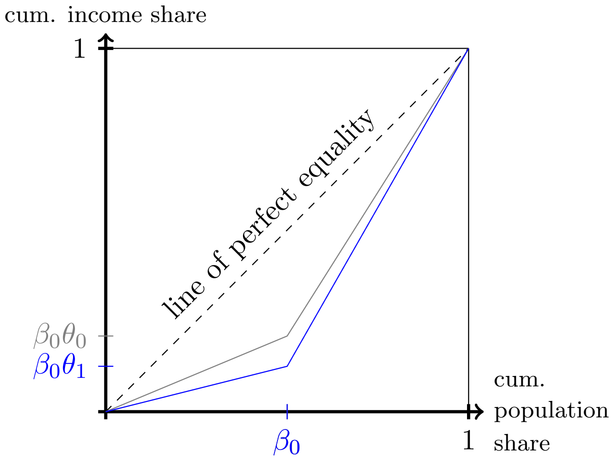
Figure 3.2: Impact of labor efficiency on labor demand



(a) Benchmark

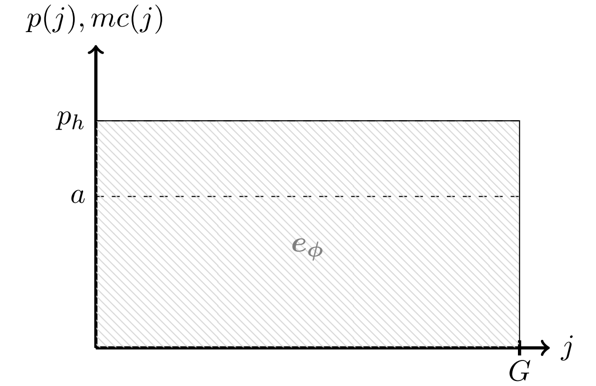


(b) Increasing *Income Concentration*

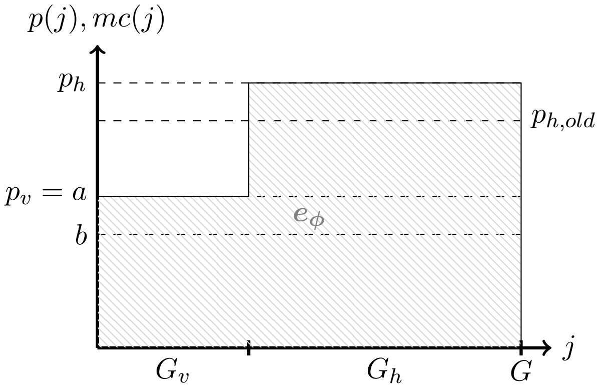


(c) Increasing *Income Gap*

Figure 3.3: Lorenz Curves

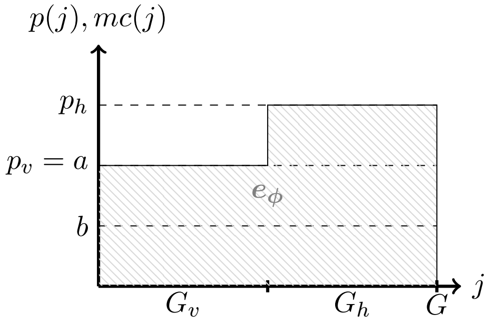


(a) Introductory Model

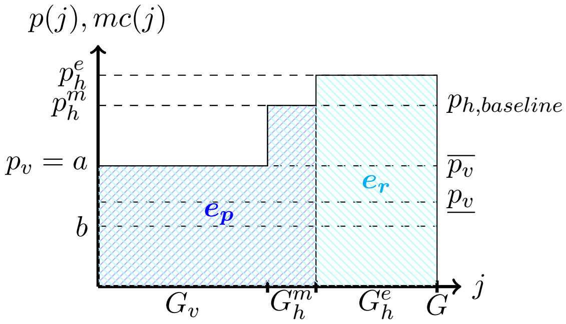


(b) Benchmark Model

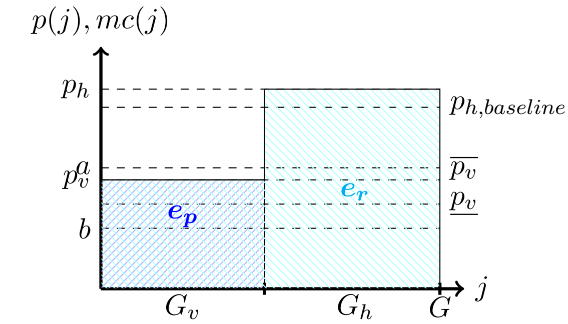
Figure 3.4: Market Structure



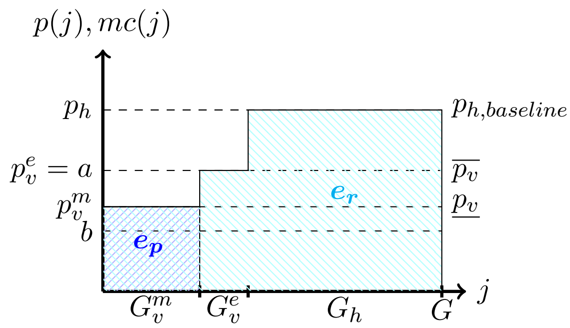
(a) benchmark model



(b) Scenario 1

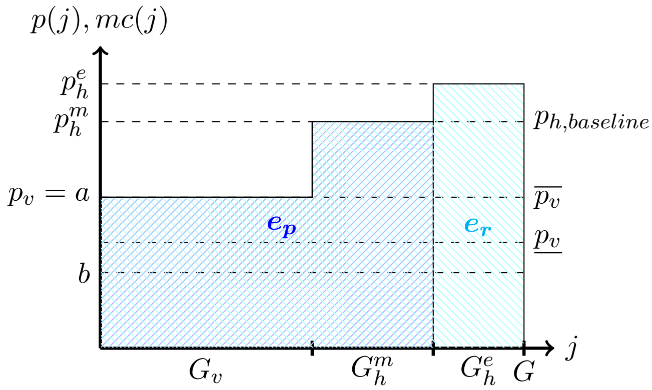


(c) Scenario 2

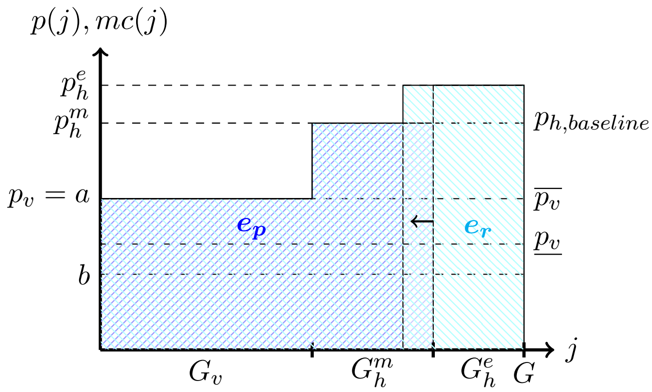


(d) Scenario 3

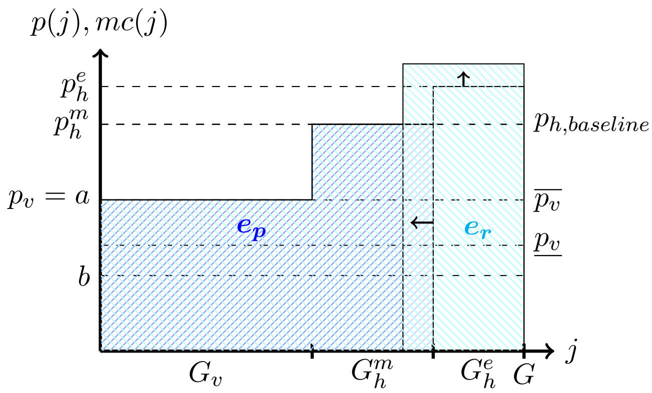
Figure 3.5: Market Structure: Inequality



(a) Basic Model

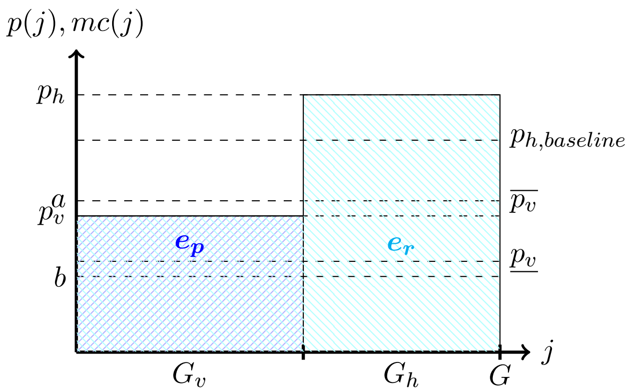


(b) Increasing Income Gap $\theta \downarrow$

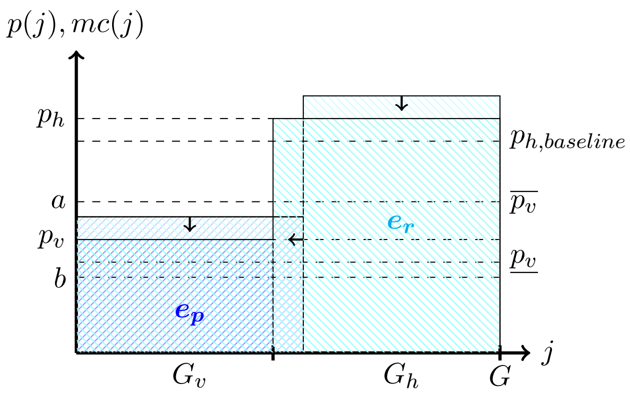


(c) Increasing Income Concentration $\beta \uparrow$

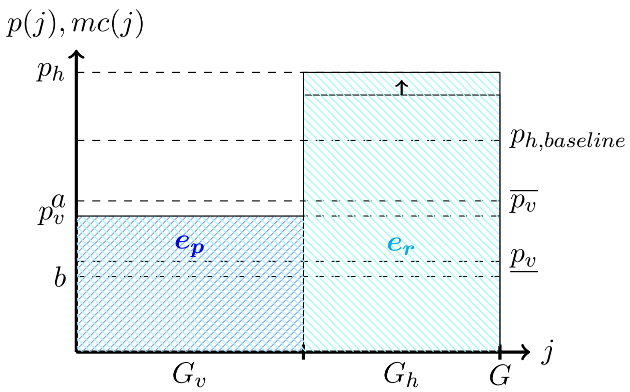
Figure 3.6: Inequality Changes: Scenario 1



(a) Basic Model

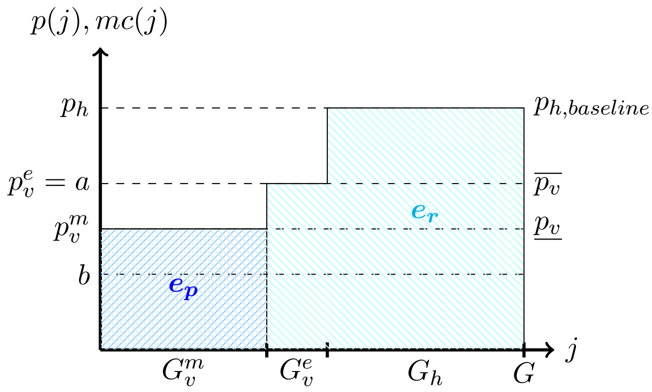


(b) Increasing Income Gap $\theta \downarrow$

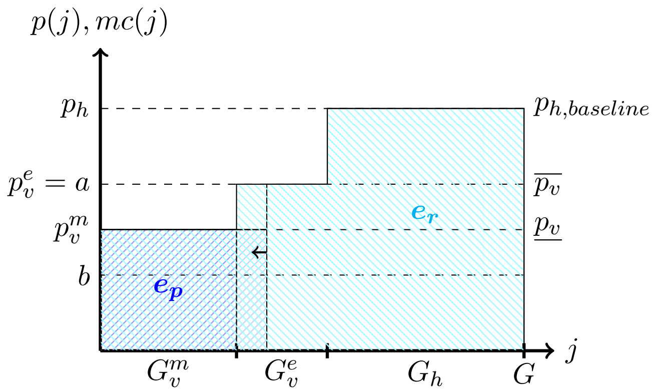


(c) Increasing Income Concentration $\beta \uparrow$

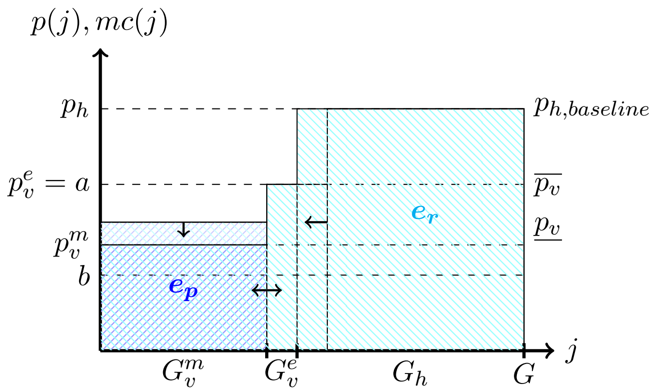
Figure 3.7: Inequality Changes: Scenario 2



(a) Basic Model



(b) Increasing Income Gap $\theta \downarrow$



(c) Increasing Income Concentration $\beta \uparrow$

Figure 3.8: Inequality Changes: Scenario 3

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Declaration of Authorship

I hereby declare that the enclosed master's thesis entitled

Inequality, Market Structure and Innovation

has been composed by myself, and describes my own work, unless otherwise acknowledged in the text. Any thoughts, quotations and models which were inferred from other sources are clearly marked as such and are cited according to established academic citation rules.

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Zurich, August 2017 Florian Hulfeld