Social Comparisons, Inequality and Demand

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Abstract

This thesis investigates the effect of inequality on the demand for status goods in presence of social comparisons. The analysis is conducted under different ways of pricing those goods. When individuals care about their ordinal rank in the distribution of income, a rise in the income of the poor forces the rich to increase their demand for status goods. When prices are exogenous, the adjustment in demand arises through an increase in the variety of status goods. When prices are endogenously determined by a small number of monopolists, it arises through an increase in the price, allowing the monopolists to receive larger profits. The most interesting result of this thesis is that, if a single monopolist supplies all the status goods, its optimal strategy is to supply a very small variety of status goods at a very high price, implying that the monopolist does not promote distortion in consumption. In an extension of the model, the definition of status is modified and consists of the cardinal rank in the distribution of income. In this case, a rise in the income of the poor leads the rich to decrease their demand for status goods if the society is affluent. This result allows a redistribution of income from the rich to the poor that improves the welfare of both the poor and the rich. Such redistribution is more easily implementable in the case of a single monopolist.

Contents

1 Introduction

Neoclassical economic theory assumes that individuals' well-being depends solely on the absolute amount of commodities they consume. However, economists and social scientists have long recognized that in addition to direct satisfaction derived from consumption, individuals care about how their consumption relates to that of others and how outsiders perceive them in terms of wealth. This theory goes back to **?**, who argues that concern for status is a characteristic of human beings and leads wealthy individuals to advertise their wealth through lavish spending on visible goods. He coined the term conspicuous consumption to describe such a behaviour. It wasn't until 50 years later that preferences taking social comparisons into account were first formally modelled by **?** (**?**). Despite this notable progress, the concept of social comparisons has been neglected thereafter. It is only in the last decades that there has been a renewed interest for it. Since then, many authors have introduced the concern for status into their utility function and investigated how this consumption behaviour affects the demand for status and normal goods, in comparison to cases where solely absolute consumption matters. It has been shown that concern for status leads individuals to spend too much money on status goods (later on also referred to as "conspicuous goods") thus diverting consumption from normal goods towards status goods and driving to suboptimal welfare levels (**??**). In most of these studies, it is assumed that any quantity of status goods can be purchased at any exogenous price. However, more recently, authors have investigated how pricing of status goods affects the demand in equilibrium. For instance, **?** shows that restricting the supply of status goods, forces individuals to pool around more expensive goods, allowing the supplier to set higher prices and to receive larger profits.

Some empirical studies show that not only rich individuals engage in conspicuous consumption, as indicated by **?**, but also poor individuals spend a surprisingly large share of their income on status goods (**??**). This consumption behaviour seems to arise at any income level and hence is harmful to all individuals in terms of welfare. In particular, some authors investigate how wealth inequality affects the extent to which individuals care about status, and therefore affects their demand for status goods (**???**). Such analyses can be used to predict how income distribution influences the distortion in consumption due to social comparisons and may lead to implementing appropriate policies to improve individuals' welfare. The introduction of micro-founded individual consumption behaviour to better predict macroeconomic issues is quite new and deserves in-depth investigation.

Until now, theoretical models addressing how inequality affects the demand for status goods have assumed that status goods are available at any price and in any quantity. However, as previously mentioned, the way of pricing status goods plays an important role in the determination of the demand in equilibrium. This thesis contributes to the field of conspicuous consumption insofar as the relationship between inequality and demand in presence of social comparisons will be addressed under different ways of pricing status goods.

In a first step, several theoretical models of conspicuous consumption will be analysed. They provide a basis, on which my model is constructed. In a second step, I develop a model in which poor and rich individuals are assumed to care about their consumption of normal and status goods and, in addition, about status. Status is defined as the individual's relative position in the distribution of income, also called ordinal rank. Consequently, individuals' decisions are strategic and individuals seek to signal their wealth through the purchase of status goods. Furthermore, individuals are assumed to prefer normal goods over status goods at each consumption level. It follows that in equilibrium, only the rich purchase status goods in order to differentiate themselves from the poor.

The relationship between inequality and demand will be addressed when prices of status goods are exogenous and the variety is unrestricted, and when prices are endogenously determined by firms. In the latter case, three ways of pricing status goods are considered. In the two first cases, the variety of status goods is fixed and determined either by a large number or a small number of monopolists, each supplying one status good. In the third case, one monopolist supplies all the status goods.

In this model, changes in equality in a given society can either come from changes in income, or from changes in population frequencies. It will be shown that, whichever the way of pricing status goods, solely a change in equality resulting from a change in the income of the poor affects the demand for status goods. More specifically, an increase in the income of the poor fosters the competition for status. This forces the rich to increase their spending on status goods in order to dissuade imitation by the poor and in order to be granted a higher status. However, depending on how status goods are priced, the adjustment in demand can either arise through an adjustment in the variety of status goods or through an adjustment in status goods prices. In summary, greater equality within the society resulting from an increase in the income of the poor fosters the distortion in consumption. On the contrary, if a single monopolist supplies all the status goods, it will be shown that its optimal strategy is to offer a very small variety of status goods at a very high price. This result constitutes the main contribution of this work and has interesting policy implications. First, preventing competition in status good markets may considerably decrease distortion in consumption and improve social welfare. Second, it suggests that innovation in such markets is bad in terms of individuals' well-being.

This thesis further seeks to emphasize the importance of the definition of status. In fact, depending on how status is specified, the impact of inequality on demand may differ considerably. For this purpose, an extended model will be developed in order to consider the case in which individuals care about how far they are from others in terms of income, also called cardinal rank. In this case, an increase in equality following a decrease in the numerosity of the poor has an impact on the demand for status goods. In fact, it decreases the status of the rich and thus the incentive of the poor to imitate the rich. Consequently, rich individuals can decrease their demand for status goods and still be perceived as rich. Moreover, an increase in equality following a rise in the income of the poor can lead to two different results. On the one hand, if the income of the poor is below a certain threshold, an increase in the income of the poor leads the rich to increase their demand for status goods. This is because the difference between the status associated to being perceived as rich, and the status associated to being perceived as poor is large. Consequently, the incentive of the poor to make the third party believe they are rich is high. This leads to a strong competition for status and forces the rich to increase their spending on status goods. On the other hand, if the income of the poor is beyond a certain threshold, an increase in the income of the poor leads to a decrease in the purchase of status goods by the rich. This is because the difference between the status of the rich and the status of the poor is small. Therefore, the incentive of the poor to imitate the rich is lower than before and the rich can decrease their purchase of status goods. The latter result suggests that a redistribution of income from the rich to the poor may improve the welfare of both groups of individuals. It will be shown that such redistribution can achieve higher social welfare under certain circumstances. In particular, we will see that it is easier to determine income redistribution that makes both the poor and the rich better off in the case of a single monopolist supplying all the status goods than in the case where prices are exogenous.

2 Literature Overview

Commodities partly exist in order to satisfy needs in terms of consumption and people experience pleasure when consuming them (**??**). In this line of thought, neoclassical economic theory assumes that the well-being of individuals depends solely on the absolute amount of commodities they consume. But this is not the only role of the existence of commodities. Economists and social scientists have long recognized that consumption of specific goods is also used as an instrument to display social status and to achieve a certain position in society (**??**). Indeed, in addition to direct satisfaction derived from consumption, individuals may care about how their consumption relates to that of others and how outsiders perceive them in terms of wealth. This theory stems at least from **?**, who argues that concern for status is a characteristic of human beings and leads wealthy individuals to advertise their wealth through lavish spending on visible goods. He coined the term conspicuous consumption to describe such a behaviour (**?**).

We may be of the view that concern about relative position involves jealousy and that this vicious character trait should not be taken into account when making decisions in terms of public policy. However, today's world functions in such a way, that it is barely possible not to consider how we relate to others. For instance, a job promotion depends on how others perform. Furthermore, parents should save for retirement but at the same time may want to guarantee their children's future. To do so, American families save less in order to purchase houses in the region of the best schools. Since families share the same purpose, the demand for such houses is large and this contributes to considerable rises in home prices. It results that children attend schools they would have attended if parents had spent less. Therefore, positional concerns may arise for reasons other than envy and hence must be taken into account (**?**).

Concern for social status has been increasingly given attention by economists in both theory and empirical works. For instance, **?** conduct an empirical study to investigate whether status consumption arises in real life. For this purpose, they choose four categories of women's cosmetics products with different brands. A sample of women was asked to rank the four categories with respect to their social visibility and then to rank the different brands of a same category with respect to their preferences. The authors find that the most visible goods have a zero price-quality correlation, whereas the least visible cosmetics have a positive price-quality correlation. This result suggests that women care more about the quality of goods that they intrinsically value. Quality seems to be less important for goods purchased to display a certain social status. They further show that income and occupational status significantly determine the propensity to buy conspicuous goods. Indeed, women with higher income are more likely to consume conspicuous goods. All in all, evidence exists that shows some cosmetics products are used to display social status and that status-buying is a reality. **?** also shows that individuals indulge in conspicuous consumption. In particular, he aims to find the explanation of the variation in the income elasticity of goods. We know that household budget shares spent on necessity goods decrease with income, whereas household budget shares spent on luxury goods increase with income. In fact, goods have different income elasticities but the reason for these differences is unknown. What is it that makes some commodities used as necessity goods and some others as luxury goods? The author conducts an empirical study and finds that income elasticities can be predicted from the visibility of consumer expenditures. More specifically, the visibility of goods, defined as the speed with which outsiders notice a household's expenditures on different commodities, can explain a substantial part of the variation in income elasticities. The most visible goods are cigarettes, cars, clothing, furniture, jewelry, while the bottom of the list mainly encompasses services. Moreover, the fact that income elasticities are largely explained by the visibility of goods, provides evidence that consumers have a strong motivation to indulge in conspicuous consumption. It supports Veblen's theory, which argues that consumption of specific goods not only provides satisfaction as an end in itself but also serves as a signal of wealth.

Some authors study whether certain populations are more likely to indulge in conspicuous consumption than others. **?** show that Blacks and Hispanics spend a larger amount of income on conspicuous goods than Whites. They conduct their analysis on three different status goods, cars, jewelry and clothing. The difference in conspicuous expenditures can be explained by racial differences in preferences but also and more interestingly, by status seeking. In fact, it may be the case that all individuals have the same preferences but some external characteristics of the economic environment make some individuals care more about social status. In fact, they find that the amount of money spent on conspicuous consumption depends on the distribution of income of the reference group. It should depend positively on own income but because being perceived as belonging to a poorer reference group has negative informational impacts, conspicuous consumption should be negatively related to the average income of the reference group. Indeed, Whites, Blacks and Hispanics belong to different reference groups with different average income. This constitutes an explanation for the difference in conspicuous expenditures, even if races have the same preferences. This result suggests that not only rich individuals engage in conspicuous consumption as argued by **?** but such a behaviour can arise at any income level. Indeed, **?** support this hypothesis. They investigate how the poor spend their money and find that poor individuals spend an unexpectedly low share of their income on food. For instance, the rural poor in Mexico spend less than half of their revenue on food. We may expect the remainder of poor individuals' income to be spent on goods they highly need. Instead, among the non-food commodities on which they devote a share of their income, spending on alcohol and cigarettes dominates. The poor also spend large amounts of money on festivals such as weddings, funerals, or religious festivals. These findings may suggest that conspicuous consumption takes place among the poor too. In fact, consumption of alcohol and cigarettes is visible and so are festivals. They can therefore be used to show that their status is not as low as one might think. This result is consistent with the findings by **?**. They collected data from households in Jaffna, Sri Lanka to conduct their study. This society that was rather collective post war, has been increasingly evolving towards an individualistic society during the last ten years. This change has led to shifts in consumer behaviour. They find that after civil war, poor individuals strive to imitate others in terms of consumption patterns. They consume conspicuous goods not only to increase their chance of coming into contact with high-ability or middle and upper class individuals, but also to display social status and show that they are better off than before. Finally, we observe that conspicuous consumption is a behaviour that also concerns poor individuals. The prevalence of consumption of these kinds of goods among poor individuals does not seem to alleviate poverty (**?**). Instead, such consumption behaviour seems very costly to the poor. They often fail to make efficient investments and savings that would eliminate the need of eating less in response to a decrease in their income. Spending on status goods and thus inefficient signaling equilibrium could lead to persistence of poverty. **?** provide an explanation for the positive relationship between income and savings, that is consistent with the consumption behaviour described previously. They argue that individuals with high human capital have discernible ability and thus need less to make use of conspicuous consumption to signal their abilities than do poorer individuals without certified success.

Note that conspicuous consumption does not only affect economic growth but also welfare. **?** state that even if real income has increased over the period for which happiness data exists, happiness has stagnated. This phenomenon cannot be explained by conventional growth models. In 1974, **?** already observed that self-reported happiness depends rather on relative income. This suggests that the consumer's desire to achieve social status may be a determinant of the above acknowledgement. In fact, **?** stresses the fact that conspicuous consumption is context dependent. It means that it strongly depends on the decisions of others. For instance, satisfaction derived from consumption of goods depends much more on the decisions of others rather than satisfaction derived from free time. This indicates that people spend too much time at work in order to purchase consumption goods and take too little time for their leisure, like family, friends and so on than would be optimal in terms of welfare. Indeed, concern for status diverts consumption towards signaling, involving suboptimal welfare levels (**?**). It is therefore interesting to investigate how the distribution of income affects conspicuous consumption, and thus welfare in order to find policies likely to improve social welfare (**???**).

Note that, up to now we have considered conspicuous goods that directly indicate a certain wealth, like luxury goods. Even if only these kinds of goods will be taken into account in the present work, it is interesting to note that there exist other indirect means used to display wealth. For instance, the purchase of pro-environmental green products is naturally associated with altruistic behaviour. However, some individuals purchase green products to show their ability to incur the cost of owning a good that benefits others but which is of inferior quality than other non-green products (**?**). It follows that not only altruistic people purchase these kinds of goods but also people who seek to achieve high status. A positive consequence of this result is that status competition can lead to pro-environmental behaviour. Similarly, **?** show that non-anonymous donations may be driven by status motives. They find that some individuals donate to be perceived as rich individuals even if they do not have warm-glow feelings.

All in all, there is strong evidence consistent with conspicuous consumption and more and more economists have introduced concern for status into the utility function when developing their model. We can distinguish two broad manners of modelling the concern for status. First, status can be understood as a characteristic of human behaviour. Human has an intrinsic motivation to achieve status. As a consequence, we can model it by introducing status directly into the utility function and a signaling game would be needed to find the reaction functions. In 1949, **?** is the first to formally model such a behaviour (**?**). He chose to enter concern for status interdependency directly into the utility function. More specifically, the individual's utility function is defined over the ratio between his own consumption and the consumption of others. According to him, an individual is less concerned with the absolute level consumption than by relative levels of income. **?** also uses interdependent preferences, in which status is defined as the rank in conspicuous consumption. He shows that if people care about their ordinal rank in the distribution of consumption of conspicuous goods, also referred to as status goods, there is an inefficiently high level of conspicuous consumption. It suggests that policy aiming to reduce spending on status goods may be welfare enhancing. Second, status can be viewed as an instrument to display one's wealth but does not provide direct satisfaction. If an agent is associated with high status, he will be rewarded with better consumption opportunities. In this case, matching models would be used to model status concern. In the next section, we will analyse theoretical models, on which my work is based.

3 Theoretical Framework

Several economists have developed different models that account for individuals' aspiration for social status. It is of great importance to analyse these models in order to understand how to specify the utility function in my model, to understand the consequences of different underlying assumptions and to get the intuition regarding the effect of inequality on demand.

3.1 Benchmark Model

? developed a model of conspicuous consumption with many status goods. In their paper, the authors analyse the aspiration of consumers to achieve social status by signaling their wealth through consumption of status goods. In particular, they explore the conditions under which the desire to reach a certain status implies "Veblen effects". "Veblen effects" arise when consumers are willing to pay a high price for a good, even if there exists a qualitatively identical good at a lower price. Their analysis is of great interest for the present thesis, since they address which signal the agents choose when there are different ways of signaling their wealth. Furthermore, the specification of the utility function used in my model is based on the specification of the agents' utility function by **?**.

In their model, households are endowed with resources y, which they can allocate between two types of goods. The first category consists of many "conspicuous" goods, which differ with respect to quality. Quality is denominated by q, with $q \in [\underline{q}, \bar{q}]$. Their characteristics and the quantity consumed are observable. The second type of good is an "inconspicuous" good. It means that it is consumed privately and not observed by others. By assumption, it is supplied in fixed quality and is used as the numeraire. Let $v(q)$ denote the quantity purchased of the conspicuous good with quality q, and let e denote the total conspicuous expenditures. Let N denote both the quantity purchased of the inconspicuous good and the total inconspicuous expenditures. This holds since the normal good is used as the numeraire.[1] We thus end up in a framework with many status goods and one normal good. Since this model is atemporal, resources might be interpreted as income, total consumption, or wealth (**?**).

The households fall into two categories: rich households (R), which are endowed

[1]To facilitate the understanding, I use the same notation as in my model in the whole analysis of the different models. In the model by **?**, resources are denoted by R, the quality-weighted quantity of conspicuous goods is denoted by x, the quantity of inconspicuous goods is denoted by z and total conspicuous expenditures are denoted by s.

with resources y_R, and poor households (P), which are endowed with resources y_P, with $y_P < y_R$. The frequency of poor and rich households is denoted by β and $(1 - \beta)$, respectively. It should be noted that the distribution of the households' income is discrete. Furthermore, their type is private information.

Each household cares about its total quality-weighted conspicuous consumption, $V \equiv \int_{\underline{q}}^{\bar{q}} \mu(q)v(q)dq$, its inconspicuous consumption, N, and an action, s, taken by the representative social contact. The variable $\mu(q)$ is assumed to be common for all households and increasing in q. It follows that every agent puts a higher weight on high-quality goods than on low-quality goods. What they mean by social contact is that individuals who are perceived as being rich by the outsiders are rewarded with preferential treatments by social contacts. In others words, those individuals achieve a higher status and they receive a greater payoff than individuals perceived as being poor. In summary, this variable captures how a representative social contact perceives the household's wealth from the observation of the household's signal, and responds by an appropriate action. It results that the status-bearing object, i.e. the object whose distribution in the population is thought to grant status, is given by the endowment of resources one is believed to own (Bilancini and Boncinelli (2012)). It is important to note that, in this case, status is modelled such that payoffs depend on the perception of one's absolute type rather than on the perceived relative position. However, what matters for individuals is their relative economic standing. Finally, total utility of household of type i= R,P is given by $U_i(V_i, N_i, s_i)$. By assumption, $U_i(\cdot)$ is increasing, continuous and strictly concave in each of its arguments. Since the utility function is increasing in inconspicuous consumption level N, consumers will spend all their residual resources that are left after having chosen the conspicuous quality-weighted quantity, V, to buy inconspicuous goods. As a result, the budget constraint holds with equality. Therefore, we can rewrite the utility function as $W_i(V_i, e_i, s_i)$. Note that with this specification of the utility function, the intrinsic value of status matters.

It is worth emphasizing that larger quantity is a perfect substitute to higher quality in the above utility function specification. Consumers value their total quality-weighted conspicuous consumption V, regardless of how they choose the variety and corresponding quantities.

Regarding the production side, conspicuous goods can be produced by a large number of firms. These firms are divided into two groups, incumbents and entrants. All firms can produce the same range of qualities, $q \in [\underline{q}, \bar{q}]$, and conspicuous goods are otherwise identical. We notice that the supply of status goods is not fixed and depends on consumer behaviour.

In the first-best allocation, firms will produce quality levels that minimize the cost-quality ratio. By assumption, there is a unique solution to this problem and the quality produced at the minimal cost is called the first-best quality level, q_F. Furthermore, each firm produces a single product, which is branded, such that social contacts can readily identify the producer of the good. Incumbents are endowed with an advantage over entrants: consumers purchase the product from an incumbent, unless they can strictly increase their utility by buying it from an entrant.

The game is divided into two phases. In the first one, Bertrand competition takes place, where firms choose qualities and prices they want to offer. In the second one, each household observes the prices announced by the firms and selects bundles. In addition to prices, social contacts observe brand labels, qualities and quantities, so that they can infer each household's quality-weighted quantity, V, and then draw inferences about the household's wealth. It follows that households can signal their wealth through two different ways, namely quality-weighted price and quality-weighted quantity. Note that the selection of brands purchased by consumers does not matter, since consumers are completely indifferent between bundles with the same quality-weighted quantity and with the same total conspicuous expenditures. If a household signals its type through buying higher quality-weighted quantities of conspicuous goods, no Veblen effects take place. If instead a household is willing to pay a higher price for a functionally equivalent conspicuous good in order to signal its wealth, Veblen effects arise. Households may be willing to pay more than the minimal amount needed to acquire conspicuous goods because social contacts observe it, and doing so increases the probability of being identified as rich households. On the contrary, it would not be rational to spend more than the minimal amount needed to obtain a given quantity of the inconspicuous good. The latter is only valuable in itself and does not help to achieve a higher status. Therefore, in order to get maximum utility, households must buy as many inconspicuous goods as possible with their residual income. It implies buying them at the lowest price. Note further that in either case does conspicuous consumption serve as a signal of wealth. It should be noted that consumers' decisions are strategic, since they have to anticipate choices of other households in making their optimal choice in order to achieve the intended status. We end up in a signaling game.

Status is defined in such a way that individuals with larger conspicuous expenditures are granted with higher status. It follows that rich households have an incentive to distinguish themselves from the poor. Hence, the equilibrium is separating and households are correctly identified (**?**). In order to achieve such an equilibrium, we need several conditions. First, the incentive compatibility constraint ensures that no household wants to deviate after having selected its optimal bundle. It can be shown that if poor individuals do not want to deviate then rich individuals do not want to deviate neither. In other words, poor households' utility when consuming their optimal bundle must be larger than their utility when imitating the consumption behaviour of rich households. Second, the household's choices of conspicuous quality-weighted quantity and total conspicuous expenditures must be feasible, i.e., they must satisfy the budget constraint. Third, to achieve a separating equilibrium, social contacts, which observe V_R (V_P) and e_R (e_P) must infer that the household is rich (poor) with certainty. Finally, the choices must be optimal given the inferences of social contacts.

In fact, social contacts also observe which brands are purchased. However, since consumers care only about the quality-weighted quantity of conspicuous goods and total conspicuous expenditures, regardless of the firm from which they purchase their status goods, observing brands does not help social contacts to infer the individuals' type.

We now address the strategy of poor households in the separating equilibrium where social contacts perfectly identify them. Poor households get the lowest treatment by social contacts and are granted the lowest status. As a result, it is useless to increase their quality-weighted quantity or their quality-weighted price to try to mislead social contacts. They only need to choose feasible bundles such as to maximize their intrinsic utility. For this purpose, they buy conspicuous goods at the lowest quality-weighted price, which allows them to consume the maximum quality-weighted quantity of conspicuous goods. It is crucial to emphasize that poor households still purchase conspicuous goods as they derive satisfaction from consuming them.

In order to analyse the equilibrium further **?** make the assumption that the single-crossing property holds. This property is primordial in signaling models, since it must be satisfied in order to end up in a separating equilibrium. More specifically, it ensures that conspicuous consumption is more costly in terms of utility for poor households than for rich households and thus rich households are more willing to signal their wealth compared to poor households.

The authors find that Veblen effects cannot arise when the single-crossing property is satisfied. As far as poor households are concerned, we have seen that it is optimal to buy conspicuous goods at the lowest quality-weighted price. Regarding rich households, they have several ways to dissuade imitation by poor households. They could choose to pay more than the minimal amount for their quality-weighted quantity. But this would not be optimal since they could increase their utility by purchasing a larger quality-weighted quantity at a lower quality-weighted price, while preventing imitation by poor households. This stems from the single-crossing property, which ensures that an increase in the quality-weighted quantity and a decrease in the quality-weighted price make rich households better off, while making poor households who imitate worse off. Thus, the poor do not want to deviate. In summary, rich households choose to signal their wealth through higher quality-weighted quantity rather than through higher quality-weighted price. Because each household purchases its conspicuous quality-weighted quantity at the lowest quality-weighted price, each firm is forced to set its product's price at the marginal cost and only conspicuous goods of first-best quality are produced. This results from Bertrand competition.

The authors also investigate the equilibrium when the single-crossing property is not satisfied and show that Veblen effects can arise in this case. More specifically, they address the case of tangency of the two indifference curves, which is unlikely to happen in reality. That is why we do not go into detail.

In summary, this model consists of many status goods modelled through different qualities or brands. Also, the authors describe their model rather as a one-status-good model with many brands. In their setup, higher quality is a perfect substitute to higher quantity and consumers are totally indifferent regarding the variety of conspicuous goods they select insofar as the quality-weighted quantity of status goods purchased remains unchanged. Therefore, the present model does not help to understand which status goods households consume, to signal their wealth. It is also important to note that the supply of conspicuous goods and prices are continuous and not fixed. They result from the consumers' choices. In fact, households can signal their wealth through the quantity of conspicuous goods they are willing to consume or through the price at which they purchase them. Therefore, their choice of quantity and price of conspicuous goods influence the supply made by the firms. In the present model, we have seen that if the single-crossing property is satisfied, all the households purchase the cheapest status goods and therefore signal their wealth through the quantity of status goods they purchase. As a consequence of Bertrand competition, firms are forced to supply their conspicuous goods at the lowest price equal to the marginal cost. They receive zero profit. This result suggests that it is difficult to rationalize the purchase of "luxury" brands, when the single-crossing property holds and when the supply of conspicuous goods is not restricted. "Luxury" brands are defined as goods sold at a price above the marginal cost and, in equilibrium those brands are of identical quality as other brands sold at a higher price. The single-crossing property seems rather realistic, and thus the result of this paper is not distorted by the latter assumption. This model is in contrast with a model where the supply of conspicuous goods is restricted.

Conspicuous goods could be supplied either at fixed prices or in fixed quantities. In such a context, individuals have only one way to signal their type. If prices are fixed, they can display their wealth through the quantity of conspicuous goods they consume. If the quantity supplied is fixed, their single choice is to differentiate themselves by purchasing status goods at a higher price. It follows that firms can charge a higher margin on their products and receive positive profits. We will investigate such cases in section 3.2.1 and in my model.

3.2 Related Literature

Several authors have introduced concern for status into utility functions. Depending on the particular aspect they want to explore, their model takes different forms.

For instance, **?** developed one of the simplest model of conspicuous consumption (**?**). In his model, consumers also value both normal and status goods and in addition, care about others' perception of their status. The individual's private utility from consumption of quantity V of the status good and quantity N of the normal good is denoted by $u(V, N)$.[2] As always in signaling models of conspicuous consumption, consumption of status goods is visible, whereas consumption of normal goods is not. The individual's status is defined as spectators' inference about his utility level and is denoted by $s = u(V, g(V))$, where $g(V)$ is the inference of the unobservable quantity of normal goods, N, consumed by the individual from the observation of the consumption of quantity V of status goods. Formally, the utility is assumed to be given by $U = F(u(V, N), s) = F(u(V, N), u(V, g(V))$. In particular, the author suggests specifying the utility function as $U = (1 - a)f(V, N) + af(V, g(V))$. This specification indicates that consumers weigh two utilities: private utility and status defined as their utility as perceived by others. The parameter a describes weights denoting the importance of status relative to private utility. It allows analysing how the intensity of the concern for status affects the demand for status goods. It is of great importance to note that $u(V, N)$ is increasing and strictly concave in V and N, and has a non-negative cross-derivative. The first property of the function $u(\cdot)$ implies that individuals spend all their residual income after the purchase of status goods to buy normal goods. The next properties of $u(\cdot)$ correspond to the conditions, under which the single-crossing property is satisfied. This means that the marginal utility of visible goods is larger for richer than for poorer individuals. In other words, consuming status goods to signal one's wealth is more costly for poor individuals than for rich ones.

[2] In the model by **?**, the quantity of the status good is denoted by v and the quantity of normal goods is denoted by w.

Note that, in contrast to the model by **?**, this setup encompasses only one status good and one normal good. We also observe that the utility function is increasing in status and that higher status is associated with higher conspicuous consumption. This is equivalent to the case where social contacts treat households perceived as being rich better in the model by **?**. In this model, to form beliefs about an individual's utility level, outsiders must first infer his consumption of normal good from the observation of his consumption of status goods in order to be able to infer his income and then his utility level. Therefore, the definition of status is similar to the paper by **?**, where social contacts draw inferences about one's wealth. In both papers, outsiders form beliefs about one's absolute type, but what matters for individuals is their relative position in the distribution of consumers' type, also called ordinal rank. Let us further emphasize that, in the paper by **?**, consumers have only one way to display their wealth, which is through the quantity of conspicuous goods they consume. **?** have shown that agents buy the cheapest visible goods when the single-crossing property holds. Therefore, providing agents with the quantity of conspicuous goods as the single signal of wealth does not distort the analysis. If agents would have the possibility to display their wealth through prices, they all would choose to buy the cheapest status goods, as the supply is unrestricted.

It should further be noted that the distribution of consumers' type consists of a continuum of consumers, who differ with respect to their income; this results in a continuous distribution of consumers' type. Therefore the definition of the fully separating equilibrium differs slightly from the definition of the separating equilibrium in the paper by **?**. The existence of a continuum of consumers implies that spectators' inferences must be correct for all income types. It is important to mention that it is possible to identify consumers' type correctly, only if the visible good can be purchased in any amount. If, however, this good could only be purchased in discrete quantities, partial-pooling equilibria would arise. This case will be analysed by **?** in section 3.2.1.

Regarding the equilibrium, the author finds that consumption is distorted due to the need of signaling one's status. It means that concern for status leads consumers to purchase too many visible goods and they therefore reach inefficient utility levels. This suggests that a tax on status goods could help increasing the utility of all consumers. **?** further addresses the extent of utility losses due to status-seeking by using different numerical values for the weights of status compared to direct utility, a. To find explicit solutions, he uses a quasi-linear direct utility function. He finds that if status effects are small (a small), the distortion in consumption is not high for consumers with middle to high income. However, for low-income consumers, who would not purchase any visible goods when they do not value status at all, the marginal loss of utility from consuming status goods is much higher. If status effects are large (a is large), the income propensity to consume the visible good is very large for low-income households. "... this explains how it is that people who cannot afford to properly feed their children may still give them expensive birthday presents" (**?**). In summary, as consumers assign higher weights to status, their conspicuous consumption increases.

3.2.1 Pricing of Conspicuous Goods in Signaling Models

When individuals have concerns about status, it automatically implies that they take into account others' beliefs about their type from the observation of their consumption decisions. For this reason, the use of a signaling model is needed. Up to now, we have analysed models in which utility functions were defined over status, and over two types of goods, i.e., normal and status goods. However, some authors have developed models with so-called reduced-form preferences. In such preferences, the price of status goods enters the utility function directly. This is in contrast to the theory of conspicuous consumption by **?**. According to him, utility should be defined over consumption and status, and not over consumption and prices. Naturally, prices affect status in equilibrium, but this effect should be derived and not assumed (**?**). The models by **?** and by **?** are faithful to the latter theory, and models in the present section disagree on that point.

Two authors are worth mentioning, since they have recently contributed to the field of status' concern. In their paper, they investigate how the willingness to achieve high status affects the pricing of conspicuous goods.

Rayo (2013) studies a signaling model, in which a single monopolist selects the set of signals available. The monopolist can thus manage the supply of conspicuous goods. Consumers' type, θ, is continuously distributed, where the type may denote the individual's wealth. The utility function is given by $U(\pi, e; \theta) = \pi v(\theta) - e$, where π denotes the public perception of the consumer's absolute type and is equal to the average type across all consumers that, in equilibrium, buy the same signal. Note that $v(\theta)$ corresponds to the marginal utility of others' beliefs and is assumed to be increasing in the type. This ensures that the single-crossing property is satisfied. It means that rich individuals value status more and are therefore willing to pay more in order to be perceived as rich individuals. Indeed, from the logic of general signaling models, separating equilibria require that wealthier individuals have a higher incentive to signal their wealth in order to benefit from preferential treatments. Let e depict the amount that the consumer pays to acquire the chosen item[3].

In contrast to the two models described earlier, we observe that individuals care about the perception of their absolute wealth and not about their relative position in the distribution of consumers' type. In other words, what matters for individuals is their cardinal rank as opposed to their ordinal rank. This distinction is relevant when analysing the effect of the income distribution on the demand for status goods. We will study this case in section 3.2.3. However, in this model, where pricing of status goods is addressed, the above specification of status does not affect the results in a specific way. Moreover, in contrast to the models by **?** and by **?**, normal goods do not appear directly in the utility function. However, $-e$ captures the fact that the larger the amount of money spent on conspicuous goods, the less income left to purchase other goods and the lower the individual's utility. Note further that only the indirect effect of status goods is taken into account. In fact, the consumption of status goods does not increase utility per se, but rather increases the level of outsiders' beliefs. It follows that conspicuous consumption takes place only for its signaling benefit.

In the present paper, a monopolist offers a menu of signals, which are defined as money amounts he is willing to advertise. The consumer selects only one item from the menu, from which price and quantity cannot be distinguished. The author addresses the effect of restricting the variety of available items on the demand for conspicuous goods and then on profit. He finds that the monopolist can increase its profit by limiting the set of goods it supplies, when consumers are willing to signal their status. Indeed, the restriction forces the continuum of consumers to pool around some more expensive items. When the supply of conspicuous expenditures is restricted, some individuals are forced to pay a larger amount to be considered as having a certain income than would have been necessary if a continuum of conspicuous expenditures would have existed. This allows the monopolist to increase its profit. The author gives a simple example to illustrate the mechanism above. Let us say that, at the beginning, the monopolist offers a menu of five rings with prices \$1, \$2, \$3, \$4, \$5. If the monopolist decides to remove the \$3 ring from its menu, there will be a positive impact on profits if consumers have a signaling motive. On the one hand, the elimination of the \$3 ring will lead to a lower status associated with the \$4 ring. Thus, some wealthier consumers will even select the \$5 ring. On the other hand, the average wealth of consumers purchasing \$2 ring increases and thus higher status is associated to this ring. This will encourage \$1 ring consumers to buy the \$2 ring. All in all, profits increase.

It is worth mentioning that, what is observed by outsiders and what matters for

[3] In the model by **?**, the amount paid to the monopolist is denoted by t.

individuals are the total conspicuous expenditures, e, and not the combination price-quantity. We have seen that, when the single-crossing property holds and the supply of status goods is not restricted, individuals purchase the cheapest status goods (**?**). As a result, only total conspicuous expenditures matter. In the present model, the single-crossing property is satisfied but the supply is restricted. That is why individuals may be forced to pay a higher price for the item in order to display their wealth when the available quantity does not suffices to achieve a certain status. Consequently, the items, i.e. amounts of money, differ not only with respect to quantity but also with respect to price. We further observe that the monopolist has the power to direct consumer choices through its pricing strategies. It can manipulate the consumers' beliefs. In fact, concern for status allows it to increase its profit by pooling customers around the desired items. In sum, unlike the paper by **?**, **?** can explain why individuals purchase "luxury" goods. When the supply of conspicuous goods is restricted, concern for status leads some individuals to buy status goods at a higher price enabling them to be assigned a certain economic standing.

? also addresses how the supply side reacts to consumers' desire to display their wealth. More specifically, in this model individuals might have an interest in quality and/or an interest in image. This results in four categories of consumers. As in the model by **?**, the distribution of consumers is discrete. Note that concern for image is a broad concept, to which concern for economic standing may belong. The form of the utility function is very close to the one by **?** and is given by $U_{\theta\rho}(q, p, s) = \theta q + \rho\lambda s - p$, where θ is equal to one if the consumer cares about quality, q, and zero otherwise. It reflects the marginal utility of quality when the consumer has an interest in it. Similarly, ρ is equal to one if the consumer cares about image, s, and zero otherwise. $\rho\lambda$ represents the marginal utility of image when the consumer accounts for image. It follows that λ describes the weight put by the individual on image compared to quality.[4] The image is specified as the outsiders' inferences about a consumer's concern for quality, i.e. a consumer's absolute type. It is defined as the expectation of an individual's quality preference parameter from the observation of his purchasing decision. Again, we observe that the utility is increasing in image, i.e. in status, and decreasing in price. The interpretation of the utility function is the same as in Rayo's model. However, individuals may care about quality in addition to concern for status.

Regarding the supply side, a single monopolist offers a menu of quality-price items, from which the consumer selects only one or none of them. In establishing its supply, the monopolist takes into account which image will be associated with each of his

[4] In the model by **?**, quality is denoted by s, image is denoted by R, the interest in quality is denoted by σ and the interest in image is denoted by ρ.

products in equilibrium. Like in Rayo's model, the consumer's type is unknown, and therefore perfect price discrimination is impossible. However, in contrast with Rayo's model, status is not signaled through an amount of money, but rather through a choice of a quality-price pair.

In my model, it will be assumed that consumers have the same utility function and therefore the same preferences regarding status. That is why we focus on Friedrichsen's benchmark case, where consumers have homogeneous image concerns. It follows that only heterogeneity in quality preferences remains and we are left with only two types, which differ with respect to their quality interest. Here, heterogeneity in quality preferences is exogenously given. However, if it was determined endogenously, we could argue that it stems from differences in income. In fact, in a macroeconomic model with non-homothetic preferences, income level affects the structure of demand. It can be illustrated through different demands for quality, where richer households purchase goods with higher quality. It results that people having an interest in quality can be interpreted as rich households, and people without any interest in quality as poor households. In the end, the framework of the analysis would be similar to the paper by **?**. Two types of households, differing with respect to their income/quality concerns, have the same preferences for status. However, in this model, individuals care about the perception of their absolute type, i.e. their quality preferences, also interpreted as their income.

? analyses separating equilibria, in which quality concern is correctly identified. Compared to the case where individuals do not have any interest in image, consumers with image concerns get a higher utility from buying a product, which is bought by high-type consumers, i.e. consumers, who care about quality. Consequently, the monopolist can charge a higher price without changing the quality allocation. In both cases, people who do not care about quality do not buy any item, since they derive no satisfaction from consuming such an item.

In this model, consumers can choose only zero or one product, with two attributes, quality and price, from the menu of items supplied by the monopolist. When people have homogeneous image concerns, people who do not care about quality do not buy any item. It suggests that poor individuals do not buy any conspicuous good, since they are correctly identified as being poor and since they do not like conspicuous goods. If instead, individuals care about quality, they are willing to pay a higher price for the conspicuous product. In fact, they want to purchase a higher-quality good when they have an interest for quality. This can be interpreted as the willingness to differentiate themselves from others by paying a higher price in order to be identified as being rich.

Note that in this model, as in Rayo's model, the supply of conspicuous products is restricted. Furthermore, it is only possible to purchase one unit of the product. That is why, it is not possible to dissuade imitation by people who do not care about quality by purchasing a greater quantity of conspicuous goods. Instead, rich individuals are forced to pay a higher price. It follows that the monopolist can increase its profit by restricting its supply of conspicuous goods when people care about status.

In this paper, consumption is conspicuous in that it displays personal characteristics like taste for quality. Moreover, the concept of image is broader than the concept of status, in the sense that it takes into consideration not only goods reflecting a certain wealth but also personality traits. For instance, people aim to buy goods associated with green or charitable acts. There exist firms that strategically supply soft drinks such as ChariTea and LemonAid in accordance with customers' willingness of being identified as a pro-social individual (**?**).

Note that using reduced-form preferences of signaling models may not be the proper preferences' specification for our study. In fact, we are working on a macroeconomic issue. It may thus not be appropriate to use such a utility function to analyse changes in the demand structure caused by changes in the income distribution.

3.2.2 Matching Models

Concern for status can be of two different natures. If, on the one hand, we argue that people seek status achievement because it is part of human beings, then status can enter the utility function directly. In other words, people value status in itself. On the other hand, we may claim that concern for relative standing is instrumental in the sense that achieving higher status leads to better consumption opportunities. In this case, we may end up in matching models where, higher status improves match quality between two individuals but does not increase utility directly. In this section, we address two papers in which matching models of conspicuous consumption are developed.

? developed an effort model, in which individuals have to make labor/leisure decisions. Moreover, individuals have concern for relative wealth, also known as concern for social status. Individuals' wealth is not observable.

Two types of agents come into play, men and women. Men are continuously distributed and indexed by $i \in [0, 1]$. Similarly, women are continuously distributed and indexed by $j \in [0, 1]$. Men are endowed with one type of good, x, while women can produce another type of good, y. Specifically, male i is endowed with i units of good x and women choose the effort level, l, they are willing to make, which allows them to produce a certain output level of goods y. Note that the higher the man's index, the larger his endowment and hence the wealthier he is. Given that trade cannot take place in this model, men and women need to match with each other in order to have access to both types of goods. The utility function over the joint consumption of x and y is identical across all individuals and is denoted by $u(y) + x$, where $u(\cdot)$ is increasing and concave in y. Women differ with respect to their productivity per unit of effort $a(j)$, which increases with their type j. Furthermore, women incur a disutility of effort, $-v(l)$. Note that the function $v(\cdot)$ is stricly convex. The output of good y produced by women is denoted by $a(j)l$, and is unobservable. It can therefore be interpreted as women's unobservable wealth. The assumption about $a(j)$ implies that richer women have a greater incentive to signal their wealth to distinguish themselves from poorer women and to achieve a better quality of match. It follows that the concept of conspicuous consumption by **?** arises and this model encompasses a signaling game. However, the reason why the agents engage in conspicuous consumption differs from before. Conspicuous consumption is now an instrument to achieve better consumption opportunities but individuals do not derive satisfaction from it. This is a simplifying assumption of the model by **?** and **?**, where conspicuous consumption increases both the utility from consumption and the utility from status.

Woman's output level y is private information. The only means to signal their output level consists in destroying an amount of wealth in conspicuous consumption. The level of output a woman destroys is denoted by V.[5] Men draw inferences about woman's consumption ($y - V$) from the observation of V in order to estimate their attractiveness in terms of mates. As a result, the match is a function of the amount of wealth destroyed. We can notice that $(y - V)$ determines the status and V illustrates conspicuous consumption. In fact, showing one's ability to incur the cost of destroying wealth for something that does not increase the satisfaction directly is a signal of high status. **?** support such a behaviour. As mentioned earlier, they analyse how signaling motives can drive towards charity acts. They find that people who care little about warm glow may still make donations because they care a lot about status. They thus incur a cost for donations, which have no intrinsic value for them. Other authors like **?** also argue that people buy green products instead of luxury goods because altruism serves as a costly signal associated with status. In fact, purchasing such products shows the ability to incur a cost for a good of inferior quality for personal use. This is in opposition to the view of **?** and of **?**, where individuals signal their wealth with

[5]**?** denote the amount of money destroyed in conspicuous consumption by d.

goods they like to consume.

Note that the wealth distribution of men is exogenously given and therefore men make no decision in this model. In other words, the role of men is reduced to a prize, used to reward women for their rank in the final wealth distribution. On their side, women care about status, i.e. about relative wealth. This stems from the competition for men. Women engage in wealth tournaments in order to attract a man as wealthy as possible to achieve better consumption opportunities. When making their choice, women take into account the decisions made by the other women, since those decisions affect their match and hence their utility.

The authors address the separating equilibrium, where wealthier women destroy a larger amount of their output level compared to less wealthy women.

Woman j's total utility depends positively on her direct utility derived from consumption of her output, negatively on her effort and positively on her match. Her problem is given by:

$$max_{l,V} u(a(j)l - V) - v(l) + m(V)$$

Given that $u(\cdot)$ is concave and $v(\cdot)$ is convex, they find that V and y are non-decreasing in ability. It means that women with higher productivity produce a larger amount of goods y and destroy a larger amount of wealth in conspicuous consumption. In equilibrium, the median woman will be matched with the median man. This constitutes the desired outcome of the signaling equilibrium. The same happens in the models by **?** and **?**, since it is more costly for poor than for rich individuals to renounce to normal goods in order to purchase status goods. In fact, rich individuals are more willing to signal their wealth than poor individuals. Consequently, $m(V(j))$ needs to be strictly increasing in $V(j)$. This can only be the case if higher V is a signal for better consumption opportunities, $(y - V)$. Note that, like in any separating equilibrium, the woman who wastes wealth the least will be matched with the poorest man. As a result, it is optimal for her not to destroy any wealth at all. The underlying logic is the same as in the models by **?** and **?**, where the poorest household chooses the optimal conspicuous consumption that maximizes its direct utility of consumption. However, in this case, because individuals signal their wealth with something from which they do not derive satisfaction, the poorest individual chooses not to signal his wealth at all. The same result is found by **?**, where people who do not like conspicuous goods do not buy any.

The authors analyse the effect of shocks in income on decisions by women. In this model, a shock in income is illustrated by a shock in wages, which are equal to the productivity of women, $a(j)$. If all women experience an increase in their productivity, the wealth distribution shifts up and the resulting matching function shifts down. It means that when all women get richer, they have to destroy more wealth in order to be granted the same match quality as before.

The above model shows results compatible with Veblen's theory. As income increases, individuals spend more resources on conspicuous consumption. Note that in this model, individuals do not gain direct satisfaction from conspicuous consumption. This stems from the fact that conspicuous consumption solely serves as an instrument to achieve better match quality but has no value in itself. We may suggest that the model by **?** is a mix between the model by **?** and the matching model by **?**. In fact, on the one hand, individuals derive satisfaction from conspicuous and normal goods, as it is the case in Ireland's model. On the other hand, in addition to the direct utility from normal and status goods, individuals derive utility from status. This is modelled through a payoff associated with status, like in Cole et al.'s matching model. In the matching model, this payoff comes from a good quality of match allowing better consumption opportunities. In the model by Bagwell and Bernheim, the payoff represents the benefit of preferential treatments associated with high status. Preferential treatments can be understood as better consumption opportunities, but encompasses a broader range of preferential treatments. Note further that this paper ignores the pricing side. This can be explained by the fact that the matching decision is not mediated by prices. Women do not pay for their match. Indeed, the wealthiest woman will be matched with the wealthiest man but she can still consume her wealth. In other words, the amount of wealth allocated to conspicuous consumption does not depend on prices. What matters is how much money they destroy compared to other women. This is similar to the concept of conspicuous expenditures in the previously presented models, where concern for status leads to overconsumption of conspicuous goods compared to what would be optimal in terms of welfare. The authors justify their choice of using matching models by their desire to illustrate adjustments in decisions by women, which are not induced by markets.

The contribution of the authors in this field is important, since standard models investigating tax policies ignore the effect of changes in income distribution following a tax on consumers' decisions. This is because they often use homothetic preferences.

Later, in 2008, **?** points out that the main authors, who have considered the importance of concern about social status to explain consumer behaviour and introduced it in their models, have always assumed that individuals care about status. He aims to demonstrate that this assumption really holds. For this purpose, he uses a matching model with a signaling game. Indeed, he argues that people engage in social interactions with each other and that the return to these interactions depends on the abilities of the matched individuals. There is a continuum of consumer types $\theta \in [0,1]$, where $\theta_i \in [0,1]$ defines the abilities of person i. The payoff of person i from his interaction with person j is denoted by $m_i = m(\theta_i, \theta_j) \geq 0$. The analysis will show that when social interactions take the form of complementary interactions, people care about social status. Formally, this complementary interaction implies that the individual's payoff increases with his own abilities, $m_1 > 0$. Furthermore, if the latter individual with given abilities interacts with a person having high-abilities, his payoff itself increases, $m_2 > 0$, and his own contribution to the payoff increases as well, $m_{12} > 0$.

Abilities are unobservable. However, the agents can invest in conspicuous goods in order to signal their abilities. The reduced-form preferences are given by $U_i = y + m_i - V(\theta_i) = N_i$, where y denotes person i's exogenous income and $V(\theta_i)$ defines the part of income invested in status goods.[6] Because the purpose of this paper is to show that people care about social status, the author assumes that people derive no direct utility from social status and status goods. As a result, each individual maximizes his total consumption of normal goods, $N_i = U_i$. This model is similar to the matching model by Cole et al.. However, we notice that, in this case, the unobservable variable is a person's abilities and not his income or wealth. The interpretation is still very close, since higher abilities help to achieve a larger payoff and hence a larger total income.

? addresses the separating equilibrium. By analogy with the model by **?**, a person with abilities zero will be matched to a person of same abilities. It follows that his optimal choice is not to invest any income in status consumption. In fact, it is assumed that people do not get direct utility from conspicuous consumption. Moreover, a separating Nash equilibrium is characterized by $V(\theta)$, which is strictly increasing in θ. A person with high abilities is more willing to invest in status goods than a person with low abilities. The author finds that people care about social status only because social status serves as an instrument to achieve higher payoff from their complementary interactions. By investing in social goods, people signal their abilities and increase their chances of being matched with an individual with high abilities. It leads to a higher payoff and has a positive effect on the utility level. However, spending money in conspicuous consumption induces lower consumption of normal goods and has a negative effect on the utility level. If an agent decides to decrease his investment in so-

[6] In the model by **?**, consumer types are denoted by r, the part of income invested in conspicuous consumption is denoted by k, the income is denoted by I and the consumption of normal goods is denoted by c.

cial status, his consumption of normal goods increases but his match quality decreases. In both cases, according to Rege, the cost exceeds the benefit. However, compared to a framework where people cannot invest in status goods and where people are matched randomly, the possibility of investing in social status improves welfare. This finding is a new contribution to this field. In fact, most of the existing papers have focused on the cost of consuming status goods. It would imply distortion in consumers' choices, where individuals consume too many status goods and too few normal goods than what would be optimal in terms of welfare. However, Rege shows that, when status goods serve as a signal for abilities and help to achieve efficient matching, investing in such goods may be welfare enhancing. It should be noted that this result differs from others, since it refers to complementary interactions.

3.2.3 Income Distribution and Demand

The purpose of the present work is to develop a model illustrating how inequality affects demand for status goods, when those goods are used as a signal of wealth. It is therefore important to address papers that develop models to analyse such effects. For instance, the paper by **?** is of great importance for my work, since it analyses in detail the effect of exogenous changes in the distribution of income on the demand for status goods and their implications in terms of welfare.

In their model, **?** assume that agents care about status. We have seen that concern for status can arise for different reasons. However, the issue whether the desire to achieve a higher status is an end in itself or is instrumental in order to benefit from preferential treatments is not quite addressed in their model.

? develop a model with a continuum of agents. They differ solely with respect to their endowed income y, which is unobservable. The distribution of income is therefore continuous and is denoted by $G(\cdot)$, which is twice continuously differentiable with a strictly positive density on the interval $[\underline{y}, \bar{y}]$. Agents can allocate their income between two different goods, status goods that are observable and normal goods that are unobservable. Let V denote the amount consumed of status goods and N the amount consumed of normal goods. As in previous papers, consumption of status goods, V, can be interpreted as conspicuous consumption, which serves as a means to display one's unobservable income.[7] Following **?**, the authors define concern for status as the ordinal rank in the distribution of conspicuous consumption. Formally, $F(\cdot)$ denotes

[7] In the paper by **?**, the endowed income is denoted by z, the amount consumed of status goods is denoted by x and the amount consumed of normal goods is denoted by y.

the distribution of conspicuous consumption in society. If an agent consumes V units of conspicuous goods, $F(V)$ corresponds to the mass of agents consuming less than or exactly V. When $F(V)$ increases, it means that more agents are consuming a quantity of status goods smaller or equal to V. It follows that the status of an individual consuming V units of conspicuous goods rises and his utility increases in $F(V)$. Specifically, status is defined as follows:

$$S(V, F(\cdot)) = \gamma F(V) + (1-\gamma)F^{-}(V) + \alpha$$

where $\gamma \in [0, 1)$ and $F^{-}(V)$ is the mass of individuals with consumption strictly less than V. The parameter $\alpha \geq 0$ represents a guaranteed minimum level of status.

The above specification of status is a modification of the one in the model by **?**. In Frank's paper, status is modelled by $F(V)$. It implies that when all agents choose the same level of consumption $\tilde{V}$, $F(\tilde{V})$ is maximal and equal to one. Agents achieve the highest level of status. However, it is more plausible that consumers value being uniquely first more than being equal first. That is why **?** introduce the parameter γ, which captures the decreased satisfaction resulting from a tie situation.

They assume that agents have identical preferences over absolute consumptions and status. Furthermore, the agents maximize the following utility function with respect to V:

$$U(V, N, S(V, F(\cdot))) = W(V, N)S(V, F(\cdot))$$

subject to $pV + N \leq y$, $V \geq 0$, $N \geq 0$.

Such preferences are called "interdependent preferences". It means that status, here defined as the ordinal rank in the distribution of conspicuous consumption, enters interdependency directly into the preferences (**?**). In this model, agents do not receive preferential treatments from outsiders if they are perceived as richer. However, achieving a higher rank directly raises their utility level. Note further that the payoff depends on the perception of one's relative position. This is in contrast to all previous papers, in which payoffs depend on status defined as the perception of one's absolute wealth. Decisions are still strategic in this case of utility function specification. Indeed, individuals are in competition for status, and therefore, their final rank in the society depends on the conspicuous consumption of others. We end up in a game for status. The benefit of such a specification for status is that it emphasizes the fact that agent consumption depends on the distribution of income. Actually, the difference in specification does not drive to different results compared to the above models. As mentioned earlier, **?** used a very similar utility function to investigate the demand for conspicuous and normal goods and finds that concern for status leads people to spend a too large amount of money in status goods compared to what would be optimal in terms of welfare.

The function $W(V, N)$ represents the direct utility from consuming both goods. It is assumed to be strictly increasing in both status and normal goods and strictly quasi-concave. As a consequence, the opportunity cost of signaling is greater for poor than for rich individuals. This constitutes the required single-crossing property. Furthermore, we can deduce from the direct utility that consuming status goods provides satisfaction and, in addition, increases total utility by allowing to achieve higher status. It should also be noted that status enters multiplicatively into the utility function. Let us emphasize that the form of the utility function and the problem to a first-price sealed-bid auction look very much alike. On the one hand, selecting a high level of conspicuous consumption increases the probability of achieving a high rank and status increases ($S(V, F(\cdot))$). On the other hand, a higher V reduces direct utility as a consequence of the decline in consumption of the normal good ($W(V, N)$). This results in a trade-off.

In a symmetric equilibrium, each agent chooses the same mapping from income y to conspicuous consumption V. It follows that an agent's position in the distribution of conspicuous consumption $F(V)$ will be equal to his position in the distribution of income $G(y)$. Indeed, the equilibrium is separating here, since status is defined in such a way that an agent who consumes more status goods receives a higher status. Consequently, agents have an incentive to consume more than the others and to differentiate themselves.

The authors first address the role of the parameter $\alpha \geq 0$, which appears in the specification of status. We may recall that the individual with the lowest income will be provided with the lowest status α. Since concern for relative position enters the utility function multiplicatively, the individual with the lowest income will receive zero utility when α is equal to zero. He is therefore indifferent regarding the amount of money to spend on status goods. However, the lowest-status individual with zero utility is deadly and he will try to avoid zero status by investing all his income in conspicuous consumption out of desperation. In contrast, if α is larger than zero, the poorest agent still has a guaranteed minimum level of status. Thus, the pressure to compete gets softer and, like in the models analysed earlier, the individual will choose the amount of V that maximizes his direct utility without taking into account the decisions of all other agents. The authors provide examples of real facts illustrating the two different specifications. To explain the first specification with $\alpha = 0$, they argue that the poorest individuals engage in conspicuous consumption even if they cannot afford it. This is driven by the fact that, even today, low status may be associated with unemployment, poor marriage prospects and social exclusion. Regarding the second specification, a positive value of α ensures a minimal level of status independent of consumption behaviour of others. It suggests that conspicuous consumption is only accessible to middle and upper classes. This picture of conspicuous consumption is less pessimistic than the first one. The two views correspond to different real situations that are interesting to investigate.

The authors analyse the comparative statics of the effect of a change in the distribution of income on the level of conspicuous consumption. It is of great importance to note that the results will refer to the change in conspicuous consumption of an individual, whose income remains unchanged as the distribution of income in the society changes. Later, in 2009, the authors investigate the behaviour of an agent who occupies the same rank in the distribution of income, before and after the change (**?**). Even if the results of the two different approaches seem contradictory at the first sight, the described phenomena are the same. They just differ in the way we are looking at them.

In their first paper, they find that conspicuous consumption is decreasing with the minimum level of status guaranteed. A higher α makes the competition for status softer and leads individuals to decrease their purchase of status goods. They further address the behaviour of a society, which experiences an increase in average income. In the situation where poor individuals want to avoid zero status at all costs ($\alpha = 0$), agents respond to the rise in average income by an increase in conspicuous consumption at all levels of income. In fact, when average income rises, each individual's new position in society corresponds to a lower rank. Consequently, each one wants to catch up with his neighbours and increases his demand for status goods. On the contrary, if $\alpha > 0$, the poor decide to reduce their expenditures in conspicuous goods. A minimal level of status is ensured and we have seen that they choose their optimal level of consumption irrespective of the decisions taken by others. Therefore, they maximize their direct utility and decide to allocate their additional income towards normal goods. However, the rich and some of the middle class decide to increase their conspicuous consumption. At those income levels, competition for relative position gets harder and they have a greater incentive to differentiate themselves. It follows that they spend a higher amount of money on status goods. Similarly, in their paper in 2009, **?** find that an increase in income at every rank in society leads individuals to spend more on conspicuous consumption. They also show that an increase in poor individuals' income and a more dispersed ex post distribution also pushes agents to raise their conspicuous consumption. The greater affluence of individuals at the bottom of the income distribution pushes those above them to spend more on conspicuous consumption. Hopkins and Kornienko (2004) suggest that this could explain the absence of improvement in happiness over time in the developed world. This phenomenon had already been identified by Easterlin in 1974.

They also look at a society, which becomes more egalitarian. More specifically, suppose that the old society's income distribution is a mean-preserving spread of the new society's income distribution. Greater equality induces a greater density of individuals in the middle class. It follows that the individuals are closer to each other and that it is easier to overtake others. This intensifies the competition for status in the middle class and agents with middle incomes increase their conspicuous consumption. Since the new distribution is a mean-preserving contraction of the old distribution, the population density in the tails is lower after the change. The competition is lower at the bottom and at the top of the distribution. Concerning the rich, because competition is softer and the mass of individuals with less income than them remains more or less the same, they reduce their conspicuous consumption. The effect on the poor is different. On the one hand, the population density with low income is reduced and social competition diminishes. On the other hand, at the bottom of the distribution, individuals' rank decreases. Indeed, less people are endowed with a lower income than them. They have been left behind. Depending on the α again, their response differs. For desperate individuals ($\alpha = 0$), the decreased status hurts them a lot. They would do anything to avoid the consequences of a lower rank in the society. As a result, they increase their consumption of status goods. If there is a positive minimum level of status guaranteed, poor individuals care less about increasing one's status. The decrease in competition leads them to reduce their conspicuous consumption. Conversely, when individuals are indexed by their rank (**?**), the increase in equality involves greater income for everyone at the lower end of the distribution and leads them to spend more on status goods. The difference found in the results stems from the difference in points of view. But in the end, the two results mean the same.

The authors later investigate the effect of a change in the income distribution on welfare. As social income increases, the utility of the reference individual is smaller or equal after the change compared to before the change. First, this individual experiences no increase in his income, whereas the income of others rises. Consequently, his status decreases, since he sees fewer individuals poorer than or with the same income as him. Second, the individual of interest is involved in a stronger competition for status. In response to this phenomenon, the individual increases his conspicuous consumption inducing a decrease in his utility from normal goods. All in all, the individual experiences a reduction in both parts of his utility; the direct utility from consumption of both goods and status. In sum, this kind of change in the income distribution decreases the utility at each level of income. However, we also know that everyone is richer in the ex-post society than in the ex-ante society. Whether this rise in income compensates the decrease in utility remains ambiguous. When individuals are indexed by their rank (**?**), as the income hierarchy becomes more affluent, the reference individual with a given rank who experiences no increase in his own income is made worse off. This comes from higher social pressure by others with higher incomes, which leads the individual to increase conspicuous expenditures.

If society becomes more equal or more affluent, for a fixed income, individuals at the lower end of the distribution of income have a lower status in the more equal society. Therefore, the poor are better off under the more unequal society for any $\alpha \geq 0$. We have seen that individuals also respond to such a change by an adjustment in their conspicuous consumption. Depending on the minimum status level ensured α, the poor will increase or decrease their level of conspicuous consumption. If $\alpha = 0$, they increase it, and the direct utility derived from consumption of normal goods decreases. Clearly, for those individuals, the total effect on welfare is negative. If $\alpha > 0$, the poor decide to reduce their consumption of status goods, their direct utility increases. However, the authors find that this positive effect on welfare is not sufficient to exceed the negative effect of status on welfare. In the end, those individuals are worse off too. Poor agents have a higher utility when there are more people around with an income as low as theirs. **?** find that if individuals are indexed by their rank, poor individuals are made better off when their income is no lower after the change and when the ex-post income distribution is more dispersed.

All in all, a greater equality in the distribution of income within the society increases competition in status. In fact, it makes it simpler to overtake other agents in terms of conspicuous consumption. The resulting stronger social competition leads to a rise in conspicuous consumption. Therefore, the distortion in consumption is amplified.

In the latter paper, social status is defined as the individual's relative conspicuous consumption compared to what others consume in terms of conspicuous goods. What matters for an individual is how many people he overtakes in the distribution of income, i.e., his ordinal rank, no matter by how much his income exceeds that of his neighbours. This means that the individual sees his utility increase by a lot if he climbs up the ladder in the distribution of income, even if his income exceeds that of the individual just below him by one monetary unit. Therefore, this specification leads to quite extreme conclusions.

? address this issue by investigating how concern for cardinal rank versus ordinal rank modifies the equilibrium outcomes. They show how crucial the definition of status is for model predictions in terms of consumption behaviour. More specifically, they address the circumstances, under which a redistributive policy from rich to poor individuals can make both the rich and the poor better off.

They develop a signaling model of conspicuous consumption. The population consists of two types of individuals, poor and rich. Their fractions in the population are β and $(1-\beta)$, respectively. Poor individuals are endowed with low resources y_P and rich individuals are endowed with high resources y_R, with $y_R > y_P$. All individuals have the choice to allocate their resources between two types of goods, a conspicuous good, whose price is p, and an inconspicuous good, whose price is normalized to one. Conspicuous consumption is denoted by V and is observable, whereas inconspicuous consumption is denoted by N and is unobservable.[8] Note that the price of status goods is exogenously given. We will focus on their model's extension, where individuals take pleasure in consuming conspicuous goods. Indeed, in my model, it will be assumed that conspicuous consumption generates intrinsic utility. It is therefore more interesting to focus on a similar framework.

The individual's utility function consists of the addition of two components, one corresponding to the direct utility from consumption of both goods, $u(N, V)$, and another representing the utility from status, s. Note that $u(\cdot)$ is increasing and strictly concave in both of its arguments. Conspicuous consumption is used to signal one's resources, which are private information. But the distribution of resources, described by β, y_R, y_P, is common knowledge. Individuals will draw inferences about each individual's type from the observation of his conspicuous consumption. $\mu(V)$ denotes the belief function and corresponds to the probability of being perceived as rich given that V is observed. They define the utility from status as $s(\mu(V))$, where $L = s(0)$ denotes the status of the poor and $H = s(1)$ denotes the status of the rich. Since the direct utility function $u(\cdot)$ is strictly increasing in both goods, the budget constraint holds with equality.

When individuals care about how much ahead or behind others they are in terms of income, i.e. about their cardinal rank, s depends on y_P and y_R. The intuition of the effect of a redistribution of income on the demand for status goods is as follows. An increase in poor individuals' income decreases the absolute difference between the income of the poor and the income of the rich. It means that the value of being consid-

[8] In the paper by **?**, the income is denoted by R, conspicuous consumption is denoted by x, inconspicuous consumption is denoted by c and the fractions of poor and rich individuals in the population are reversed.

ered rich relatively to being considered poor decreases. As the prize for competition falls, poor individuals have a lower incentive to waste money in signaling and this allows rich individuals to decrease their conspicuous expenditures. As a result, the utility of the rich may increase. This phenomenon suggests that a redistribution of income from the rich to the poor may be welfare enhancing for both types of individuals. In fact, the redistribution makes the poor better off. The increase in their income allows them to consume more goods, enhancing their utility. Moreover, it increases the value of being perceived as being poor, since the absolute difference of incomes is smaller. It follows that poor houdeholds' total utility increases. Regarding the rich, they experience a decrease in their income. Therefore, for a given amount of money spent on conspicuous goods, they have to reduce their consumption of normal goods and they are made worse off. But at the same time, the prize for competition decreases, reducing the incentive of the poor to invest in signaling. As a result, rich households can reduce their spending on status goods leading to an increase in utility. If the second effect dominates the first one, a redistributive policy can improve the welfare of both rich and poor individuals. In fact, **?** discuss the circumstances under which a balanced-budget redistribution is likely to make poor and rich individuals better off.

We notice that the definition of status has a great impact on the relationship between inequality and demand for status goods. Whether people care about their ordinal rank, cardinal rank, or both can lead to very different policy implications. It is therefore of great importance to explicitly state how status is defined in models.

4 The Model

Consider a population comprised of two types of individuals, poor (P) and rich (R) individuals, differing only with respect to their income. Poor individuals are endowed with income y_P, whereas rich individuals are endowed with income y_R, with $y_R > y_P$. The fractions of poor and rich people in the population are denoted by β and $(1-\beta)$, respectively. Individuals' wealth is private information.

Each individual has the choice to allocate his income between the purchase of two types of goods, i.e. normal goods and status goods. Let N_i be the amount of normal goods purchased by individual i, and V_i the variety of status goods purchased by individual i, with $i = P, R$. Normal goods are "inconspicuous" in the sense that they are unobservable, whereas status goods are "conspicuous" in the sense that they are observable. Given the above assumptions about observability, only status goods' consumption can serve as a signal of wealth. The third party plays the same role as social contacts in the model by **?**. It infers the wealth of the individuals from the observation of their consumption of status goods. If one individual is perceived as being rich by the third party, he will be attributed a higher status than in the case where he would be perceived as being poor. This phenomenon is called preferential treatments in the paper by **?**. To be more specific, the outsider can observe the variety and the quantity of each conspicuous good the individual chooses to consume, and the price at which the individual purchases them.

Each individual cares about his consumption of both, normal and status goods. In addition, individuals care about their relative position in the distribution of income. The utility function takes the following form:

$$U_i = \gamma \frac{1}{\alpha} N_i^{\alpha} + N_i + \int_{j=0}^{V_i} c_{ij} dj + \begin{cases} s & \text{if individual } i \text{ is perceived as being rich} \\ 0 & \text{if individual } i \text{ is perceived as being poor.} \end{cases}, \quad (1)$$

with $\alpha \in (0,1)$, $\gamma > 0$ and $c_{ij} = \{0,1\}$. Note that individual i's status, $s\left(c_{ij}, c_{-ij}, p_j\right)$, depends on his own conspicuous consumption, on the conspicuous consumption of all other individuals, and in addition, on the price of conspicuous goods. In fact, from the observation of all individuals' conspicuous consumption, the third party will infer each individual's absolute type, i.e. each individual's income. I use (0-1)-preferences to model preferences of status goods. More specifically, an individual chooses whether to consume the first good $j = 0$, or not. If the individual decides to consume it, then only one unit of it suffices to achieve satiety. Once the individual has consumed one unit of good $j = 0$, he decides whether to consume the next good with j slightly larger than zero or not, and so on. In the end, the individual consumes a variety of different status goods rather than a quantity of one status good. Such a specification allows modelling many status goods. It should be noted that this specification of utility function is a special case of the utility function by **?**. Here, the supply of status goods is infinite instead of being defined on a certain range of qualities. As mentioned earlier, individuals choose to purchase the status good with the first-best quality in the model by **?**. As a result, both utility functions are linear in the quantity of status goods. We further notice that consuming status goods provides direct satisfaction and thus is an end in itself. This is in opposition to both matching models studied previously, where the consumption of conspicuous goods constitutes solely a mean to signal wealth in order to get better consumption opportunities, but provides no intrinsic utility. As mentioned before, the third party makes an inference about the individual's absolute type, but what matters for individuals is their relative economic standing. The third party forms beliefs about each individual's wealth from the observation of the variety, the quantity and the price of the status goods he consumes. If the third party perceives the individual as being rich, the latter will be granted with a high status represented by a payoff equal to $s > 0$. If, on the contrary, the individual is perceived as being poor, the status granted is low and the payoff associated with this belief is equal to zero.

It is of great importance to first emphasize that the individuals' decisions are strategic. They have to take into account the decisions of others in making their optimal choice, since the perception of their type by the third party depends on how their conspicuous consumption relates to that of the others. It results in a signaling game, in which individuals compete for status. Second, the specification of the utility function is such that individuals prefer to consume normal goods than status goods at each consumption level. More specifically, the marginal utility of normal goods is always larger than one, whereas the marginal utility of status goods is always equal to one. In fact, it is necessary that individuals do not experience more satisfaction in consuming conspicuous goods than normal goods, otherwise they would decide to purchase conspicuous goods exclusively and there would be no place for signaling anymore. Specifically, if the direct utility from consumption of status goods plus the utility from the status associated to the observation of conspicuous consumption was higher than the total utility when some normal goods are consumed, each individual would choose to consume solely status goods. Because conspicuous consumption is observable and individuals would spend their entire income on status goods, the third party would observe their income directly. Accordingly, the consumption choice of individuals is not strategic anymore and the necessity of signaling one's wealth disappears. All in all, as the purpose of this study is to investigate how the concern for status affects the demand for normal and status goods, it is necessary that status goods are not more desirable than normal goods. The above specification of the utility function satisfies this condition. Third, the utility function is increasing in both types of goods. However, the utility function is strictly concave in the purchase of normal goods. This constitutes the condition under which the single-crossing property is satisfied. For this particular property to hold, it is required that renouncing normal goods in order to consume status goods is more costly for poor than for rich individuals. For this purpose, the marginal utility of normal goods has to be decreasing in the quantity of normal goods. Because rich individuals are endowed with a higher income than poor individuals, they can consume a larger amount of normal goods and therefore the marginal utility of normal goods is smaller. As a result, decreasing the consumption of normal goods to increase conspicuous consumption has a smaller negative effect on total utility of the rich than on total utility of the poor.

In the following sections, we will analyse the separating equilibrium in two different cases. In the first one, prices are exogenous (section 4.1), whereas in the second one, prices are endogenous (section 4.2).

4.1 Exogenous Prices

In this section, it is assumed that prices are exogenously given. We will use the normal good as the numeraire throughout the whole analysis. For simplicity, we will investigate symmetric equilibria, where all status goods are supplied at the same price. Moreover, we will study the case, where status goods are either more expensive than or as expensive as normal goods. Formally, $p_j = p \geq 1$, $\forall j$. In fact, it is sufficient to only consider this case, since status goods should never be more desirable than normal goods and since the price of normal goods is equal to one.

Each individual's problem consists in choosing the variety of conspicuous goods that maximizes their utility while taking into account the decisions of others. Because the utility function is increasing in normal goods, the budget constraint, $N_i + \int_{j=0}^{V_i} p_j c_{ij} \leq y_i$, holds with equality. This means that individuals use all their residual income after having purchased status goods to consume normal goods. Let us consider the separating equilibrium, in which the rich and the poor spend different amounts of money on signaling, and in which the two types are correctly identified by the third party. In fact, the only equilibrium here is separating. Status is defined such that an individual with higher conspicuous consumption is associated with a higher status. Therefore, rich individuals have a great incentive to increase their spending on status goods in order to distinguish themselves and this results in a separating equilibrium (?). The varieties of status goods chosen by the two types of individuals (V_P, V_R) constitute a separating equilibrium, if V_P and V_R, with $V_P \neq V_R$, satisfy the following incentive compatibility constraint:

$$\gamma \frac{1}{\alpha} N_P^{\alpha} + N_P + \int_{j=0}^{V_P} c_{Pj} dj \geq \gamma \frac{1}{\alpha} \left(y_P - \int_{j=0}^{V_R} p_j c_{Rj} dj \right)^{\alpha} + \left(y_P - \int_{j=0}^{V_R} p_j c_{Rj} dj \right) + \int_{j=0}^{V_R} c_{Rj} dj + s, \quad \text{(IC)}$$

meaning that the poor do not want to deviate from their optimal choice to make the third party believe they are of the other type. Consumption choices also have to be feasible. Moreover, if the third party observes the signal $\int_{j=0}^{V_P} p_j c_{Pj} dj$, it should infer that the individual is poor and should grant him a low status. The individual perceived as being poor therefore receives zero payoff by the third party. If the third party observes the signal $\int_{j=0}^{V_R} p_j c_{Rj} dj$, it should infer that the individual is rich and should grant him a high status. The individual perceived as being rich receives a payoff $s > 0$ by the third party. Finally, taking into account the relationship between inference and status granted by the third party, the choices (V_P, V_R) must be optimal (?).

It is important to note that any belief about the individuals income can be assigned to signals off the equilibrium path (?). That is why multiple separating equilibria exist. In order to get a unique prediction of the separating equilibrium, we can apply an equilibrium refinement called "Intuitive Criterion" (?). It states that an equilibrium signal, $\int_{j=0}^{V_i} p_j c_{ij} dj$, is equilibrium dominated if individual i can get a greater utility than the highest utility associated with the signal $\int_{j=0}^{V_i} p_j c_{ij} dj$ by using another signal. In fact, in the separating equilibrium, poor individuals are correctly identified. It is thus useless for them to increase their signal $\int_{j=0}^{V_P} p_j c_{Pj} dj$ to attempt to mislead the third party, since they will be perceived as being poor and will be granted the lowest status anyway. Consequently, poor individuals choose the variety of status goods that maximizes their direct utility. As noted earlier, the marginal utility of normal goods is greater than the marginal utility of status goods at each consumption level. In the end, poor individuals decide to devote all their income to purchase goods that provide the highest utility, i.e. to purchase normal goods. We get $N_P{}^* = y_P$.

LEMMA 1: *In any separating equilibrium, $V_P{}^* = 0$; a poor individual chooses not to purchase status goods.*

According to the Intuitive Criterion, the optimal choice of rich individuals is to choose the minimum variety of status goods that dissuade imitation by poor individuals. In other words, their utility is maximal when they spend a minimum amount of money on status goods that just ensures that they will be perceived as rich without wasting too much resources in signaling and keeping the maximal residual income to purchase normal goods. Formally, the incentive compatibility constraint (IC) must hold with equality. Taking into account that ${N_P}^* = y_P$, ${V_P}^* = 0$ and $p_j = p$, $\forall j$ with $p \geq 1$, the problem of a rich individual can be written as follows:

$$\begin{gathered} max_{V_R}\, U_R = \gamma \frac{1}{\alpha}\left(y_R - pV_R\right)^{\alpha} + \left(y_R - pV_R\right) + V_R + s \\ s.t. \\ \gamma \frac{1}{\alpha} {y_P}^{\alpha} + y_P = \gamma \frac{1}{\alpha}\left(y_P - pV_R\right)^{\alpha} + \left(y_P - pV_R\right) + V_R + s. \end{gathered} \tag{2}$$

To find the optimal variety a rich individual decides to purchase, we need to solve the incentive compatibility constraint for V_R. To find an explicit solution for V_R, we make the assumption that $\alpha = \frac{1}{2}$.

LEMMA 2: *In any separating equilibrium,*

$$\begin{aligned} {V_R}^* = min \Bigg\{ & \frac{y_P}{p} - \frac{p}{(p-1)^2}\left(2\gamma^2 - 2\gamma \left\{\gamma^2 - \frac{p-1}{p}\left(s - 2\gamma {y_P}^{\frac{1}{2}} - \frac{p-1}{p} y_P\right)\right\}^{\frac{1}{2}}\right) \\ & + \left(\frac{1}{p-1}\right)\left(s - 2\gamma {y_P}^{\frac{1}{2}} - \frac{p-1}{p} y_P\right), \frac{y_P}{p}\Bigg\}. \end{aligned}$$

(Proof in Appendix, A1)

It is necessary to emphasize that it makes no sense for rich individuals to spend more than the income of the poor on status goods for the sake of discouraging them. It would be a waste of money to have $p{V_R}^* > y_P$. Hence, the condition $p{V_R}^* \leq y_P$ must hold. We will restrict our attention to the set of parameters for which the above condition holds with strict inequality. Therefore, the set of parameters which satisfy the latter condition with strict inequality is given by:

$$2\gamma {y_P}^{\frac{1}{2}} + \frac{p-1}{p} y_P > s. \tag{3}$$

(Proof in Appendix, A2)

It is interesting to investigate the behaviour of the variety of status goods purchased by the rich in response to different shocks. We thus turn to the comparative statics of ${V_R}^*$. We analyse how the parameter s affects the variety of status goods demanded by the rich and also the effect of the distribution of income, y_P and β, on the demand for status goods by the rich.

PROPOSITION 1: *The equilibrium variety of conspicuous goods consumed by rich individuals, ${V_R}^*$, (a) increases with the reward of being perceived as being rich, s, (b) increases with the income of poor individuals, y_P, and (c) is not affected by a change in the proportion of poor individuals in the population, β.*[9]

(Proof in Appendix, A3)

Result (a) in Proposition 1 implies that rich individuals increase their spending on conspicuous goods, as the reward of being perceived as a rich individual, s, rises. If the status associated to being perceived as wealthy increases, poor individuals have a greater incentive to imitate the rich. Therefore, the rich respond by raising their conspicuous consumption to dissuade imitation by the poor.

Result (b) suggests that as poor individuals become richer, the competition for status is magnified and rich individuals have to spend more money on status goods in order to render imitation inconvenient for the poor. In fact, after an increase in y_P, and thus an increase in equality within the society, it is easier for the poor to pretend to be rich, making them more willing to purchase status goods. As a response to these rivalries, the rich increase their conspicuous consumption in order to distinguish themselves from the poor. However, note that in equilibrium, poor individuals still do not purchase status goods.

Result (c) implies that if the fraction of poor individuals in the population increases, the residual income, $y - \beta y_P$, is divided among fewer rich individuals, for a given total income, y, and a given income of poor individuals, y_P. In the end, each rich individual is endowed with a higher income. However, we observe from Lemma 2 that the optimal variety of status goods purchased by the rich is independent of the income of the rich. Hence, the incentive of the poor to imitate the rich is not affected, since poor individuals still receive the same income. As a result, there is no reason for the rich to adjust their spending on status goods, as the same variety as before the change suffices to discourage the poor. All in all, because all individuals prefer normal

[9] It can be shown that ${V_R}^*$ decreases in both utility function's parameters, α and γ. Note that, since these results are not of great importance for the present study, we will not conduct such an analysis in the following sections.

goods over conspicuous goods, the rich spend all their additional revenue on normal goods.

It should be noted that the variety of conspicuous goods the rich are willing to consume depends on the price of these goods.

PROPOSITION 2: *The variety of status goods consumed by rich individuals, ${V_R}^*$, decreases with the price of status goods, p.*

(Proof in Appendix, A4)

When conspicuous goods become more expensive, they become even less desirable. Therefore poor individuals are less willing to purchase them than before the change in prices. As a result, rich individuals need to buy a smaller variety of status goods to make the poor not willing to spend as much money on conspicuous goods as them.

In contrast with the models by **?** and **?**, we have found that poor individuals do not consume any status good. However, this result is supported in both matching models by **?** and **?**, and also in the model by **?**. In fact, **?** develop a model, in which individuals like to consume both normal and status goods. Individuals value status goods as much as normal goods. That is why, when poor individuals maximize their direct utility from both goods, they decide to consume status goods in addition to normal goods. In my model, each individual prefers to consume normal goods. Status goods are purchased only for their signaling characteristic. Therefore, as the third party correctly identifies poor individuals, it is of no value for the latter to buy any status goods. Exactly the same phenomenon occurs in the two matching models, where individuals signal their wealth with something they do not like. Specifically, individuals destroy a part of their income to display their wealth but do not experience satisfaction from conspicuous consumption. This interpretation also holds for the present model, in which individuals prefer normal goods over status goods. Similarly, in the paper by **?**, individuals do not derive direct satisfaction from conspicuous consumption. As a result, individuals who do not have any interest in quality do not buy any conspicuous goods.

It is worth mentioning that, in both the model by **?** in the case where $\alpha = 0$ and in my model, the poorest individuals are granted with zero status. However, the consumers' choice differs a lot. Consumers decide to spend all their revenue in conspicuous consumption in the former model, whereas consumers choose not to consume status goods at all in the latter one. This is explained by the difference in utility function's specification. In the model by **?**, status enters multiplicatively into the utility function. It follows that the utility of the poorest individuals with zero status is equal to zero. Therefore, they desperately attempt to avoid this event by spending all their income on signaling. By contrast, in my model, status enters additively into the utility function. Consequently, zero status assigned to poor individuals affects only the utility derived from status but keeps the direct utility from both goods unaffected. Therefore, they maximize their direct utility from both goods, which results in consuming zero status goods.

Proposition 1 states that an increase in poor individuals' income leads to an increase in the purchase of status goods by rich individuals. **??** find a similar result: as a society's average income increases, rich individuals increase their spending on status goods. Note that the two above-mentioned papers differ from each other provided the analysis in the former paper is made from the point of view of an individual, whose income is not affected, and the analysis in the latter is made from the point of view of an individual, whose rank does not change. As in my model, the distribution of income is discrete and consists only of two groups of individuals, this distinction is not necessary. Note that, in my model, an increase in the society's average income affects the demand for conspicuous goods by the rich only insofar as the income of the poor rises. A change in the income of the rich does not affect their spending on signaling (Result (c) in Proposition 1). It is worth emphasizing that this result stems from the fact that, in the three models, individuals care only about their relative economic standing.

However, as opposed to the two papers by **?**, a rise in equality that comes from a decrease in the proportion of poor individuals in the population does not affect the demand for status goods by the rich. Again, this stems from the distribution of income in my model, which is discrete and consists only of two classes of individuals. In the paper by **?**, if a more unequal society is a mean-preserving spread of the more equal society, then in the latter society, the competition for status increases in the middle class but decreases in both the lower and upper classes. It follows that rich and poor individuals decrease their conspicuous expenditures, whereas the middle class raises them. If we take the point of view of an individual, whose rank remains unchanged (**?**), as society becomes more equal, individuals ranked as poor are now richer and thus increase their purchase of conspicuous goods. The effect on the rich is not analysed. Consequently, depending on the point of view from which the analysis is conducted, the behaviour of the poor in response to a rise in equality differs. However, in both cases, they adjust their conspicuous consumption. On the contrary, in my model, as the society becomes more equal in response to a decrease in β, the incentives of poor individuals to increase their spending on signaling are not affected, since their income remains unchanged. Therefore, it is not necessary for rich individuals to adjust their spending on signaling. This is explained by the absence of a middle class in my model, and by the fact that the lower and upper classes consist only of two groups of individuals with the same income. No competition for status exists within the same class and the competition between the two classes is not affected, since the cost of imitation of the poor remains unchanged.

4.2 Endogenous Prices

So far, we have assumed that prices were exogenously given and that individuals had to decide the extent of the variety of status goods they want to purchase, for given prices of normal and status goods. In this section, we aim to analyse the case, in which prices are endogenously determined by firms. Let us assume that the process by which normal goods are produced is perfect competition. This allows a stronger focus on the different mechanisms resulting from various production processes of status goods (**?**).

Let us recall that individuals can signal their wealth through the variety or the price of status goods. In the previous section, prices were exogenous. As a result, individuals had only one possibility to signal their wealth, i.e. through the variety. Suppose that both the variety and the price of conspicuous goods are endogenous and unrestricted. It means that individuals have the choice between advertising their wealth by purchasing a large variety of status goods or advertising their wealth by purchasing them at a higher price. In the latter case, Veblen effects would arise (**?**). It can be shown that individuals choose to signal their wealth through the variety and not through the price. Only total conspicuous expenditures matter in order to signal one's wealth, not the combination price-variety.

PROPOSITION 3: *Individuals prefer to signal their wealth through the variety rather than the price of status goods, and therefore prefer to buy the cheapest status goods.*

(Proof in Appendix, A5)

In this section, we investigate three cases in which prices are endogenously determined by firms. In the first two cases, the variety of status goods, V_R, is fixed and supplied by monopolists. Each monopolist supplies one status good. In the first case, the given variety of conspicuous goods supplied by all monopolists is large. In the second case, the exogenous variety of conspicuous goods supplied by all monopolists is small. As in section 4.1, we will investigate solely symmetric equilibria, in which all status goods are supplied at the same price. Furthermore, the price of status goods must be larger than or equal to the price of normal goods, i.e. one, to make normal goods more desirable than conspicuous goods. Formally, the symmetric equilibrium is characterized by $p_j = p \geq 1$, $\forall j$. In the third and last case, the whole supply of status goods is offered by a single monopolist. In all cases, the marginal cost of production is equal to one. This is feasible, since the price of status goods must be larger than or equal to one.

4.2.1 Large Number of Monopolists Supplying Status Goods

In this section, suppose that there is a large number of monopolists supplying status goods. Each monopolist supplies a single status good. It follows that the exogenous supply of status goods, V_R, is large. The problem of each monopolist consists in maximizing profit, given that the incentive compatibility constraint in the case of a symmetric equilibrium is binding. It can be written as follows:

$$\begin{gathered} max_p \, \pi_j = \pi = (p-1)(1-\beta), \quad \forall j \\ s.t. \\ \gamma \frac{1}{\alpha} {y_P}^{\alpha} + y_P = \gamma \frac{1}{\alpha} \left(y_P - pV_R \right)^{\alpha} + \left(y_P - pV_R \right) + V_R + s. \end{gathered} \tag{4}$$

In fact, we already know from before that only rich individuals purchase status goods, and their fraction of the population is denoted by $(1-\beta)$. Furthermore, each rich individual purchases only one unit of good j. It results that the profit of one monopolist is equal to the profit per unit, $(p-1)$, times the number of rich individuals, $(1-\beta)$.

In this case, the variety of status goods supplied is large. Therefore, for any price supplied by all monopolists, rich individuals can choose the variety that maximizes their utility, while satisfying the incentive compatibility constraint with equality. It means that the supply of status goods allows rich individuals to dissuade imitation by the poor by purchasing a large enough variety of status goods for any price offered. Consequently, monopolists do not need to adjust their price in order to make the incentive compatibility constraint binding. Hence, each monopolist chooses the price that simply maximizes its profit. Due to the large number of monopolists, the competition in prices is very strong and we end up in a situation designated as Bertrand Competition. If one monopolist sets its price at a higher level than other monopolists, it will get no demand for its good and is constrained to decrease its price. Each monopolist is willing to sell its good and thus lowers its price. The competition in price continues up to the point, where profits are equal to zero. Therefore, in equilibrium, each monopolist sets its price equal to the marginal cost of production. More specifically, $p_j = p = 1$, $\forall j$. We notice that we find ourselves in a special case of section 4.1, where normal and status goods cost the same.

This result is supported by **?**. First, individuals prefer to display their wealth by purchasing a large quantity of conspicuous goods rather than purchasing them at a higher price. Second, when the supply of status goods is not restricted, individuals can use the quantity as a signal of wealth and monopolists end up in a situation of Bertrand Competition. It follows that monopolists are pricing at the marginal cost.

4.2.2 Small Number of Monopolists Supplying Status Goods

In the present section, suppose that there is a small number of monopolists, each supplying one status good. It follows that the supply of status goods is small. The problem of each monopolist is the same as in section 4.2.1. To find an explicit solution for p, we insert $\alpha = \frac{1}{2}$ into the incentive compatibility constraint.

LEMMA 3: *In any separating equilibrium,*

$$p^* = min \left\{ \frac{-2\gamma^2 + V_R + s - 2\gamma y_P^{\frac{1}{2}} + 2\gamma \left(\gamma^2 - V_R - s + 2\gamma y_P^{\frac{1}{2}} + y_P \right)^{\frac{1}{2}}}{V_R}, \frac{y_P}{V_R} \right\},$$

where $p^* > 1$.

(Proof in Appendix, A6)

We know from section 4.1 that a negative relationship exists between V_R and p. More specifically, we have found that $\frac{dV_R}{dp} < 0$, for $p \geq 1$. It follows that a small variety of status goods offered is associated with a large price of status goods. That is, in the present section, the common price of conspicuous goods is strictly greater than one. Moreover, as in section 4.1, it is important to note that the condition $p^* V_R \leq y_P$ must hold. We will restrict our attention to the set of parameters for which this condition holds with strict inequality. Therefore, we consider the following set of parameters:

$$2\gamma y_P^{\frac{1}{2}} + y_P > s + V_R \,. \tag{5}$$

(Proof in Appendix, A7)

In this section, unlike section 4.1, we consider the variety of status goods offered by all monopolists, V_R, as given. Hence, each monopolist chooses the price of its status good by taking into account the incentive compatibility constraint. That is why condition (5) is a function of V_R rather than p, as opposed to condition (3). In fact, in this case, p adjusts to V_R in order to satisfy the incentive compatibility constraint with equality, when V_R does not allow it. In practical terms, it is possible that, for a given price, rich individuals would need to consume a larger variety of status goods than the variety actually supplied by all monopolists to distinguish themselves from the poor. This forces rich individuals to buy status goods at a higher price to prevent imitation, even if they prefer to purchase the cheapest status goods, as stated previously. As a result, the monopolists can increase their price, and thus their profit up to the point where the incentive compatibility constraint is binding. Consequently, the common price of status goods is larger than one. We end up in the case where status goods are more expensive than normal goods.

We now turn to the comparative statics concerning p^*. We will observe exactly the same dynamics as for ${V_R}^*$, when prices are exogenous.

PROPOSITION 4: *The equilibrium price of status goods set by all monopolists, p^*, (a) increases with the reward of being perceived as being rich, s, (b) increases with the income of poor individuals, y_P, and (c) is not affected by a change in the proportion of poor individuals in the population, β.*

(Proof in Appendix, A8)

Result (a) suggests that the higher the reward of being perceived as being rich, the greater the incentive of poor individuals to spend money on signaling. Because the variety supplied is fixed, rich individuals are forced to purchase status goods at a higher price. Thus, monopolists can set a higher price in response to a rise in the payoffs associated to a high status.

Regarding result (b), an increase in the income of the poor increases their incentive to signal their wealth. Therefore, for a fixed and small variety of status goods supplied, as poor individuals become richer, status goods have to get more expensive in order to allow rich individuals to differentiate themselves from the poor. Monopolists can therefore charge a higher price for their conspicuous good and receive a greater profit.

Result (c) shows that monopolists do not adjust their price in response to a change in the numerosity of poor individuals. If the population encompasses a higher fraction of poor individuals, the income left for rich individuals is divided among fewer people. As a result, each rich individual is endowed with a higher income. However, this change does not alter the incentive of poor individuals to purchase status goods. Therefore, rich individuals do not experience a higher competition for status and do not need to adjust their spending on status goods. Their additional income is simply spent on normal goods.

The mechanisms are the same as in section 4.1. However, p instead of V_R adjusts in order to make the incentive compatibility constraint binding. As a result, the same comparisons with the theoretical papers of section 3 can be made regarding the effect of change in the income distribution on the demand for status goods (p here). A new result appears compared to section 4.1. Restricting the supply of status goods allows monopolists to charge a higher price and to receive greater profits. This result is supported by **?** and **?**. Even if, in both models, a single monopolist supplies all the status goods as opposed to the present section, we can still compare them to my model. In their model, consumers have to select only one item from a fixed supply of different amounts of money (**?**) or from a fixed supply of quality-price offers (**?**). **?** finds that restricting the supply of conspicuous goods drives consumers to pool around more expensive items in order to be associated with the desired status. The monopolist can therefore extract more profit by supplying a small variety of status goods. Similarly, **?** finds that concern for image allows the monopolist to set a higher price for its status goods and receive higher profit.

4.2.3 One Monopolist

In this section, we investigate the case of a single monopolist supplying all the status goods. The monopolist can now decide on both, the variety and the price of conspicuous goods to be offered. We analyse the symmetric equilibrium, in which the price of all status goods is equal. It is interesting to analyse whether the monopolist chooses to supply a large variety of status goods at a low price, or to supply a small variety at a higher price. The problem of the monopolist can be written as follows:

$$\begin{gathered} max_{V_R}\, \pi = V_R\left(p(V_R) - 1\right)(1-\beta) \\ s.t. \\ \gamma\frac{1}{\alpha}{y_P}^{\alpha} + y_P = \gamma\frac{1}{\alpha}\left(y_P - pV_R\right)^{\alpha} + \left(y_P - pV_R\right) + V_R + s. \end{gathered} \tag{6}$$

Note that we take the price as a function of the variety of status goods supplied, since we are interested in analysing which variety the monopolist is willing to supply. In order to find which strategy maximizes its profit, we can study whether the profit is increasing or decreasing in the variety of status goods, V_R. If the derivative of the profit with respect to the variety is positive, the optimal strategy is to offer the largest variety of status goods compatible with the incentive compatibility constraint, at a low price. If it is negative, the optimal strategy consists in supplying the lowest possible variety at a very high price.

PROPOSITION 5: *The single monopolist prefers to supply a very small variety, $V_R = \varepsilon > 0$, at a very high price, $p \to \infty$, rather than to supply a large variety at a small price.*

Proof. The derivative of the profit with respect to the variety is given by:

$$\frac{d\pi}{dV_R} = (1-\beta)\left(p(V_R) - 1 + V_R \frac{dp(V_R)}{dV_R}\right)$$

From Lemma 3, we can calculate the derivative of the price with respect to the variety of status goods by using the implicit function theorem, with $Q(V_R, p) = 2\gamma \left(y_P - pV_R\right)^{\frac{1}{2}} + \left(y_P - pV_R\right) + V_R + s - 2\gamma {y_P}^{\frac{1}{2}} - y_P = 0$. We obtain:

$$\frac{dp(V_R)}{dV_R} = \frac{-\gamma \left(y_P - p(V_R)V_R\right)^{-\frac{1}{2}} p(V_R) - p(V_R) + 1}{\gamma \left(y_P - p(V_R)V_R\right)^{-\frac{1}{2}} V_R + V_R}.$$

Given that condition (5) must hold, the above derivative is strictly smaller than zero for $p \geq 1$.

We now replace $p(V_R)$ by its expression (Lemma 3) into the derivative of the price with respect to the variety of status goods:

$$\frac{dp(V_R)}{dV_R} =$$

$$\frac{-\gamma\left[y_P+2\gamma^2-V_R-s+2\gamma{y_P}^{\frac{1}{2}}-2\gamma\left(\gamma^2-V_R-s+2\gamma{y_P}^{\frac{1}{2}}+y_P\right)^{\frac{1}{2}}\right]^{-\frac{1}{2}}}{\gamma\left[y_P+2\gamma^2-V_R-s+2\gamma{y_P}^{\frac{1}{2}}-2\gamma\left(\gamma^2-V_R-s+2\gamma{y_P}^{\frac{1}{2}}+y_P\right)^{\frac{1}{2}}\right]^{-\frac{1}{2}}{V_R}^2+{V_R}^2}$$

$$\times\frac{\left[-2\gamma^2+V_R+s-2\gamma{y_p}^{\frac{1}{2}}+2\gamma\left(\gamma^2-V_R-s+2\gamma{y_P}^{\frac{1}{2}}+y_P\right)^{\frac{1}{2}}\right]}{\gamma\left[y_P+2\gamma^2-V_R-s+2\gamma{y_P}^{\frac{1}{2}}-2\gamma\left(\gamma^2-V_R-s+2\gamma{y_P}^{\frac{1}{2}}+y_P\right)^{\frac{1}{2}}\right]^{-\frac{1}{2}}{V_R}^2+{V_R}^2}$$

$$+\frac{2\gamma^2-V_R-s+2\gamma{y_P}^{\frac{1}{2}}-2\gamma\left(\gamma^2-V_R-s+2\gamma{y_P}^{\frac{1}{2}}+y_P\right)^{\frac{1}{2}}+V_R}{\gamma\left[y_P+2\gamma^2-V_R-s+2\gamma{y_P}^{\frac{1}{2}}-2\gamma\left(\gamma^2-V_R-s+2\gamma{y_P}^{\frac{1}{2}}+y_P\right)^{\frac{1}{2}}\right]^{-\frac{1}{2}}{V_R}^2+{V_R}^2}$$

We now insert $\frac{dp(V_R)}{dV_R}$ into the derivative of the profit with respect to the variety, replace $p(V_R)$ by its expression (Lemma 3), simplify, and get:

$$\frac{d\pi}{dV_R} =$$

$$(1-\beta)\frac{\left\{1+\gamma\left(y_P+2\gamma^2-V_R-s+2\gamma{y_P}^{\frac{1}{2}}-2\gamma\left(\gamma^2-V_R-s+2\gamma{y_P}^{\frac{1}{2}}+y_P\right)^{\frac{1}{2}}\right)^{-\frac{1}{2}}\right\}}{\gamma\left(y_P+2\gamma^2-V_R-s+2\gamma{y_P}^{\frac{1}{2}}-2\gamma\left(\gamma^2-V_R-s+2\gamma{y_P}^{\frac{1}{2}}+y_P\right)^{\frac{1}{2}}\right)^{-\frac{1}{2}}V_R+V_R}$$

$$\times\frac{-V_R}{\gamma\left(y_P+2\gamma^2-V_R-s+2\gamma{y_P}^{\frac{1}{2}}-2\gamma\left(\gamma^2-V_R-s+2\gamma{y_P}^{\frac{1}{2}}+y_P\right)^{\frac{1}{2}}\right)^{-\frac{1}{2}}V_R+V_R}$$

$$+\frac{V_R}{\gamma\left(y_P+2\gamma^2-V_R-s+2\gamma{y_P}^{\frac{1}{2}}-2\gamma\left(\gamma^2-V_R-s+2\gamma{y_P}^{\frac{1}{2}}+y_P\right)^{\frac{1}{2}}\right)^{-\frac{1}{2}}V_R+V_R}$$

We then divide both the numerator and the denominator by V_R, rearrange and obtain:

$$\lim_{V_R \to 0} \frac{d\pi}{dV_R} = -\frac{1-\beta}{1+\frac{1}{\gamma}\left(y_P + 2\gamma^2 - s + 2\gamma y_P{}^{\frac{1}{2}} - 2\gamma\left(\gamma^2 - s + 2\gamma y_P{}^{\frac{1}{2}} + y_P\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} < 0$$

This derivative is negative.[10] To determine whether this result is a global or a local maximum, different values of V_R were entered in an Excel table to calculate the profit. The results concluded that, as V_R increases, the profit becomes negative and falls continuously. Therefore, we have a global maximum of the profit at $V_R = \varepsilon$ and $p \to \infty$. □

The above-demonstrated proof is not formal yet shows a very interesting result. In fact, nobody likes status goods as such. Normal goods are preferred by everybody. Therefore, the rich purchase conspicuous goods only for signaling purposes. Hence, if a valid signal can be provided by consuming a small variety of status goods, the monopolist cannot increase its profit by supplying a larger variety. This is because each unit of a status good provides less utility than one unit of a normal good. Hence, once the rich have bought a very small variety of status goods sufficient to be correctly identified, they spend their residual income on normal goods. It is therefore not optimal for the monopolist to produce a large variety of status goods, since only a limited part will be sold. We can reinterpret this result by saying that the rich pay a fix amount of money to the monopolist that is sufficient to be perceived as being wealthy. All in all, the monopolist provides the signal but is not willing to create any additional distortion in consumption. It supplies the minimal variety compatible with a separating equilibrium and, in doing so, it allows the rich to spend the minimal amount of their income on signaling. It follows that as large a part of their income as possible is kept for spending on normal goods. This result constitutes the main contribution of this thesis and suggests important policy implications. First, preventing competition in status good markets would considerably decrease distortion in consumption. Second, it suggests that innovation in such markets is bad for social welfare.

We are now able to calculate the maximum profit.

LEMMA 4: *The maximum profit is equal to:*

$$\pi_{max} = \lim_{V_R \to 0} \pi = \left\{-2\gamma^2 + s - 2\gamma y_P{}^{\frac{1}{2}} + 2\gamma\left(\gamma^2 - s + 2\gamma y_P{}^{\frac{1}{2}} + y_P\right)^{\frac{1}{2}}\right\}(1-\beta)$$

[10] We must have $\gamma^2 - s + 2\gamma y_P{}^{\frac{1}{2}} + y_P > 0$ in order to have a positive number under the root. This condition is less restrictive than condition (5) and is therefore satisfied.

(Proof in Appendix, A9)

We now investigate how it reacts in response to a change in the income of poor individuals.

PROPOSITION 6: *The maximum profit of the single monopolist increases as the income of the poor rises.*

(Proof in Appendix, A10)

In fact, if all the status goods are supplied by the same monopolist, the solution to the monopolist's problem is, for rich individuals, to pay the monopolist a fix amount of money in order to display their wealth and to be correctly identified by the third party. As poor individuals become richer, the incentive compatibility constraint implies that an appropriate dissuasion of imitation is, for the rich, to pay a larger amount of money to the monopolist.

In **?**'s model, the single monopolist supplies a restricted menu of items to a continuum of consumers, from which consumers have to select only one. Concern for status together with the continuous distribution of income, implies that in equilibrium, consumers pool around some more expensive items in response to a restriction of the supplied menu. The monopolist can therefore coordinate the individuals on the desired items. In my model, the distribution of income is dicrete and only rich individuals purchase status goods. Hence, partial pooling does not take place in my model. The monopolist can only restrict the variety of status goods to offer rich individuals. Because status goods are purchased merely for their signaling benefit, the monopolist's optimal strategy is to supply the lowest possible variety of conspicuous goods at a very high price. This result is not present in the previous literature and constitutes the main contribution of this thesis.

5 Extension of the Model: Cardinal Rank

So far, individuals were assumed to care about their relative position in the distribution of income. That is, what matters for individuals is whether they are perceived as being rich or poor, whatever the difference between their wealth and the wealth of others. However, status can be defined in different ways. In particular, as stated in the paper by **?**, people may take into account how much ahead or behind others they are. The absolute difference of income, also called cardinal rank, might matter.

In this section, we investigate how the distribution of income affects the demand for status goods, when people care only about the perception of their absolute income. This gives rise to an interest in cardinal rank. Apart from that, the model remains unchanged. As a consequence of the model's modification, instead of getting a payoff s associated to being perceived as rich and a payoff of zero associated to being perceived as poor, individuals get a payoff equal to the inference of their income by the third party. Formally, the utility becomes:

$$U_i = \gamma \frac{1}{\alpha} N_i^{\alpha} + N_i + \int_{j=0}^{V_i} c_{ij} dj + \pi_i, \tag{7}$$

where $\pi_i = E\left[y : \int_{j=0}^{V_i} p_j c_{ij} dj, \int_{j=0}^{V_{-i}} p_j c_{-ij} dj\right]$. The third party forms an average perception, π_i, about consumers' wealth, according to Bayes rule and prior knowledge of the distribution of income (**?**). We assume that poor individuals are endowed with a non-zero income, i.e. $y_P > 0$.

5.1 Exogenous Prices

As in the previous model, status is defined such that a higher status is granted to individuals with greater conspicuous consumption. It follows that the equilibrium is a separating equilibrium, in which individuals are correctly identified. More specifically, in the present setup, we have $\pi_P = y_P$ and $\pi_R = y_R$. Similarly to section 4.1, we first assume that prices are exogenously given. In section 4.1, we have solved the problem for $p_j = p \geq 1 \, \forall j$. In this section, for the sake of simplicity, we will focus on the case where the price of status goods and the price of normal goods are identical. Formally, $p_j = 1$, $\forall j$.

In the present case, where cardinal rank matters, Lemma 1 still holds. Because the third party makes the correct inference regarding the income of the poor, the latter have no incentive to increase their spending on status goods in order to mislead the third party. They thus maximize their direct utility and decide to consume no status goods. We get ${V_P}^* = 0$ and ${N_P}^* = y_P$.

The problem of the rich for $p_j = 1$, $\forall j$ becomes:

$$\begin{gathered} max_{V_R} \, U_R = \gamma \tfrac{1}{\alpha} (y_R - V_R)^{\alpha} + 2y_R \\ s.t. \\ \gamma \tfrac{1}{\alpha} {y_P}^{\alpha} + y_P = \gamma \tfrac{1}{\alpha} \left(y_P - V_R\right)^{\alpha} + y_R \end{gathered} \tag{8}$$

We then solve the incentive compatibility constraint of problem (8) for V_R.

LEMMA 5: *In any separating equilibrium,*

$$V_R{}^* = min\left\{ y_P - \left(y_P^\alpha + \frac{\alpha}{\gamma}\left(y_P - y_R\right)\right)^{\frac{1}{\alpha}}, y_P \right\}.$$

Again, the condition $pV_R{}^* \leq y_P$ must hold. As before, we will restrict our attention to the set of parameters for which this condition holds with strict inequality. From Lemma 5, it can be shown that the set of parameters is equal to:

$$y_R < y_P + \frac{\gamma}{\alpha} y_P{}^\alpha . \tag{9}$$

We now turn to the comparative statics of the variety of conspicuous goods purchased by the rich. For simplicity, we use $\alpha = \frac{1}{2}$.

PROPOSITION 7: *The equilibrium variety of status goods consumed by rich individuals, $V_R{}^*$, (a) increases with the reward of being perceived as being rich, y_R, (b) increases with y_P if y_P is low, such that $\frac{(3-\beta)\gamma + y_P{}^{\frac{1}{2}}}{(1-\beta)\gamma y_P{}^{-1} + y_P{}^{-\frac{1}{2}}} < y_R < y_P + 2\gamma y_P{}^{\frac{1}{2}}$, decreases with y_P, if y_P is high, such that $y_R < \frac{(3-\beta)\gamma + y_P{}^{\frac{1}{2}}}{(1-\beta)\gamma y_P{}^{-1} + y_P{}^{-\frac{1}{2}}}$, and (c) increases with β.*

Formally, result (b) can be written as:

$$\frac{dV_R{}^*}{dy_P} = \begin{cases} > 0 \quad if \quad y_R > \dfrac{(3-\beta)\gamma + y_P{}^{\frac{1}{2}}}{(1-\beta)\gamma y_P{}^{-1} + y_P{}^{-\frac{1}{2}}} \\ < 0 \quad if \quad y_R < \dfrac{(3-\beta)\gamma + y_P{}^{\frac{1}{2}}}{(1-\beta)\gamma y_P{}^{-1} + y_P{}^{-\frac{1}{2}}} \end{cases}$$

(Proof in Appendix, A11)

We have previously argued that the higher the reward associated to being perceived as rich, the greater the incentive of the poor to purchase status goods and the more rich individuals spend money on conspicuous goods. In the present case, it means that the higher the income of the rich, the larger the value of being considered rich relatively to being considered poor, and the greater the incentive of poor individuals to attempt to be perceived as rich. Therefore, rich individuals need to increase their spending on status goods to dissuade imitation by the poor.

Result (b) points out that two mechanisms come into play. On the one hand, if the income of the poor is low but still sufficiently large to ensure that rich individuals do not waste money (condition (9)), then the difference between the reward of being perceived as rich and the reward of being perceived as poor is large. Consequently, the poor have a greater incentive to spend a large amount of money on signaling, since the gain of misleading the third party and being perceived as wealthy is high. Formally, if $\frac{(3-\beta)\gamma+y_P^{\frac{1}{2}}}{(1-\beta)\gamma y_P{}^{-1}+y_P{}^{-\frac{1}{2}}} < y_R < y_P + 2\gamma y_P{}^{\frac{1}{2}}$, then an increase in the income of the poor is insufficient to catch up with the rich and therefore the incentive of poor individuals to make the third party believe they are rich is high. As a result, a rise in y_P leads to strong competition for status and rich individuals need to increase their purchase of status goods to dissuade imitation by the poor. On the other hand, if the income of the poor is large, such that, $y_R < \frac{(3-\beta)\gamma+y_P^{\frac{1}{2}}}{(1-\beta)\gamma y_P{}^{-1}+y_P{}^{-\frac{1}{2}}}$, the difference between status associated to being perceived as rich versus poor is small. It follows that the incentive of the poor to imitate the rich is lower than in the previous case. So, an increase in y_P allows the rich to decrease their purchase of status goods.

Result (c) differs from previous sections too. An increase in the numerosity of poor individuals in the population leads rich individuals to purchase a larger variety of status goods. This effect stems from the indirect effect of β on y_R. The reason of the increase in the variety of status goods demanded by the rich in response to an increase in the fraction of the poor is the higher status associated to being perceived as a rich individual. This mechanism has been explained in result (a).

We have seen that greater equality in the distribution of income increases competition for status and thus leads to a higher demand for status goods under ordinal ranking (**??**). However, the opposite may happen when people care about cardinal ranking and when the society is affluent. As an affluent society becomes more equal, i.e. either y_P increases or β falls, poor and rich individuals are closer to each other in terms of income. The value of being considered rich relatively to being considered poor gets smaller, and the incentive of the poor to imitate the rich is weakened. Therefore, rich individuals respond by an decrease in conspicuous expenditures. This result is in accordance with the paper by **?**, in which individuals care about their cardinal rank.

5.2 Endogenous Prices

As in section 4.2, three production processes are possible. We also consider solely the symmetric equilibrium in which all monopolists supply their status good at the same price. In the first case, the exogenous variety of status goods supplied by all monopolists is large. The solution to this problem is the same as in section 4.2.1. As mentioned previously, rich individuals prefer to signal their wealth by buying a large variety of status goods, rather than purchasing more expensive ones. Because the supply of status goods by monopolists is large, rich individuals can choose the variety that satisfies the incentive compatibility constraint with equality for any price offered by monopolists. As a result, a rise in the price of status goods by monopolists is unnecessary, as the incentive compatibility constraint is binding for all prices. It follows that monopolists' optimal choice is to maximize their profit. The large number of firms leads to a strong competition in prices and forces monopolists to set their price equal to the marginal cost of production. We are back to the situation where $p_j = 1$, $\forall j$ (section 5.1). In this section, we will investigate the two remaining cases in which either the variety supplied by all monopolists is small, or in which all the status goods are offered by a single monopolist.

5.2.1 Small Number of Monopolists Supplying Status Goods

In this section, the exogenous variety offered by all monopolists is small. For a given price set by monopolists, it is likely that rich individuals do not manage to purchase a large enough variety of status goods that makes poor individuals unwilling to imitate them. Monopolists can therefore raise their price and thus their profit up to the point where the incentive compatibility constraint is binding. This results in a situation of monopolistic competition. The problem of each monopolist for $p_j = p$, $\forall j$ is:

$$\begin{gathered} max_p\, \pi_j = \pi = (p-1)(1-\beta), \quad \forall j \\ s.t. \\ \gamma \tfrac{1}{\alpha} {y_P}^{\alpha} + 2y_P = \gamma \tfrac{1}{\alpha} \left(y_P - pV_R\right)^{\alpha} + (y_P - pV_R) + V_R + y_R. \end{gathered} \tag{10}$$

To simplify calculations, we set $\alpha = \frac{1}{2}$ and then solve the incentive compatibility constraint of problem (10) for p.

LEMMA 6: *In any separating equilibrium,*

$$p^* = min\left\{ \frac{-2\gamma^2 + V_R + y_R - 2\gamma {y_P}^{\frac{1}{2}} - y_P + 2\gamma\left(\gamma^2 - V_R - y_R + 2\gamma {y_P}^{\frac{1}{2}} + 2y_P\right)^{\frac{1}{2}}}{V_R}, \frac{y_P}{V_R} \right\}.$$

(Proof in Appendix, A12)

Again, we know from section 4.1 that a negative relationship exists between V_R and p. It follows that a small variety of status goods offered implies that the price of status goods is strictly greater than one. The latter result also applies to the extension of the model, since only the utility derived from status undergoes a change. As in previous sections, it is also important to note that condition $p^* V_R \leq y_P$ must hold. Again, we will restrict our attention to the set of parameters for which this condition holds with strict inequality. This condition is satisfied with strict inequality, if and only if:

$$2\gamma {y_P}^{\frac{1}{2}} + 2y_P > V_R + y_R \tag{11}$$

holds.

(Proof in Appendix, A13)

We are now ready to study the comparative statics of the price set by all monopolists.

PROPOSITION 8: *The equilibrium price of status goods set by all monopolists, p^*, (a) increases with the reward of being perceived as being rich, y_R, (b) increases with y_P if y_P is low, such that $\frac{(3-\beta)(1-\beta)\gamma^2+\gamma(6-4\beta){y_P}^{\frac{1}{2}}+2y_P}{(1-\beta)^2\gamma^2{y_P}^{-1}+2(1-\beta)\gamma{y_P}^{-\frac{1}{2}}+1} - V_R < y_R < 2\gamma{y_P}^{\frac{1}{2}} + 2y_P - V_R$, decreases with y_P if y_P is large, such that $y_R < \frac{(3-\beta)(1-\beta)\gamma^2+\gamma(6-4\beta){y_P}^{\frac{1}{2}}+2y_P}{(1-\beta)^2\gamma^2{y_P}^{-1}+2(1-\beta)\gamma{y_P}^{-\frac{1}{2}}+1} - V_R$, and (c) increases with β.*

Formally, result (b) can be written as:

$$\frac{dp^*}{dy_P} = \begin{cases} > 0 \quad if \quad y_R > \dfrac{(3-\beta)(1-\beta)\gamma^2+\gamma(6-4\beta){y_P}^{\frac{1}{2}}+2y_P}{(1-\beta)^2\gamma^2{y_P}^{-1}+2(1-\beta)\gamma{y_P}^{-\frac{1}{2}}+1} - V_R \\ < 0 \quad if \quad y_R < \dfrac{(3-\beta)(1-\beta)\gamma^2+\gamma(6-4\beta){y_P}^{\frac{1}{2}}+2y_P}{(1-\beta)^2\gamma^2{y_P}^{-1}+2(1-\beta)\gamma{y_P}^{-\frac{1}{2}}+1} - V_R \end{cases}$$

(Proof in Appendix, A14)

Again, we observe exactly the same dynamics as for ${V_R}^*$, when prices are exogenous. Result (a) in the above proposition shows that a higher status associated to being perceived as rich gives poor individuals a greater incentive to indulge in conspicuous consumption. Therefore, rich individuals must spend a larger amount of money for each available status good in order to end up in a separating equilibrium and monopolists can increase their profit by raising their price.

Depending on whether the absolute difference between the status associated to being perceived as rich and the status associated to being perceived as poor is below or above a certain threshold, the behaviour of the rich in response to an increase in the income of the poor changes. The intuition is exactly the same as in section 5.1. However, in the present case, rich individuals cannot buy the variety of status goods that would be optimal given the price set by all monopolists, since monopolists supply just a small variety of conspicuous goods. Rich individuals buy all the status goods offered and the monopolists can increase their profit by increasing their prices, up to the point where the incentive compatibility constraint holds with equality. Thus, in this case, the price set by the monopolists adjusts in response to a change in the income of the poor. If the poor are very poor, the prize for competition in status is very large. As a consequence, an increase in the revenue of the poor does not decrease the difference between both incomes by a large enough amount to allow the poor to catch up with the rich. Therefore, it does not decrease competition for status. On the contrary, because the reward of being perceived as rich versus poor is large, an increase in poor individuals' income strengthens the competition for status and forces the rich to pay a higher price for status goods in order to dissuade imitation by the poor. In this situation, monopolists can increase their prices. If instead the poor are rich enough, such that the prize for competition in status is small, the incentive of the rich to differentiate themselves from the poor and the incentive of the poor to catch up with the rich are lower, as the income of the poor rises. Consequently, an increase in the income of the poor implies a fall in competition for status and a decrease in the willingness of the rich to invest in signaling. As a result, the rich are not willing to pay a price as large as before and monopolists must lower their price.

The intuition for result (c) is the same as in section 5.1. The increase in prices set by monopolists as a consequence of an increase in the numerosity of poor individuals stems from the increase in the "prize" of being perceived as rich.

As in section 4, we find similar mechanisms where the price is exogenous and where the price is endogenous with a small variety supplied by all monopolists. However, in the latter case, p adjusts to changes instead of V_R. Therefore, the same comparisons with the papers by **?** as in section 5.1 can be made regarding the effect of a change in the income distribution on the demand for status goods (p here).

5.2.2 One Monopolist

We now turn to the case where a single monopolist supplies all the status goods. The problem of the single monopolist is:

$$\begin{aligned} max_{V_R}\, \pi &= V_R\left(p(V_R)-1\right)(1-\beta) \\ &s.t. \\ \gamma\tfrac{1}{\alpha}y_P{}^{\alpha}+2y_P &= \gamma\tfrac{1}{\alpha}\left(y_P-pV_R\right)^{\alpha}+\left(y_P-pV_R\right)+V_R+y_R. \end{aligned} \tag{12}$$

As stated earlier, the optimal strategy for the single monopolist is to offer a very small variety of conspicuous goods at a very high price. In fact, in the extension of the model, the specification of the utility function is still such that normal goods are preferred over status goods. Rich individuals consume status goods only for their signaling characteristic. Consequently, supplying a limited part of status goods is the best strategy for the monopolist. The rich are willing to consume a variety of status goods that just allows them to differentiate themselves from the poor. Once this goal is achieved, they spend their residual income on normal goods, whose consumption makes them happier. We can calculate the maximum profit in this case.

LEMMA 7: *The maximum profit is equal to:*

$$\pi_{max} = \left(-2\gamma^2 + y_R - 2\gamma y_P{}^{\frac{1}{2}} - y_P + 2\gamma\left(\gamma^2 - y_R + 2\gamma y_P{}^{\frac{1}{2}} + 2y_P\right)^{\frac{1}{2}}\right)(1-\beta).$$

(Proof in Appendix, A15)

We are now interested in analysing how the maximum profit, i.e. the limit of the profit when the variety approaches zero, reacts to a change in the income of the poor. From the results of the previous sections, we expect the profit to first increase in the income of the poor but beyond a given threshold of that income, the trend would be reversed.

PROPOSITION 9: *If y_P is low, such that $\frac{(3-\beta)(1-\beta)\gamma^2+\gamma(6-4\beta)y_P{}^{\frac{1}{2}}+2y_P}{(1-\beta)^2\gamma^2 y_P{}^{-1}+2(1-\beta)\gamma y_P{}^{-\frac{1}{2}}+1} < y_R < 2\gamma y_P{}^{\frac{1}{2}} + 2y_P$, then the maximum profit of the single monopolist, π_{max}, increases with the income of the poor, y_P. If y_P is large, such that $y_R < \frac{(3-\beta)(1-\beta)\gamma^2+\gamma(6-4\beta)y_P{}^{\frac{1}{2}}+2y_P}{(1-\beta)^2\gamma^2 y_P{}^{-1}+2(1-\beta)\gamma y_P{}^{-\frac{1}{2}}+1}$, then the maximum profit of the single monopolist, π_{max}, decreases with the income of the poor, y_P.*

Formally, we have:

$$\frac{d\pi}{dy_P} = \begin{cases} > 0 \quad if \quad y_R > \dfrac{(3-\beta)(1-\beta)\gamma^2 + \gamma(6-4\beta){y_P}^{\frac{1}{2}} + 2y_P}{(1-\beta)^2\gamma^2{y_P}^{-1} + 2(1-\beta)\gamma{y_P}^{-\frac{1}{2}} + 1} \\ < 0 \quad if \quad y_R < \dfrac{(3-\beta)(1-\beta)\gamma^2 + \gamma(6-4\beta){y_P}^{\frac{1}{2}} + 2y_P}{(1-\beta)^2\gamma^2{y_P}^{-1} + 2(1-\beta)\gamma{y_P}^{-\frac{1}{2}} + 1} \end{cases}$$

(Proof in Appendix, A16)

We observe that the condition under which the profit of the single monopolist is increasing (decreasing) with the income of the poor is the same as the condition under which the price is increasing (decreasing) with the income of the poor, for $V_R = 0$. This result is therefore convincing. The interpretation is identical to the interpretation of result (b) in Proposition 8.

We have already argued that the result concerning the optimal strategy of the single monopolist is new. Moreover, the response of the maximum profit to a change in the income distribution is consistent with the behaviour of the price in response to a change in poor individuals' income. Therefore, the intuition of this result is consistent with the intuition in the paper by **?** where the case of a single monopolist is not addressed.

5.3 Redistribution

We have seen that when individuals care about their cardinal rank, a rise in the income of the poor can lead to a decrease in the spending on conspicuous goods by the rich, if the poor are rich enough. This result suggests that if the poor are endowed with an income exceeding a certain threshold, an increase in the latter may decrease the distortion in consumption, and thus may have a welfare enhancing effect on both, the rich and the poor. In this section, we investigate the condition(s) under which a balanced-budget redistribution from the rich to the poor makes the poor and the rich better off, when all individuals care about the perception of their absolute income. Formally, a balanced-budget redistribution is defined as: $\beta \Delta y_P + (1-\beta)\,\Delta y_R = 0$. In this section, we will address two cases. In the first case, the price of all goods is exogenous and equal to one. In the second case, a single monopolist supplies all the status goods. It is sufficient to investigate these two cases, since the mechanisms when all prices are equal to one are similar to all other cases except the case of the single monopolist. More specifically, when prices are exogenous or endogenous, ${V_R}^*$ and

p^* behave the same in response to a change in the income of the poor y_P. The only difference is whether the variety or the price adjusts in response to the change.

5.3.1 Exogenous Prices

Let us begin with the first case. The purpose of this section is to find the condition(s) under which poor and rich individuals are made better off by redistribution. To do so, we need to calculate the derivative of their utility with respect to y_P. Note that the present analysis is a special case of the analysis by **?**.

From section 5.1, we know that $V_P{}^* = 0$ and $V_R{}^* = y_P - \left(y_P{}^\alpha + \frac{\alpha}{\gamma}(y_P - y_R)\right)^{\frac{1}{\alpha}}$ with $y_R < y_P + \frac{\gamma}{\alpha} y_P{}^\alpha$. We insert the optimal varieties of status goods consumed by the poor and the rich into the utility function to find their respective utility. We obtain:

$$\begin{aligned} U_P &= \gamma \frac{1}{\alpha} y_P{}^\alpha + 2y_P && (13) \\ U_R &= \gamma \frac{1}{\alpha}\left(y_R - y_P + \left(y_P{}^\alpha + \frac{\alpha}{\gamma}(y_P - y_R)\right)^{\frac{1}{\alpha}}\right)^\alpha + y_R - y_P \\ &+ \left(y_P{}^\alpha + \frac{\alpha}{\gamma}(y_P - y_R)\right)^{\frac{1}{\alpha}} + y_P - \left(y_P{}^\alpha + \frac{\alpha}{\gamma}(y_P - y_R)\right)^{\frac{1}{\alpha}} \\ &+ y_R && (14) \end{aligned}$$

Taking the derivative of the utility of the poor with respect to their income, we find:

$$\frac{dU_P}{y_P} = \gamma y_P{}^{\alpha-1} + 2 > 0 \qquad (15)$$

Because $\gamma > 0$, and $y_P > 0$, we find that a redistribution of income from the rich to the poor always makes the poor better off.

We now calculate the derivative of the utility function of the rich with respect to the income of the poor. It can be shown that it is equal to:

$$
\begin{aligned}
\frac{dU_R}{dy_P} &= -\frac{\beta}{1-\beta}\left\{\gamma\left(y_R - y_P + \left({y_P}^{\alpha} + \frac{\alpha}{\gamma}\left(y_P - y_R\right)\right)^{\frac{1}{\alpha}}\right)^{\alpha-1} + 1\right\} \\
&+ \left\{\gamma\left(y_R - y_P + \left({y_P}^{\alpha} + \frac{\alpha}{\gamma}\left(y_P - y_R\right)\right)^{\frac{1}{\alpha}}\right)^{\alpha-1}\right\} \\
&\times \left\{\frac{\gamma {y_P}^{\alpha-1} - \gamma\left({y_P}^{\alpha} + \frac{\alpha}{\gamma}\left(y_P - y_R\right)\right)^{\frac{\alpha-1}{\alpha}} + 1 + \frac{\beta}{1-\beta}}{\gamma\left({y_P}^{\alpha} + \frac{\alpha}{\gamma}\left(y_P - y_R\right)\right)^{\frac{\alpha-1}{\alpha}}}\right\} - \frac{\beta}{1-\beta}
\end{aligned}
\tag{16}
$$

We are now able to investigate under what circumstances $\frac{dU_R}{dy_P}$ is likely to be positive. First, let us analyse how status should depend on the income levels to make equation (16) likely to be positive. We know that $\frac{dH}{dy_P} = \frac{dy_R}{dy_P} = -\frac{\beta}{1-\beta}$ and that $\frac{dL}{dy_P} = \frac{dy_P}{dy_P} = 1$. From the last two lines of equation (16), we observe that $\frac{dH}{dy_P} = -\frac{\beta}{1-\beta}$ is multiplied by:

$$
\left(\frac{\gamma\left(y_R - y_P + \left({y_P}^{\alpha} + \frac{\alpha}{\gamma}\left(y_P - y_R\right)\right)^{\frac{1}{\alpha}}\right)^{\alpha-1}}{\gamma\left({y_P}^{\alpha} + \frac{\alpha}{\gamma}\left(y_P - y_R\right)\right)^{\frac{\alpha-1}{\alpha}}} - 1\right).
$$

Because $y_R - y_P > 0$ and $\alpha \in (0,1)$, the latter expression is smaller than zero. It follows that $H = y_R$ needs to be insensitive to a change in y_P. In other words, the closer $\frac{dH}{dy_P} = -\frac{\beta}{1-\beta}$ is to zero, the more likely it is to make the rich better off. As a result, β should be close to zero. In fact, if there are only a few poor people but many rich people in the population, a small amount of money taken from the rich is sufficient to finance the redistributive policy. As a result, rich individuals do not see their income fall by a large amount. Second, we observe that β also enters equation (16) in the first line. This term corresponds to $\frac{dy_R}{dy_P}$. Note that β affects the utility of the rich in two ways. On the one hand, as mentioned before, an increase in the numerosity of the poor requires a greater reduction in the income of the rich in order to finance the redistributive policy. On the other hand, an increase in the fraction of the poor increases the endowment of income for every rich individual, for a fixed total income. It follows that the status assigned to rich individuals increases. However, we know from condition (9) that $\left\{\gamma\left(y_R - y_P + \left({y_P}^{\alpha} + \frac{\alpha}{\gamma}\left(y_P - y_R\right)\right)^{\frac{1}{\alpha}}\right)^{\alpha-1} + 1\right\}$ is larger than zero. The overall effect of β, i.e. its effect on $\frac{dH}{dy_P}$ in addition to its effect on $\frac{dy_R}{dy_P}$, is therefore negative.

We now turn to the shape of the utility function. We notice that the term $\gamma {y_P}^{\alpha-1}$ $-\gamma \left({y_P}^{\alpha} + \frac{\alpha}{\gamma}\left(y_P - y_R\right)\right)^{\frac{\alpha-1}{\alpha}}$ is smaller than zero. Therefore, the closer to zero it is, the more likely it is to improve the welfare of the rich. This can be achieved when y_P and y_R are close to each other. It means that the status associated to being perceived as poor should be close to the status associated to being perceived as rich. If this is the case, the poor are not willing to spend a large amount of money on signaling, since the gain on being perceived as rich compared to the gain on being perceived as poor is low. Similarly, rich individuals do not fear the likelihood of being perceived as poor. Consequently, the competition for status is low and rich individuals decrease their consumption of status goods ($\frac{d{V_R}^*}{dy_P} < 0$, if y_P is large). Another possibility of making the above expression close to zero is to have a very flat utility function. Again, this can be achieved when y_P is very large, i.e. when the society is affluent. In such a society, the cost of renouncing to normal goods in order to spend more money on signaling is low for both, the rich and the poor.

In summary, rich individuals experience a fall in their income, and consequently of their status assigned by the third party following the redistribution. However, their utility decreases only slightly, since the marginal utility from normal goods is small in an affluent society. At the same time, rich individuals decrease their spending on status goods in response to the rise in the wealth of the poor. This leads to an increase in their utility. The second effect is larger than the first one. In fact, the incentive compatibility constraint must be satisfied. It means that the left-hand side, i.e. the utility of the poor when they consume no status goods, must be equal to the right-hand side, i.e their utility when they consume the optimal consumption basket of the rich. On the left-hand side of the incentive compatibility constraint, the increase in y_P leads to an increase in the purchase of normal goods and in the status. Consequently, poor individuals are better off. On the right-hand side of the incentive compatibility constraint, the variety of status goods consumed by the rich decreases and more normal goods are purchased, increasing the utility. Moreover, the decrease in y_R drives the status of the individuals perceived as being rich down. However, because the incentive compatibility constraint must hold with equality, the first effect, i.e. the increase in utility due to the fall in the spending on signaling, dominates. We thus may conclude that it is possible to make the rich better off.

By this time, we have not depicted the exact conditions under which redistribution improves the welfare of rich individuals yet. To simplify the analysis, we address special cases, in which the redistribution from the rich to the poor can improve the welfare of both types of individuals. Let us keep in mind that the welfare of the poor is enhanced in any case.

PROPOSITION 10: *If* $\beta = 0$ *and* $y_R < \frac{3\gamma + y_P^{\frac{1}{2}}}{\gamma y_P^{-1} + y_P^{-\frac{1}{2}}}$ *or if* $y_P = y_R$ *and* $y_P > \left(\frac{\beta\gamma}{1-2\beta}\right)^2$, *the balanced-budget redistribution from the rich to the poor makes both the poor and the rich better off.*

(Proof in Appendix, A17)

From the above interpretation, we know that a small proportion of poor individuals will increase the likelihood of making the rich better off. It leads us to try the special case $\beta = 0$. Another circumstance under which it is likely to enhance the welfare of rich individuals is when the status assigned by the third party to rich individuals and to poor individuals are close to each other. This justifies a special case in which $y_R = y_P$.

For the case where $\beta = 0$, the condition under which rich individuals are made better off, corresponds exactly to the condition under which the variety of status goods consumed by the rich decreases with the income of the poor for $\beta = 0$ (Result (b) in Proposition 7). It follows that if the proportion of poor individuals in the population approaches zero, and if the income of the poor is large enough such that rich individuals decrease their spending on status goods as the redistribution of income occurs, the redistribution of income improves the welfare of rich individuals. In the case where $y_P = y_R$, the necessary condition for redistribution to improve the welfare of the rich is more restrictive than the condition under which rich individuals decrease their conspicuous expenditures in response to a rise in the income of the poor. In fact, the latter condition is reduced to $y_P > 0$ for $y_P = y_R$.

? investigate the circumstances under which it is likely to make rich individuals better off. The direction taken by the parameters should be the same as in the present study, however, my model is a specific case of their paper.

5.3.2 One Monopolist

Let us now turn to the second case, where all the status goods are supplied by the same monopolist. We already know that the strategy maximizing its profit is to offer a small variety of status goods at a very high price. In other words, rich individuals pay a fix amount of money to the monopolist and spend all their residual income on normal goods. We denote the fix amount of money paid by rich individuals to the single monopolist by F. From the maximum profit of the single monopolist in Lemma 7, we deduce that:

$$\begin{aligned} F &= \frac{\pi_{max}}{1-\beta} \\ &= -2\gamma^2 + y_R - 2\gamma y_P{}^{\frac{1}{2}} - y_P + 2\gamma\left(\gamma^2 - y_R + 2\gamma y_P{}^{\frac{1}{2}} + 2y_P\right)^{\frac{1}{2}}. \end{aligned}$$

We are interested in investigating, whether the redistribution of income from rich to poor individuals can improve their welfare. That is, we want to calculate whether the utility of the poor and the rich depends positively on a change in y_P.

Note that the utility of poor individuals remains unchanged compared to the first case. Therefore, we know that redistribution enhances their welfare. We are now ready to address the circumstances, under which the rich can be made better off. We start by calculating the utility of a rich individual when $\alpha = \frac{1}{2}$, and get:

$$\begin{aligned} U_R &= 2\gamma\left\{2\gamma^2 + y_P + 2\gamma y_P{}^{\frac{1}{2}} - 2\gamma\left(\gamma^2 - y_R + 2y_P + 2\gamma y_P{}^{\frac{1}{2}}\right)^{\frac{1}{2}}\right\}^{\frac{1}{2}} + 2\gamma^2 \\ &+ y_P + 2\gamma y_P{}^{\frac{1}{2}} - 2\gamma\left(\gamma^2 - y_R + 2y_P + 2\gamma y_P{}^{\frac{1}{2}}\right)^{\frac{1}{2}} + y_R \end{aligned}$$

It can be shown that the derivative of U_R with respect to y_P is equal to:

$$\begin{aligned} \frac{dU_R}{dy_P} &= \gamma\left\{2\gamma^2 + y_P + 2\gamma y_P{}^{\frac{1}{2}} - 2\gamma\left(\gamma^2 - y_R + 2y_P + 2\gamma y_P{}^{\frac{1}{2}}\right)^{\frac{1}{2}}\right\}^{-\frac{1}{2}} \\ &\times \left\{1 + \gamma y_P{}^{-\frac{1}{2}} - \gamma\left(\gamma^2 - y_R + 2y_P + 2\gamma y_P{}^{\frac{1}{2}}\right)^{-\frac{1}{2}}\left(\frac{\beta}{1-\beta} + 2 + \gamma y_P{}^{-\frac{1}{2}}\right)\right\} \\ &+ 1 + \gamma y_P{}^{-\frac{1}{2}} - \gamma\left(\gamma^2 - y_R + 2y_P + 2\gamma y_P{}^{\frac{1}{2}}\right)^{-\frac{1}{2}}\left(\frac{\beta}{1-\beta} + 2 + \gamma y_P{}^{-\frac{1}{2}}\right) \\ &- \frac{\beta}{1-\beta} \end{aligned} \tag{17}$$

Again, we try to find special cases for which equation (17) is positive.

PROPOSITION 11: *If* $\beta = 0$ *and* $y_R < \frac{3\gamma^2 + 6\gamma y_P{}^{\frac{1}{2}} + 2y_P}{\gamma^2 y_P{}^{-1} + 2\gamma y_P{}^{-\frac{1}{2}} + 1}$, *the redistribution improves the welfare of rich individuals.*

(Proof in Appendix, A18)

We find that the condition that must hold besides $\beta = 0$ is equal to the condition, under which p^* decreases with y_P for $V_R = 0$. Therefore, it corresponds to the case in which the profit of the monopolist decreases with the income of poor individuals. It means that redistribution improves the welfare of both rich and poor individuals at the expense of the welfare of the monopolist. It seems convincing that not everybody can benefit from the redistribution of income. Moreover, it can be shown that the condition that must hold besides $\beta = 0$ is more restrictive in the case of exogenous prices than in the case of a single monopolist. It means that it is easier to make rich individuals better off in the present case. In fact, rich individuals pay a fix amount of money corresponding the a small variety of status goods purchased at a very high price to the monopolist to be correctly identified. It follows that the decrease in the small conspicuous consumption following the redistribution of income increases the utility of the rich considerably. It is easy to improve their welfare by preventing signaling.

? have not investigated the case of the single monopolist. Therefore, the latter result showing that a redistribution of income from the rich to the poor can make both classes better off more easily in the case where all the status goods are supplied by a single monopolist is new and constitutes the main contribution of this thesis.

We have seen that preventing competition in the market of status goods reduces distortion in consumption. Moreover, it has been shown that when individuals care about their cardinal rank in the income distribution, it is possible to implement a redistribution policy that improves the welfare of both the poor and the rich. But the conditions under which the redistribution policy achieves higher individuals' welfare are softer if the status goods are supplied by a single monopolist. Again, preventing competition in the market of status goods is good for the welfare of the individuals.

6 Summary of Results

In the present thesis we have investigated the effect of inequality on the demand for status goods under different pricing methods. In this model, the population studied consists of two groups of individuals, the poor and the rich. They are assumed to care about their consumption of normal and status goods and, in addition, about status. Consequently, individuals' decisions are strategic and individuals seek to signal their wealth through the purchase of status goods. Furthermore, individuals are assumed to prefer normal goods over status goods at each consumption level. It follows that in equilibrium, only the rich purchase status goods in order to differentiate themselves from the poor. It is important to note that, if status goods are available in any variety and at any price, individuals prefer to signal their wealth through the variety of status they purchase rather than through the price.

First, the above relationship is addressed when prices of status goods are exogenous and when status goods are available in an unrestricted variety. It is important to note that the effect of inequality on rich individuals' demand for status goods depends strongly on how we define status. When individuals care about whether they are behind or ahead others, no matter how far they are from them, they are told to care about their ordinal rank. In this case, an increase in equality resulting from an increase in the income of the poor increases the competition for status, since it makes it easier for the poor to imitate the rich. Therefore, in order to dissuade imitation by the poor, the rich must purchase a larger variety of status goods. However, when individuals care about how much ahead or behind others they are, i.e. about their cardinal rank, an increase in the income of the poor may have two effects. If the poor are endowed with a very low income, such that the difference between the status associated to being perceived as poor and the status associated to being perceived as rich is large, a rise in the income of the poor has the same effect as in the previous case. If instead, the poor are rich enough, such that the difference between the status of the rich and the status of the poor is low, an increase in the income of the poor decreases the incentive of the poor to imitate the rich. As a result, the rich can decrease their spending on conspicuous goods and still be perceived as rich.

If equality increases following a fall in the frequency of poor individuals in the population, the rich will not adjust their demand for status goods if all individuals care about their ordinal rank, whereas they will decrease their spending on status goods if all individuals care about their cardinal rank. The latter result stems from the decrease in the reward of being perceived as rich involving a lower incentive for the poor to imitate the rich. Therefore, rich individuals need to spend a lower amount of money on status goods to distinguish themselves.

The relationship between inequality and demand is also addressed when prices are endogenously determined by firms and when the supply of status goods is fixed. It has been found that the same mechanisms as in the case in which prices are exogenous take place, in both cases when individuals care about their ordinal and cardinal rank. If the variety supplied by all monopolists is small, the change in the demand arises through an adjustment of the price. If the rich cannot purchase a large enough variety of status goods to dissuade imitation by the poor, the rich are forced to pay a higher price for those goods to display their wealth. Therefore, monopolists can extract profit by charging a higher price for their status good. But the response of the price to a change in inequality is identical to the above-described response of the variety of status goods to such a change.

The most surprising and interesting result of the thesis is that when a single monopolist supplies all the status goods, its optimal strategy is to supply a very small variety of those goods at a very high price. This holds for both, concerns for ordinal and cardinal rank. It stems from the fact that nobody likes status goods as such. Therefore, the rich purchase conspicuous goods only for signaling purposes. It indicates that rich individuals just have to pay a fix amount of money to the monopolist in order to be considered as rich and spend their residual income on normal goods, which provide a greater utility than status goods. It is worth emphasizing that the single monopolist does not promote distortion in consumption. This result suggests important policy implications. First, preventing competition in status good markets would considerably decrease distortion in consumption. Second, it suggests that innovation in those markets is bad for social welfare.

It has been found that an increase in equality following a rise in the income of the poor leads the rich to decrease their spending on status goods when individuals care about their cardinal rank and live in an affluent society. This suggests that a redistribution of income from the rich to the poor may improve social welfare. In fact, we have seen that under certain circumstances, redistribution can make both the poor and the rich better off. In particular, it has been shown that it is easier to improve the welfare of all individuals in the case where one monopolist supplies all the status goods than in other cases. Of course, this occurs at the expense of the monopolist.

7 Conclusion

In this thesis, we have investigated the relationship between inequality and demand for status goods under different pricing methods. In a first step, several models of conspicuous consumption have been analysed to help modelling the utility function, to understand the consequences of different underlying assumptions and to get the intuition regarding the effect of inequality on demand. In a second step, a model has been developed to analyse how the relationship between inequality and demand is affected by the way of pricing status goods.

When status is defined as the ordinal rank in the distribution of income, an increase in equality following an increase in the income of the poor leads the rich to increase their demand for status goods in order to dissuade imitation by the poor. It follows that greater equality within the society resulting from an increase in the income of the poor fosters the distortion in consumption. However, when status is defined as the cardinal rank in the distribution of income, a rise in the income of the poor may allow the rich to decrease their spending on conspicuous goods and still be perceived as rich. This holds if the poor are rich enough, such that the difference between the status of the rich and the status of the poor is low. In sum, if society is affluent, an increase in equality following an increase in the income of the poor may reduce distortion in consumption. These results hold for all pricing methods except for the case of a single monopolist. However, depending on how status goods are priced, the adjustment in demand can either arise through an adjustment in the variety of status goods or through an adjustment in their price.

The most surprising and interesting result of the thesis is that when a single monopolist supplies all the status goods, its optimal strategy is to supply a very small variety of those goods at a very high price. Therefore, it is easier to find a redistribution of income that makes the rich and the poor better off in the case of a single monopolist than in other cases. This suggests that the single monopolist does not promote distortion in consumption. It is important to note that this result stems from the assumptions that normal goods are strictly preferred over status goods, that there exist only two groups of individuals, the rich and the poor, and that individuals care about their status either defined as their ordinal rank or their cardinal rank in the distribution of income. Indeed, the first assumption together with the second assumption implies that only rich individuals consume status goods and that they purchase them solely to signal their wealth. Therefore, the single monopolist cannot increase profits by increasing the variety of status goods it supplies, if a small variety of status goods is sufficient for the rich to be correctly identified. Note that the assumption that normal goods are always preferred over status goods may be a little bit extreme. For example, it seems more realistic that beyond a certain level of consumption of normal goods, rich individuals value an additional watch more than additional toothpaste. Hence, further study may use preferences exhibiting a larger marginal utility of status goods than normal goods beyond a certain consumption level of normal goods. It may also be interesting to introduce the middle-class into the model and analyse how the optimal strategy of the single monopolist changes. The third assumption implies that the status-bearing object, i.e. the object whose distribution in the population is thought to grant status, is the individuals' income. However, the ways used by consumers to display their status evolves continuously. If in the 1950s individuals sought to keep up with their neighbours by purchasing as many goods as them, today's consumers rather seek to flaunt their hipness or virtue. Markets are shaped to offer products allowing individuals to express their personalities, their willingness to be perceived as concerned about world's problems. Furthermore, today's consumers increasingly realize that time is scarce, whereas material possessions are abundant. Consequently, individuals seek to boast about their activities in leisure time and companies try to supply experiences for them in addition to material goods (**?**). This indicates that depending on the object thought to grant status, the result to the above problem may differ. For instance, we may take into account that individuals care about differentiating themselves not only in terms of income but also that they strive to purchase products that reflect their personality. In this case, the strategy of the single monopolist consisting in supplying a small variety of status goods would not be optimal. Individuals would be better off if a large variety of status goods was supplied and the monopolist could increase its profit by supplying a larger variety of status goods. It is therefore important to keep in mind that the present thesis solely accounts for status defined as the perception of one's relative or absolute income, since it aims to address the impact of income inequality on demand.

Furthermore, it has been argued earlier that even if concerns for status can be interpreted as a vicious character trait, it must be taken into account when thinking about the implementation of policies. However, policies aiming to decrease the distortion in consumption due to social comparisons should be well-considered. Indeed, we have observed that the effect of inequality on the demand for status goods depends strongly on the definition of status. This demonstrates how important it is, to deeply analyse how the population of interest cares about status before designing and implementing policies that aim to decrease distortion in consumption and improve social welfare.

This thesis leaves scope for future research. As mentioned earlier, it would be interesting to introduce the middle class into this model. We expect the effect of inequality on demand and the supply of the single monopolist to change if the population consists of three groups of individuals instead of two, since the competition between groups would be amended.

8 Appendix

A1. Proof of Lemma 2

Proof. We plug $\alpha = \frac{1}{2}$ into the incentive compatibility constraint of problem (2), and get:

$$2\gamma {y_P}^{\frac{1}{2}} + y_P = 2\gamma \left(y_P - pV_R\right)^{\frac{1}{2}} + \left(y_P - pV_R\right) + V_R + s$$

Let us define $x \equiv (y_P - pV_R)^{\frac{1}{2}}$ and $x^2 \equiv (y_P - pV_R)$, with $V_R \equiv \frac{y_P - x^2}{p}$. Then, we substitute x and x^2 into the incentive compatibility constraint, put all the terms on the right-hand side, and obtain the following quadratic equation:

$$\left(\frac{p-1}{p}\right) x^2 + 2\gamma x + s - 2\gamma {y_P}^{\frac{1}{2}} - \left(\frac{p-1}{p}\right) y_P = 0$$

Solving for x, we get:

$$x = \frac{-2\gamma \pm \sqrt{4\gamma^2 - 4\left(\frac{p-1}{p}\right)\left(s - 2\gamma {y_P}^{\frac{1}{2}} - \frac{p-1}{p} y_P\right)}}{2\frac{p-1}{p}}$$

We know that $p{V_R}^* \leq y_P$. This implies that $x \equiv (y_P - p{V_R}^*)^{\frac{1}{2}} \geq 0$, and thus $x^2 \equiv (y_P - p{V_R}^*) \geq 0$. Therefore, we choose the solution for x that makes it larger than or equal to zero. After simplification, we get:

$$x = \left(\frac{p}{p-1}\right)\left(-\gamma + \left\{\gamma^2 - \left(\frac{p-1}{p}\right)\left(s - 2\gamma {y_P}^{\frac{1}{2}} - \frac{p-1}{p} y_P\right)\right\}^{\frac{1}{2}}\right)$$

To find ${V_R}^*$, we need to calculate x^2:

$$\begin{aligned} x^2 &= \left(\frac{p}{p-1}\right)^2 \left(2\gamma^2 - 2\gamma\left\{\gamma^2 - \left(\frac{p-1}{p}\right)\left(s - 2\gamma {y_P}^{\frac{1}{2}} - \frac{p-1}{p} y_P\right)\right\}^{\frac{1}{2}}\right) \\ &- \left(\frac{p}{p-1}\right)^2 \left(\left(\frac{p-1}{p}\right)\left(s - 2\gamma {y_P}^{\frac{1}{2}} - \frac{p-1}{p} y_P\right)\right) \end{aligned}$$

Using ${V_R}^* = \frac{y_P - x^2}{p}$, we finally get:

$$\begin{aligned} {V_R}^* &= \frac{y_P}{p} - \frac{p}{(p-1)^2}\left(2\gamma^2 - 2\gamma\left\{\gamma^2 - \frac{p-1}{p}\left(s - 2\gamma {y_P}^{\frac{1}{2}} - \frac{p-1}{p}y_P\right)\right\}^{\frac{1}{2}}\right) \\ &+ \frac{p}{(p-1)^2}\left(\left(\frac{p-1}{p}\right)\left(s - 2\gamma {y_P}^{\frac{1}{2}} - \frac{p-1}{p}y_P\right)\right) \end{aligned}$$

□

A2. Proof of condition (3)

Proof.

$$x \equiv (y_P - p{V_R}^*)^{\frac{1}{2}} \;=\; \left(\frac{p}{p-1}\right)\left(-\gamma + \left\{\gamma^2 - \left(\frac{p-1}{p}\right)\left(s - 2\gamma {y_P}^{\frac{1}{2}} - \frac{p-1}{p}y_P\right)\right\}^{\frac{1}{2}}\right) > 0$$

Because $p \geq 1$, we can divide the above equation by $\left(\frac{p}{p-1}\right)$ and keep the same sign of inequality:

$$\begin{aligned} &\left\{\gamma^2 - \left(\frac{p-1}{p}\right)\left(s - 2\gamma {y_P}^{\frac{1}{2}} - \frac{p-1}{p}y_P\right)\right\}^{\frac{1}{2}} > \gamma \\ \Leftrightarrow\;& \gamma^2 - \left(\frac{p-1}{p}\right)\left(s - 2\gamma {y_P}^{\frac{1}{2}} - \frac{p-1}{p}y_P\right) > \gamma^2 \end{aligned}$$

We then substract γ^2 on both sides, again divide the above expression by $\left(\frac{p}{p-1}\right)$, and get:

$$2\gamma {y_P}^{\frac{1}{2}} + \frac{p-1}{p}y_P > s$$

□

A3. Proof of Proposition 1

Proof of result (a)

Proof. In order to find the derivative of ${V_R}^*$ with respect to s, we use the implicit function theorem which says that:

If $Q(x,y) = 0$, the derivative of the implicit function $y(x)$ with respect to x is given by:

$$\frac{dy}{dx} = -\frac{\partial Q/\partial x}{\partial Q/\partial y}$$

In this case, $Q(s, V_R{}^*) = 2\gamma \left(y_P - pV_R{}^*\right)^{\frac{1}{2}} - pV_R{}^* + V_R{}^* + s - 2\gamma y_P{}^{\frac{1}{2}} = 0$. We thus get:

$$\frac{dV_R{}^*}{ds} = -\frac{\partial Q/\partial s}{\partial Q/\partial V_R{}^*} = \frac{1}{\gamma \left(y_P - pV_R{}^*\right)^{-\frac{1}{2}} p + p - 1} > 0$$

The above derivative is positive, if and only if $\left(y_P - pV_R{}^*\right)^{-\frac{1}{2}} > 0$, given that $p \geq 1$. Since condition (3) must hold, the above derivative is positive. □

Proof of result (b)

Proof. We use the implicit function theorem to find the derivative of $V_R{}^*$ with respect to y_P, where $Q(y_P, V_R{}^*) = 2\gamma \left(y_P - pV_R{}^*\right)^{\frac{1}{2}} - pV_R{}^* + V_R{}^* + s - 2\gamma y_P{}^{\frac{1}{2}}$ $= 0$.

$$\frac{dV_R{}^*}{dy_P} = -\frac{\partial Q/\partial y_P}{\partial Q/\partial V_R{}^*} = \frac{\gamma \left(y_P - pV_R{}^*\right)^{-\frac{1}{2}} - \gamma y_P{}^{-\frac{1}{2}}}{\gamma \left(y_P - pV_R{}^*\right)^{-\frac{1}{2}} p + p - 1} > 0$$

Because condition (3) must hold, $p \geq 1$ and $\gamma \left(y_P - pV_R{}^*\right)^{-\frac{1}{2}} - \gamma y_P{}^{-\frac{1}{2}}$ is larger than zero, we can conclude that $V_R{}^*$ depends positively on y_P. □

Proof of result (c)

Proof. We first have a look at how the income of rich individuals is affected by a change in β. Note that y_P is assumed to remain unchanged and recall that total income, y, is fixed. From the equation of total revenue, we get $y_R = \frac{y - \beta y_P}{1-\beta}$.

$$\frac{dy_R}{d\beta} = \frac{y - y_P}{(1-\beta)^2} > 0$$

If the fraction of poor individuals in the population increases, the income of poor individuals remaining unchanged, the residual income, $y - \beta y_P$ is divided among fewer rich individuals. In the end, each rich individual gets a higher revenue. However, we observe from Lemma 2 that the optimal variety of status goods purchased by the rich is independent of the income of the rich. □

A4. Proof of Proposition 2

Proof. We again use the implicit function theorem with $Q(p, V_R{}^*) = 2\gamma\left(y_P - pV_R{}^*\right)^{\frac{1}{2}}$ $- pV_R{}^* + V_R{}^* + s - 2\gamma y_P{}^{\frac{1}{2}} = 0$ in order to find the derivative of $V_R{}^*$ with respect to p.

$$\frac{dV_R{}^*}{dp} = -\frac{\partial Q/\partial p}{\partial Q/\partial V_R{}^*} = \frac{\gamma\left(y_P - pV_R{}^*\right)^{-\frac{1}{2}} V_R{}^* + V_R{}^*}{-\gamma\left(y_P - pV_R{}^*\right)^{-\frac{1}{2}} p - p + 1} < 0$$

Because condition (3) has to be satisfied and because $p \geq 1$, we know that the denominator is smaller than zero and that the numerator is greater than zero. It follows that the variety of status goods consumed by the rich gets smaller as the price of those goods increases. □

A5. Proof of Proposition 3

Proof. To prove the above proposition, I follow exactly the same steps as in Bagwell and Bernheim (1996).

We know that poor individuals choose not to purchase any status good. It remains to be demonstrated that rich individuals buy status goods at the lowest price. Suppose that the price of status goods is increasing in j, with $p_0 \geq 1$. Proposition 3 can be restated as follows. Rich individuals start to consume status good $j = 0$ and go on consuming status goods until good $j = V_R$. They end up with conspicuous expenditures equal to $\int_{j=0}^{V_R} p_j dj$. Let us remind that individuals' utility function depends on the consumption of normal goods, the consumption of status goods and on status. The consumption of normal goods corresponds to the residual income left after the purchase of status goods. It is thus a function of conspicuous expenditures. We can write the individual's utility in general terms as $U\left(\int_{j=0}^{V_i} p_j dj, V_i, \rho\right)$, where $\rho = 0$ ($\rho = s$) if the individual is perceived as poor (rich).

The proof by contradiction will be used. That is, we assume that instead of spending conspicuous expenditures $\int_{j=0}^{V_R} p_j dj$, rich individuals spend a larger amount of money on status goods equal to $\int_{j=x}^{x+V_R{}'} p_j dj$, with $x \in \mathbb{R}^+$.

First, we need to show that the optimal variety and prices of status goods chosen by rich individuals satisfy the incentive compatibility constraint with equality. If this would not be the case, they could increase their utility by selecting a cheaper bundle that just makes poor individuals indifferent between their own optimal bundle and the bundle of the rich. It would involve that the initial optimal bundle would not be optimal at all (step 1). Second, we have to show that, if poor individuals are indifferent between a bundle, in which status goods are the cheapest ones, and a bundle, in which status goods are purchased at higher prices, then the variety of status goods in the former bundle is larger than the variety of status goods in the latter bundle (step 2). Third, it has to be shown that, resulting from the single-crossing property, rich individuals prefer the former bundle to the latter one (step 3). Fourth, we show that there exists a bundle that satisfies all conditions and this bundle encompasses the cheapest status goods (step 4).

Step 1: We show that the incentive compatibility constraint binds. In general terms, we can write it as follows:

$$U_P{}^*\left(0,0,0\right) = U_P\left(\int_{j=x}^{x+V_R{}'} p_j dj, V_R{}', s\right) \tag{A1}$$

To prove this, we also use the proof by contradiction. Consider an x' with $\int_{j=x'}^{x'+V_R{}'} p_j dj$ slightly smaller than $\int_{j=x}^{x+V_R{}'} p_j dj$, where $V_R{}'$ remains unchanged. (It follows that $x' < x$.) Suppose that x' is such that conspicuous expenditures are not minimal, i.e. $\int_{j=x'}^{x'+V_R{}'} p_j dj \geq \int_{j=0}^{V_R{}'} p_j dj$ and that the incentive compatibility constraint of the poor holds with strict inequality:

$$U_P{}^*\left(0,0,0\right) > U_P\left(\int_{j=x'}^{x'+V_R{}'} p_j dj, V_R{}', s\right)$$

It follows that poor individuals still prefer their equilibrium payoff over $\left(\int_{j=x'}^{x'+V_R{}'} p_j dj, V_R{}'\right)$, for any $s \in [0, s]$.

However, regarding the rich, there are some values of s for which they prefer $\left(\int_{j=x'}^{x'+V_R{}'} p_j dj, V_R{}'\right)$ to their equilibrium outcome, $\left(\int_{j=x}^{x+V_R{}'} p_j dj, V_R{}'\right)$. Using the intuitive criterion, if the third party observes $\left(\int_{j=x'}^{x'+V_R{}'} p_j dj, V_R{}'\right)$, it will infer that those individuals are rich and will assign them a status s. However, the rich prefer $\left(\int_{j=x'}^{x'+V_R{}'} p_j dj, V_R{}', s\right)$ to their equilibrium outcome $\left(\int_{j=x}^{x+V_R{}'} p_j dj, V_R{}', s\right)$, since they consume the same variety of status goods at a lower price. It follows that the equilibrium payoff $\left(\int_{j=x}^{x+V_R{}'} p_j dj, V_R{}', s\right)$ fails the intuitive criterion. As a result, x must be such that (A1) holds. This means that the rich must choose prices of status goods that make the incentive compatibility constraint of the poor hold with equality. In other words, rich individuals choose prices of status goods that just allow to prevent imitation by the poor.

Step 2: Let us define $\hat{V}$ as the largest value of V satisfying:

$$U_P\left(\int_{j=0}^{V} p_j dj, V, s\right) = U_P{}^*\left(0,0,0\right), \tag{A2}$$

To show: $\hat{V} > V_R{}'$. Note that:

$$U_P\left(\int_{j=0}^{V_R{}'} p_j dj, V_R{}', s\right) > U_P\left(\int_{j=x}^{x+V_R{}'} p_j dj, V_R{}', s\right) \overset{(A1)}{=} U_P{}^*\left(0,0,0\right) \tag{A3}$$

From (A2) and (A3), it follows that:

$$U_P\left(\int_{j=0}^{\hat{V}} p_j dj, \hat{V}, s\right) < U_P\left(\int_{j=0}^{V_R{}'} p_j dj, V_R{}', s\right)$$

We deduce that $\hat{V} \neq V_R{}'$. Since $\hat{V}$ is the maximal value of V satisfying (A2), if rich consume less than $\hat{V}$, the incentive compatibility constraint of the poor is not satisfied, and therefore, $\hat{V} > V_R{}'$ follows from (A3).

Step 3: To show:

$$U_R\left(\int_{j=0}^{\hat{V}} p_j dj, \hat{V}, s\right) > U_R\left(\int_{j=x}^{x+V_R{}'} p_j dj, V_R{}', s\right)$$

To prove that rich individuals prefer $\left(\int_{j=0}^{\hat{V}} p_j dj, \hat{V}, s\right)$ to $\left(\int_{j=x}^{x+V_R{}'} p_j dj, V_R{}', s\right)$, we use the single-crossing property. This property says that, for any $(\tilde{V}, V, \int_{j=\tilde{x}}^{\tilde{x}+\tilde{V}} p_j dj,$ $\int_{j=x}^{x+V} p_j dj, s)$, with $\tilde{V} > V$ and $\int_{j=\tilde{x}}^{\tilde{x}+\tilde{V}} p_j dj > \int_{j=x}^{x+V} p_j dj, s \in [0, s]$,

$$U_P\left(\int_{j=\tilde{x}}^{\tilde{x}+\tilde{V}} p_j dj, \tilde{V}, s\right) \geq U_P\left(\int_{j=x}^{x+V} p_j dj, V, s\right)$$

implies

$$U_R\left(\int_{j=\tilde{x}}^{\tilde{x}+\tilde{V}} p_j dj, \tilde{V}, s\right) > U_R\left(\int_{j=x}^{x+V} p_j dj, V, s\right).$$

Let us remind that in the present demonstration, the incentive compatibility constraint holds with equality:

$$U_P\left(\int_{j=0}^{\hat{V}} p_j dj, \hat{V}, s\right) = U_P\left(\int_{j=x}^{x+V_R{}'} p_j dj, V_R{}', s\right) = U_P{}^*(0,0,0)$$

We also know that $\hat{V} > V_R{}'$. Using the single-crossing property, it follows that:

$$U_R\left(\int_{j=0}^{\hat{V}} p_j dj, \hat{V}, s\right) > U_R\left(\int_{j=x}^{x+V_R{}'} p_j dj, V_R{}', s\right),$$

and thus $\int_{j=0}^{\hat{V}} p_j dj > \int_{j=x}^{x+V_R{}'} p_j dj$.

Step 4: There exists a V_R and $\int_{j=0}^{V_R} p_j dj$ such that:

$$U_P\left(\int_{j=0}^{V_R} p_j dj, V_R, s\right) < U_P{}^*(0,0,0)$$

and

$$U_R\left(\int_{j=0}^{V_R} p_j dj, V_R, s\right) > U_R\left(\int_{j=x}^{x+V_R{}'} p_j dj, V_R{}', s\right)$$

To see this, let $V_R = \hat{V} + \varepsilon$. For $\varepsilon > 0$ sufficiently small, $\left(\int_{j=0}^{V_R} p_j dj, V_R\right)$ necessarily satisfies the required properties. Again using the logic of the intuitive criterion, we argue that observing $\left(\int_{j=0}^{V_R} p_j dj, V_R\right)$, the third party infers that those individuals are rich and respond by $s = s$. However, rich individuals prefer $\left(\int_{j=0}^{V_R} p_j dj, V_R, s\right)$ to $\left(\int_{j=x}^{x+V_R{}'} p_j dj, V_R{}', s\right)$. As a result, the equilibrium candidate $\left(\int_{j=x}^{x+V_R{}'} p_j dj, V_R{}', s\right)$ fails the intuitive criterion. Therefore, we have the desired contradiction and can conclude that rich individuals start to consume the cheapest status goods first. □

A6. Proof of Lemma 3

Proof. We first replace $\alpha = \frac{1}{2}$ into the incentive compatibility constraint, and obtain:

$$2\gamma y_P{}^{\frac{1}{2}} + y_P = 2\gamma\left(y_P - pV_R\right)^{\frac{1}{2}} + y_P - pV_R + V_R + s$$

Let us define $x \equiv (y_P - pV_R)^{\frac{1}{2}}$ and $x^2 \equiv (y_P - pV_R)$. It results that $p \equiv \frac{y_P - x^2}{V_R}$. We then substitute x and x^2 into the above incentive compatibility constraint, put all the terms on the righthand side, and get:

$$x^2 + 2\gamma x + V_R + s - 2\gamma {y_P}^{\frac{1}{2}} - y_P = 0$$

Solving for x, we get:

$$x = \frac{-2\gamma \pm \sqrt{4\gamma^2 - 4\left(V_R + s - 2\gamma {y_P}^{\frac{1}{2}} - y_P\right)}}{2}$$

Again, we know that $p^* V_R \leq y_P$ must hold. This implies that $x \equiv (y_P - p^* V_R)^{\frac{1}{2}} \geq 0$, and thus $x^2 \equiv (y_P - p^* V_R) \geq 0$. Therefore, we choose the solution for x that makes it larger than or equal to zero. After simplification, we get:

$$x = -\gamma + \left\{\gamma^2 - \left(V_R + s - 2\gamma {y_P}^{\frac{1}{2}} - y_P\right)\right\}^{\frac{1}{2}}$$

To find p, we need to calculate x^2:

$$x^2 = 2\gamma^2 - V_R - s + 2\gamma {y_P}^{\frac{1}{2}} + y_P - 2\gamma\left(\gamma^2 - V_R - s + 2\gamma {y_P}^{\frac{1}{2}} + y_P\right)^{\frac{1}{2}}$$

We are now able to calculate p, using $p \equiv \frac{y_P - x2}{V_R}$:

$$p^* = \frac{-2\gamma^2 + V_R + s - 2\gamma {y_P}^{\frac{1}{2}} + 2\gamma\left(\gamma^2 - V_R - s + 2\gamma {y_P}^{\frac{1}{2}} + y_P\right)^{\frac{1}{2}}}{V_R}$$

□

A7. Proof of condition (5)

Proof.

$$x \equiv (y_P - p^* V_R)^{\frac{1}{2}} > 0$$
$$\Leftrightarrow \quad -\gamma + \left(\gamma^2 - V_R - s + 2\gamma {y_P}^{\frac{1}{2}} + y_P\right)^{\frac{1}{2}} > 0$$
$$\Leftrightarrow \quad 2\gamma {y_P}^{\frac{1}{2}} + y_P > s + V_R$$

□

A8. Proof of Proposition 4

Proof of result (a)

Proof. We use the implicit function theorem, where the function $Q(p^*, s) = 2\gamma\left(y_P - p^*V\right)^{\frac{1}{2}}$ $- p^*V_R + V_R + s - 2\gamma y_P{}^{\frac{1}{2}} = 0$:

$$\frac{dp^*}{ds} = -\frac{\partial Q/\partial s}{\partial Q/\partial p^*} = \frac{1}{\gamma\left(y_P - p^*V_R\right)^{-\frac{1}{2}} V_R + V_R} > 0$$

Since, $\gamma > 0$ and $V_R > 0$, the above derivative is positive, if and only if $\left(y_P - p^*V_R\right)^{-\frac{1}{2}}$ is greater than zero. The latter term is positive, as condition (5) must hold. Hence, we can conclude that the relationship between p^* and s is positive. □

Proof of result (b)

Proof. Again, we use the implicit function theorem with $Q(y_P, p^*) = 2\gamma\left(y_P - p^*V\right)^{\frac{1}{2}}$ $- p^*V_R + V_R + s - 2\gamma y_P{}^{\frac{1}{2}} = 0$ to calculate the derivative of the price with respect to the income of poor individuals.

$$\frac{dp^*}{dy_P} = -\frac{\partial Q/\partial y_P}{\partial Q/\partial p^*} = \frac{\gamma\left(y_P - p^*V_R\right)^{-\frac{1}{2}} - \gamma y_P{}^{-\frac{1}{2}}}{\gamma\left(y_P - p^*V_R\right)^{\frac{1}{2}} V_R + V_R} > 0$$

Because condition (5) must be satisfied, and because $\gamma > 0$, $V_R > 0$ and $\left(y_P - p^*V_R\right)^{-\frac{1}{2}}$ $> y_P{}^{-\frac{1}{2}}$, we can state that the above derivative is larger than zero. □

Proof of result (c)

Proof. We have already shown in appendix A3 result (c) that, for a given total income y, an increase in the fraction of poor individuals, β, leads to a rise in the income of rich individuals, y_R. Furthermore, we can see from Lemma 3 that the income of rich individuals does not appear in the expression of the price. It implies that the price is insensitive to a change in the population's composition. □

A9. Proof of Lemma 4

Proof. In general terms, we know that the profit of the single monopolist can be written as:

$$\pi = V_R\left(p(V_R) - 1\right)(1 - \beta)$$

We plug the expression of the price (Lemma 3) into the above equation, and get:

$$\pi = \left\{ -2\gamma^2 + V_R + s - 2\gamma {y_P}^{\frac{1}{2}} + 2\gamma \left(\gamma^2 - V_R - s + 2\gamma {y_P}^{\frac{1}{2}} + y_P \right)^{\frac{1}{2}} - V_R \right\} (1 - \beta)$$

We know that the profit is maximal when $V_R \to 0$ and $p \to \infty$. Taking the limit, we obtain:

$$\pi_{max} = \lim_{V_R \to 0} \pi = \left\{ -2\gamma^2 + s - 2\gamma {y_P}^{\frac{1}{2}} + 2\gamma \left(\gamma^2 - s + 2\gamma {y_P}^{\frac{1}{2}} + y_P \right)^{\frac{1}{2}} \right\} (1 - \beta)$$

□

A10. Proof of Proposition 6

Proof. We take the derivative of the maximum profit (Lemma 4) with respect to y_P, and get:

$$\frac{d\pi_{max}}{dy_P} = \left\{ -\gamma {y_P}^{-\frac{1}{2}} + \gamma \left(\gamma^2 - s + 2\gamma {y_P}^{-\frac{1}{2}} + y_P \right)^{-\frac{1}{2}} \left(\gamma {y_P}^{-\frac{1}{2}} + 1 \right) \right\} (1 - \beta) > 0$$

The derivative is positive, if and only if $s > 0$. As it is the case, the profit increases when the poor become richer. □

A11. Proof of Proposition 7

Proof of result (a)

Proof.

$$\begin{aligned} \frac{d{V_R}^*}{dy_R} &= -\frac{1}{\alpha} \left({y_P}^{\alpha} + \frac{\alpha}{\gamma} (y_P - y_R) \right)^{\frac{1-\alpha}{\alpha}} (-1) > 0 \\ &\Leftrightarrow \; y_R < y_P + \frac{\gamma}{\alpha} {y_P}^{\alpha} \end{aligned}$$

And we know that this corresponds exactly to condition (9) that holds in any case. □

Proof of result (b)

Proof.

$$\frac{d{V_R}^*}{dy_P} = 1 - \frac{1}{\alpha} \left(y_P^{\alpha} + \frac{\alpha}{\gamma} (y_P - y_R) \right)^{\frac{1-\alpha}{\alpha}} \left(\alpha {y_P}^{\alpha - 1} + \frac{\alpha}{\gamma} + \frac{\alpha}{\gamma} \frac{\beta}{1 - \beta} \right)$$

To simplify, we take $\alpha = \frac{1}{2}$, and get:

$$\frac{dV_R{}^*}{dy_P} = 1 - \left(y_P{}^{\frac{1}{2}} + \frac{1}{2\gamma} (y_P - y_R) \right) \left(\frac{1}{\gamma} \frac{1}{1-\beta} + y_P{}^{-\frac{1}{2}} \right) \gtrless 0$$

We then simplify:

$$\begin{aligned}
& 1 \gtrless \left(y_P{}^{\frac{1}{2}} + \frac{1}{2\gamma} (y_P - y_R) \right) \left(\frac{1}{\gamma} \frac{1}{1-\beta} + y_P{}^{-\frac{1}{2}} \right) \\
\Leftrightarrow \quad & 1 \gtrless 1 + \frac{1}{2\gamma} y_P{}^{-\frac{1}{2}} (y_P - y_R) + \frac{1}{\gamma} \frac{1}{1-\beta} y_P{}^{\frac{1}{2}} + \frac{1}{2\gamma^2} \frac{1}{1-\beta} (y_P - y_R) \\
\Leftrightarrow \quad & 0 \gtrless \frac{1}{2\gamma} (y_P - y_R) + \frac{1}{\gamma} \frac{1}{1-\beta} y_P + \frac{1}{2\gamma^2} \frac{1}{1-\beta} y_P{}^{\frac{3}{2}} - \frac{1}{2\gamma^2} \frac{1}{1-\beta} y_P{}^{\frac{1}{2}} y_R \\
\Leftrightarrow \quad & 0 \gtrless \frac{1}{\gamma} \frac{1}{1-\beta} y_P{}^{\frac{3}{2}} + \frac{3-\beta}{1-\beta} y_P - y_R \left(1 + \frac{1}{\gamma} \frac{1}{1-\beta} y_P{}^{\frac{1}{2}} \right) \\
\Leftrightarrow \quad & y_R \left(1 + \frac{1}{\gamma} \frac{1}{1-\beta} y_P{}^{\frac{1}{2}} \right) \gtrless \frac{3-\beta}{1-\beta} y_P + \frac{1}{\gamma} \frac{1}{1-\beta} y_P{}^{\frac{3}{2}} \\
\Leftrightarrow \quad & y_R \gtrless \frac{\frac{3-\beta}{1-\beta} y_P + \frac{1}{\gamma} \frac{1}{1-\beta} y_P{}^{\frac{3}{2}}}{\left(1 + \frac{1}{\gamma} \frac{1}{1-\beta} y_P{}^{\frac{1}{2}} \right)} = \frac{(3-\beta)\gamma + y_P{}^{\frac{1}{2}}}{(1-\beta)\gamma y_P{}^{-1} + y_P{}^{-\frac{1}{2}}}
\end{aligned}$$

All in all, we obtain:

$$\frac{dV_R{}^*}{dy_P} = \begin{cases} > 0 \quad if \quad y_R > \dfrac{(3-\beta)\gamma + y_P{}^{\frac{1}{2}}}{(1-\beta)\gamma y_P{}^{-1} + y_P{}^{-\frac{1}{2}}} \\ < 0 \quad if \quad y_R < \dfrac{(3-\beta)\gamma + y_P{}^{\frac{1}{2}}}{(1-\beta)\gamma y_P{}^{-1} + y_P{}^{-\frac{1}{2}}} \end{cases}$$

□

Proof of result (c)

Proof. We know that a rise in the proportion of poor individuals leads to an increase in the income of the rich, for a given total income. Indeed, the first result of proposition 7 is that the variety consumed by rich individuals increases, as the income of the rich increases. As a result, a increase in β leads to an increase in $V_R{}^*$. □

A12. Proof of Lemma 6

Proof. We first plug $\alpha = \frac{1}{2}$ into the incentive compatibility constraint (16):

$$2\gamma {y_P}^{\frac{1}{2}} + 2y_P = 2\gamma \left(y_P - pV_R\right)^{\frac{1}{2}} + \left(y_P - pV_R\right) + V_R + y_R$$

Let us define $x \equiv (y_P - pV_R)^{\frac{1}{2}}$ and $x^2 \equiv y_P - pV_R$, plug them into the above constraint, put all the terms on the righthand side, and get:

$$x^2 + 2\gamma x + V_R + y_R - 2\gamma {y_P}^{\frac{1}{2}} - 2y_P = 0$$

We solve for x, and find:

$$x = \frac{-2\gamma \pm \sqrt{4\gamma^2 - 4\left(V_R + y_R - 2\gamma {y_P}^{\frac{1}{2}} - 2y_P\right)}}{2}$$

For the same reason as in section 4.1, $p^* V_R \leq y_P$. This implies that $x = (y_P - p^* V_R)^{\frac{1}{2}} \geq 0$, and thus $x^2 = (y_P - p^* V_R) \geq 0$. Therefore, we choose the solution for x that makes it larger or equal than zero. After simplification, we get:

$$x = -\gamma + \sqrt{\gamma^2 - V_R - y_R + 2\gamma {y_P}^{\frac{1}{2}} + 2y_P}$$

To find p, we need to calculate x^2:

$$x^2 = 2\gamma^2 - 2\gamma\sqrt{\gamma^2 - V_R - y_R + 2\gamma {y_P}^{\frac{1}{2}} + 2y_P} - V_R - y_R + 2\gamma y_P \frac{1}{2} + 2y_P$$

From the definition of x, we can calculate p:

$$p^* = \frac{-2\gamma^2 + V_R + y_R - 2\gamma {y_P}^{\frac{1}{2}} - y_P + 2\gamma\left(\gamma^2 - V_R - y_R + 2\gamma {y_P}^{\frac{1}{2}} + 2y_P\right)^{\frac{1}{2}}}{V_R}$$

□

A13. Proof of condition (11)

Proof.

$$
\begin{aligned}
& (y_P - pV_R)^{\frac{1}{2}} > 0 \\
\Leftrightarrow\ & x = -\gamma + \left(\gamma^2 - V_R - y_R + 2\gamma {y_P}^{\frac{1}{2}} + 2y_P\right)^{\frac{1}{2}} > 0 \\
\Leftrightarrow\ & -V_R - y_R + 2\gamma {y_P}^{\frac{1}{2}} + 2y_P > 0 \\
\Leftrightarrow\ & 2\gamma {y_P}^{\frac{1}{2}} + 2y_P > V_R + y_R
\end{aligned}
$$

□

A14. Proof of Proposition 8

Proof of result (a)

Proof. We use the implicit function theorem with $Q(y_R, p^*) = 2\gamma\left(y_P - p^* V_R\right)^{\frac{1}{2}} - p^* V_R + V_R + y_R - 2\gamma {y_P}^{\frac{1}{2}} - y_P = 0$.

$$
\frac{dp^*}{dy_R} = -\frac{\partial Q/\partial y_R}{\partial Q/\partial p^*} = \frac{1}{\gamma\left(y_P - p^* V_R\right)^{-\frac{1}{2}} V_R + V_R} > 0
$$

Because $V_R > 0$ and condition (11) holds in any case, the derivative of the price with respect to rich individuals' absolute wealth is positive. □

Proof of result (b)

Proof.

$$
\begin{aligned}
\frac{dp^*}{dy_P} = & \frac{1}{V_R}\left\{\gamma\left(\gamma^2 - V_R - y_R + 2\gamma {y_P}^{\frac{1}{2}} + 2y_P\right)^{-\frac{1}{2}}\left(\frac{\beta}{1-\beta} + \gamma {y_P}^{-\frac{1}{2}} + 2\right) - \frac{\beta}{1-\beta}\right. \\
& \left. -\ \gamma {y_P}^{-\frac{1}{2}} - 1\right\} \gtrless 0 \\
& \Leftrightarrow \gamma\left(\gamma^2 - V_R - y_R + 2\gamma {y_P}^{\frac{1}{2}} + 2y_P\right)^{-\frac{1}{2}}\left(\frac{1}{1-\beta} + \gamma {y_P}^{-\frac{1}{2}} + 1\right) \gtrless \gamma {y_P}^{-\frac{1}{2}} + \frac{1}{1-\beta} \\
& \Leftrightarrow \gamma^2\left(\gamma^2 - V_R - y_R + 2\gamma {y_P}^{\frac{1}{2}} + 2y_P\right)^{-1}\left(\frac{1}{1-\beta} + \gamma {y_P}^{-\frac{1}{2}} + 1\right)^2 \gtrless \gamma^2 {y_P}^{-1} \\
& + \frac{2}{1-\beta}\gamma {y_P}^{-\frac{1}{2}} + \frac{1}{(1-\beta)^2}
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow\quad & \frac{\gamma^2}{(1-\beta)^2}+\frac{2}{1-\beta}\gamma^3 {y_P}^{-\frac{1}{2}}+\frac{2\gamma^2}{1-\beta}+\gamma^4 {y_P}^{-1}+2\gamma^3 {y_P}^{-\frac{1}{2}}+\gamma^2 \gtrless \gamma^4 {y_P}^{-1}\\
+\quad & \frac{2}{1-\beta}\gamma^3 {y_P}^{-\frac{1}{2}}+\frac{\gamma^2}{(1-\beta)^2}-(V_R+y_R)\left(\gamma^2 {y_P}^{-1}+\frac{2}{1-\beta}\gamma {y_P}^{-\frac{1}{2}}+\frac{1}{(1-\beta)^2}\right)\\
+\quad & 2\gamma^3 {y_P}^{-\frac{1}{2}}+\frac{4\gamma^2}{1-\beta}+\frac{2\gamma}{(1-\beta)^2}{y_P}^{\frac{1}{2}}+2\gamma^2+\frac{4\gamma}{1-\beta}{y_P}^{\frac{1}{2}}+\frac{2}{(1-\beta)^2}y_P\\
\Leftrightarrow\quad & y_R\left(\gamma^2 {y_P}^{-1}+\frac{2}{1-\beta}\gamma {y_P}^{-\frac{1}{2}}+\frac{1}{(1-\beta)^2}\right)\gtrless \frac{\gamma(6-4\beta)}{(1-\beta)^2}{y_P}^{\frac{1}{2}}+\frac{3-\beta}{1-\beta}\gamma^2+\frac{2}{(1-\beta)^2}y_P\\
-\quad & V_R\left(\gamma^2 {y_P}^{-1}+\frac{2}{1-\beta}\gamma {y_P}^{-\frac{1}{2}}+\frac{1}{(1-\beta)^2}\right)\\
\Leftrightarrow\quad & y_R \gtrless \frac{(3-\beta)(1-\beta)\gamma^2+\gamma(6-4\beta){y_P}^{\frac{1}{2}}+2y_P}{(1-\beta)^2\gamma^2 {y_P}^{-1}+2(1-\beta)\gamma {y_P}^{-\frac{1}{2}}+1}-V_R
\end{aligned}
$$

All in all, we obtain:

$$
\frac{dp^*}{dy_P}=\begin{cases} >0 \quad if \quad y_R > \dfrac{(3-\beta)(1-\beta)\gamma^2+\gamma(6-4\beta){y_P}^{\frac{1}{2}}+2y_P}{(1-\beta)^2\gamma^2 {y_P}^{-1}+2(1-\beta)\gamma {y_P}^{-\frac{1}{2}}+1}-V_R\\[2ex] <0 \quad if \quad y_R < \dfrac{(3-\beta)(1-\beta)\gamma^2+\gamma(6-4\beta){y_P}^{\frac{1}{2}}+2y_P}{(1-\beta)^2\gamma^2 {y_P}^{-1}+2(1-\beta)\gamma {y_P}^{-\frac{1}{2}}+1}-V_R \end{cases}
$$

□

Proof of result (c)

Proof. As always, an increase in the fraction of the poor raises the income received by every rich individual, for a given total income. As a result, y_R rises. And we have already shown that the derivative of p^* with respect to y_R is positive. □

A15. Proof of Lemma 7

Proof. We replace the expression of the price (Lemma 6) into the profit, and get:

$$
\begin{aligned}
\pi \; &= V_R\left(p(V_R)-1\right)(1-\beta)\\
= \; &\left(-2\gamma^2+V_R+y_R-2\gamma {y_P}^{\frac{1}{2}}-y_P+2\gamma\left(\gamma^2-V_R-y_R+2\gamma {y_P}^{\frac{1}{2}}+2y_P\right)^{\frac{1}{2}}-V_R\right)\cdot\\
&(1-\beta)
\end{aligned}
$$

To obtain the maximal profit, we have to take the limit of the profit, when V_R is going to zero. The above expression becomes:

$$\pi_{max} = \lim_{V_R \to 0} \pi = \left(-2\gamma^2 + y_R - 2\gamma {y_P}^{\frac{1}{2}} - y_P + 2\gamma\left(\gamma^2 - y_R + 2\gamma {y_P}^{\frac{1}{2}} + 2y_P\right)^{\frac{1}{2}}\right)(1-\beta)$$

□

A16. Proof of Proposition 9

Proof. We take the derivative of the maximum profit (Lemma 7) with respect to y_P, and get:

$$\begin{aligned}
\tfrac{d\pi}{dy_P} \quad & = (1-\beta)\left(-\frac{\beta}{1-\beta} - \gamma {y_P}^{-\frac{1}{2}} - 1\right) \\
+ \quad & (1-\beta)\left\{\gamma\left(\gamma^2 - y_R + 2\gamma {y_P}^{\frac{1}{2}} + 2y_P\right)^{-\frac{1}{2}}\left(\frac{\beta}{1-\beta} + \gamma {y_P}^{-\frac{1}{2}} + 2\right)\right\} \gtrless 0 \\
\Leftrightarrow \quad & \gamma\left(\gamma^2 - y_R + 2\gamma {y_P}^{\frac{1}{2}} + 2y_P\right)^{-\frac{1}{2}}\left(\frac{1}{1-\beta} + \gamma {y_P}^{-\frac{1}{2}} + 1\right) \gtrless \gamma {y_P}^{-\frac{1}{2}} + \frac{1}{1-\beta} \\
\Leftrightarrow \quad & \gamma^2\left(\gamma^2 - y_R + 2\gamma {y_P}^{\frac{1}{2}} + 2y_P\right)^{-1}\left(\frac{1}{1-\beta} + \gamma {y_P}^{-\frac{1}{2}} + 1\right)^2 \gtrless \gamma^2 {y_P}^{-1} \\
+ \quad & \frac{2}{1-\beta}\gamma {y_P}^{-\frac{1}{2}} + \frac{1}{(1-\beta)^2} \\
\Leftrightarrow \quad & \frac{\gamma^2}{(1-\beta)^2} + \frac{2}{1-\beta}\gamma^3 {y_P}^{-\frac{1}{2}} + \frac{2\gamma^2}{1-\beta} + \gamma^4 {y_P}^{-1} + 2\gamma^3 {y_P}^{-\frac{1}{2}} + \gamma^2 \gtrless \gamma^4 {y_P}^{-1} \\
+ \quad & \frac{2}{1-\beta}\gamma^3 {y_P}^{-\frac{1}{2}} + \frac{\gamma^2}{(1-\beta)^2} - y_R\left(\gamma^2 {y_P}^{-1} + \frac{2}{1-\beta}\gamma {y_P}^{-\frac{1}{2}} + \frac{1}{(1-\beta)^2}\right) \\
+ \quad & 2\gamma^3 {y_P}^{-\frac{1}{2}} + \frac{4\gamma^2}{1-\beta} + \frac{2\gamma}{(1-\beta)^2}{y_P}^{\frac{1}{2}} + 2\gamma^2 + \frac{4\gamma}{1-\beta}{y_P}^{\frac{1}{2}} + \frac{2}{(1-\beta)^2}y_P \\
\Leftrightarrow \quad & y_R\left(\gamma^2 {y_P}^{-1} + \frac{2}{1-\beta}\gamma {y_P}^{-\frac{1}{2}} + \frac{1}{(1-\beta)^2}\right) \gtrless \frac{\gamma(6-4\beta)}{(1-\beta)^2}{y_P}^{\frac{1}{2}} + \frac{3-\beta}{1-\beta}\gamma^2 \\
+ \quad & \frac{2}{(1-\beta)^2}y_P \\
\Leftrightarrow \quad & y_R \gtrless \frac{(3-\beta)(1-\beta)\gamma^2 + \gamma(6-4\beta){y_P}^{\frac{1}{2}} + 2y_P}{(1-\beta)^2\gamma^2 {y_P}^{-1} + 2(1-\beta)\gamma {y_P}^{-\frac{1}{2}} + 1}
\end{aligned}$$

All in all, we have:

$$\frac{d\pi}{dy_P} = \begin{cases} > 0 \quad if \quad y_R > \dfrac{(3-\beta)(1-\beta)\gamma^2 + \gamma(6-4\beta){y_P}^{\frac{1}{2}} + 2y_P}{(1-\beta)^2\gamma^2{y_P}^{-1} + 2(1-\beta)\gamma{y_P}^{-\frac{1}{2}} + 1} \\ < 0 \quad if \quad y_R < \dfrac{(3-\beta)(1-\beta)\gamma^2 + \gamma(6-4\beta){y_P}^{\frac{1}{2}} + 2y_P}{(1-\beta)^2\gamma^2{y_P}^{-1} + 2(1-\beta)\gamma{y_P}^{-\frac{1}{2}} + 1} \end{cases}$$

□

A17. Proof of Proposition 10

Proof. Let us first remind that condition (9) implies that:

$$\left\{\gamma\left(y_R - y_P + \left({y_P}^{\alpha} + \frac{\alpha}{\gamma}(y_P - y_R)\right)^{\frac{1}{\alpha}}\right)^{\alpha-1} + 1\right\} > 0$$

It follows that $\gamma{y_P}^{\alpha-1} - \gamma\left({y_P}^{\alpha} + \frac{\alpha}{\gamma}(y_P - y_R)\right)^{\frac{\alpha-1}{\alpha}} + 1 + \frac{\beta}{1-\beta}$ must be larger than zero. To simplify calculations, we replace $\alpha = \frac{1}{2}$. We thus get:

$$\begin{aligned} &\gamma{y_P}^{-\frac{1}{2}} - \gamma\left(y^{\frac{1}{2}} + \frac{1}{2\gamma}(y_P - y_R)\right)^{-1} + \frac{1}{1-\beta} > 0 \\ \Leftrightarrow \quad &\gamma{y_P}^{-\frac{1}{2}} + \frac{1}{1-\beta} > \gamma\left(y^{\frac{1}{2}} + \frac{1}{2\gamma}(y_P - y_R)\right)^{-1} \end{aligned}$$

We then multiply both sides by ${y_P}^{\frac{1}{2}}\left({y_P}^{\frac{1}{2}} + \frac{1}{2\gamma}(y_P - y_R)\right)$, and obtain:

$$\gamma{y_P}^{\frac{1}{2}} + \frac{1}{2}(y_P - y_R) + \frac{1}{1-\beta}y_P + \frac{1}{1-\beta}\frac{1}{2\gamma}(y_P - y_R){y_P}^{\frac{1}{2}} > \gamma{y_P}^{\frac{1}{2}}$$

After simplification, we get:

$$\begin{aligned} &y_R\left(\frac{1}{2} + \frac{1}{1-\beta}\frac{1}{2\gamma}{y_P}^{\frac{1}{2}}\right) < y_P\left(\frac{1}{2} + \frac{1}{1-\beta}\frac{1}{2\gamma}{y_P}^{\frac{1}{2}} + \frac{1}{1-\beta}\right) \\ \Leftrightarrow \quad &y_R < \frac{\gamma(3-\beta) + {y_P}^{\frac{1}{2}}}{\gamma(1-\beta){y_P}^{-1} + {y_P}^{-\frac{1}{2}}} \qquad (1) \end{aligned}$$

Naturally, we also need naturally is that the whole derivative (16) is larger than zero. In order to simplify calculations, let us first rewrite equation (16) as follows:

$$\begin{aligned}
\frac{dU_R}{dy_P} &= -\frac{\beta}{1-\beta}\left\{\gamma\left(y_R - y_P + \left({y_P}^{\alpha} + \frac{\alpha}{\gamma}\left(y_P - y_R\right)\right)^{\frac{1}{\alpha}}\right)^{\alpha-1} + 1\right\} \\
&+ \left\{\gamma\left(y_R - y_P + \left({y_P}^{\alpha} + \frac{\alpha}{\gamma}\left(y_P - y_R\right)\right)^{\frac{1}{\alpha}}\right)^{\alpha-1}\right\} \\
&\times \left\{-1 + \left({y_P}^{\alpha} + \frac{\alpha}{\gamma}\left(y_P - y_R\right)\right)^{\frac{1-\alpha}{\alpha}}\left({y_P}^{\alpha-1} + \frac{1}{\gamma}\left(\frac{1}{1-\beta}\right)\right)\right\} \\
&- \frac{\beta}{1-\beta} > 0
\end{aligned}$$

To further simplify the analysis, we set $\alpha = \frac{1}{2}$ and get:

$$\begin{aligned}
\frac{dU_R}{dy_P} &= -\frac{\beta}{1-\beta}\left\{\gamma\left(y_R - y_P + \left({y_P}^{\frac{1}{2}} + \frac{1}{2\gamma}\left(y_P - y_R\right)\right)^{2}\right)^{-\frac{1}{2}} + 1\right\} \\
&+ \left\{\gamma\left(y_R - y_P + \left({y_P}^{\frac{1}{2}} + \frac{1}{2\gamma}\left(y_P - y_R\right)\right)^{2}\right)^{-\frac{1}{2}}\right\} \\
&\times \left\{-1 + \left({y_P}^{\frac{1}{2}} + \frac{1}{2\gamma}\left(y_P - y_R\right)\right)\left({y_P}^{-\frac{1}{2}} + \frac{1}{\gamma}\left(\frac{1}{1-\beta}\right)\right)\right\} \\
&- \frac{\beta}{1-\beta} > 0
\end{aligned} \tag{2}$$

□

Proof for $\beta = 0$

Proof. When $\beta = 0$, equation (2) becomes:

$$\begin{aligned}
\frac{dU_R}{dy_P} &= \gamma\left(y_R - y_P + \left({y_P}^{\frac{1}{2}} + \frac{1}{2\gamma}\left(y_P - y_R\right)\right)^{2}\right)^{-\frac{1}{2}} \\
&\times \left\{-1 + \left({y_P}^{\frac{1}{2}} + \frac{1}{2\gamma}\left(y_P - y_R\right)\right)\left({y_P}^{-\frac{1}{2}} + \frac{1}{\gamma}\right)\right\} > 0
\end{aligned}$$

We then solve for y_R, and obtain:

$$y_R < \frac{3\gamma + {y_P}^{\frac{1}{2}}}{\gamma {y_P}^{-1} + {y_P}^{-\frac{1}{2}}} \tag{3}$$

Conditions (3) is identical to condition (1) when $\beta = 0$. □

Proof for $y_P = y_R$

Proof. We plug $y_R = y_P$ into equation (2), and find:

$$\begin{aligned}\frac{dU_R}{dy_P} &= -\frac{\beta}{1-\beta}\gamma {y_P}^{-\frac{1}{2}} - \frac{\beta}{1-\beta} - \gamma {y_P}^{-\frac{1}{2}} + \gamma {y_P}^{-\frac{1}{2}} {y_P}^{\frac{1}{2}} \left({y_P}^{-\frac{1}{2}} + \frac{1}{\gamma}\left(\frac{1}{1-\beta}\right)\right) \\ &- \frac{\beta}{1-\beta} > 0\end{aligned}$$

After simplification, we find:

$$y_P > \left(\frac{\beta\gamma}{1-2\beta}\right)^2 \tag{4}$$

Note that if $y_P = y_R$, condition (9) becomes $y_P > 0$. Furthermore, the condition under which the variety of status goods consumed by the rich decreases in the income of the poor (Proposition 7) becomes $y_P > 0$ too. Therefore, condition (4) is more restrictive. □

A18. Proof of Proposition 11

Proof. We first insert $\beta = 0$ into equation (17), simplify, and get:

$$\begin{aligned}\frac{dU_R}{dy_P} &= \left\{1 + \gamma {y_P}^{-\frac{1}{2}} - \gamma\left(\gamma^2 - y_R + 2y_P + 2\gamma {y_P}^{\frac{1}{2}}\right)^{-\frac{1}{2}}\left(2 + \gamma {y_P}^{-\frac{1}{2}}\right)\right\} \\ &\times \left\{1 + \gamma\left(2\gamma^2 + y_P + 2\gamma {y_P}^{\frac{1}{2}} - 2\gamma\left(\gamma^2 - y_R + 2y_P + 2\gamma {y_P}^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{-\frac{1}{2}}\right\}\end{aligned}$$

We know that $\gamma\left(2\gamma^2 + y_P + 2\gamma {y_P}^{\frac{1}{2}} - 2\gamma\left(\gamma^2 - y_R + 2y_P + 2\gamma {y_P}^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)$ is equal to $y_R - F$. Since $y_P - F$ is positive, it follows that $y_R - F$ is positive too. It implies that the whole expression $\left\{1 + \gamma\left(2\gamma^2 + y_P + 2\gamma {y_P}^{\frac{1}{2}} - 2\gamma\left(\gamma^2 - y_R + 2y_P + 2\gamma {y_P}^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{-\frac{1}{2}}\right\}$ is positive and we can divide the above derivative by this latter expression. We end up with:

$$1 + \gamma {y_P}^{-\frac{1}{2}} - \gamma\left(\gamma^2 - y_R + 2y_P + 2\gamma {y_P}^{\frac{1}{2}}\right)^{-\frac{1}{2}}\left(2 + \gamma {y_P}^{-\frac{1}{2}}\right) > 0$$

After simplification, we obtain:

$$y_R < \frac{3\gamma^2 + 6\gamma {y_P}^{\frac{1}{2}} + 2y_P}{\gamma^2 {y_P}^{-1} + 2\gamma {y_P}^{-\frac{1}{2}} + 1}$$

□

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November, 2015 Juliette Cattin