ONLINE APPENDIX TO "LONG-TERM RELATIONSHIPS: STATIC GAINS AND DYNAMIC INEFFICIENCIES"

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B. Online Appendix

B.1. Cooperative equilibrium characterization: complements

In this appendix we complete the proof of Proposition A.1 in Appendix A.1 by going through steps 3 and 4 in all possible cases and showing that they cover the full range of possibilities.

B.1.1. Step 3: deriving the IC constraint in all cases. Whether a producer would rather stick to a non-cooperating good match (a good match playing the Nash level because a deviation has occurred) or keep looking for a new supplier will affect the IC constraint. We derive it in all the possible cases:

- case 1, when the producer will choose the non-cooperating good match in any circumstances,

- case 2, when the producer will choose the innovator over the non-cooperating good match, but stick to the non-cooperating good match otherwise,

- case 3, when the producer will choose the non-cooperating good match in period without innovation, but in period with innovation the non-cooperating good match is worse than even an outdated supplier,

- case 4, when the producer will choose a new match in periods without innovation, but in period with innovation, the non-cooperating good match is better than trying an outdated supplier,

- case 5, when the non-cooperating good match is never one of the two best options (which is the special case studied in Appendix A.1).

Moreover, in cases 1, 2 and 3, the non-cooperating good match could be chosen one step away from the equilibrium path, we then need to check that whether the producer knows only one non-cooperating good match or more matters.

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Case 1.1. We consider the case where the non-cooperating good match is always better than starting a new relationship. We consider a producer who only knows one non-cooperating good match (and no other good match), we derive the conditions under which this case applies, and the incentive constraint of a producer who would be in a good match relationship and would not know any non-cooperating good match. We still denote by V_N^T the joint value of the producer and the non-cooperating good match in periods without innovation and we define W_N^T as the corresponding value in periods with innovation. We then get:

$$V_N^T = \Pi(n) + \frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_N^T + \delta^I\gamma W_N^T\right), \tag{B.1}$$

$$W_N^T = \gamma^{-1}\Pi(n) + \frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_N^T + \delta^I\gamma W_N^T\right). \tag{B.2}$$

Now recall that $V_0^{T,n}$ denotes the joint value when a producer starts a new relationship (n indicates that the producer knows a non-cooperating good match) in periods without innovation, we denote by $V_I^{T,n}$ the same value in periods with innovation, and we get:

$$V_I^{T,n} = V_0^{T,n} = (1-b)V_1^T + b\theta\Pi(n) + b\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_N^p + \delta^I\gamma W_N^p\right), \tag{B.3}$$

with probability $(1-b)$ the new supplier is a good match and the joint value becomes V_1^T, with probability b the new supplier is a bad match, in which case the producer should revert back to the non-cooperating good match in the following period. Bertrand competition ensures that:

$$V_N^p = V_0^{T,n} \text{ and } V_N^s = V_N^T - V_0^{T,n}, \tag{B.4}$$

$$W_N^p = V_I^{T,n} \text{ and } W_N^s = W_N^T - V_I^{T,n}. \tag{B.5}$$

The condition to be in that case is that in periods with innovation the producer would rather stick to the non-cooperating good match than choose the innovator, note that if the producer chooses the innovator, the value of the non-cooperating good match is not null, instead it is given by:

$$V_{AN}^s = \frac{1-\delta^D}{1+\rho}b\left(\left(1-\delta^I\right)V_N^s + \delta^I\gamma W_N^s\right), \tag{B.6}$$

as with probability b the innovator will be a bad match and the producer would revert back to the non-cooperating good match in the following period. The condition to be in that case can then be expressed as:

$$W_N^T \geq V_0^T + V_{AN}^s. \tag{B.7}$$

Combining (B.1) and (B.2), we get:

$$W_N^T = V_N^T - \left(1-\gamma^{-1}\right)\Pi(n), \tag{B.8}$$

so that:

$$V_N^T = \frac{1+\rho-\left(1-\delta^D\right)\delta^I\left(\gamma-1\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}\Pi\left(n\right). \tag{B.9}$$

Combining (B.3), (B.6) and (B.8), we can rewrite (B.7) as:

$$V_N^T-\left(1-\gamma^{-1}\right)\Pi\left(n\right)\geq\left(1-b\right)V_1^T+b\theta\Pi\left(n\right)+b\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_N^T+\delta^I\gamma W_N^T\right),$$

which using (B.9) translates into:

$$\begin{aligned}&\frac{\left(\gamma^{-1}\left(1+\rho\right)-b\left(1-\delta^D\right)+\left(1-\delta^D\right)\left(1-\delta^I\right)\left(1-\gamma^{-1}\right)\right)\Pi\left(n\right)}{\left(1+\rho\right)\left(1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)\right)}\\&\geq\left(\left(1-b\right)V_1^T+b\theta\Pi\left(n\right)\right).\end{aligned} \tag{B.10}$$

Now we want to express the IC constraint of a producer in a good match who does not know any non-cooperating good match. To do so, we first need to compute the expected value of a non-cooperating good match. Combining (B.3), (B.4) and (B.5) we get:

$$V_I^{T,n}=V_0^{T,n}=\left(1-b\right)V_1^T+b\theta\Pi\left(n\right)+b\frac{1-\delta^D}{1+\rho}\left(1-\delta^I+\delta^I\gamma\right)V_0^{T,n},$$

so that:

$$V_I^{T,n}=V_0^{T,n}=\frac{1+\rho}{\left(1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)\right)}\left(\left(1-b\right)V_1^T+b\theta\Pi\left(n\right)\right), \tag{B.11}$$

which combined with (B.9) gives:

$$\begin{aligned}V_N^s &= \frac{1+\rho-\left(1-\delta^D\right)\delta^I\left(\gamma-1\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}\Pi\left(n\right)\\&\quad-\frac{1+\rho}{\left(1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)\right)}\left(\left(1-b\right)V_1^{T\prime}+b\theta\Pi\left(n\right)\right),\end{aligned}$$

$$\begin{aligned}W_N^s &= \frac{\gamma^{-1}\left(1+\rho\right)+\left(1-\gamma^{-1}\right)\left(1-\delta^D\right)\left(1-\delta^I\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}\Pi\left(n\right)\\&\quad-\frac{1+\rho}{\left(1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)\right)}\left(\left(1-b\right)V_1^{T\prime}+b\theta\Pi\left(n\right)\right).\end{aligned}$$

Therefore we can write:

$$\begin{aligned}&\left(1-\delta^I\right)V_N^s+\delta^I\gamma W_N^s\\&=\frac{1+\rho}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}\Pi\left(n\right)\\&\quad-\frac{\left(1+\rho\right)\left(1-\delta^I+\delta^I\gamma\right)}{\left(1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)\right)}\left(\left(1-b\right)V_1^T+b\theta\Pi\left(n\right)\right).\end{aligned} \tag{B.12}$$

Using (A.12) and (B.11) we get:

$$V_1^s = V_1^T - \frac{1+\rho}{\left(1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)\right)}\Big((1-b)\,V_1^T + b\theta\Pi\,(n)\Big). \tag{B.13}$$

Combining (A.8), (A.10), (B.12), (B.13) and (A.11), and knowing that if the non-cooperating good match is better than the innovator, then a good match supplier is also necessarily better than the innovator, we get:

$$I = \frac{(1+\rho)\left(\left(1-\delta^I\right)\Pi\,(x^*) + \delta^I\Pi\,(y^*) - \Pi\,(n)\right)}{1+\rho-\left(1-\delta^D\right)\left(1+\delta^I\,(\gamma-1)\right)}.$$

Case 1.2. We consider now the same situation except that the producer already knows at least two non-cooperating good match suppliers. Note that Bertrand competition implies that the producer can then capture the entire value of the relationship so that:

$$V_N^s = W_N^s = V_{AN}^s = 0,\ V_N^p = V_N^T \text{ and } W_N^p = W_N^T \tag{B.14}$$

(B.1), (B.2) and therefore (B.9) still hold. However (B.3) combined with (B.14) now gives:

$$V_0^{T,n} = V_I^{T,n} = (1-b)\,V_1^T + b\theta\Pi\,(n) + b\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_N^T + \delta^I\gamma W_N^T\right), \tag{B.15}$$

and the condition to be in that case is now

$$W_N^T \geq V_0^{T,n},$$

instead of (B.7) (as the value of the non-cooperating good match is always null). This condition then leads to:

$$\begin{aligned}&\frac{\left(\gamma^{-1}(1+\rho)-b\left(1-\delta^D\right)+\left(1-\delta^D\right)\left(1-\delta^I\right)\left(1-\gamma^{-1}\right)\right)\Pi\,(n)}{(1+\rho)\left(1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)\right)}\\ &\geq \Big((1-b)\,V_1^T + b\theta\Pi\,(n)\Big).\end{aligned}$$

which is the same condition as in case 1.

(A.13) gives:

$$V_1^s = V_1^T - V_N^T.$$

Now note that we are precisely in the case where the analysis leading to (A.9) may not apply. If the producer would rather switch to the non-cooperating good match than the innovator, we get that:

$$W_1^p = W_N^T \text{ and } W_1^s = W_1^T - W_N^T,$$

so that we get:

$$I = \frac{(1+\rho)\left(\left(1-\delta^I\right)\Pi\,(x^*) + \delta^I\Pi\,(y^*) - \Pi\,(n)\right)}{1+\rho-\left(1-\delta^D\right)\left(1+\delta^I\,(\gamma-1)\right)}.$$

The incentive constraint for a supplier with a producer who knows a non-cooperating good match is the same as in case 1, which is a necessary requirement for the existence of the equilibrium (because condition 1 requires that the profile of investment with a good match supplier is always the same even if the good match supplier is not the first good match supplier).

We need however to check that switching to the innovator when one is in a good match remains worse than switching to the non-cooperating good match (this is conceptually not equivalent as saying that a producer with a non-cooperating good match would not switch to the innovator, indeed for a producer in a good match, switching to the innovator does not necessarily lead to punishment in the following period, whereas switching to the non-cooperating good match does so[1]). We prove this by contradiction, assume that a producer in a good match would rather deviate by switching to the innovator than to the non-cooperating good match. We would then get:

$$W_1^p = V_I^{T,g} = (1-b)\, V_1^T + b\theta\Pi(n) + \frac{b\left(1-\delta^D\right)}{1+\rho}\left(\left(1-\delta^I\right)V_N^T + \delta^I\gamma V_I^{T,g}\right),$$

with the condition $V_I^{T,g} > W_N^T$, however this condition leads to the reverse of condition (B.10).

Case 2.1. We now consider again a producer who knows only a single non-cooperating good match. We assume that sticking to the non-cooperating good match is preferred to trying out a new supplier in periods without innovation, or an outdated supplier in periods with innovation, but remains worse than switching to the innovator in periods with innovation. In other words, we assume:

$$V_N^T \geq V_0^{T,n} \text{ and } W_0^{T,n} \leq W_N^T - V_{AN}^s \leq V_I^{T,n}. \tag{B.16}$$

Bertrand competition leads to (B.4) and to:

$$W_N^s = V_{AN}^s \text{ and } W_N^p = V_I^{p,n} = W_N^T - V_{AN}^s, \tag{B.17}$$

the value of the non-cooperating good match in periods with innovation is not null because if the innovator turns out to be a bad match, the producer would come back to the non-cooperating good match in the following periods.

(B.1), (B.2) (and therefore (B.8) and (B.9)), (B.3) and (B.6) still hold. (B.6), (B.17) and (B.4) give:

$$V_{AN}^s = \frac{b\left(1-\delta^D\right)\left(1-\delta^I\right)}{1+\rho-\left(1-\delta^D\right)b\delta^I\gamma}\left(V_N^T - V_0^{T,n}\right). \tag{B.18}$$

1. We will not have to worry about this in the following cases because there the non-cooperating good match will be worse than the innovator for a producer not in a good match (by assumption), which therefore implies that the non-cooperating good match will be worse than the innovator also for a producer in a good match.

Moreover, we get (using (B.8), (B.17) and (B.18)):

$$\begin{aligned} W_N^p = V_I^{p,n} &= \frac{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}{1+\rho-\left(1-\delta^D\right)b\delta^I\gamma}V_N^T \\ &\quad -\left(1-\gamma^{-1}\right)\Pi\left(n\right)+\frac{b\left(1-\delta^D\right)\left(1-\delta^I\right)}{1+\rho-\left(1-\delta^D\right)b\delta^I\gamma}V_0^{T,n}. \end{aligned}$$

Plugging this into (B.3) leads to:

$$\begin{aligned} V_0^{T,n} &= \frac{\left(1+\rho-\left(1-\delta^D\right)b\delta^I\gamma\right)\left(1-b\right)V_1^T}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}+b\frac{1-\delta^D}{1+\rho}\delta^I\gamma V_N^T \\ &\quad+\frac{1+\rho-\left(1-\delta^D\right)b\delta^I\gamma}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}b\theta\Pi\left(n\right) \\ &\quad-\frac{1+\rho-\left(1-\delta^D\right)b\delta^I\gamma}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}b\frac{1-\delta^D}{1+\rho}\delta^I\gamma\left(1-\gamma^{-1}\right)\Pi\left(n\right). \end{aligned}$$

Using (B.9), we further get:

$$\begin{aligned} V_0^{T,n} &= \frac{\left(1+\rho-\left(1-\delta^D\right)b\delta^I\gamma\right)\left(\left(1-b\right)V_1^T+b\theta\Pi\left(n\right)\right)}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)} \\ &\quad+b\frac{1-\delta^D}{1+\rho}\delta^I\gamma\Pi\left(n\right) \\ &\quad\times\left(\frac{1+\rho-\left(1-\delta^D\right)\delta^I\gamma\left(1-\gamma^{-1}\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}-\frac{\left(1+\rho-\left(1-\delta^D\right)b\delta^I\gamma\right)\left(1-\gamma^{-1}\right)}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}\right). \end{aligned} \tag{B.19}$$

We can then express the conditions of (B.16) as:

$$\begin{aligned} &\frac{\gamma^{-1}\left(1+\rho\right)-b\left(1-\delta^D\right)+\left(1-\delta^D\right)\left(1-\delta^I\right)\left(1-\gamma^{-1}\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}\Pi\left(n\right) \\ &\leq\left(1-b\right)V_1^T+b\theta\Pi\left(n\right) \\ &\leq\frac{\left(1+\rho-b\left(1-\delta^D\right)-\left(1-\delta^D\right)\delta^I\left(\gamma-1\right)\right)\Pi\left(n\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)} \\ &-\left(\left(1-\gamma^{-1}\right)\left(1-b\theta\right)\Pi\left(n\right)-\left(1-b\right)\left(\Pi\left(x^*\right)-\gamma^{-1}\Pi\left(y^*\right)\right)\right)^+. \end{aligned} \tag{B.20}$$

We now move on to compute the incentive constraint of a producer in a good match. Using (B.17) and (B.6), we get:

$$\left(1-\delta^I\right)V_N^s+\delta^I\gamma W_N^s=\frac{\left(1+\rho\right)\left(1-\delta^I\right)V_N^s}{1+\rho-\left(1-\delta^D\right)b\delta^I\gamma}. \tag{B.21}$$

In this case (A.10) and (A.12) apply, combining them with (A.11), (B.4) and (B.21) we can express the reward from cooperation I as:

$$I = \frac{(1+\rho)\left(\left(1-\delta^I\right)V_1^s + \delta^I\gamma\left(\frac{1}{\gamma}\Pi\left(y^*\right) - \left(\left(1-b\right)\Pi\left(x^*\right) + b\theta\Pi\left(n\right)\right)\right)^+\right)}{1+\rho-b\left(1-\delta^D\right)\delta^I\gamma} - \frac{(1+\rho)\left(1-\delta^I\right)V_N^s}{1+\rho-\left(1-\delta^D\right)b\delta^I\gamma}.$$

Hence, we get:

$$I = \frac{(1+\rho)\left(\left(1-\delta^I\right)\left(V_1^T - V_N^T\right) + \delta^I\gamma\left(\frac{1}{\gamma}\Pi\left(y^*\right) - \left(\left(1-b\right)\Pi\left(x^*\right) + b\theta\Pi\left(n\right)\right)\right)^+\right)}{1+\rho-b\left(1-\delta^D\right)\delta^I\gamma} \tag{B.22}$$

$$= \frac{1+\rho}{1+\rho-b\left(1-\delta^D\right)\delta^I\gamma}\times \left(\begin{array}{c} \frac{\left(1-\delta^I\right)\left(\left(1+\rho-\left(1-\delta^D\right)\delta^I\gamma\right)\Pi(x^*)+\left(1-\delta^D\right)\delta^I\Pi(y^*)-\left(1+\rho-\left(1-\delta^D\right)\delta^I\gamma\left(1-\gamma^{-1}\right)\right)\Pi(n)\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)} \\ +\delta^I\gamma\left(\frac{1}{\gamma}\Pi\left(y^*\right) - \left(\left(1-b\right)\Pi\left(x^*\right) + b\theta\Pi\left(n\right)\right)\right)^+ \end{array}\right).$$

Recall that we needed to check that when a producer in a good match switches to the innovator, sticking to the old supplier remains better than trying out a new outdated supplier, that is we need to check that $W_1^p = W_1^T - V_A^s > W_0^{T,n}$. Using that $V_1^p = V_n^p = V_0^{T,n}$, we get:

$$\begin{aligned} W_1^p &= \frac{1}{\gamma}\Pi\left(y^*\right) \\ &\quad + \frac{1-\delta^D}{1+\rho}\left(\left(1-b\right)\left(\left(1-\delta^I\right)V_1^T + \delta^I\gamma W_1^T\right) + b\left(\left(1-\delta^I\right)V_0^{T,n} + \delta^I\gamma W_1^p\right)\right) \\ W_0^{T,n} &= \frac{1-b}{\gamma}\Pi\left(y^*\right) + \frac{b\theta\Pi\left(n\right)}{\gamma} \\ &\quad + \frac{1-\delta^D}{1+\rho}\left(\left(1-b\right)\left(\left(1-\delta^I\right)V_1^T + \delta^I\gamma W_1^T\right) + b\left(\left(1-\delta^I\right)V_0^{T,n} + \delta^I\gamma W_0^{T,n}\right)\right) \end{aligned}$$

so the inequality is satisfied.

Case 2.2. As for case 1 2, we now consider the same situation as in case 2.1 except that the producer knows two non-cooperating good match suppliers. To ensure the existence of the equilibrium we need that the conditions to be in case 2.2 are the same as the conditions to be in case 2.1, and that the IC constraint that we derive here (the IC constraint for a producer in a good match who knows a non-cooperating good match) is the same as the incentive constraint derived in case 2..1. (B.1), (B.2) (and therefore

(B.8) and (B.9)) still hold. Bertrand competition now leads to (B.14), so that (B.3) gives (B.15) as in case 1 2. The conditions to be in that case now writes as

$$V_N^T \geq V_0^{T,n} \text{ and } W_0^{T,n} \leq W_N^T \leq V_I^{T,n}, \tag{B.23}$$

as $V_{AN}^s = 0$. Using equations (B.15), (B.8) and (B.9) we get that these conditions are equivalent to (B.20) as it should.

For the IC constraint, (A.13) gives $V_1^s = V_1^T - V_N^T$, (A.10) still holds, so using (B.14) we directly get that I is given by (B.22) as it should.

Note that we need to check that when the producer does not switch to the innovator, sticking to a good match supplier remains a better option than going for the innovator, that is we need to check that $W_1^p = W_1^T - V_A^s$ remains greater than W_N^T this is direct because:

$$\begin{aligned} W_1^p = &\frac{1}{\gamma}\Pi\left(y^*\right) + \frac{1-\delta^D}{1+\rho}\Bigg((1-b)\left(\left(1-\delta^I\right)V_1^T + \delta^I\gamma W_1^T\right) \\ &+ b\left(\left(1-\delta^I\right)V_N^T + \delta^I\gamma W_1^p\right)\Bigg) \\ W_N^T = &\frac{1}{\gamma}\Pi\left(y^*\right) + \frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_N^T + \delta^I\gamma W_N^T\right), \end{aligned}$$

and $\left(1-\delta^I\right)V_1^T + \delta^I\gamma W_1^T > \left(1-\delta^I\right)V_N^T + \delta^I\gamma W_1^p$.

Case 3.1. We now consider the case where the producer would rather stick to the non-cooperating good match in periods without innovation, but, in periods with innovations, the non-cooperating good match is worse than even a new outdated supplier. We consider a producer who only knows one non-cooperating good match. The conditions to be in that case can then be expressed as:

$$V_N^T \geq V_0^{T,n} \text{ and } \left(W_N^T - V_{AN}^s\right) \leq W_0^{T,n}.$$

As a consequence, the value of a producer in a period without innovation when he knows a non-cooperating good match is given by

$$V_I^{p,n} = W_0^{T,n}. \tag{B.24}$$

We get that (B.1) must be replaced by:

$$V_N^T = \Pi(n) + \frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_N^T + \delta^I\gamma\left(W_0^{T,n} + V_{AN}^s\right)\right), \tag{B.25}$$

in a period with innovation, the value of the non-cooperating good match supplier is indeed not null and given by V_{AN}^s, where V_{AN}^s is given by

$$V_{AN}^s = \frac{1-\delta^D}{1+\rho}b\left(\left(1-\delta^I\right)V_N^s + \delta^I\gamma V_{AN}^s\right). \tag{B.26}$$

Note that (B.8) still holds. (B.3) is replaced by:

$$V_0^{T,n} = (1-b)\,V_1^T + b\theta\Pi(n) + b\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_N^p + \delta^I\gamma V_I^{p,n}\right), \quad \text{(B.27)}$$

while the value of starting a relationship with an outdated supplier is given by:

$$W_0^{T,n} = V_0^{T,n} - (1-b)\left(\Pi\left(x^*\right) - \gamma^{-1}\Pi(y)\right) - b\theta\left(1-\gamma^{-1}\right)\Pi(n). \quad \text{(B.28)}$$

Bertrand competition still leads to (B.4), which, together with (B.24), (B.27) and (B.28) gives:

$$\begin{aligned} V_0^{T,n} \;=\; & \frac{(1+\rho)\left((1-b)\,V_1^T + b\theta\Pi(n)\right)}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)} \\ & -\frac{\delta^I\gamma b\left(1-\delta^D\right)(1-b)\left(\Pi(x^*)-\gamma^{-1}\Pi(y^*)\right)}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)} \\ & -\frac{b\left(1-\delta^D\right)\delta^I\gamma b\theta\left(1-\gamma^{-1}\right)\Pi(n)}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}. \end{aligned} \quad \text{(B.29)}$$

Now (B.4) and (B.26) give:

$$V_{AN}^s = \frac{b\left(1-\delta^D\right)\left(1-\delta^I\right)}{1+\rho-\left(1-\delta^D\right)b\delta^I\gamma}\left(V_N^T - V_0^{T,n}\right). \quad \text{(B.30)}$$

Combining (B.25), (B.28), (B.29) and (B.30), we get:

$$\begin{aligned} V_N^T = & \frac{1+\rho-\left(1-\delta^D\right)b\delta^I\gamma}{1+\rho-\left(1-\delta^I\right)\left(1-\delta^D\right)-\left(1-\delta^D\right)b\delta^I\gamma}\Pi(n) \\ & + \frac{\left(1-\delta^D\right)\delta^I\gamma\left((1-b)\,V_1^T + b\theta\Pi(n)\right)}{1+\rho-\left(1-\delta^I\right)\left(1-\delta^D\right)-\left(1-\delta^D\right)b\delta^I\gamma} \\ & - \frac{\left(1-\delta^D\right)\delta^I\gamma(1-b)\left(\Pi(x^*)-\gamma^{-1}\Pi(y^*)\right)}{1+\rho-\left(1-\delta^I\right)\left(1-\delta^D\right)-\left(1-\delta^D\right)b\delta^I\gamma} \\ & - \frac{\left(1-\delta^D\right)\delta^I\gamma b\theta\left(1-\gamma^{-1}\right)\Pi(n)}{1+\rho-\left(1-\delta^I\right)\left(1-\delta^D\right)-\left(1-\delta^D\right)b\delta^I\gamma}. \end{aligned} \quad \text{(B.31)}$$

The condition $V_N^T \geq V_0^{T,n}$ and $\left(W_N^T - V_{AN}^s\right) \leq W_0^{T,n}$ then translate into:

$$\begin{aligned}
&\frac{\left(1+\rho-b\left(1-\delta^D\right)-\left(1-\delta^D\right)\delta^I\left(\gamma-1\right)\right)\Pi\left(n\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)} \\
&-\left(\left(1-b\theta\right)\left(1-\gamma^{-1}\right)\Pi\left(n\right)-\left(1-b\right)\left(\Pi\left(x^*\right)-\gamma^{-1}\Pi\left(y^*\right)\right)\right) \\
&\leq\left(1-b\right)V_1^T+b\theta\Pi\left(n\right) \\
&\leq\frac{1+\rho-b\left(1-\delta^D\right)-\left(1-\delta^D\right)\delta^I\left(\gamma-1\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}\Pi\left(n\right) \\
&+\left(\frac{\left(1-b\right)\delta^I\gamma\left(1-\delta^D\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}\right) \\
&\times\left(\left(1-b\theta\right)\left(1-\gamma^{-1}\right)\Pi\left(n\right)-\left(1-b\right)\left(\Pi\left(x^*\right)-\gamma^{-1}\Pi\left(y^*\right)\right)\right),
\end{aligned} \tag{B.32}$$

this case exists only when $\left(1-b\theta\right)\left(1-\gamma^{-1}\right)\Pi\left(n\right)-\left(1-b\right)\left(\Pi\left(x^*\right)-\gamma^{-1}\Pi\left(y^*\right)\right)\geq 0$.

We now move to express the IC constraint. First note that (B.4) and (B.30) lead to

$$\left(1-\delta^I\right)V_N^s+\delta^I\gamma V_{AN}^s=\frac{\left(1+\rho\right)\left(1-\delta^I\right)}{1+\rho-\left(1-\delta^D\right)b\delta^I\gamma}\left(V_N^T-V_0^{T,n}\right).$$

Combining this with (A.10), (A.12), (A.11) and (B.31), we get:

$$\begin{aligned}
I=&\frac{1+\rho}{1+\rho-b\left(1-\delta^D\right)\delta^I\gamma} \\
&\times\left(\begin{array}{c}\frac{\left(1-\delta^I\right)\left(\left(1+\rho-b\left(1-\delta^D\right)\delta^I\gamma\right)\left(\Pi\left(x^*\right)-\Pi\left(n\right)\right)+b\left(1-\delta^D\right)\delta^I\left(\Pi\left(y^*\right)-\theta\Pi\left(n\right)\right)\right)}{\left(1+\rho-\left(1-\delta^I\right)\left(1-\delta^D\right)-\left(1-\delta^D\right)b\delta^I\gamma\right)} \\ +\delta^I\gamma\left(\frac{1}{\gamma}\Pi\left(y^*\right)-\left(\left(1-b\right)\Pi\left(x^*\right)+b\theta\Pi\left(n\right)\right)\right)\end{array}\right).
\end{aligned} \tag{B.33}$$

Checking that trying a new outdated supplier is worse than staying with a good match supplier for a producer when innovation occurs proceeds as in case 2.

Case 3.2. As before we redo this case assuming that there are several non-cooperating good match suppliers. The condition now writes as:

$$V_N^T > V_0^{T,n} \text{ and } W_N^T < W_0^{T,n}. \tag{B.34}$$

Bertrand competition leads to

$$V_N^s = W_N^s = V_{AN}^s = 0,\ V_N^p = V_N^T \text{ and } V_I^{p,n} = W_0^{T,n}. \tag{B.35}$$

(B.25) is replaced by

$$V_N^T = \Pi\left(n\right)+\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_N^T+\delta^I\gamma W_0^{T,n}\right), \tag{B.36}$$

and (B.27) by:

$$V_0^{T,n} = (1-b)V_1^T + b\theta\Pi(n) + b\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_N^T + \delta^I\gamma W_0^{T,n}\right), \quad \text{(B.37)}$$

while (B.8) and (B.28) still hold. Using (B.28) and (B.37) we can now write:

$$V_0^{T,n} = \frac{(1+\rho)\left((1-b)V_1^T + b\theta\Pi(n)\right)}{1+\rho-b\left(1-\delta^D\right)\delta^I\gamma} + \frac{b\left(1-\delta^D\right)}{1+\rho-b\left(1-\delta^D\right)\delta^I\gamma}\left(1-\delta^I\right)V_N^T \quad \text{(B.38)}$$
$$-\frac{b\left(1-\delta^D\right)\delta^I\gamma\left((1-b)\left(\Pi(x^*)-\gamma^{-1}\Pi(y^*)\right)+b\theta\left(1-\gamma^{-1}\right)\Pi(n)\right)}{1+\rho-b\left(1-\delta^D\right)\delta^I\gamma},$$

which combined with (B.8) and (B.36) gives:

$$V_N^T = \frac{1+\rho-b\left(1-\delta^D\right)\delta^I\gamma}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I\right)-b\left(1-\delta^D\right)\delta^I\gamma}\Pi(n) \quad \text{(B.39)}$$
$$+\frac{\delta^I\gamma\left(1-\delta^D\right)\left((1-b)V_1^T+b\theta\Pi(n)\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I\right)-b\left(1-\delta^D\right)\delta^I\gamma}$$
$$-\frac{\delta^I\gamma\left(1-\delta^D\right)\left((1-b)\left(\Pi(x^*)-\gamma^{-1}\Pi(y^*)\right)+b\theta\left(1-\gamma^{-1}\right)\Pi(n)\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I\right)-b\left(1-\delta^D\right)\delta^I\gamma},$$

which plugged back in (B.38) leads to:

$$V_0^{T,n} = \frac{\left(1+\rho-\left(1-\delta^D\right)\left(1-\delta^I\right)\right)\left((1-b)V_1^T+b\theta\Pi(n)\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I\right)-b\left(1-\delta^D\right)\delta^I\gamma}$$
$$+\frac{b\left(1-\delta^D\right)\left(1-\delta^I\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I\right)-b\left(1-\delta^D\right)\delta^I\gamma}\Pi(n)$$
$$-\frac{b\left(1-\delta^D\right)\delta^I\gamma\left((1-b)\left(\Pi(x^*)-\gamma^{-1}\Pi(y^*)\right)+b\theta\left(1-\gamma^{-1}\right)\Pi(n)\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I\right)-b\left(1-\delta^D\right)\delta^I\gamma}$$

Using these last expressions, we can rewrite (B.34) as (B.32) (which is necessary to get the equilibrium in the first place).

Finally (A.13) gives $V_1^s = V_1^T - V_N^T$, (A.10) still holds, so using (A.11), (B.35) and (B.39), we can express I exactly as in (B.33).

Case 4. We now consider the case where a producer not in a good match would rather look for a new supplier than stick to a non-cooperating good match in periods without innovations, while in periods with innovation he tries out the innovator but the non-cooperating good match represents a better alternative than trying out an outdated supplier. Note that no matter what, the non-cooperating good match actually never works with the producer, his value is then always null and it does not matter whether the producer knows only one non-cooperating good match or more. The conditions to be in that case can then be expressed as:

$$V_N^T < V_0^{T,n} \text{ and } W_0^{T,n} < W_N^T. \tag{B.40}$$

Bertrand competition implies that

$$V_I^{p,n} = W_N^T.$$

We then get

$$V_N^T = \Pi(n) + \frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_0^{T,n} + \delta^I \gamma W_N^T\right), \tag{B.41}$$

and

$$V_0^{T,n} = (1-b)V_1^T + b\theta\Pi(n) + b\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_0^{T,n} + \delta^I\gamma W_N^T\right), \tag{B.42}$$

while (B.8) and (B.28) still hold.

Using (B.8), (B.41) and (B.42), we can write $V_0^{T,n}$ as:

$$V_0^{T,n} = \frac{\left(1+\rho-\left(1-\delta^D\right)\delta^I\gamma\right)\left((1-b)V_1^T + b\theta\Pi(n)\right) + b\delta^I\left(1-\delta^D\right)\Pi(n)}{1+\rho-\left(1-\delta^D\right)\delta^I\gamma - b\left(1-\delta^D\right)\left(1-\delta^I\right)}, \tag{B.43}$$

plugging this back in (B.41) and using (B.8), we get:

$$\begin{aligned}
&V_N^T \\
&= \frac{\left(1+\rho-\left(1-\delta^D\right)\delta^I\gamma\left(1-\gamma^{-1}\right)\right)\left(1+\rho-\left(1-\delta^D\right)\delta^I\gamma - b\left(1-\delta^D\right)\left(1-\delta^I\right)\right)\Pi(n)}{\left(1+\rho-\left(1-\delta^D\right)\delta^I\gamma\right)\left(1+\rho-\left(1-\delta^D\right)\delta^I\gamma - b\left(1-\delta^D\right)\left(1-\delta^I\right)\right)} \\
&= \frac{-b\delta^I\left(1-\delta^D\right)^2\left(1-\delta^I\right)\Pi(n)}{\left(1+\rho-\left(1-\delta^D\right)\delta^I\gamma\right)\left(1+\rho-\left(1-\delta^D\right)\delta^I\gamma - b\left(1-\delta^D\right)\left(1-\delta^I\right)\right)} \\
&+ \frac{\left(1-\delta^D\right)\left(1-\delta^I\right)}{1+\rho-\left(1-\delta^D\right)\left(\delta^I\gamma\right) - b\left(1-\delta^D\right)\left(1-\delta^I\right)}\left((1-b)V_1^T + b\theta\Pi(n)\right).
\end{aligned}$$

Now combining these two last expressions with (B.8) and (B.28) we can rewrite (B.40) as:

$$\begin{aligned}
&\frac{\left(1+\rho-b\left(1-\delta^D\right)-\left(1-\delta^D\right)\delta^I(\gamma-1)\right)\Pi(n)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)} \\
&< (1-b)V_1^T + b\theta\Pi(n) \\
&< \frac{\left(1+\rho-\left(1-\delta^D\right)\delta^I(\gamma-1) - b\left(1-\delta^D\right)\right)\Pi(n)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)} \\
&+ \frac{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I\right)-\left(1-\delta^D\right)\delta^I\gamma}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)} \\
&\times\left((1-b)\left(\Pi\left(x^*\right)-\gamma^{-1}\Pi\left(y^*\right)\right)-(1-b\theta)\left(1-\gamma^{-1}\right)\theta\Pi(n)\right),
\end{aligned} \tag{B.44}$$

note that this case requires that $(1-b)\left(\Pi\left(x^*\right)-\gamma^{-1}\Pi\left(y^*\right)\right)>(1-b\theta)\left(1-\gamma^{-1}\right)\theta\Pi\left(n\right).$

Finally to express the incentive constraint, first note that (A.12) holds so combining (A.11) and (B.43), we get:

$$V_1^s=\frac{b\left(\left(1+\rho-\left(1-\delta^D\right)\delta^I\gamma\right)\left(\Pi\left(x^*\right)-\theta\Pi\left(n\right)\right)+\left(1-\delta^D\right)\delta^I\left(\Pi\left(y^*\right)-\Pi\left(n\right)\right)\right)}{1+\rho-\left(1-\delta^D\right)\left(\delta^I\gamma\right)-b\left(1-\delta^D\right)\left(1-\delta^I\right)},$$

now, as (A.10) holds, we get:

$$I=\frac{1+\rho}{1+\rho-b\left(1-\delta^D\right)\delta^I\gamma}\times\left(\begin{array}{c}\left(1-\delta^I\right)b\frac{\left(1+\rho-\left(1-\delta^D\right)\delta^I\gamma\right)\left(\Pi\left(x^*\right)-\theta\Pi\left(n\right)\right)+\left(1-\delta^D\right)\delta^I\left(\Pi\left(y^*\right)-\Pi\left(n\right)\right)}{1+\rho-\left(1-\delta^D\right)\left(\delta^I\gamma\right)-b\left(1-\delta^D\right)\left(1-\delta^I\right)}\\+\delta^I\gamma\left(\frac{1}{\gamma}\Pi\left(y^*\right)-\left(\left(1-b\right)\Pi\left(x^*\right)+b\theta\Pi\left(n\right)\right)\right)^+\end{array}\right).$$

Further, note that when a producer in a good match switches to the innovator, we do get that staying with the good match supplier is indeed the second best option and not switching to the non-cooperating good match, that is $W_1^p=W_1^T-V_A^s>W_N^T$.

Case 5. We treated that case in Appendix A.1, except that we did not derive the conditions to be in it. Case 5 occurs when $V_N^T<V_0^T$ and $W_N^T<W_0^T$.

In a period without innovation, the joint value of a relationship with the non-cooperating good match now obeys

$$V_N^T=\Pi\left(n\right)+\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_0^T+\delta^I\gamma W_0^T\right). \tag{B.45}$$

Indeed, after one period, the producer will look for a new supplier in a period without innovation, and in a period with innovation a new outdated supplier will be his second best option (after the innovator). Similarly in a period with innovation, the joint value of a relationship with a non-cooperating good match (necessarily outdated) is given by:

$$W_N^T=\frac{1}{\gamma}\Pi\left(n\right)+\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_0^T+\delta^I\gamma W_0^T\right). \tag{B.46}$$

Combining (B.45) and (B.46) with (6) and (A.15), we obtain that:

$$W_0^T-W_N^T=$$
$$V_0^T-V_N^T-\left(\left(1-b\right)\left(\Pi\left(x^*\right)-\gamma^{-1}\Pi\left(y^*\right)\right)-\left(1-b\theta\right)\left(1-\gamma^{-1}\right)\Pi\left(n\right)\right) \tag{B.47}$$

Therefore if $(1-b)\left(\Pi\left(x^*\right)-\gamma^{-1}\Pi\left(y^*\right)\right)>(1-b\theta)\left(1-\gamma^{-1}\right)\Pi\left(n\right)$, then $W_N^T<W_0^T$ is the stricter constraint and otherwise $V_N^T<V_0^T$ is the stricter one.

Combining (B.45), (6) and (A.15), we further get:

$$
\begin{aligned}
V_0^T - V_N^T =& \frac{\left(1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)\right)\left((1-b)V_1^T+b\theta\Pi(n)\right)}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)} \\
&-\frac{\left(1+\rho-b\left(1-\delta^D\right)-\left(1-\delta^D\right)\delta^I(\gamma-1)\right)\Pi(n)}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)} \\
&-\frac{(1-b)\left(1-\delta^D\right)\delta^I\gamma\left[(1-b\theta)\left(1-\gamma^{-1}\right)\Pi(n)-(1-b)\left(\Pi(x^*)-\frac{1}{\gamma}\Pi(y^*)\right)\right]}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}
\end{aligned}
\tag{B.48}
$$

Therefore $V_0^T > V_N^T$ is equivalent to

$$
\begin{aligned}
&(1-b)V_1^T+b\theta\Pi(n) \\
>\;& \frac{1+\rho-b\left(1-\delta^D\right)-\left(1-\delta^D\right)\delta^I(\gamma-1)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}\Pi(n) \\
&+\frac{(1-b)\left(1-\delta^D\right)\delta^I\gamma\left[(1-b\theta)\left(1-\gamma^{-1}\right)\Pi(n)-(1-b)\left(\Pi(x^*)-\frac{1}{\gamma}\Pi(y^*)\right)\right]}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}
\end{aligned}
\tag{B.49}
$$

Combining (B.47) with (B.48), we obtain that $W_0^T > W_N^T$ is equivalent to:

$$
\begin{aligned}
&(1-b)V_1^T+b\theta\Pi(n) \\
>\;& \frac{1+\rho-b\left(1-\delta^D\right)-\left(1-\delta^D\right)\delta^I(\gamma-1)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}\Pi(n) \\
&+\frac{\left(1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I\right)-\left(1-\delta^D\right)\delta^I\gamma\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)} \\
&\times\left((1-b)\left(\Pi\left(x^*\right)-\gamma^{-1}\Pi\left(y^*\right)\right)-(1-b\theta)\left(1-\gamma^{-1}\right)\Pi(n)\right).
\end{aligned}
\tag{B.50}
$$

Recalling that $W_N^T < W_0^T$ is the stricter constraint if and only if $(1-b)\times\left(\Pi(x^*)-\gamma^{-1}\Pi(y^*)\right) > (1-b\theta)\left(1-\gamma^{-1}\right)\Pi(n)$, we can combine (B.49) and (B.50), to get that the equilibrium is in case 5 if and only if:

$$
\begin{aligned}
&(1-b)V_1^T+b\theta\Pi(n) \\
>\;& \frac{1+\rho-b\left(1-\delta^D\right)-\left(1-\delta^D\right)\delta^I(\gamma-1)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}\Pi(n) \\
&+\frac{(1-b)\left(1-\delta^D\right)\delta^I\gamma}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}\times \\
&\left((1-b\theta)\left(1-\gamma^{-1}\right)\theta\Pi(n)-(1-b)\left(\Pi\left(x^*\right)-\gamma^{-1}\Pi\left(y^*\right)\right)\right)^+ \\
&+\frac{1+\rho-\left(1-\delta^D\right)\left(b\left(1-\delta^I\right)+\delta^I\gamma\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}\times \\
&\left((1-b)\left(\Pi\left(x^*\right)-\gamma^{-1}\Pi\left(y^*\right)\right)-(1-b\theta)\left(1-\gamma^{-1}\right)\theta\Pi(n)\right)^+.
\end{aligned}
\tag{B.51}
$$

Summary. Overall conditions (B.10), (B.20), (B.32), (B.44) and (B.51) span all the possibilities. Moreover, a producer who knows a non-cooperating good match and one who does not always face the same incentive constraint, so that the profile of investment levels played by a new supplier can indeed be the same no matter whether the producer knows other good matches or not. The IC constraint of a good match supplier only takes two forms depending on whether the supplier has access to the frontier technology or not.

B.1.2. Part 4. Therefore to prove the existence of the equilibrium, the last step is to show that there always exists a solution to x^* and y^* such that IC constraint binds or the first best is achieved. As argued in Appendix A.1, we simply need to show the IC constraints do not bind for (x, y) just above n, and since n minimizes φ, it is enough to show that I is positive at the first order in $(x-n)$ and $(y-n)$ when x and y are greater than n.

Note that as $\left((1-b)\left(\Pi(x)-\gamma^{-1}\Pi(y)\right)-(1-b\theta)\left(1-\gamma^{-1}\right)\Pi(n)\right)=-(1-\theta)\times\left(1-\gamma^{-1}\right)b\Pi(n)+O(x-n)+O(y-n)$, the only possible cases when x, y are close to n are 1, 2, 3 and 5. We have already checked that I is positive for x and y just above n in case 5. In case 1, we get

$$I=\Pi'(n)\frac{(1+\rho)\left(\left(1-\delta^I\right)(x-n)+\delta^I(y-n)\right)}{1+\rho-\left(1-\delta^D\right)\left(1+\delta^I(\gamma-1)\right)}+o(x-n)+o(y-n),$$

which is positive at first order in $(x-n)$, $(y-n)$ when x and y approach n by superior values. In case 2, we get

$$I=\frac{1+\rho}{1+\rho-b\left(1-\delta^D\right)\delta^I\gamma}\left(\begin{array}{c}\Pi'(n)\frac{\left(1-\delta^I\right)\left(1+\rho-\left(1-\delta^D\right)\delta^I\gamma\right)(x-n)+\left(1-\delta^D\right)\delta^I(y-n)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}\\+\delta^I\gamma\left(\frac{1}{\gamma}\Pi(y)-\left((1-b)\Pi(x)+b\theta\Pi(n)\right)\right)^+\end{array}\right)\tag{B.52}$$

$$+o(x-n)+o(y-n),$$

also positive at first order in $(x-n)$, $(y-n)$ when x and y approach n by superior values. In case 3, we get

$$\begin{aligned}I\ =\ &\frac{1+\rho}{1+\rho-b\left(1-\delta^D\right)\delta^I\gamma}\frac{\left(1-\delta^I\right)b\left(1-\delta^D\right)\delta^I(1-\theta)\Pi(n)}{1+\rho-\left(1-\delta^I\right)\left(1-\delta^D\right)-\left(1-\delta^D\right)b\delta^I\gamma}\\&+\frac{(1+\rho)\delta^I\gamma}{1+\rho-b\left(1-\delta^D\right)\delta^I\gamma}\left(\frac{1}{\gamma}\Pi(y)-\left((1-b)\Pi(x)+b\theta\Pi(n)\right)\right)^+\\&+O(x-n)+O(y-n),\end{aligned}$$

which is positive at first order in $(x-n)$, $(y-n)$. Therefore, there always exist x^* and y^* solutions to the problem above n. This achieves the proof.

B.2. Proof of Proposition A.1

Here we prove Proposition A.1.

Step 1. [Incentive constraint]

The incentive constraint must be of the following form. After a history of h_t when a good match supplier makes her investment decision she can invest n instead of the prescribed $z(h_t)$, which would increase ex-post profits this period by $\varphi(z(h_t)) A_k(h_t)$, where $A_k(h_t)$ is the technology of supplier and

$$\varphi(z) \equiv \beta R(n) - n - (\beta R(z) - z)$$

Denoting by $I \in \{0,1\}$ either no new innovation ($I = 0$) or a new innovation ($I = 1$) we can express the incentive constraint as:

$$\varphi(z(h_t)) A_k(h_t) \leq \frac{1-\delta^D}{1+\rho} \times \tag{B.53}$$

$$\left(\begin{array}{c} \left(1-\delta^I\right) V^{s,k}(h_t \cup \{z(h_t)\} \cup \{I=0\}) + \delta^I V^{s,k}(h_t \cup \{z(h_t)\} \cup \{I=1\}) \\ -\left(\left(1-\delta^I\right) V^{s,k}(h_t \cup \{n\} \cup \{I=0\}) + \delta^I V^{s,k}(h_t \cup \{n\} \cup \{I=1\})\right) \end{array} \right),$$

where $V^{s,k}(h)$ denotes the value of the supplier after history h (The continuation value after a deviation other than n could be different, but the producer has no reason not to punish any deviation in the same way so we focus on the incentive not to play n).

Step 2. [Producers in a good match do not switch suppliers in periods without innovation]

Let us consider the first time the producer meets a good match supplier. Then this good match supplier has an advantage over any other supplier in the future except for a possible innovator, as a consequence any payoff achievable with another supplier is achievable with the good match supplier in periods without innovation. In order to maximize the joint value of the producer and the supplier, the producer should then stick to the supplier at least as long as no innovation occurs. Moreover if an innovation occurs, and the equilibrium is such that the producer should not switch to the innovator, the argument carries through. Note that the argument applies no matter whether the first good match supplier happens to be outdated or not, moreover, because of condition 1, this must be true for any relationship not only the first time the producer starts a relationship with a good match supplier.

Step 3. [Bertrand competition]

Condition 1 imposes that strategies once a producer has chosen a supplier are independent of the ex-ante transfer that was made, so that the ex-ante transfer does not affect the joint value of a relationship if the producer and the supplier end up working together. As a consequence the supplier whom the producer ends up working with, must offer an ex-ante transfer low enough that the value of the producer is the same as it would have been if he had chosen another supplier. In return, the supplier with whom the producer ends up being just indifferent to start working with or not must make an ex-ante transfer sufficiently high that he is himself just indifferent between working with the producer or not. A new supplier will then just break-even (as his value is zero if the producer does not choose him); and, the value of a good match supplier when an innovation occurs and for parameters such that the producer switches to the innovator, $V^{s,k}\left(h_t \cup \{z\left(h_t\right)\} \cup \{I=1\}\right)$, is not zero because the producer may go back to the old supplier after having tried the innovator.

Without condition 1 it is possible to build equilibria where the value of the producer is strictly higher than the value he gets with his second best option by conditioning normalized investment levels on the ex-ante transfer offered.

Step 4. [The joint value of a producer and a supplier is the value on the path where they never stop working together]

This is obvious in the case where the producer does not switch to the innovator. In the case where the producer does switch to the innovator, the take-it or leave-it offer of the innovator implies that the value of the producer should be equal to the value of the producer had he stayed with his old supplier, as specified by step 3. Now the old supplier is willing to offer an ex-ante transfer that leaves the producer with the entire joint value of a relationship with him had they stayed together minus what the supplier still gets when the producer switches to the innovator. Therefore the joint value of the producer and the old supplier is the joint value of the producer and the supplier on the path where the producer and the supplier do not break-up their relationships when there is an innovation. Note that on this path the argument of step 2 applies and the producer and the supplier never break up.

Step 5. [The value of the supplier is also determined by the value on the path where the producer and the supplier never stop working together]

Combining steps 3 and 4, the value of the supplier in periods without innovation is simply given by the joint value of the producer and the supplier on the path where they relationship never breaks down minus the value of the second best relationship that the producer can get. This reasoning extends to periods with innovations when the producer does not switch. When the producer switches, the value of the supplier is given by the rents he can capture in the future if the innovator turns out to be a bad match, so that the producer comes back to the supplier, and by condition 2, we know that the strategies must then be identical to the strategies if the producer had not switched. Therefore in this case too, the value of the supplier is ultimately determined by the joint value of the producer and the supplier in the future, on the path where they never stop working together.

Step 6. [Higher investment levels in the future increase the RHS of the IC constraint]

Higher investment levels on the path where the relationship does not break down then necessarily increases the joint value of the relationship of the producer and the supplier (step 4), as a consequence, they also increase the value of the supplier and make the IC constraint looser (step 5).

Note that condition 2 stipulates that if the producer switches and the innovator has turned out to be a bad match, the behavior of the producer and the supplier is identical to what they would have done had they stick together, so that the investment levels in that case are in fact the investment levels in the path where the producer and the supplier never break up. On the contrary, if the innovator turns out to be a good match, we know from step 2 that the producer would then stay with the innovator. As a consequence, the previous claim (the higher the investment on the path where the relationships does not break, the looser the IC constraints in previous periods) implies that the higher the investment in the relationship on the equilibrium path, the looser the IC constraints in the previous period.[2]

Step 7. [Investments are at first best or the IC constraint binds]

From step 6, it is then direct that investment in a good match relationship should be as high as possible (on the equilibrium path and on the path where the relationship never breaks down), therefore either the first best must be achieved or the IC constraint binds.

Step 8. [Punishment strategies]

To achieve the highest possible investment level, the RHS of the incentive constraint must in return be the highest possible. Therefore, the value of the supplier in case a deviation occurs must be as low as possible, this occurs if the supplier plays the Nash level of investment n in any future interaction between the producer and the supplier. The value of the supplier if no deviation occurs must be as high as possible.

Let us first consider the case of periods without innovation. Step 5 already ensures that the suppliers gets the largest possible value out of the relationship, so that the producer is just indifferent between staying in the current relationship or starting a new one (from which he would capture the entire benefit).[3] Now, to still ensure the highest value for the supplier it is then necessary that the value the producer can get with his second best option must be as low as possible. If the producer switches to a new supplier who turns out to be a bad match, then the producer may be willing to come back to the old supplier; as long as the first best is not achieved, the strategy of the old supplier should then be never to cooperate in that case, as cooperation in the future increases the value that the producer could capture by switching. If the producer switches to a new supplier and this supplier turns out to be a good match, then condition 1 specifies what the outcome is (and we come back to that case to discuss what the strategy of the old supplier must be in step 9). If the producer switches to a good match with whom a deviation has occurred, then the strategy of the old supplier should be such that - on that path- the producer does not cooperate again with him (for the same reason). In periods where innovation occurs, the same reasoning applies: Bertrand competition ensures that the supplier already gets the largest possible value of the relationship

2. Without condition 2 this would not necessarily be the case, lower investment levels if the producer switches to the innovator and comes back could reduce the incentive for the innovator to switch and therefore increase the joint value of the relationship.

3. One could then dispense with the assumption that normalized investment levels do not depend on the ex-ante transfers if the first best is not reached: condition 3 would then ensures that the supplier would capture the entire benefit of the relationship.

if the producer does not switch, while, otherwise, the value of the supplier is fixed according to step 5; if the producer deviates to an old good match supplier with whom a deviation has occurred, the strategy of the supplier must be such that he does not cooperate with the producer again in the future (a producer will necessarily prefer to deviate by switching to the innovator than a new outdated good match, so we don't have to specify what happens in that case); however condition 2 stipulates that if the producer switches to the innovator and the innovator turns out to be a good match, the old supplier must forgive the producer.

Step 9. [Investment by good match suppliers when a deviation has occurred]

Step 8 already specified that if a supplier deviates, his investment in the future must be at the Nash level. It also specified the behavior of the supplier if the producer has deviated but only found bad matches. Now let us focus on the case where the producer deviates and finds a new good match, we denote by k the previous supplier and by k' the new supplier. Condition 1 stipulates that the outcome should be identical to the outcome in the first interaction between the producer and a good match supplier. As explained in step 8, the investment level in the relationship with supplier k' is going to directly depend on the outside option of the producer. A better outside option for the producer implies a lower lower value for supplier k', and therefore (unless the first best is achieved), lower investment levels. When the producer met a good match supplier for the first time, however, there was no other good match supplier that the producer knew, so his outside option was a priori worse. To satisfy condition 1, we must then ensure that the value of the producer once he has started a relationship with supplier k' must be as low as possible if he is to switch to a different supplier. If switching to a new match is better than resuming working with the old good match supplier, then what the old good match supplier would do does not matter, however, if resuming working with supplier k is the best option, the value of a relationship with the supplier k must be as low as possible, which is achieved if supplier k plays the Nash level of investment in any possible future interaction.[4]

Therefore, as soon as the producer switches suppliers[5] (except if it is the innovator and the innovator turns out to be a bad match), or the supplier under-invests, investment in any future interaction between the producer and the supplier leads to the Nash level of normalized investment.

4. Note that without condition 1, the value of the supplier k could be increased if when the producer switches to the supplier k', some cooperation were to arise if the producer comes back to the supplier k in the future. This does not contradict condition 3 though, because condition 3 takes as given the strategy of the producer once the producer has started working with a new good match supplier. Note also that even if the old good match supplier keeps playing the Nash level in any future interaction there is no guarantee that it is possible to satisfy condition 1, part 3 of the proof of Proposition 1 showed that there is no contradiction though.

5. Technically this needs to be true only if working again with a good match supplier with whom a deviation has occurred is a better second option than starting over a relationship, for a producer who is in a new good match relationship. Of course, in the other case, the strategy of the old good match does not matter, so we can assume that he plays the Nash level without affecting the analysis.

Step 10. [Excepted strategies of the other players are identical in all periods]

Condition 1 stipulates how new good match suppliers are expected to behave in the future. Condition 4 stipulates that the investment level of bad matches is n, and we derived that the investment level of good match suppliers with whom a deviation has occurred must be at the Nash level n too. Moreover Bertrand competition determines the form of the ex-ante transfer that these suppliers are willing to offer: in periods without innovation they should all break even, in periods with innovation the innovator can capture the surplus from his innovation if the producer switches. Therefore when the supplier makes his investment decision all periods are identical in term of the strategies played by the other players, the only difference is that in periods where the supplier has access only to an outdated technology he knows that he will get access to the frontier technology in the next period.

Step 11. [Investment levels are constant]

The only thing that remains to be proved is that investment levels are the same in good matches in all periods when the supplier has access to the frontier technology and when the supplier has access only to an outdated technology, step 8 has already proved that the situation was symmetric in all periods where the supplier has access to the frontier technology and in all periods where he does not, so if a path of investment is sustainable after one given history, it is also sustainable after any other history (provided that the access to the frontier technology for the supplier is the same).

Let us then consider a history h_{t_0}, where a producer and a good match supplier starts working together for the first time. Let us denote by $x(h_t)$ and $y(h_t)$ the investment levels in all histories h_t belonging to the set of histories H_{t_0} of histories following h_{t_0} on the path where the relationship between the producer and the supplier never break-up their relationship, where $x(h_t)$ is used for histories where the supplier has access to the frontier technology and $y(h_t)$.for when he does not. The joint value of the producer and a supplier at an history $\widetilde{h_t} \in H_{t_0}$ is then simply the discounted sum of the expected values of profits on the path following $\widetilde{h_t}$ where the producer and the supplier never break up their relationship, that is it a discounted sum of the $\Pi(x(h_t))$ and $\Pi(y(h_t))$ for h_t following $\widetilde{h_t}$ in H_{t_0} Therefore, there is a M_1 such that if for all h_t following $\widetilde{h_t}$ in H_{t_0}, $|\Pi(x(h_t)) - \Pi(\widehat{x})| < \nu/M_1$ and $[\Pi(y(h_t)) - \Pi(\widehat{y})] < \nu/M_1$, then the joint value of the relationship is within ν of what the joint value of the relationship would have been if the investments levels where $\widehat{x}$ at all histories where the producer has access to the frontier technology and $\widehat{y}$ in histories where he does not, and by symmetry between the different periods, we can choose the same M_1 for all histories $h_t \in H_{t_0}$.

Now because of step 5, the value of the supplier (in all cases) is determined by the joint value of the producer and the supplier on the history path where the relationship does not break down, and, because of step 10 the strategies of the other players are the same over time, therefore there exists a M_2, such that if the joint value of the producer and the supplier is within v/M_2 of the joint value of the producer and the supplier if investment levels had been $\widehat{x}$ in periods where the supplier has the frontier technology and $\widehat{y}$ in the other periods, the value of the supplier is within ν of the value of the producer and the supplier if investment levels had followed the same alternative (and this M_2 can be the same for all histories $h_t \in H_{t0}$). Finally because the RHS of the IC constraint is just the discounted expectation of the value of the supplier in the next period, there is therefore a M_3 such that if the value of the supplier is (in both the case with innovation and the case without innovation) within ν/M_3 of the value of the supplier if the investment levels were given by $\widehat{x}$ in periods where the supplier has the frontier technology and $\widehat{y}$ otherwise, then the RHS of the IC constraint is within ν of the RHS of the IC constraint under the alternative profile.

Let us define $\overline{x} = \sup(x(h_t)|h_t \in H_{t_0})$ and $\overline{y} = \sup(x(y_t)|h_t \in H_{t_0})$. Then, for any $\varepsilon > 0$, there exists some $\eta > 0$ such that if $x(h_t) \in [\overline{x}-\eta, \overline{x}]$ and $y(h_t) \in [\overline{y}-\eta, \overline{y}]$, for all histories $h_t \in H_{t_0}$, the RHS of the IC constraint after any history $h_t \in H_{t_0}$ when the profile of normalized investment is given by $x\left(\widetilde{h_t}\right)$ $y\left(\widetilde{h_t}\right)$ where $\widetilde{h_t}$ are the histories following h_t in the set H_{t_0}, is weakly smaller than ε + the RHS of the IC constraint if the profile of normalized investment was given by $\overline{x}-\eta$, $\overline{y}-\eta$ (we just have to choose η such that if $x \in [\overline{x}-\eta, \overline{x}]$, and $y \in [\overline{y}-\eta, \overline{y}]$, then $|\Pi(x) - \Pi(\overline{x})| < \varepsilon/(M_1M_2M_3)$ and $|\Pi(y) - \Pi(\overline{y})| < \varepsilon/(M_1M_2M_3)$). Let us then define $\varphi_\varepsilon \equiv \varphi - \varepsilon$.

Moreover there exists a history $h_t^1 \in H_{t_0}$, where $x\left(h_t^1\right) \in [\overline{x} - \min(\eta, \varepsilon), \overline{x}]$, so that $x\left(h_t^1\right)$ must satisfy the IC constraint at history h_t^1, therefore it necessarily satisfies the IC constraint if the normalized investment in the future were given by $max\left(x\left(\widetilde{h_t}\right), \overline{x} - \min(\eta, \varepsilon)\right)$ and $max\left(y\left(\widetilde{h_t}\right), \overline{y} - \min(\eta, \varepsilon)\right)$ instead of the actual path $x\left(\widetilde{h_t}\right)$ $y\left(\widetilde{h_t}\right)$ where $\widetilde{h_t}$ are the histories following h_t^1 in the set H_{t_0}. Note then that, by the definition of η, $x\left(h_t^1\right)$ would satisfy the IC constraint if the incentive to deviate was given by φ_ε instead of ε and the profile of normalized investment levels was given by $\overline{x} - \min(\eta, \varepsilon)$ and $\overline{y} - \min(\eta, \varepsilon)$ in any future histories. Similarly we can find a history $h_t^2 \in H_{t_0}$, where $y\left(h_t^2\right) \in [\overline{y} - \min(\eta, \varepsilon), \overline{y}]$, and the same property arises.

Now let us consider the profile of normalized investment where for all histories $h_t \in h_{t_0}$, we replace $x(h_t)$ by $max(x(h_t), \overline{x} - \min(\eta, \varepsilon))$ and $y(h_t)$ by $max(y(h_t), \overline{y} - \min(\eta, \varepsilon))$, then this alternative profile necessarily leads to a strictly higher investment joint value (as long as $x(h_t)$ is not always equal to $\overline{x}$, and $y(h_t)$ is not always equal to $\overline{y}$) and for any history where the normalized investment level has not changed, the IC constraint remains satisfied. Let us now consider a history h_t' where the investment level has changed under the alternative profile and the supplier has access to the frontier technology, the profile of future investment levels is within $[\overline{x} - \min(\eta, \varepsilon), \overline{x}]$ and $[\overline{y} - \min(\eta, \varepsilon), \overline{y}]$, and we know that $x\left(h_t^1\right)$ (which is weakly larger than $\overline{x} - \min(\eta, \varepsilon)$) satisfies the IC constraint if the profile of future investment is given by $\overline{x} - \min(\eta, \varepsilon)$ and $\overline{y} - \min(\eta, \varepsilon)$ and φ is replaced by φ_ε, therefore the investment level $\overline{x} - \min(\eta, \varepsilon)$ also satisfies the IC constraint at history h_t' under the alternative profile provided that φ is replaced by φ_ε. The same logic applies to periods where the supplier does not have access to the frontier technology.

Thus, the alternative profile leads to a higher joint value and is sustainable up to replacing φ by φ_ε, letting ε goes to 0, we get that a profile with constant investment $\overline{x}$, $\overline{y}$ satisfies the IC constraint and yields a higher joint value. As a consequence, normalized investment must take two values only one for when the supplier has access to the frontier technology and one for when he has access to the outdated technology.

Furthermore the situation is symmetric whether during their first interaction the supplier has access to the frontier technology or not, so if he does not, investment levels are also determined by the same two constants. Finally, condition 1 stipulates that the profile of investment levels needs to be the same in any new good match relationship.

Step 12. [Summary]

So far we have then shown that in a SPNE equilibrium satisfying conditions 1-4:

1. investment levels in all good matches are given by two constant x^* and y^*, where the former is undertaken when the supplier has access to the best technology and the latter when he does not, as long as no deviation has occurred in the relationship between the producer and the supplier;
2. investment levels are at the first best level if possible and otherwise the IC constraint binds;
3. producers stay with the same supplier until an innovation or a deviation occurs, if an innovation occurs, the producer may or may not switch, but if he switches and the innovator turned out to be a bad match, he goes back to his old supplier;
4. producers are just indifferent between choosing the supplier they are supposed to work with on equilibrium path and choosing the "second best" supplier, the "second best" supplier is just indifferent between being chosen and not being chosen by the producer;
5. if a supplier deviates once, investment is at the Nash level in any further interaction, and - without loss of generality-[6] if the producer deviates (by switching to another supplier except if it is the innovator and the innovator turned out to be a bad match) investment is also at the Nash level in any future interaction.

These conditions correspond to the strategies described in Proposition 1.

B.3. Proof of Proposition A.2 and Remark A.1

We seek to demonstrate that both x^* and y^* are (weakly) increasing in γ and the conditions under which x^* and y^* are decreasing in δ^I. The proof of the rest of Proposition A.2 follows along the same lines and is omitted. We prove the proposition in the same special case as considered in the main text. The proof for the remaining cases is analogous and is omitted. Trivially, the effect is zero when x and y are equal

6. Stricto sensu, this is not necessary if a good match supplier with whom a deviation has occurred does not count, in the sense that he represents a worse alternative than trying a new supplier.

to the first best m. When they are not, equations (10) and (11) deliver:[7]

$$\varphi(x) - f(x,y,\gamma) = 0, \tag{B.54}$$

$$\varphi(y) - \gamma f(x,y,\gamma) = 0, \tag{B.55}$$

where

$$\varphi(x) = (\beta R(n) - n) - (\beta R(x) - x),$$

$$f(x,y,\gamma) = \frac{\left(1-\delta^D\right) b \left(1-\delta^I\right)}{1+\rho - b\left(1-\delta^D\right)\left(1-\delta^I\right) - b\left(1-\delta^D\right)\delta^I\gamma} \times \left(\begin{array}{c} \Pi(x) - \theta\Pi(n) \\ + \frac{b(1-\delta^D)\delta^I}{1+\rho-b(1-\delta^D)\delta^I\gamma}\left(\Pi(y) - \theta\Pi(n)\right) \end{array}\right).$$

We define $g_0(x,y,\gamma) \equiv \varphi(x) - f(x,y,\gamma)$ and $g_1(x,y,\gamma) \equiv \varphi(y) - \gamma f(x,y,\gamma)$ and note that $g_{0,\gamma} < 0$ and $g_{1,\gamma} < 0$, where a subscript denotes a partial derivative.[8] Then there are three cases i) both x and y equal to m if $g_0(m,m) \leq 0$ and $g_1(m,m) \leq 0$, ii) $y = m$ and $x < m$ as a solution to $g_0(x,m) = 0$ if $g_1(x,m) \leq 0$ and iii) x and y are solutions to $g_0(x,y) = 0$ if $g_1(x,y) = 0$.

As $\varphi(y)$ is convex and $f(x,y)$ is concave in y (as $\Pi(y)$ is concave in y) on $y\varepsilon\,[n,m]$ it follows that $g_1(x,y)$ is convex in y. Let us define $y = h(x)$ such that $h(x) = m$ if $g_1(x,m) < 0$ and otherwise $g_1(x,h(x)) = 0$ ($h(x)$ is single-valued as $g_1(x,h(x)) = 0$ has a unique solution from $g_1(x,n) < 0$ and convexity of $g_1(x,y)$ in y). Note that $h(x)$ is trivially increasing in x. Further, define $\widetilde{g}(x) = g_0(x,h(x))$ such that either i) $x = m$ and $\widetilde{g}(m) \leq 0$ ii) x is a solution to $\widetilde{g}(x) = 0$. Note that $\widetilde{g}(n) < 0$ and that $\widetilde{g}(x)$ is convex when $h(x)(= y)$ is constant and equal to m.

We first seek to demonstrate that $h(x)'' < 0$ (when not equal to m). Note first, that by concavity of $\Pi(x)$ it follows that $g_{1,x} < 0$ and $g_{1,xx} > 0$ and it has already been argued that g_1 is convex with $g_{1,y} > 0$ and $g_{1,yy} > 0$ (for $y < m$). Differentiate $g_1(x,h(x)) = 0$ twice and get:

$$g_{1,xx} + 2g_{1,xy}h'(x) + g_{1,yy}\left(h'(x)\right)^2 + g_{1,y}h''(x) = 0.$$

By inspection $g_{1,xy} = 0$ and hence $h''(x) < 0$ when $h(x) \neq m$. Along the same lines and using the properties of $h(x)$ we can show that $\widetilde{g}(x)$ is increasing and strictly convex in $x\varepsilon(n,m)$. Hence, if $\widetilde{g}(m) \leq 0$ then $x = m$ is optimal. As $\widetilde{g}(x)$ is decreasing in γ this implies that x is decreasing in γ and so is y.

To study the impact of an increase in the innovation rate, δ^I, note that we can rewrite $f(x,y,\gamma)$ as:

$$\frac{\left(1-\delta^D\right) b \left(1-\delta^I\right)}{\left[1+\rho - b\left(1-\delta^D\right)\left(1+\delta^I(\gamma-1)\right)\right]\left(1+\rho - b(1-\delta^D)\delta^I\gamma\right)} \times \tag{B.56}$$

7. To be consistent we should use (x^*, y^*) as these are the equilibrium values. This omission will not lead to confusion.

8. In order to avoid cluttering the notation we will suppress the dependence of g_0 and g_1 on γ

$$\left(\begin{array}{c}(1+\rho)\left(\Pi(x)-\theta\Pi(n)\right)\\-\delta^{I}(1-\delta^{D})b\left(\gamma\left(\Pi(x)-\theta\Pi(n)\right)-\left(\Pi(y)-\theta\Pi(n)\right)\right)\end{array}\right).$$

Below, we demonstrate that$\gamma\left(\Pi(x)-\theta\Pi(n)\right)>\Pi(y)-\Pi(n)$. Under this condition, a sufficient condition for the expression in equation (B.56) to be decreasing in δ^I (and thereby for x and y to be decreasing in δ^I) is that $\gamma<(1+\rho)/\left(b(1-\delta^D)(2-\delta^I)\right)$ as written in the proposition. All we need is therefore $\gamma\left(\Pi(x)-\theta\Pi(n)\right)>\Pi(y)-\Pi(n)$ which we now demonstrate.

First, define a function $\kappa(x)$ which is $\kappa(x)=\varphi^{-1}(\gamma\varphi(x))$ if $\gamma\varphi(x)\leq\varphi(m)$ and $\kappa(x)=m$ otherwise. We then define the function:

$$\lambda(x)\equiv\gamma\left(\Pi(x)-\theta\Pi(n)\right)-\left(\Pi(\kappa(x))-\theta\Pi(n)\right). \tag{B.57}$$

Note that as $\kappa(n)=n$ (as $\varphi(n)=0$), $\lambda(n)>0$, and

$$\lambda'(x)=\gamma\Pi'(x)-\Pi'(\kappa(x))\kappa'(x). \tag{B.58}$$

Obviously, on parts where κ is constant λ will be increasing. Where κ is not constant, $\kappa'(x)=\gamma\varphi'(x)/\varphi'(\kappa'(x))$. Using this in equation (B.58) returns:

$$\lambda'(x)=\gamma\Pi'(x)\left[1-\Pi'(\kappa(x))\varphi'(x)/\left(\Pi'(x)\varphi'(\kappa'(x))\right)\right].$$

With $\kappa(x)\geq x,\Pi(x)$ concave and $\varphi(x)$ convex, we get that $\lambda'(x)\geq0$, hence for any pair $x\varepsilon[n,m]$, $\lambda(x)>0$, in particular $\lambda(x)>0$ for the equilibrium investment pair (x,y), which completes the proof.

B.4. Proof of Propositions 3 and Remark 1

We derive necessary and sufficient conditions under which $\delta^{Nash}>\delta^{coop}$ in each of the three cases ($\gamma\leq\gamma^{Nash}$, $\gamma\in(\gamma^{Nash},\gamma^{coop}]$ and $\gamma>\gamma^{coop}$). Then we combine them to prove Propositions 3 and Remark 1.

First case. [Assume $\gamma<\gamma^{Nash}$] As part of Appendix B.3 we demonstrated that the function $\lambda(x)$ defined in (B.57) was increasing in x (note that this held regardless of whether after a deviation a producer preferred a non-cooperating good match to a new supplier or not). This directly implies that $\Pi(x)-\gamma^{-1}\Pi(\kappa(x))$, where κ is defined as in Appendix B.3, is also increasing in x. Therefore we must always have

$$\left(1-\gamma^{-1}\right)\Pi(n)<\Pi\left(x^*\right)-\gamma^{-1}\Pi\left(y^*\right)\leq\left(1-\gamma^{-1}\right)\Pi(m).$$

This shows directly that $Z_{Nash}-Z_{coop}$ in (20) is strictly negative, and that if $\gamma\leq\gamma^{Nash}$ then $\delta^{Nash}<\delta^{coop}$ (which proves the first part of Part b)).

Third case. [Assume $\gamma>\gamma^{coop}$] Then using (20), we obtain that

$$Z_{Nash}>Z_{coop}\Leftrightarrow\gamma<\frac{1}{1-b\left(1-\delta^D\right)}\frac{\Pi\left(y^*\right)-\Pi\left(n\right)}{\Pi\left(x^*\right)-\Pi\left(n\right)}. \tag{B.59}$$

Since $\Pi\left(y^*\right)\geq\Pi\left(x^*\right)$, then $\gamma<\left(1-b\left(1-\delta^D\right)\right)^{-1}$ is a sufficient condition to achieve $Z_{Nash}>Z_{coop}$. On the other hand, this equality must be violated for γ large enough (since $\Pi\left(x^*\right)>\Pi\left(n\right)$), proving the second part of Part b).

Second case. [Assume $\gamma^{Nash} < \gamma \leq \gamma^{coop}$] Then using the definition of ω and (21), we can rewrite:

$$Z_{Nash} - Z_{coop} = \frac{1-b}{1-\left(1-\delta^D\right)b} \times$$
$$\left[\left(1-\delta^D\right)\left(1-b+b\theta-\gamma^{-1}\right)\Pi(n) + \delta^D\left(\left(1-\gamma^{-1}\right)\Pi(n) - \left(\Pi\left(x^*\right) - \gamma^{-1}\Pi\left(y^*\right)\right)\right)\right].$$

Hence we can rewrite that in this case:

$$Z_{Nash} > Z_{coop} \Leftrightarrow \delta^D < \frac{1-b+b\theta-\gamma^{-1}}{\frac{\Pi(x^*)}{\Pi(n)} - b + b\theta - \gamma^{-1}\frac{\Pi(y^*)}{\Pi(n)}}.$$

To derive this we used that since $\gamma^{-1} < 1-b+b\theta$, and $\Pi(x^*)/\Pi(n) - \gamma^{-1}\Pi(y^*)/\Pi(n) > 1-\gamma^{-1}$, then both the numerator and the denominator are positive. We can also rewrite this equivalence as

$$Z_{Nash} > Z_{coop} \Leftrightarrow \gamma^{-1}\left(1-\delta^D\frac{\Pi(y^*)}{\Pi(n)}\right) < 1-\delta^D\frac{\Pi(x^*)}{\Pi(n)} - b(1-\theta)\left(1-\delta^D\right).$$

Hence we have that if $1-\delta^D\Pi(x^*)/\Pi(n) - b(1-\theta)\left(1-\delta^D\right) > 0$, then $Z_{Nash} > Z_{coop}$ is equivalent to:

$$\gamma > \frac{1-\delta^D\frac{\Pi(y^*)}{\Pi(n)}}{1-\delta^D\frac{\Pi(x^*)}{\Pi(n)} - b(1-\theta)\left(1-\delta^D\right)}. \tag{B.60}$$

Proof of Part (c). Therefore we get that $\delta^{coop} < \delta^{Nash}$ if $\gamma > \gamma^{coop}$ and $\gamma < \left(1-b\left(1-\delta^D\right)\right)^{-1}$ or if $\gamma^{Nash} < \gamma \leq \gamma^{coop}$ and

$$\gamma > \left(1-\delta^D\frac{\Pi(y^*)}{\Pi(n)}\right) / \left(1-\delta^D\frac{\Pi(x^*)}{\Pi(n)} - b(1-\theta)\left(1-\delta^D\right)\right)$$

with $1-\delta^D\frac{\Pi(x^*)}{\Pi(n)} - b(1-\theta)\left(1-\delta^D\right) > 0$. Assume that $\delta^D < \theta\Pi(n)/\Pi(m)$, this ensures that for any x^*, y^* we have $1-\delta^D\Pi(x^*)/\Pi(n) - b(1-\theta)\left(1-\delta^D\right) > 0$ and $1-\delta^D\Pi(y^*)/\Pi(n) > 0$. Moreover we get that

$$\frac{1-\delta^D\frac{\Pi(y^*)}{\Pi(n)}}{1-\delta^D\frac{\Pi(x^*)}{\Pi(n)} - b(1-\theta)\left(1-\delta^D\right)} < \frac{1-\delta^D\frac{\Pi(m)}{\Pi(n)}}{1-\delta^D\frac{\Pi(m)}{\Pi(n)} - b(1-\theta)\left(1-\delta^D\right)}$$

Hence $\gamma > \left(1-\delta^D\frac{\Pi(m)}{\Pi(n)}\right) / \left(1-\delta^D\frac{\Pi(m)}{\Pi(n)} - b(1-\theta)\left(1-\delta^D\right)\right)$ is a stricter condition than $\gamma > \left(1-\delta^D\frac{\Pi(y^*)}{\Pi(n)}\right) / \left(1-\delta^D\frac{\Pi(x^*)}{\Pi(n)} - b(1-\theta)\left(1-\delta^D\right)\right)$. In addition we have:

$$\frac{1-\delta^D\frac{\Pi(m)}{\Pi(n)}}{1-\delta^D\frac{\Pi(m)}{\Pi(n)} - b(1-\theta)\left(1-\delta^D\right)} = \frac{1}{1-b(1-\theta)\frac{1-\delta^D}{1-\delta^D\frac{\Pi(m)}{\Pi(n)}}} > \frac{1}{1-b(1-\theta)} = \gamma^{Nash}$$

since $\Pi(m) > \Pi(n)$.

Hence we have that $\delta^{coop} < \delta^{Nash}$ if

$$\gamma \in \left(\frac{1-\delta^D \frac{\Pi(m)}{\Pi(n)}}{1-\delta^D \frac{\Pi(m)}{\Pi(n)} - b(1-\theta)\left(1-\delta^D\right)}, \max\left(\gamma^{coop}, \left(1-b\left(1-\delta^D\right)\right)^{-1}\right)\right),$$

which implies that $\delta^{coop} < \delta^{Nash}$ if

$$\gamma \in \left(\frac{1-\delta^D \frac{\Pi(m)}{\Pi(n)}}{1-\delta^D \frac{\Pi(m)}{\Pi(n)} - b(1-\theta)\left(1-\delta^D\right)}, \left(1-b\left(1-\delta^D\right)\right)^{-1}\right).$$

This interval is non-empty as long as $\delta^D < \frac{\theta\Pi(n)}{\Pi(m)}$.

Proof of Remark 1. We now assume that we are in the first best, so that $x^* = y^* = m$ and $\gamma^{coop} = \left(1-b+b\theta\frac{\Pi(n)}{\Pi(m)}\right)^{-1}$, further we still have that $\delta^D < \theta\frac{\Pi(n)}{\Pi(m)}$. Then we get that as long as $\gamma \in \left(\gamma^{Nash}, \gamma^{coop}\right)$, then $\delta^{Nash} > \delta^{coop}$ if and only if $\gamma > \left(1-b(1-\theta)\frac{1-\delta^D}{1-\delta^D\frac{\Pi(m)}{\Pi(n)}}\right)^{-1}$ (using (B.60)). Furthermore if $\gamma > \gamma^{coop}$, then $\delta^{Nash} > \delta^{coop}$ if and only if $\gamma < \left(1-b\left(1-\delta^D\right)\right)^{-1}$ (using (B.59)). Further with $\delta^D < \theta\frac{\Pi(n)}{\Pi(m)}$, we must have that $\left(1-b(1-\theta)\frac{1-\delta^D}{1-\delta^D\frac{\Pi(m)}{\Pi(n)}}\right)^{-1} < \gamma^{coop} < \left(1-b\left(1-\delta^D\right)\right)^{-1}$, so that we obtain:

$$\delta^{Nash} > \delta^{coop} \iff \left(1-b\left(1-\delta^D\right)\frac{1-\theta}{1-\delta^D\frac{\Pi(m)}{\Pi(n)}}\right)^{-1} < \gamma < \left(1-b\left(1-\delta^D\right)\right)^{-1}.$$

B.5. Proof of Remark 2

Denote by B_t the number of producers who do not know a good match at the start of period. We obtain the law of motion

$$B_{t+1} = \left(1-\delta^D\right)bB_t + \delta^D N_t + N_{t+1} - N_t.$$

Indeed, among the producers that were in the same situation in the last period, only a fraction $1-\delta^D$ survived and of those a fraction b met a bad match. Moreover, the new producers, namely those that correspond to new products plus those that replace producers who died, also count as producers who do not know a good match at the beginning of the period. The share of producers who do not know a good match then obeys:

$$\omega_{t+1} = 1 - \frac{\left(1-\delta^D\right)(1-b\omega_t)}{1+g_N},$$

so that its steady-state value is given by:

$$\omega = \frac{g_N + \delta^D}{1+g_N-b\left(1-\delta^D\right)}.$$

Using this expression in (21), we get that for $\gamma \in \left(\gamma^{Nash}, \gamma^{coop}\right)$,

$$\begin{aligned} Z_{Nash} - Z_{coop} = & \frac{1-b}{1+g_N - b\left(1-\delta^D\right)} \left(1-\delta^D\right)\left(1-b+b\theta-\gamma^{-1}\right)\Pi(n) \\ & + \frac{g_N + \delta^D}{1+g_N - b\left(1-\delta^D\right)} (1-b)\left(\left(1-\gamma^{-1}\right)\Pi(n) - \left(\Pi\left(x^*\right) - \gamma^{-1}\Pi\left(y^*\right)\right)\right). \end{aligned}$$

Therefore if $x^* = y^* = m$, we have:

$$\begin{gathered} Z_{Nash} > Z_{coop} \iff \\ \left(1-\delta^D\right)\left(1-b+b\theta-\gamma^{-1}\right)\Pi(n) - \left(1-\gamma^{-1}\right)\delta^D\left(\Pi(m) - (\Pi(n))\right) \\ > g_N\left(1-\gamma^{-1}\right)\left(\Pi(m) - \Pi(n)\right). \end{gathered}$$

This expression clearly shows that the higher is g_N, the more difficult is it to get $\delta^{Nash} > \delta^{coop}$.

Similarly, if $\gamma > \gamma^{coop}$, then using (15), (16), 17) and the new expression of ω, we obtain:

$$\begin{gathered} Z_{Nash} - Z_{coop} = \\ \frac{1-b}{1+g_N-b\left(1-\delta^D\right)} \left(\begin{array}{c} \left(\gamma^{-1}\Pi(y^*) - \left(1-b\left(1-\delta^D\right)\right)\Pi(x^*)\right) - \left(\gamma^{-1} - 1 + b\left(1-\delta^D\right)\right)\Pi(n) \\ - \left(\left(\Pi(x^*) - \gamma^{-1}\Pi(y^*)\right) - \left(1-\gamma^{-1}\right)\Pi(n)\right) g_N \end{array} \right) \end{gathered}$$

In the special case where $x^* = y^* = m$, this translates into

$$Z_{Nash} > Z_{coop} \iff \gamma^{-1} - 1 + b\left(1-\delta^D\right) - g_N\left(1-\gamma^{-1}\right) > 0,$$

so that in that case too, a higher growth rate g_N makes it more difficult to get $\delta^{Nash} > \delta^{coop}$.

B.6. Proof of Proposition A.3

In this appendix we consider the case where the strategy of suppliers is to punish the producer - by playing the Nash strategy - if we he switches to an innovator that turns out to be a bad match. We derive expression (A.18) in the special case in which the expected value of a new relationship is higher than remaining with a non-cooperating good match, such that if the innovator turns out to be a bad match the producer will seek out a new supplier rather than stick with the old one.

Compare to the situation in Appendix A.1, if the producer switches the old supplier loses all its value, hence $V_A^s = 0$. The producer will now switch if and only if:

$$V_I^{T,g} > W_1^T, \tag{B.61}$$

that is the total value of a new relationship with the innovator is higher than the total value of a relationship with the old supplier instead of (A.3). If the innovator turns out to be a bad match, the producer will try another new supplier in the following period, so the total value of the relationship with the innovator does not depend on whether the producer already knew a good match or not:

$$V_I^{T,g} = V_I^{T,b} = V_0.$$

Equation (A.4) is replaced by:

$$V_I^{T,g} = V_0^T = (1-b)\,\Pi\left(x^*\right) + (1-b)\,\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_1^T + \delta^I\gamma W_1^T\right)$$
$$+b\theta\Pi(n) + b\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_0^T + \delta^I\gamma W_0^T\right). \qquad \text{(B.62)}$$

Using that (5) still holds, we get:

$$V_I^{T,g} - W_1^T = (1-b)\,\Pi\left(x^*\right) + b\theta\Pi(n) - \gamma^{-1}\Pi\left(y^*\right)$$
$$-b\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)\left(V_1^T - V_0^T\right) + \delta^I\gamma\left(W_1^T - W_0^T\right)\right).$$

We use (5), (B.62) and (A.15) and (A.11) which both still hold to obtain

$$\left(1-\delta^I\right)\left(V_1^T - V_0^T\right) + \delta^I\gamma\left(W_1^T - W_0^T\right) \qquad \text{(B.63)}$$
$$= \frac{b(1+\rho)\left[\left(1-\delta^I\right)\left(\Pi\left(x^*\right) - \theta\Pi(n)\right) + \delta^I\left(\Pi\left(y^*\right) - \theta\Pi(n)\right)\right]}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\gamma\delta^I\right)}.$$

Therefore, a producer in a good match will switch to the innovator if and only if (A.18) holds, which defines a γ^{coop2}. Note, that equation (A.18) differs from equation (13) only in the last term, it then follows that $\gamma^{coop2} > \gamma^{con} = \gamma^{Nash}$.

To show that the incentive to innovate is lower we need the fraction of the firms that are in good matches. In all cases, a producer in a bad match switch. If $\gamma < \gamma^{coop2}$ then only producers in bad matches in the cooperate equilibrium will switch, implying that in steady state (weakly) more producers will be in good matches in the cooperative equilibrium than in the contractible equilibrium. As the extra benefit for the innovator from contractibility is higher for good matches than bad matches, it follows that the incentive to innovate is higher in the contractible case, $\delta^{coop2} < \delta^{con}$.

Now, consider the case where $\gamma > \gamma^{coop2}$, such that good matches switch to the innovator. In the contractible case a fraction $\tilde{\omega}^{coop} = \delta^D / \left(1-b\left(1-\delta^D\right)\right)$ of producers will not be in good relationships, whereas in the cooperative case this fraction is given by $\tilde{\omega}^{coop} = \left(\delta^D + b\delta^I\left(1-\delta^D\right)\right) / \left(1-b\left(1-\delta^D\right)\left(1-\delta^I\right)\right)$. In the contractible case, the expected value for the innovator is given by:

$$Z_{cont} = \left(\tilde{\omega}^{cont}(\gamma-1)(1-b+b\theta) + \left(1-\tilde{\omega}^{cont}\right)\left((1-b+b\theta)\gamma-1\right)\right)\Pi(m).$$

In the cooperative case, the expected value for the innovator is given by:

$$Z_{coop} = \tilde{\omega}^{coop}\left((1-b)\left(\gamma\Pi\left(x^*\right) - \Pi\left(y^*\right)\right) + b\theta(\gamma-1)\Pi(n)\right)$$
$$+\left(1-\tilde{\omega}^{coop}\right)\left(\gamma(1-b)\Pi\left(x^*\right) + \gamma b\theta\Pi(n) - \Pi\left(y^*\right)\right)$$
$$-\frac{\left(1-\tilde{\omega}^{coop}\right)\gamma\left(1-\delta^D\right)b^2\left(\left(1-\delta^I\right)\Pi\left(x^*\right) + \delta^I\Pi\left(y^*\right) - \theta\Pi(n)\right)}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)}.$$

Straightforward, but somewhat tedious algebra demonstrates that the condition $\delta^{coop2}\left(1+\rho-b\left(1-\delta^D\right)\left(1+\delta^{coop2}(\gamma-1)\right)\right) < \gamma\left(1-b\left(1-\delta^D\right)\right)$ is sufficient to ensure that the incentive to innovate is lower in the cooperative case: $\delta^{coop2} < \delta^{con}$.

B.7. Appendix: Proofs for Section 4

B.7.1. Contractible and Nash cases. Here we show that the solution must satisfy $\delta_0^I = \delta_1^I$ and $V_0^T = V_1^T$. Taking the difference between (23) and (25) and using (24) one obtains that:

$$V_1^T - V_0^T = \frac{(1+\rho)\left(\psi\left(\delta_0^I\right) - \psi\left(\delta_1^I\right) + \left(\delta_1^I - \delta_0^I\right)\psi'\left(\delta_1^I\right)\right)}{1+\rho-\left(1-\delta_0^I\right)\left(1-\delta^D\right)}.$$

Next taking the difference between (26) and (24), one gets:

$$\psi'\left(\delta_0^I\right) - \psi'\left(\delta_1^I\right) = \frac{\left(1-\delta^D\right)\left(\psi\left(\delta_0^I\right) - \psi\left(\delta_1^I\right) + \left(\delta_1^I - \delta_0^I\right)\psi'\left(\delta_1^I\right)\right)}{1+\rho-\left(1-\delta_0^I\right)\left(1-\delta^D\right)}$$

The LHS is increasing in δ_0^I (since ψ is convex). On the RHS, the denominator is positive and increasing in δ_0^I, while the numerator is positive and decreasing in δ_0^I (once again thanks to the convexity of ψ), therefore the RHS is decreasing in δ_0^I. As a result this equation has a unique solution: $\delta_0^I = \delta_1^I$. In return, this ensures that $V_0^T = V_1^T$.

B.7.2. Cooperative case. Here, we describe the cooperative equilibrium in more details. To do so, we first spell out the strategies followed by the different agents, then we derive the results written in the main text, characterize the equilibrium level of cooperation and prove the existence of the equilibrium.

Strategies. The strategies are characterized as follows:

- An augmented supplier refers to a supplier who has access to a technology which is higher than the fringe (if deviations have occurred there could be more than one augmented supplier).
- An augmented supplier with whom no deviation ever occurred offers an ex-ante transfer which allows her to capture the full surplus of the relationship over the second best option for the producer (namely going with a new supplier or choosing one with whom a deviation has occurred). If she is chosen by the producer, she invests x^*.
- Non-augmented suppliers with whom no deviation ever occurred, offers an ex-ante transfer which make them break even. If they are chosen, they invest x^* if there is an innovation and n otherwise.
- An augmented supplier with whom a deviation occurred, offers an ex-ante transfer that allows her to break even if either there are several suppliers in her situation, or she cannot offer the highest value for their joint relationship. She offers an ex-ante transfer that allows her to capture the surplus of a relationship with her over starting a new relationship if the producer does not know any other augmented supplier and if that surplus is positive. She invests n.
- A non-augmented supplier with whom a deviation occurred, offers an ex-ante transfer that allows her to break even and invests n.

- A producer chooses the supplier that offers him the highest value. If several suppliers offer the same value, he chooses one with whom the joint value is the highest.
- x^* is the highest level of cooperation within $(n, m]$ which does not violate the incentive constraint of the supplier.
- The innovation rate is chosen so as to maximize the joint value of the relationship.

Proof of Proposition 4. We first need to prove that $V_0^T < V_1^T$. To show that, we first use (27) to derive

$$V_0^T = \frac{(1+\rho)\left[\left(1-\delta_0^I\right)\Pi(n)+\delta_0^I\gamma\Pi(x^*)-\psi\left(\delta_0^I\right)\right]+\delta_0^I\gamma\left(1-\delta^D\right)V_1^T}{1+\rho-\left(1-\delta_0^I\right)\left(1-\delta^D\right)}. \tag{B.64}$$

Since $x^* > n$, we obtain:

$$V_0^T < \frac{(1+\rho)\left[\left(1-\delta_0^I+\delta_0^I\gamma\right)\Pi(x^*)-\psi\left(\delta_0^I\right)\right]+\delta_0^I\gamma\left(1-\delta^D\right)V_1^T}{1+\rho-\left(1-\delta_0^I\right)\left(1-\delta^D\right)}.$$

We denote the right hand side as $f\left(\delta_0^I\right)$. We then get that

$$V_0^T < f\left(\delta_0^I\right) \leq \max_{\delta} f(\delta).$$

To find $\max_{\delta} f(\delta)$, we take a first order condition and obtain that the solution $\widetilde{\delta}$ must satisfy:

$$\begin{aligned}
&\psi'\left(\widetilde{\delta}\right)\left(1+\rho-\left(1-\widetilde{\delta}\right)\left(1-\delta^D\right)\right)-\left(1-\delta^D\right)\psi\left(\widetilde{\delta}\right)\\
&=\left((\gamma-1)\left(\rho+\delta^D\right)-\left(1-\delta^D\right)\right)\Pi\left(x^*\right)+\gamma\frac{1-\delta^D}{1+\rho}V_1^T\left(\rho+\delta^D\right).
\end{aligned}$$

Since the LHS is increasing in $\widetilde{\delta}$ and the RHS is independent of it, this uniquely defines $\widetilde{\delta}$. Using (23) and (24) with $z = x^*$, we can check that $\widetilde{\delta} = \delta_1^I$ satisfies the previous equation. Then, using (23), one gets

$$V_0^T < f\left(\delta_1^I\right) = V_1^T.$$

Comparing (28) and (24) with $z = m, n$ or x^*, it is then direct that $\delta_0^{I,coop} > \delta_1^{I,coop}$ and that $\delta_0^{I,coop} > \delta^{I,Nash}$. Further, we get that if x^* is close to n, $\delta_0^{I,coop}$ is close to $\delta^{I,Nash}$ (and lower than $\delta^{I,cont}$), and if x^* is close to m, $\delta_0^{I,coop} > \delta^{I,cont}$.

The growth rate of the economy in the cooperative case depends on the share of producers who know an augmented supplier and their average productivity. Nevertheless, this growth rate must be larger in the cooperative case than in the Nash case (since the innovation rate is larger whether the producer knows an innovator or not). Similarly, as long as $\delta_0^{I,coop} < \delta^{I,cont}$ (which is true if x^* is close to n), growth is lower in the cooperative than contractible case. If on the other hand $\delta_0^{I,coop} > \delta^{I,cont}$

and $\delta_1^{I,coop} = \delta^{I,cont}$ (which is obtained if $x^* = m$), then the innovation rate is weakly higher (and in some line strictly higher) in the cooperative case than in the contractible one, leading to a higher growth rate in the former.

Characterizing the equilibrium. Here we first characterize the equilibrium and then prove its existence. There are two possible cases, either after a deviation the producer stays with an augmented supplier which has deviated, or he ignores that firm and looks for a new supplier. Depending on whether the producer knows one or more deviating firms, his behavior may be different, because he can capture a different value from a relationship with such a non-cooperating good match. However, we demonstrate below that the number of known non-cooperating good match suppliers in fact does not matter for the producer's decision.

Case where the producer does not stick with a non-cooperating good match. In this section, we consider the case where a producer does not stick with a non-cooperating good match (regardless of the number of known non-cooperating good match suppliers). We characterize the level of cooperation x^*, the condition under which this scenario applies and demonstrate that if a producer does not stick with a non-cooperating good match if he only knows one of them, then he will not do so if he knows more than one of them either.

Characterizing x^*. Consider a producer and supplier on equilibrium path, then the incentive compatibility constraint for the supplier is given by

$$\varphi(x) \leq \frac{1-\delta^D}{1+\rho}\left(V_1^s - V_N^s\right), \tag{B.65}$$

where V_N^s is the value the innovator would capture at the beginning of the following period should a deviation occurs (so that the supplier would play the Nash level). Note that no γ term appear here because the technology level at the time of investment in x is the same as the technology level at the beginning of the following period. Since we have assumed that the producer would not want to work with the supplier after the deviation, we obtain $V_N^s = 0$, such that the incentive constraint is

$$\varphi(x) \leq \frac{1-\delta^D}{1+\rho} V_1^s.$$

Following the innovator's strategy, the value she captures from a relationship with the producer corresponds to the surplus over the producer's second best option. Namely we have

$$V_1^p = \gamma^{-1} V_0^{T,n} \text{ and } V_1^s = V_1^T - \gamma^{-1} V_0^{T,n}. \tag{B.66}$$

Indeed, at the beginning of a period, the second best option of the producer is to look for another supplier, whose technology is γ lower (recall that the V's are normalized by the supplier's technology here, which is why γ^{-1} appears). Importantly, if the producer where to switch to a new supplier, he would now be off-equilibrium path and knowing one innovator-non-cooperating good match. The value from a relationship in that situation $V_{0,n}^T$ may differ from the value V_0^T on equilibrium path.

In fact, we can write the law of motion:

$$V_0^{T,n} = -\psi\left(\delta_0^{I,n}\right) + \left(1-\delta_0^{I,n}\right)\left(\Pi(n) + \frac{1-\delta^D}{1+\rho}V_0^{T,n}\right) + \delta_0^{I,n}\gamma\left(\Pi\left(x_n^*\right) + \frac{1-\delta^D}{1+\rho}V_1^{T,n}\right). \tag{B.67}$$

$V_1^{T,n}$ denotes the joint value of a relationship with an innovator (non non-cooperating good match) when the producer knows an innovator-non-cooperating good match and x_n^* is the cooperation level in that case. If the new supplier fails to innovate, then the producer's situation does not change and by assumption he would then prefer to stay away from the innovator-non-cooperating good match (who plays Nash); so that the continuation value in that case is $V_0^{T,n}$. If on the other hand, an innovation occurs, the producer knows both an innovator-non non-cooperating good match and an innovator-non-cooperating good match and the joint value of the relationship is $V_1^{T,n}$. $\delta_0^{I,n}$ maximizes the joint value $V_0^{T,n}$ (when deriving the law of motion for any V_X^T below, the notation δ_X^I denotes the equilibrium innovation rate, which must maximize V_X^T).

To go further, we need to think about the IC constraint of an innovator when the producer knows an innovator who deviated. This can be written as

$$\varphi\left(x_n\right) \le \frac{1-\delta^D}{1+\rho}\left(V_1^{s,n} - V_{2N}^s\right), \tag{B.68}$$

where the index n in $V_1^{s,n}$ indicates that the producer knows a non-cooperating good match and in V_{2N}^s that he knows at least 2. When the producer knows at least 2 innovator non-cooperating good match suppliers, Bertrand competition ensures that he captures the whole value, hence, in all cases we will have that $V_{2N}^s = 0$. Further with Bertrand competition, $V_1^{s,n} = V_1^{T,n} - \gamma^{-1}V_0^{T,2n}$, where $V_0^{T,2n}$ indicates the joint value of a new relationship when the producer knows at least 2 non-cooperating good match suppliers (indeed whether a producer knows two or more non-cooperating good match suppliers does not matter since with Bertrand competition he would receive the same offers of ex-ante transfers by the non-cooperating good match—namely, one which allows him to capture the whole value). This ensures that $V_1^{T,n}$ and x_n^* also apply when the producer knows more than 1 non-cooperating good match.

Then, since we have assumed that regardless of the number of non-cooperating good match suppliers the producer would rather keep looking for new suppliers, $V_0^{T,2n}$ must obey the same law of motion as $V_0^{T,n}$ given by (B.67). This ensures that $V_1^{s,n} = V_1^s$ so that $x_n^* = x^*$. In return we then obtain $V_0^{T,n} = V_0^T$ and $\delta_0^{I,n} = \delta_0^I$.

Using (23) with $z = x^*$, we get that

$$V_1^T = \frac{(1+\rho)\left(\left(1-\delta_1^I+\delta_1^I\gamma\right)\Pi\left(x^*\right) - \psi\left(\delta_1^I\right)\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta_1^I+\delta_1^I\gamma\right)}, \tag{B.69}$$

while using (27) we get (B.64). Combining this equation with (B.69), we obtain:

$$V_1^s = V_1^T - \gamma^{-1} V_0^T$$
$$= \frac{1+\rho}{1+\rho-\left(1-\delta^D\right)\left(1-\delta_0^I\right)} \begin{pmatrix} \frac{(\rho+\delta^D)\left(\left(1-\delta_1^I+\delta_1^I\gamma\right)\Pi(x^*)-\psi\left(\delta_1^I\right)\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta_1^I+\delta_1^I\gamma\right)} \\ -\gamma^{-1}\left(\left(1-\delta_0^I\right)\Pi(n)+\delta_0^I\gamma\Pi(x^*)-\psi\left(\delta_0^I\right)\right) \end{pmatrix}$$

Therefore we have that the IC constraint can be written as

$$\varphi\left(x^*\right) \leq \frac{1-\delta^D}{1+\rho-\left(1-\delta^D\right)\left(1-\delta_0^I\right)} \times \begin{pmatrix} \frac{(\rho+\delta^D)\left(\left(1-\delta_1^I+\delta_1^I\gamma\right)\Pi(x^*)-\psi\left(\delta_1^I\right)\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta_1^I+\delta_1^I\gamma\right)} \\ -\gamma^{-1}\left(\left(1-\delta_0^I\right)\Pi(n)+\delta_0^I\gamma\Pi(x^*)-\psi\left(\delta_0^I\right)\right) \end{pmatrix}. \quad \text{(B.70)}$$

$x^* = m$ if (B.70) holds in that case, or x^* is such that (B.70) holds with equality

Condition under which the producer does not stay with the non-cooperating good match. . We have assumed that the innovator would rather try a new supplier than stay with an innovator who has deviated. We need to check under which conditions, this is an equilibrium. To do that, we derive the joint value of a producer who knows a non-cooperating good match and decides to stay with her. This joint value obeys:

$$V_N^T = -\psi\left(\delta_N^I\right)+\left(1-\delta_N^I\right)\left(\Pi(n)+\frac{1-\delta^D}{1+\rho}\gamma^{-1}V_0^T\right) + \delta_N^I\left(\gamma\Pi(n)+\frac{1-\delta^D}{1+\rho}V_0^T\right). \quad \text{(B.71)}$$

If there is no innovation then in the following period, by assumption, the producer would rather try a new supplier (with a lower technology). If innovation occurs, the producer would rather try a new supplier as well (and the technology of that new supplier is the same as today). Moreover the strategy of a non-cooperating good match is to invest the Nash level n. The innovation rate must satisfy:

$$\psi'\left(\delta_N^I\right) = (\gamma-1)\left(\Pi(n)+\frac{1-\delta^D}{1+\rho}\gamma^{-1}V_0^T\right).$$

Since $x^* > n$ and $\gamma V_1^T - V_0^T > (\gamma-1)\gamma^{-1}V_0^T$, then it must be that $\delta_0^I > \delta_N^I$.

Using (B.71) and (B.64), we find that

$$\gamma^{-1}V_0^T - V_N^T$$
$$= \frac{\left(1+\rho-\left(1-\delta^D\right)\left(1-\delta_N^I+\delta_N^I\gamma\right)\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta_0^I\right)} \times\left(\left(1-\delta_0^I\right)\gamma^{-1}\Pi(n)+\delta_0^I\Pi\left(x^*\right)-\gamma^{-1}\psi\left(\delta_0^I\right)+\delta_0^I\frac{1-\delta^D}{1+\rho}V_1^T\right) - \left(\left(1-\delta_N^I+\delta_N^I\gamma\right)\Pi(n)-\psi\left(\delta_N^I\right)\right).$$

Therefore the strategies described form an equilibrium if x^* satisfies the IC constraint (with equality unless $x^* = m$) and the following condition is satisfied

$$\begin{aligned} & \frac{\left(1-\delta_0^I\right)\gamma^{-1}\Pi\left(n\right)+\delta_0^I\Pi\left(x^*\right)-\gamma^{-1}\psi\left(\delta_0^I\right)+\delta_0^I\frac{1-\delta^D}{1+\rho}V_1^T}{1+\rho-\left(1-\delta^D\right)\left(1-\delta_0^I\right)} \\ \geq \; & \frac{\left(1-\delta_N^I+\delta_N^I\gamma\right)\Pi\left(n\right)-\psi\left(\delta_N^I\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta_N^I+\delta_N^I\gamma\right)}. \end{aligned} \tag{B.72}$$

Case where the producer would stay with a non-cooperating good match if he knows at least 2 of them but not if he only knows one of them. Here, we show that this case is impossible. Assume otherwise, then we still have $\gamma^{-1}V_0^{T,n} > V_N^T$ (a producer would rather try a new supplier than stick with a single non-cooperating good match) but $\gamma^{-1}V_0^{T,2n} < V_{2N}^T$: a producer would rather work with an innovator-non-cooperating good match than a new supplier when he knows at least two non-cooperating good match suppliers. Then, the value of starting a relationship with a new supplier for a producer who knows at least 2 innovator-non-cooperating good match suppliers, $V_0^{T,2n}$, obeys the following law of motion:

$$\begin{aligned} V_0^{T,2n} = & -\psi\left(\delta_0^{I,2n}\right)+\left(1-\delta_0^{I,2n}\right)\left(\Pi\left(n\right)+\frac{1-\delta^D}{1+\rho}\gamma V_{2N}^T\right) \\ & +\delta_0^{I,2n}\gamma\left(\Pi\left(x_{2n}^*\right)+\frac{1-\delta^D}{1+\rho}V_1^{T,2n}\right). \end{aligned} \tag{B.73}$$

If there is no innovation, then in the following period, the producer should revert back to choosing one of the two non-cooperating good match suppliers (by assumption). In that case, he will capture the full joint value of the relationship (because the two non-cooperating good match suppliers Bertrand compete). If there is an innovation, then the producer will now be working with an augmented supplier, while simultaneously knowing two innovator-non-cooperating good match suppliers. The level of cooperation x_{2n}^* could in principle be different from x_n^*.

The IC constraint that determines x_n^* is given by (B.68) with $V_N^{s,2n} = 0$ and $V_1^{s,n} = V_1^{T,n} - V_{2N}^T$: indeed, should the producer not stick with an innovator who has not deviated he could either try a new supplier or go to the non-cooperating good match he knows, but he would now know two non-cooperating good match suppliers. The latter is by assumption the second best option here.

The IC constraint that determines x_{2n}^* is:

$$\varphi\left(x_{2n}^*\right) \leq \frac{1-\delta^D}{1+\rho}\left(V_1^{s,2n}-V_{2N}^s\right).$$

We still have $V_{2N}^s = 0$ and $V_1^{s,2n} = V_1^{T,2n} - V_{2N}^T$ as the second best option of the producer is to go with a non-cooperating good match now that he knows at least two of them. Hence we must have that $x_{2n}^* = x_n^*$, $\delta_0^{I,2n} = \delta_0^{I,n}$ and $V_1^{T,2n} = V_1^{T,n}$. This allows us to rewrite (B.73) as:

$$V_0^{T,2n} = -\psi\left(\delta_0^{I,2n}\right) + \left(1-\delta_0^{I,2n}\right)\left(\Pi(n) + \frac{1-\delta^D}{1+\rho}\gamma V_{2N}^T\right) \quad \text{(B.74)}$$
$$+\delta_0^{I,2n}\gamma\left(\Pi\left(x_n^*\right) + \frac{1-\delta^D}{1+\rho}V_1^{T,n}\right).$$

The joint value of a relationship between a producer and an innovator-non-cooperating good match when the producer only knows one such non-cooperating good match, still obeys (B.71) but with $V_0^{T,n}$ instead of V_0^T (as we cannot establish that they are the same here), hence:

$$V_N^T = -\psi\left(\delta_N^I\right) + \left(1-\delta_N^I\right)\left(\Pi(n) + \frac{1-\delta^D}{1+\rho}\gamma^{-1}V_0^{T,n}\right) \quad \text{(B.75)}$$
$$+\delta_N^I\left(\gamma\Pi(n) + \frac{1-\delta^D}{1+\rho}V_0^{T,n}\right).$$

Finally, the joint value of a relationship between a producer and an innovator-non-cooperating good match, when the producer knows at least 2 such non-cooperating good match suppliers is given by:

$$V_{2N}^T = -\psi\left(\delta_{2N}^I\right) + \left(1-\delta_{2N}^I\right)\left(\Pi(n) + \frac{1-\delta^D}{1+\rho}V_{2N}^T\right) \quad \text{(B.76)}$$
$$+\delta_{2N}^I\left(\gamma\Pi(n) + \frac{1-\delta^D}{1+\rho}V_0^{T,n}\right).$$

If no innovation occurs then the best option is by assumption to stick with a non-cooperating good match. If innovation occurs though, the producer would now know only 1 innovator-non-cooperating good match (the one with whom he has just worked), indeed the other non-cooperating good match suppliers will not have access to the frontier technology. By assumption, in the following period, the producer should then try a new supplier (whose technology predates the last innovation) instead of staying with the single innovator-non-cooperating good match.

From (B.67) and (B.74) and using that $\delta_0^{I,2n}$ maximizes $V_0^{T,2n}$, we have

$$V_0^{T,2n} - V_0^{T,n} \geq \left(1-\delta_0^{I,n}\right)\frac{1-\delta^D}{1+\rho}\left(\gamma V_{2N}^T - V_0^{T,n}\right) > 0, \quad \text{(B.77)}$$

by assumption. Using that $\gamma^{-1}V_0^{T,n} > V_N^T$ and that $V_{2N}^T > \gamma^{-1}V_0^{T,2n}$, we get $V_{2N}^T > V_N^T$.

Moreover, using that δ_N^I maximizes the RHS in (B.75), we get:

$$\begin{aligned} V_N^T &\geq -\psi\left(\delta_{2N}^I\right)+\left(1-\delta_{2N}^I\right)\left(\Pi(n)+\frac{1-\delta^D}{1+\rho}\gamma^{-1}V_0^{T,n}\right) \\ &+\delta_{2N}^I\left(\gamma\Pi(n)+\frac{1-\delta^D}{1+\rho}V_0^{T,n}\right). \end{aligned} \tag{B.78}$$

Further using that $\gamma^{-1}V_0^{T,n} > V_N^T$, we obtain:

$$\begin{aligned} V_N^T &> -\psi\left(\delta_{2N}^I\right)+\left(1-\delta_{2N}^I\right)\left(\Pi(n)+\frac{1-\delta^D}{1+\rho}V_N^T\right) \\ &+\delta_{2N}^I\left(\gamma\Pi(n)+\frac{1-\delta^D}{1+\rho}V_0^{T,n}\right). \end{aligned} \tag{B.79}$$

Take the difference between (B.76) and (B.79) to get

$$V_N^T - V_{2N}^T > 0. \tag{B.80}$$

This contradicts the result we obtained above, which shows that this case is impossible: if a producer would rather look for a new supplier when he knows one innovator-non-cooperating good match, he should also do so if he knows more than one innovator-non-cooperating good match.

Case where the producer stays with the non-cooperating good match. In this section, we consider the case where a producer sticks with an innovator-non-cooperating good match if he does not know a non-non-cooperating good match innovator. We assume that the producer does so regardless of the number of known non-cooperating good match suppliers. As above, we first characterize the level of cooperation x^* and the condition under which this scenario applies. Then we show that if a producer prefers a non-cooperating good match to trying a new supplier who does not have access to the latest technology when he knows one non-cooperating good match, then he must also prefer doing so when he knows several non-cooperating good match suppliers.

Characterizing the level of cooperation x^*. Consider a producer who is matched with an innovator with whom no deviation ever occurred and further assume that the producer does not know any innovator-non-cooperating good match (this corresponds to what happens after the first successful innovation on equilibrium path). The IC constraint faced by the innovator still obeys (B.65). V_1^p and V_1^s are still determined by (B.66) since the second best option of the producer is to start a new relationship with a firm with an inferior technology but now knowing one non-cooperating good match. The difference is that $V_N^s \neq 0$: by assumption in case of a deviation the producer would rather stick with an innovator who has deviated than try a new supplier with an inferior technology (the only outside option here). Hence we obtain that

$$V_N^s = V_N^T - \gamma^{-1}V_{0,n}^T \text{ and } V_N^p = \gamma^{-1}V_0^{T,n}.$$

Therefore the IC constraint can be written as

$$\varphi\left(x^*\right) \leq \frac{1-\delta^D}{1+\rho}\left(V_1^T - V_N^T\right).$$

Note that V_N^T obeys (23) with $z = n$, and that the joint value of a relationship with a non-cooperating good match when the producer knows at least 2 non-cooperating good match suppliers (V_{2N}^T) also obeys the same law of motion (since the producer would always prefer to stick with an innovator-non-cooperating good match rather than trying a new supplier with an inferior technology). Hence we get $V_N^T = V_{2N}^T = V_1^{T,Nash}$ with $\delta_N^I = \delta_{2N}^I = \delta^{I,Nash}$, so that

$$V_N^T = \frac{(1+\rho)\left(\left(1-\delta_N^I+\delta_N^I\gamma\right)\Pi\left(n\right)-\psi\left(\delta_N^I\right)\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta_N^I+\delta_N^I\gamma\right)} \tag{B.81}$$

Further V_1^T is still given by (B.69), combined with (B.81), we obtain that the level of cooperation x^* is characterized by the IC constraint:

$$\varphi\left(x^*\right) \leq \left(1-\delta^D\right)\left(\frac{\left(1-\delta_1^I+\delta_1^I\gamma\right)\Pi\left(x^*\right)-\psi\left(\delta_1^I\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta_1^I+\delta_1^I\gamma\right)} - \frac{\left(1-\delta_N^I+\delta_N^I\gamma\right)\Pi\left(n\right)-\psi\left(\delta_N^I\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta_N^I+\delta_N^I\gamma\right)}\right). \tag{B.82}$$

Therefore $x^* = m$ if (B.82) holds in that case and otherwise x^* is such that (B.82) holds with equality.

Condition under which the producer prefers an innovator-non-cooperating good match to a new supplier. We want to derive the conditions under which it is indeed the case that $V_N^T > \gamma^{-1}V_0^{T,n}$. To do that, we first need to characterize $V_0^{T,n}$. The incentive constraint faced by an augmented supplier who is in a relationship with a producer that already knows a non-cooperating good match is given by (B.68). Since the producer knows already one innovator-non-cooperating good match, then $V_{2N}^s = 0$. Furthermore $V_1^{s,n} = V_1^{T,n} - V_N^T$, since the producer's outside option next period is to start with one of the non-cooperating good match (with whom he would capture the entire surplus), and as explained above $V_{2N}^T = V_N^T$. Therefore, the IC constraint is still given by (B.82) with x_n^* instead of x^*, which implies that $V_1^{T,n} = V_1^T$, $\delta_1^{I,n} = \delta_1^I$ and $x_n^* = x^*$.

The value of a new relationship when a producer already knows exactly one innovator-non-cooperating good match obeys the following law of motion:

$$V_0^{T,n} = -\psi\left(\delta_0^{I,n}\right) + \left(1-\delta_0^{I,n}\right)\left(\Pi\left(n\right) + \frac{1-\delta^D}{1+\rho}V_0^{T,n}\right) + \delta_0^{I,n}\gamma\left(\Pi\left(x^*\right) + \frac{1-\delta^D}{1+\rho}V_1^T\right). \tag{B.83}$$

Indeed, the continuation value of the producer if there is no innovation is $V_0^{T,n}$ only since the non-cooperating good match would capture the surplus of the relationship; while, following an innovation, the level of cooperation is given by x^*. We then directly obtain that $\delta_0^{I,n} = \delta_0^I$ and that $V_0^{T,n} = V_0^T$.

Using (B.64) and (B.81), one gets

$$\gamma^{-1} V_0^T - V_N^T = (1+\rho) \times \tag{B.84}$$
$$\left(\frac{\left(\left(1-\delta_0^I\right)\gamma^{-1}\Pi(n) + \delta_0^I \Pi(x^*) - \gamma^{-1}\psi\left(\delta_0^I\right)\right) + \delta_0^I \frac{1-\delta^D}{1+\rho} V_1^T}{1+\rho - \left(1-\delta_0^I\right)\left(1-\delta^D\right)} - \frac{\left(1-\delta_N^I + \delta_N^I \gamma\right)\Pi(n) - \psi\left(\delta_N^I\right)}{1+\rho - \left(1-\delta_N^I + \delta_N^I\gamma\right)\left(1-\delta^D\right)} \right)$$

To ensure that producers want to stick with the innovator-non-cooperating good match, the RHS of this equation must be weakly negative. In other words, we obtain an equilibrium provided that the weak opposite of (B.72) holds.

Ruling out the possibility for the producer not to stay with a non-cooperating good match if he knows at least 2 of them. Furthermore, the value of starting a relationship with a new supplier when the producer knows at least two innovator-non-cooperating good match suppliers obeys the following law of motion:

$$\begin{aligned} V_0^{T,2n} &= -\psi\left(\delta_0^{I,2n}\right) + \left(1-\delta_0^{I,2n}\right)\left(\Pi(n) + \frac{1-\delta^D}{1+\rho}\max\left(\gamma V_N^T, V_0^{T,2n}\right)\right) \\ &\quad + \delta_0^{I,2n}\gamma\left(\Pi\left(x^*\right) + \frac{1-\delta^D}{1+\rho}V_1^T\right). \end{aligned}$$

Indeed, if no innovation occurs the producer will then decide whether to try a supplier without the frontier technology or a non-cooperating good match. Since there are several of each, the producer captures the whole value of the relationship. We either have $V_N^T > \gamma^{-1} V_0^{T,2n}$, that is a non-cooperating good match is preferred to a new supplier with an outdated technology regardless of the number of non-cooperating good match suppliers—which is what we have assumed; or $V_N^T < \gamma^{-1} V_0^{T,2n}$. In that case though, $V_0^{T,2n}$ obeys the same law of motion as $V_0^{T,n}$, namely (B.83), we then get $V_0^{T,2n} = V_0^{T,n} < \gamma V_N^T$: which is a contradiction. If the producer prefers a non-cooperating good match to a relationship with a supplier with a non-frontier technology, he will do so regardless of the number of non-cooperating good match suppliers (still we will have $V_0^{T,2n} \neq V_0^{T,n}$ and $\delta_0^{I,2n} \neq \delta_0^{I,n}$).

Existence. We have derived necessary conditions for the existence of an equilibrium obtained with the strategies we described. It is direct to check that these are also sufficient conditions. Therefore, the last thing to do is to ensure that there exists a x^* such that all conditions are satisfied. That is we must show that either i) m satisfies the IC constraint (B.70) together with (B.72) or m satisfies (B.82) together with the opposite of (B.72); or ii) the IC constraint (B.70) binds and (B.72) holds or the IC constraint (B.82) binds and the opposite of (B.72) holds.

To do that we first show that the IC constraint does not bind when x^* is close to n. For x^* close to n, $\delta_1^I \approx \delta_0^I \approx \delta^{I,Nash}$ and we obtain that $V_1^T \approx V_0^T \approx V_1^{T,Nash} = V_N^T$.

Therefore the opposite of (B.72) holds. Further for x^* close to but above n, (B.82) holds as

$$\frac{\left(1-\delta_1^I+\delta_1^I\gamma\right)\Pi\left(x^*\right)-\psi\left(\delta_1^I\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta_1^I+\delta_1^I\gamma\right)} \geq \frac{\left(1-\delta_N^I+\delta_N^I\gamma\right)\Pi\left(x^*\right)-\psi\left(\delta_N^I\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta_N^I+\delta_N^I\gamma\right)}$$
$$> \frac{\left(1-\delta_N^I+\delta_N^I\gamma\right)\Pi\left(n\right)-\psi\left(\delta_N^I\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta_N^I+\delta_N^I\gamma\right)}.$$

The first inequality uses that δ_1^I maximizes $\frac{(1-\delta+\delta\gamma)\Pi(x^*)-\psi(\delta)}{1+\rho-\left(1-\delta^D\right)(1-\delta+\delta\gamma)}$ and the second that $\Pi\left(x\right)$ is increasing over (n,m).

If there is a x^* such that (B.82) binds while the opposite of (B.72) holds, then an equilibrium exists. Otherwise, there must exist a $\overline{x}$ such that (B.72) holds with equality at $\overline{x}$ (and holds strictly above $\overline{x}$) with (B.82) not binding over $[n,\overline{x}]$. Note that at $\overline{x}$, $V_N^T=\gamma^{-1}V_0^T$, and as result (B.70) and (B.82) are identical. Therefore, by continuity (B.70) still does not bind for x just above $\overline{x}$. By continuity, an equilibrium exists: either the IC constraint never binds and the appropriate equilibrium condition at m ((B.72) or its opposite) is satisfied, or the IC constraint binds and the appropriate equilibrium condition holds.

B.8. Appendix: Combined model

B.8.1. Model description. The model combined the baseline model of general innovation of Section 2 with the relationship-specific innovation model of Section 4. As in the latter model, the frontier technology can now vary in each line. A general innovation pushes the frontier by a factor γ^A in each line, but it is imitated after one period (so that all suppliers get access to the frontier technology in that line at the beginning of the next period). A relationship specific innovation pushes the frontier by a factor γ^B in the line in which it occurs. The innovator is the only one with this technology until a further general or relationship specific innovation, in which case the innovator gets access to the new frontier technology and all other firms get access to the previous frontier technology. The two types of innovations do not occur in the same period, instead a period is either one where general innovation may happen (with probability ν) or one where relationship specific innovation may happen (with probability $1-\nu$). For relationship specific innovation the innovation cost is $\psi^B\left(\delta^B\right)A_j$ where ψ^B is a convex function of the innovation rate δ^B and A_j is the pre-innovation frontier technology in line j. For general innovation, the innovation cost is $\psi^A\left(\delta^A\right)\widetilde{A}$ where ψ^A is a convex function of the innovation rate δ^B and $\widetilde{A}$ is the average pre-innovation frontier technology in the economy. To ensure a steady-state, we assume that the potential innovator cannot observe when the last general innovation occurs.

As in the baseline model, there are good and bad matches. Cooperation is only possible in good matches, moreover, we also assume that relationship-specific innovation is impossible in bad matches.[9] The nature of a match is revealed before relationship-specific innovation or investment are undertaken but after a producer has decided to start working with a supplier (so after a potential general innovation). If a relationship-specific innovation occurs in a good match supplier, we will refer to the supplier as an "augmented" good match until she is not the frontier supplier (by opposition we will talk of a "regular" good match otherwise). As in the relationship-specific model, if a producer dies, he is replaced by a new producer and for that line the technology level (pre-innovation) is equal to the average technology level in the economy (pre-innovation). Note that the baseline model is obtained in the specific case where $\nu = 1$ and the relationship-specific innovation only model is obtained for $\nu = 0$ and $b = 0$.

We look at a cooperative equilibrium which has the same characteristics as that of Sections 2 and 4. In bad matches (or after a deviation occurred in the personal history of the producer and the supplier), normalized investment level is n. Producers with a good match supplier can be in four different scenarios: 1) the good match supplier has access to the frontier technology and she is a "regular" good match; 2) the good match supplier has access to a frontier technology and is an "augmented" good match; 3) in a period where a general innovation occurred, the good match supplier does not have access to the frontier technology and she is an "outdated" good match. In case 1), the investment level in equilibrium is the same and denoted x_1^*, in case 2) the investment level is denoted x_2^* and in case 3) it is denoted y^*. The baseline model made it clear why we needed two different levels of investment depending on whether the supplier had access to the frontier technology or not. In addition, the level of cooperation may differ depending on whether a relationship-specific innovation was the most recent innovation in the line or not, since the producer outside option is different. This was the case in Section 4 already: before the relationship-specific investment there was no cooperation and afterwards some cooperation, the difference is that here because of the presence of bad matches, some cooperation is possible right away; in line with that section we assume that $x_1^* \leq x_2^*$. Finally, we denote δ_1^B the relationship-specific innovation rate in periods where such innovations are possible in a "regular" good match, while δ_2^B denotes the same rate for an "augmented" good match (in Section 4 the corresponding notations were δ_0^I and δ_1^I, here we move the subscripts to 1 and 2 to have notations that mirror those of the value functions).

We still assume that a supplier forgives a producer who tries out a general innovator if that innovator turns out to be bad match. Finally, we consider parameters for which off-equilibrium path the producer would rather try out a new supplier rather than staying with a non-cooperating good match supplier playing the Nash level of investment (or equivalently, we assume that a producer forgets the identity of previous good matches if he starts working with one). Figure 1 summarizes the model by providing a timeline.

9. This is a simplifying assumption, without it the innovation rate would simply be lower in bad matches and we would still obtain very similar results.

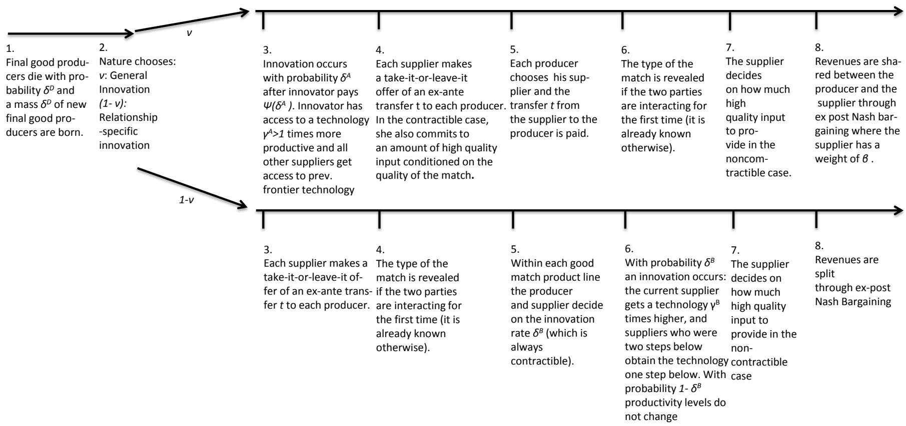

FIGURE 1. Timeline

To use the same notations as in both Sections 2 and 4, I normalize value functions by the frontier technology just after a general innovation has occurred in general innovation periods but before a relationship-specific innovation has occurred in relationship-specific innovation periods. I use the notations V, W with indexes 0, 1 or A and superscripts s, p, T or g and b exactly as in the baseline model for periods where only general innovation is possible. The superscript 2 is used to denote an augmented good match (while 1 is for a regular good match). I denote the value functions by U in periods where relationship-specific innovations are possible. The superscripts s, p, T are used as before and the indexes 0, 1 and 2 are used to denote a new match, a regular good match and an augmented good match.

B.8.2. Value functions and equilibrium description. We derive the value functions to show that the cooperative equilibrium exists and eventually describe the innovation incentives.

Relationship-specific innovation period. First consider a relationship-specific innovation period. Then the (normalized) joint value of a producer together with a new supplier, U_0^T, obeys:

$$U_0^T = (1-b)\,U_1^T + b\theta\Pi\,(n) + b\frac{1-\delta^D}{1+\rho}\left[\nu\left(\left(1-\delta^A\right)V_0^T + \delta^A\gamma^A W_0^T\right) + (1-\nu)\,U_0^T\right]. \tag{B.85}$$

With probability $1-b$, the new supplier is a good match (which cannot have been augmented yet), leading to the joint value U_1^T. With probability b, it is a bad match. The flow of profits is then given by $\theta\Pi\,(n)$. In the next period, if the producer survives, there are two cases. With probability $1-\nu$, the next period is also one of relationship-specific innovations and the situation is the same as today (the producer value is then the full joint value U_0^T). With probability ν, the next period is one of potential general innovation leading to a producer value of V_0^T, with probability $1-\delta^A$, or $\gamma^A W_0^T$, with probability δ^A: if a general innovation does indeed occur the frontier moves by a factor γ^A but the producer only captures the value of a relationship with his second-best option available, namely starting a new relationship with an outdated good match.

Similarly the joint value of a relationship with a regular good match supplier obeys:

$$\begin{aligned} U_1^T &= -\psi^B\left(\delta_1^B\right) \\ &+\left(1-\delta_1^B\right)\left(\Pi\left(x_1^*\right)+\frac{1-\delta^D}{1+\rho}\left[\nu\left(\left(1-\delta^A\right)V_1^T+\delta^A\gamma^A W_1^T\right)+(1-\nu)U_1^T\right]\right) \\ &+\delta_1^B\gamma^B\left(\Pi\left(x_2^*\right)+\frac{1-\delta^D}{1+\rho}\left(\nu\left(\left(1-\delta^A\right)V_2^T+\delta^A\gamma^A W_1^T\right)+(1-\nu)U_2^T\right)\right). \end{aligned} \tag{B.86}$$

In the current period, the supplier invests $\psi^B\left(\delta_1^B\right)$ (after having learned the nature of the match). The innovation fails with probability $1-\delta_1^B$, in which case the investment level will be x_1^*. With probability δ_1^B, the innovation succeeds, the frontier moves by a factor γ^B, the investment level is x_2^* and the match becomes an augmented good match. Note that if in the next period a general innovation does occur (which happens with probability $\nu\delta^A$), then the augmented good match supplier loses her advantage since she ceases to be the frontier supplier for that line and the joint value is $\gamma^B\gamma^A W_1^T$ or $\gamma^A W_1^T$ depending on whether the relationship specific innovation occurs today or not.

The joint value of a relationship with an augmented good match obeys:

$$\begin{aligned} U_2^T &= -\psi\left(\delta_2^B\right)+\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\times \\ &\left(\Pi\left(x_2^*\right)+\frac{1-\delta^D}{1+\rho}\left(\nu\left(\left(1-\delta^A\right)V_2^T+\delta^A\gamma^A W_1^T\right)+(1-\nu)U_2^T\right)\right), \end{aligned} \tag{B.87}$$

if the innovation succeeds the frontier moves by a factor γ^B but nothing else changes.

Within a regular good match, the incentive compatibility constraint can be written as:

$$\varphi\left(x_1^*\right)\leq\frac{1-\delta^D}{1+\rho}\left(\nu\left(\left(1-\delta^A\right)V_1^s+\delta^A\gamma^A W_1^s\right)+(1-\nu)U_1^s\right). \tag{B.88}$$

In the next period, a supplier who cooperate captures U_1^s if it is a period where relationship-specific innovation may occurs, while she captures V_1^s in a general innovation period without an actual general innovation and W_1^s (with a higher technology level) in a period where a general innovation did occur. Similarly in an augmented good match, the incentive compatibility constraint can be written as:

$$\varphi\left(x_2^*\right)\leq\frac{1-\delta^D}{1+\rho}\left(\nu\left(\left(1-\delta^A\right)V_2^s+\delta^A\gamma^A W_1^s\right)+(1-\nu)U_2^s\right). \tag{B.89}$$

In equilibrium, the innovation rates δ_1^B and δ_2^B maximize U_1^T and U_2^T so that the following first order conditions must hold:

$$\psi'\left(\delta_1^B\right) = \gamma^B\left(\Pi\left(x_2^*\right) + \frac{1-\delta^D}{1+\rho}\left(\nu\left(\left(1-\delta^A\right)V_2^T + \delta^A\gamma^A W_1^T\right) + (1-\nu)U_2^T\right)\right)$$
$$-\left(\Pi\left(x_1^*\right) + \frac{1-\delta^D}{1+\rho}\left(\nu\left(\left(1-\delta^A\right)V_1^T + \delta^A\gamma^A W_1^T\right) + (1-\nu)U_1^T\right)\right). \tag{B.90}$$

$$\psi'\left(\delta_2^B\right) = \left(\gamma^B-1\right)\left(\Pi\left(x_2^*\right) + \frac{1-\delta^D}{1+\rho}\left(\nu\left(\left(1-\delta^A\right)V_2^T + \delta^A\gamma^A W_1^T\right) + (1-\nu)U_2^T\right)\right). \tag{B.91}$$

General innovation period where innovation failed. We turn to the case of a general innovation period where innovation failed. Such a period is identical to the previous case except that relationship specific innovations are ruled out. Yet since such an innovation is undertaken before the relationship specific investment is undertaken, its absence does not affect investment levels (conditional on the match being regular or augmented at the time of the investment level). Therefore we get that the joint values with a new supplier, a regular good match and an augmented good match obey (these equations are the pendant to (B.85), (B.86) and (B.87) in the previous case):

$$V_0^T = (1-b)V_1^T + b\theta\Pi(n) + b\frac{1-\delta^D}{1+\rho}\left[\nu\left(\left(1-\delta^A\right)V_0^T + \delta^A\gamma^A W_0^T\right) + (1-\nu)U_0^T\right]; \tag{B.92}$$
$$V_1^T = \Pi\left(x_1^*\right) + \frac{1-\delta^D}{1+\rho}\left(\nu\left(\left(1-\delta^A\right)V_1^T + \delta^A\gamma^A W_1^T\right) + (1-\nu)U_1^T\right); \tag{B.93}$$
$$V_2^T = \Pi\left(x_2^*\right) + \frac{1-\delta^D}{1+\rho}\left(\nu\left(\left(1-\delta^A\right)V_2^T + \delta^A\gamma^A W_1^T\right) + (1-\nu)U_2^T\right). \tag{B.94}$$

Furthermore, the incentive compatibility constraints are still given by (B.88) and (B.89).

Equilibrium properties: $x_2^* \geq x_1^*$ and $\delta_2^B \leq \delta_1^B$. We are here going to show that the equilibrium can indeed feature $x_2^* \geq x_1^*$ and that we must have $\delta_2^B \leq \delta_1^B$. To do that first note that Bertrand competition implies:

$$U_1^s = U_1^T - U_0^T \text{ and } U_2^s = U_2^T - \frac{1}{\gamma^B}U_0^T. \tag{B.95}$$

$$V_1^s = V_1^T - V_0^T \text{ and } V_2^s = V_2^T - \frac{1}{\gamma^B}V_0^T. \tag{B.96}$$

Moreover, combining (B.86) with (B.93) and (B.87) with (B.94), we get:

$$U_1^T = -\psi\left(\delta_1^B\right) + \left(1-\delta_1^B\right)V_1^T + \delta_1^B\gamma^B V_2^T, \tag{B.97}$$

$$U_2^T = -\psi\left(\delta_2^B\right) + \left(1 - \delta_2^B + \delta_2^B \gamma^B\right) V_2^T. \tag{B.98}$$

Using (B.93) and (B.94), we can rewrite (B.90) and (B.91) as:

$$\psi'\left(\delta_1^B\right) = \gamma^B V_2^T - V_1^T \text{ and } \psi'\left(\delta_2^B\right) = \left(\gamma^B - 1\right) V_2^T. \tag{B.99}$$

Using (B.93), (B.94) and (B.97), (B.98) and then (B.99), we can derive:

$$\left(V_2^T - V_1^T\right)\left(1 - \frac{1-\delta^D}{1+\rho}\left(\nu\left(1-\delta^A\right) + (1-\nu)\left(1-\delta_1^B\right)\right)\right) \tag{B.100}$$
$$= \Pi\left(x_2^*\right) - \Pi\left(x_1^*\right) + (1-\nu)\frac{1-\delta^D}{1+\rho}\left(\psi\left(\delta_1^B\right) - \psi\left(\delta_2^B\right) - \left(\delta_1^B - \delta_2^B\right)\psi'\left(\delta_2^B\right)\right).$$

Since ψ is convex, $\psi\left(\delta_1^B\right) - \psi\left(\delta_2^B\right) - \left(\delta_1^B - \delta_2^B\right)\psi'\left(\delta_2^B\right) \geq 0$, and since $x_2^* \geq x_1^*$, then we get that $V_2^T \geq V_1^T$. From (B.99), we then obtain that $\delta_1^B \geq \delta_2^B$ and the inequality is strict unless $x_1^* = x_2^*$.

Further, using (B.95) and (B.96), we get

$$\nu\left(\left(1-\delta^A\right)V_2^s + \delta^A\gamma^A W_1^s\right) + (1-\nu)U_2^s - \left(\nu\left(\left(1-\delta^A\right)V_1^s + \delta^A\gamma^A W_1^s\right) + (1-\nu)U_1^s\right)$$
$$= \nu\left(1-\delta^A\right)\left(V_2^T - \frac{1}{\gamma^B}V_0^T\right) + (1-\nu)\left(U_2^T - \frac{1}{\gamma^B}U_0^T\right)$$
$$-\left[\nu\left(1-\delta^A\right)\left(V_1^T - V_0^T\right) + (1-\nu)\left(U_1^T - U_0^T\right)\right]$$

Then plug in (B.97), (B.98) and (B.99) to get:

$$\nu\left(\left(1-\delta^A\right)V_2^s + \delta^A\gamma^A W_1^s\right) + (1-\nu)U_2^s - \left(\nu\left(\left(1-\delta^A\right)V_1^s + \delta^A\gamma^A W_1^s\right) + (1-\nu)U_1^s\right)$$
$$= \left(\nu\left(1-\delta^A\right) + (1-\nu)\left(1-\delta_1^B\right)\right)\left(V_2^T - V_1^T\right)$$
$$+ (1-\nu)\left(\psi\left(\delta_1^B\right) - \psi\left(\delta_2^B\right) - \left(\delta_1^B - \delta_2^B\right)\psi'\left(\delta_2^B\right)\right).$$

As established before the last line is weakly positive as $V_2^T \geq V_1^T$, this in return implies that the IC constraint faced by suppliers in an augmented match is laxer than that faced by suppliers in a regular good match, which justifies that the equilibrium features $x_2^B \geq x_1^B$ (with equality if and only if $x_1^B = x_2^B = m$).

General innovation period when innovation succeeded. Finally, we look at the value functions in a general innovation period when innovation succeeded. The joint value with a new outdated producer is given by:

$$W_0^T = (1-b)W_1^T + b\theta\frac{1}{\gamma^A}\Pi(n) + b\frac{1-\delta^D}{1+\rho}\left[\nu\left(\left(1-\delta^A\right)V_0^T + \delta^A\gamma^A W_0^T\right) + (1-\nu)U_0^T\right]. \tag{B.101}$$

The logic is the same as in the baseline model except that the continuation value must take into account that with probability $1-\nu$ the following period is one where relationship specific innovations are possible (so that n case of bad match this period, the continuation value is U_0^T). The joint value with an outdated good match is then given by:

$$W_1^T = \frac{1}{\gamma^A}\Pi\left(y^*\right) + \frac{1-\delta^D}{1+\rho}\left[\nu\left(\left(1-\delta^A\right)V_1^T + \delta^A\gamma^A W_1^T\right) + (1-\nu)\,U_1^T\right]. \tag{B.102}$$

Note that there are no outdated augmented good matches, because the last relationship specific innovation becomes freely available when the next general innovation becomes available. Then, using (B.93), we get:

$$V_1^T - W_1^T = \Pi\left(x_1^*\right) - \frac{1}{\gamma^A}\Pi\left(y^*\right). \tag{B.103}$$

The joint value of a relationship with the innovator when the producer does not know a good match obeys:

$$V_I^{T,b} = V_0^T$$

as in the baseline model. Therefore we still have through Bertrand Competition, using (B.92), (B.101) and (B.103) that:

$$V_I^{s,b} = V_0^T - W_0^T = (1-b)\left(\Pi\left(x_1^*\right) - \frac{1}{\gamma^A}\Pi\left(y^*\right)\right) + b\theta\left(1-\frac{1}{\gamma^A}\right)\Pi\left(n\right). \tag{B.104}$$

The joint value of a relationship with the innovator when the producer knows a good match is given by:

$$V_I^{T,g} = (1-b)\,V_1^T + b\theta\Pi\left(n\right) + b\frac{1-\delta^D}{1+\rho}\left(\nu\left(\left(1-\delta^A\right)V_1^p + \delta^A\gamma^A W_1^p\right) + (1-\nu)\,U_1^p\right). \tag{B.105}$$

The logic is the same as in the baseline model, if the innovator turns out to be a bad match, then the producer can return to the previous good match supplier and earns V_1^p, W_1^p or U_1^p depending on the situation (note that the next period the outdated producer starts with the new frontier technology but cannot be an augmented good match).

As in the baseline model, the value of an outdated good match producer who is not picked by the producer is still positive and given by:

$$V_A^s = b\frac{1-\delta^D}{1+\rho}\left(\nu\left(\left(1-\delta^A\right)V_1^s + \delta^A\gamma^A W_1^s\right) + (1-\nu)\,U_1^s\right), \tag{B.106}$$

so that we must have $W_1^s \geq V_A^s$ and $W_1^p \leq W_1^T - V_A^s$. The producer will then switch suppliers whenever $V_I^{T,g} > W_1^T - V_A^s$. Combining (B.102), (B.105) and (B.106), we get:

$$V_I^{s,g} = \left((1-b)\,\Pi\left(x_1^*\right) + b\theta\Pi\left(n\right) - \frac{1}{\gamma^A}\Pi\left(y^*\right)\right)^+, \tag{B.107}$$

exactly as in the baseline model.

Further note that the incentive constraint in an outdated good match obeys:

$$\frac{1}{\gamma^A}\varphi\left(y^*\right) \leq \frac{1-\delta^D}{1+\rho}\left(\nu\left(\left(1-\delta^A\right)V_1^s+\delta^A\gamma^A W_1^s\right)+(1-\nu)U_1^s\right),$$

so that $y^* \geq x_1^*$ with equality if and only if the first best is achievable.

Recall that an innovator does not observe when the last general innovation took place. The expected value of a general innovator, normalized by the pre-innovation average technology ($\widetilde{A}_t$) in the economy is given by

$$Z = \omega V_I^{s,b}\frac{E\left(\widetilde{A}_{jt}|j\in b\right)}{\widetilde{A}_t}+(1-\omega)V_I^{s,g}\frac{E\left(\widetilde{A}_{jt}|j\in b\right)}{\widetilde{A}_t},$$

where $\widetilde{A}_{jt}$ denotes the pre-innovation frontier technology in line j and $j\in b$ means that the producer j does not know a (cooperating) good match. When a producer does not know a good match, then he cannot enjoy relationship specific innovations in his line. Therefore there exists a function $\lambda\left(\delta_1^B,\delta_2^B,\delta^I\right)\geq 1$ such that in expectation

$$E\left(\widetilde{A}_{jt}|j\in g\right)=\lambda\left(\delta_1^B,\delta_2^B,\delta^A\right)E\left(\widetilde{A}_{jt}|j\in b\right),$$

and this function is increasing in δ_1^B,δ_2^B, since the more relationship specific innovation occur, the more the average technology level of good match producers will pull ahead. We can then rewrite

$$Z=\frac{\omega}{\omega+(1-\omega)\lambda\left(\delta_1^B,\delta_2^B,\delta^I\right)}V_I^{s,b}+\frac{(1-\omega)\lambda\left(\delta_1^B,\delta_2^B,\delta^I\right)}{\omega+(1-\omega)\lambda\left(\delta_1^B,\delta_2^B,\delta^I\right)}V_I^{s,g}, \quad \text{(B.108)}$$

and the innovation rate solves:

$$\psi'\left(\delta^A\right)=\gamma^A Z. \quad \text{(B.109)}$$

Relationship-specific innovation rate. Here we rewrite the first order condition (B.99) in function of profits only, which will be useful to prove Proposition 5. Use (B.98) and (B.103) in (B.94) to get:

$$V_2^T=\frac{(1+\rho)\Pi\left(x_2^*\right)+\left(1-\delta^D\right)\left(\nu\delta^A\gamma^A\left(V_1^T-\left(\Pi\left(x_1^*\right)-\frac{1}{\gamma^A}\Pi\left(y^*\right)\right)\right)-(1-\nu)\psi\left(\delta_2^B\right)\right)}{1+\rho-\left(1-\delta^D\right)\left(\nu\left(1-\delta^A\right)+(1-\nu)\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\right)}, \quad \text{(B.110)}$$

where we defined

$$\widetilde{\rho}\equiv\rho-\left(1-\delta^D\right)\nu\left(1-\delta^A\right),$$

to simplify our expressions ($1+\widetilde{\rho}>0$ otherwise the value functions are not well defined).

Then use this equation together with (B.103) and (B.97) in (B.93) to get (after rearranging terms):

$$V_1^T \tag{B.111}$$
$$= \frac{\left[\begin{array}{c}\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\right)\left((1+\rho)\,\Pi\left(x_1^*\right)-\left(1-\delta^D\right)(1-\nu)\,\psi\left(\delta_1^B\right)\right)\\ -\left(1-\delta^D\right)\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_2^B+\left(\delta_2^B-\delta_1^B\right)\gamma^B\right)\right)\nu\delta^A\gamma^A\left(\Pi\left(x_1^*\right)-\frac{1}{\gamma^A}\Pi\left(y^*\right)\right)\\ +\left(1-\delta^D\right)(1-\nu)\,\delta_1^B\gamma^B\left((1+\rho)\,\Pi\left(x_2^*\right)-\left(1-\delta^D\right)(1-\nu)\,\psi\left(\delta_2^B\right)\right)\end{array}\right]}{\left[\begin{array}{c}\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[(1-\nu)\left(1-\delta_1^B\right)+\nu\delta^A\gamma^A\right]\right)\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\right)\\ -(1-\nu)\,\delta_1^B\gamma^B\left(1-\delta^D\right)^2\nu\delta^A\gamma^A\end{array}\right]}.$$

Combining this expression with (B.110), we get:

$$V_2^T \tag{B.112}$$
$$= \frac{\left[\begin{array}{c}\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[(1-\nu)\left(1-\delta_1^B\right)+\nu\delta^A\gamma^A\right]\right)\left((1+\rho)\,\Pi\left(x_2^*\right)-\left(1-\delta^D\right)(1-\nu)\,\psi\left(\delta_2^B\right)\right)\\ +\left(1-\delta^D\right)\nu\delta^A\gamma^A\left[\begin{array}{c}\left((1+\rho)\,\Pi\left(x_1^*\right)-\left(1-\delta^D\right)(1-\nu)\,\psi\left(\delta_1^B\right)\right)\\ -\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_1^B\right)\right)\left(\Pi\left(x_1^*\right)-\frac{1}{\gamma^A}\Pi\left(y^*\right)\right)\end{array}\right]\end{array}\right]}{\left[\begin{array}{c}\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[(1-\nu)\left(1-\delta_1^B\right)+\nu\delta^A\gamma^A\right]\right)\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\right)\\ -(1-\nu)\,\delta_1^B\gamma^B\left(1-\delta^D\right)^2\nu\delta^A\gamma^A\end{array}\right]}.$$

Combining this with (B.99), we get

$$\psi'\left(\delta_2^B\right)/\left(\gamma^B-1\right) \tag{B.113}$$
$$= \frac{\left[\begin{array}{c}\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[(1-\nu)\left(1-\delta_1^B\right)+\nu\delta^A\gamma^A\right]\right)\left((1+\rho)\,\Pi\left(x_2^*\right)-\left(1-\delta^D\right)(1-\nu)\,\psi\left(\delta_2^B\right)\right)\\ +\left(1-\delta^D\right)\nu\delta^A\gamma^A\left[\begin{array}{c}\left((1+\rho)\,\Pi\left(x_1^*\right)-\left(1-\delta^D\right)(1-\nu)\,\psi\left(\delta_1^B\right)\right)\\ -\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_1^B\right)\right)\left(\Pi\left(x_1^*\right)-\frac{1}{\gamma^A}\Pi\left(y^*\right)\right)\end{array}\right]\end{array}\right]}{\left[\begin{array}{c}\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[(1-\nu)\left(1-\delta_1^B\right)+\nu\delta^A\gamma^A\right]\right)\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\right)\\ -(1-\nu)\,\delta_1^B\gamma^B\left(1-\delta^D\right)^2\nu\delta^A\gamma^A\end{array}\right]}$$

B.8.3. Proof of Proposition 5.

Nash case. The Nash equilibrium obeys the same equation except that the investment level is always n. Combining (B.99) and (B.100) implies that in the Nash equilibrium $\delta_1^{B,Nash}=\delta_2^{B,Nash}=\delta^{B,Nash}$ and $V_1^{T,Nash}=V_2^{T,Nash}$. Moreover, we get (using (B.113) but replacing x_1^*, x_2^* and y^* by n and δ_1^B, δ_2^B by a single $\delta^{B,Nash}$):

$$\frac{\psi'\left(\delta^{B,Nash}\right)}{\gamma^B-1}=$$
$$\frac{\left(1+\rho-\left(1-\delta^D\right)\nu\delta^{A,Nash}\left(\gamma^A-1\right)\right)\Pi\left(n\right)-\left(1-\delta^D\right)(1-\nu)\,\psi\left(\delta^{B,Nash}\right)}{1+\rho-\left(1-\delta^D\right)\left(\nu\left(1-\delta^{A,Nash}+\delta^{A,Nash}\gamma^A\right)+(1-\nu)\left(1-\delta^{B,Nash}+\delta^{B,Nash}\gamma^B\right)\right)}.$$

Establishing part b). We first establish part b) of the proposition. (B.99) implies that:

$$\psi'\left(\delta_1^B\right)=\psi'\left(\delta_2^B\right)+V_2^T-V_1^T.$$

Using (B.100), we then get:

$$\psi'\left(\delta_1^B\right) = \frac{\left[\begin{array}{c} \psi'\left(\delta_2^B\right)\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_2^B\right)\right) \\ +\left(\Pi\left(x_2^*\right)-\Pi\left(x_1^*\right)\right)(1+\rho)+(1-\nu)\left(1-\delta^D\right)\left(\psi\left(\delta_1^B\right)-\psi\left(\delta_2^B\right)\right) \end{array}\right]}{1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_1^B\right)}. \tag{B.114}$$

Because of the convexity of ψ, we have $\psi\left(\delta_1^B\right)-\psi\left(\delta_2^B\right) \leq \left(\delta_1^B-\delta_2^B\right)\psi'\left(\delta_1^B\right)$, using the expression above we then get:

$$\psi\left(\delta_1^B\right)-\psi\left(\delta_2^B\right) \leq \left(\delta_1^B-\delta_2^B\right)\psi'\left(\delta_2^B\right)+\frac{(1+\rho)\left(\delta_1^B-\delta_2^B\right)\left(\Pi\left(x_2^*\right)-\Pi\left(x_1^*\right)\right)}{1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_2^B\right)}. \tag{B.115}$$

Rearrange (B.113) to obtain:

$$\begin{aligned} &\frac{\psi'\left(\delta_2^B\right)}{\gamma^B-1}\left(\begin{array}{c} \left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[(1-\nu)\left(1-\delta_1^B\right)+\nu\delta^A\gamma^A\right]\right)\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\right) \\ -(1-\nu)\delta_1^B\gamma^B\left(1-\delta^D\right)^2\nu\delta^A\gamma^A \end{array}\right) \\ &= \left[\begin{array}{c} \left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[(1-\nu)\left(1-\delta_1^B\right)+\nu\delta^A\gamma^A\right]\right)(1+\rho)\Pi\left(x_2^*\right) \\ +\left(1-\delta^D\right)\nu\delta^A\gamma^A\left[(1+\rho)\Pi\left(x_1^*\right)-\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_1^B\right)\right)\left(\Pi\left(x_1^*\right)-\frac{1}{\gamma^A}\Pi\left(y^*\right)\right)\right] \\ -\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_1^B\right)\right)\left(1-\delta^D\right)(1-\nu)\psi\left(\delta_2^B\right) \\ -\left(1-\delta^D\right)\nu\delta^A\gamma^A\left(1-\delta^D\right)(1-\nu)\left(\psi\left(\delta_1^B\right)-\psi\left(\delta_2^B\right)\right) \end{array}\right] \end{aligned}$$

Then use (B.115) and $y^* \geq x_1^*$ to get:

$$\begin{aligned} &\frac{\psi'\left(\delta_2^B\right)}{\gamma^B-1}\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_1^B\right)\right)\left(\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left(\nu\delta^A\gamma+(1-\nu)\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\right)\right)\right) \\ &\geq \left[\begin{array}{c} \left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[(1-\nu)\left(1-\delta_1^B\right)+\nu\delta^A\gamma^A\right]\right)(1+\rho)\Pi\left(x_2^*\right) \\ +\left(1-\delta^D\right)\nu\delta^A\gamma^A\left[1+\rho-\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_1^B\right)\right)\left(1-\frac{1}{\gamma^A}\right)\right]\Pi\left(x_1^*\right) \\ -\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_1^B\right)\right)\left(1-\delta^D\right)(1-\nu)\psi\left(\delta_2^B\right) \\ -\nu\delta^A\gamma^A\left(1-\delta^D\right)^2(1-\nu)\frac{(1+\rho)\left(\delta_1^B-\delta_2^B\right)\left(\Pi\left(x_2^*\right)-\Pi\left(x_1^*\right)\right)}{1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_2^B\right)} \end{array}\right]. \end{aligned}$$

Further reorder terms (and use that $1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_1^B\right)>0$) to obtain:

$$\begin{aligned} &\psi'\left(\delta_2^B\right)\left(\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left(\nu\delta^A\gamma+(1-\nu)\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\right)\right)\right) \\ &\geq \left(\gamma^B-1\right)\left[\begin{array}{c} \left(1+\rho-\left(1-\delta^D\right)\nu\delta^A\left(\gamma^A-1\right)\right)\Pi\left(x_1^*\right)-\left(1-\delta^D\right)(1-\nu)\psi\left(\delta_2^B\right) \\ +\frac{\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left(\nu\delta^A\gamma^A+(1-\nu)\left(1-\delta_2^B\right)\right)\right)(1+\rho)\left(\Pi\left(x_2^*\right)-\Pi\left(x_1^*\right)\right)}{1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_2^B\right)} \end{array}\right]. \end{aligned}$$

Using that $x_2^* \geq x_1^*$ and that $1+\widetilde{\rho}-\left(1-\delta^D\right)\left(\nu\delta^A\gamma^A+(1-\nu)\left(1-\delta_2^B\right)\right)>0$ for value functions to be defined, we get:

$$\psi'\left(\delta_2^B\right) \geq \frac{\left(\gamma^B-1\right)\left[\left(1+\rho-\left(1-\delta^D\right)\nu\delta^A\left(\gamma^A-1\right)\right)\Pi\left(x_1^*\right)-\left(1-\delta^D\right)(1-\nu)\psi\left(\delta_2^B\right)\right]}{1+\rho-\left(1-\delta^D\right)\left(\nu\left(1-\delta^A+\delta^A\gamma^A\right)+(1-\nu)\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\right)}.$$

Further, note that

$$\begin{aligned} &\frac{\partial}{\partial\delta^A}\frac{\left[\left(1+\rho-\left(1-\delta^D\right)\nu\delta^A\left(\gamma^A-1\right)\right)\Pi\left(x_1^*\right)-\left(1-\delta^D\right)(1-\nu)\psi\left(\delta_2^B\right)\right]}{1+\rho-\left(1-\delta^D\right)\left(\nu\left(1-\delta^A+\delta^A\gamma\right)+(1-\nu)\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\right)} \\ &= \frac{\left(\Pi\left(x_1^*\right)\left(\nu+(1-\nu)\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\right)-(1-\nu)\psi\left(\delta_2^B\right)\right)\left(1-\delta^D\right)^2\nu\left(\gamma^A-1\right)}{\left[1+\rho-\left(1-\delta^D\right)\left(\nu\left(1-\delta^A+\delta^A\gamma^A\right)+(1-\nu)\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\right)\right]^2}>0 \end{aligned}$$

under the assumption that $\psi\left(\delta_2^B\right) < \left(\nu/\left(1-\nu\right)+1-\delta_2^B+\delta_2^B\gamma^B\right)$. Therefore, if $\delta^{A,coop} \geq \delta^{A,Nash}$, we get $\psi'\left(\delta_2^B\right) \geq \frac{(\gamma^B-1)[(1+\rho-(1-\delta^D)\nu\delta^{A,Nash}(\gamma^A-1))\Pi(x_1^*)-(1-\delta^D)(1-\nu)\psi(\delta_2^B)]}{1+\rho-(1-\delta^D)(\nu(1-\delta^{A,Nash}+\delta^{A,Nash}\gamma^A)+(1-\nu)(1-\delta_2^B+\delta_2^B\gamma^B))}$. Finally, $x_1^* > n$, therefore one gets:

$$\begin{aligned}&\psi'\left(\delta_2^B\right)\left(1+\rho-\left(1-\delta^D\right)\left(\nu\left(1-\delta^{A,Nash}+\delta^{A,Nash}\gamma^A\right)+(1-\nu)\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\right)\right)\\&-\left(\gamma^B-1\right)\left[\left(1+\rho-\left(1-\delta^D\right)\nu\delta^{A,Nash}\left(\gamma^A-1\right)\right)\Pi(n)-\left(1-\delta^D\right)(1-\nu)\psi\left(\delta_2^B\right)\right]\\&>0.\end{aligned}$$

Noting that the left-hand side is an increasing function of δ_2^B and that $\delta^{B,Nash}$ is the solution to the left-hand side being equal to 0, we obtain

$$\delta_1^{B,coop} \geq \delta_2^{B,coop} > \delta^{B,Nash}.$$

Proof of Part a). Assume that $\delta_1^{B,coop} \geq \delta_2^{B,coop} > \delta^{B,Nash}$ (otherwise we know for sure that we must have $\delta^{A,coop} < \delta^{A,Nash}$ per part b)). Then one gets that $\lambda\left(\delta_1^{B,coop},\delta_2^{B,coop},\delta^A\right) > \lambda\left(\delta^{B,Nash},\delta^{B,Nash},\delta^A\right)$, since $V_I^{s,b} > V_I^{s,g}$, this factor pushes towards relatively less general innovation in the cooperative than in the Nash case. Other than that the expressions for the incentive to innovate is the same as in the baseline model, therefore sufficient conditions under which $\delta^{A,coop} < \delta^{A,Nash}$ in the baseline model are still sufficient now (but necessary conditions need not be so any more).

B.8.4. Proof of Remark 3. Assume the same exogenous innovation rates in the Nash and the cooperative cases with $\delta_2^B \leq \delta_1^B$. Then one can use (B.113) and obtain:

$$\frac{\psi'\left(\delta_{2,0}^{B,coop}\right)}{\gamma^B-1} = \frac{\left[\begin{array}{c}\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[(1-\nu)\left(1-\delta_1^B\right)+\nu\delta^A\gamma^A\right]\right)\left((1+\rho)\Pi\left(x_2^*\right)-\left(1-\delta^D\right)(1-\nu)\psi\left(\delta_2^B\right)\right)\\+\left(1-\delta^D\right)\nu\delta^A\gamma^A\left[\begin{array}{c}\left((1+\rho)\Pi\left(x_1^*\right)-\left(1-\delta^D\right)(1-\nu)\psi\left(\delta_1^B\right)\right)\\-\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_1^B\right)\right)\left(\Pi\left(x_1^*\right)-\frac{1}{\gamma^A}\Pi\left(y^*\right)\right)\end{array}\right]\end{array}\right]}{\left[\begin{array}{c}\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[(1-\nu)\left(1-\delta_1^B\right)+\nu\delta^A\gamma^A\right]\right)\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\right)\\-(1-\nu)\delta_1^B\gamma^B\left(1-\delta^D\right)^2\nu\delta^A\gamma^A\end{array}\right]}$$

where $\widetilde{\rho}$ is defined as above with the exogenous innovation rate δ^A, and similarly,

$$\frac{\psi'\left(\delta_{2,0}^{B,Nash}\right)}{\gamma^B-1} = \frac{\left[\begin{array}{c}\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[(1-\nu)\left(1-\delta_1^B\right)+\nu\delta^A\gamma^A\right]\right)\left((1+\rho)\Pi(n)-\left(1-\delta^D\right)(1-\nu)\psi\left(\delta_2^B\right)\right)\\+\left(1-\delta^D\right)\nu\delta^A\gamma^A\left[\begin{array}{c}\left((1+\rho)\Pi(n)-\left(1-\delta^D\right)(1-\nu)\psi\left(\delta_1^B\right)\right)\\-\left(1+\rho-\left(1-\delta^D\right)\left[(1-\nu)\left(1-\delta_1^B\right)+\nu\left(1-\delta^A\right)\right]\right)\left(1-\frac{1}{\gamma^A}\right)\Pi(n)\end{array}\right]\end{array}\right]}{\left[\begin{array}{c}\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[(1-\nu)\left(1-\delta_1^B\right)+\nu\delta^A\gamma^A\right]\right)\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\right)\\-(1-\nu)\delta_1^B\gamma^B\left(1-\delta^D\right)^2\nu\delta^A\gamma^A\end{array}\right]}$$

At the same time (B.104), (B.107), (B.108) and (B.109) give:

$$\begin{aligned}&\psi'\left(\delta_0^{A,coop}\right)\\ =\;&\gamma^A\left(\begin{array}{c}\frac{(1-\omega)\lambda\left(\delta_1^B,\delta_2^B,\delta^A\right)}{\omega+(1-\omega)\lambda\left(\delta_1^B,\delta_2^B,\delta^A\right)}\left((1-b)\,\Pi\left(x_1^*\right)+b\theta\Pi\left(n\right)-\frac{1}{\gamma^A}\Pi\left(y^*\right)\right)^+\\ +\frac{\omega}{\omega+(1-\omega)\lambda\left(\delta_1^B,\delta_2^B,\delta^I\right)}\left[(1-b)\left(\Pi\left(x_1^*\right)-\frac{\Pi(y^*)}{\gamma^A}\right)+b\theta\left(1-\frac{1}{\gamma^A}\right)\Pi\left(n\right)\right]\end{array}\right).\end{aligned}$$

$$\psi'\left(\delta_0^{A,Nash}\right)=\gamma^A\left(\begin{array}{c}\frac{(1-\omega)\lambda\left(\delta_1^B,\delta_2^B,\delta^A\right)}{\omega+(1-\omega)\lambda\left(\delta_1^B,\delta_2^B,\delta^A\right)}\left(1-b+b\theta-\frac{1}{\gamma^A}\right)^+\\ +\frac{\omega}{\omega+(1-\omega)\lambda\left(\delta_1^B,\delta_2^B,\delta^I\right)}\left(1-\frac{1}{\gamma^A}\right)[1-b+b\theta]\end{array}\right)\Pi\left(n\right).$$

We then obtain

$$\left(\frac{\delta_0^{A,Nash}/\delta_{2,0}^{B,Nash}}{\delta_0^{A,coop}/\delta_{2,0}^{B,coop}}\right)^{\psi-1}=\frac{\psi'\left(\delta_0^{A,Nash}\right)/\psi'\left(\delta_{2,0}^{B,Nash}\right)}{\psi'\left(\delta_0^{A,coop}\right)/\psi'\left(\delta_{2,0}^{B,coop}\right)}=\frac{A}{B}$$

with $A\equiv\dfrac{\left(\frac{(1-\omega)\lambda\left(\delta_1^B,\delta_2^B,\delta^A\right)}{\omega+(1-\omega)\lambda\left(\delta_1^B,\delta_2^B,\delta^A\right)}\left(1-b+b\theta-\frac{1}{\gamma^A}\right)^++\frac{\omega}{\omega+(1-\omega)\lambda\left(\delta_1^B,\delta_2^B,\delta^I\right)}\left(1-\frac{1}{\gamma^A}\right)[1-b+b\theta]\right)}{\left(\begin{array}{c}\frac{(1-\omega)\lambda\left(\delta_1^B,\delta_2^B,\delta^A\right)}{\omega+(1-\omega)\lambda\left(\delta_1^B,\delta_2^B,\delta^A\right)}\left((1-b)+b\theta\frac{\Pi(n)}{\Pi\left(x_1^*\right)}-\frac{1}{\gamma^A}\frac{\Pi(y^*)}{\Pi\left(x_1^*\right)}\right)^+\\ +\frac{\omega}{\omega+(1-\omega)\lambda\left(\delta_1^B,\delta_2^B,\delta^I\right)}\left[(1-b)\left(1-\frac{\Pi(y^*)}{\gamma^A\Pi\left(x_1^*\right)}\right)+b\theta\left(1-\frac{1}{\gamma^A}\right)\frac{\Pi(n)}{\Pi\left(x_1^*\right)}\right]\end{array}\right)}$; and

$$B\equiv\frac{\left[\begin{array}{c}\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[(1-\nu)\left(1-\delta_1^B\right)+\nu\delta^A\gamma^A\right]\right)\left((1+\rho)-\left(1-\delta^D\right)(1-\nu)\frac{\psi\left(\delta_2^B\right)}{\Pi(n)}\right)\\ +\left(1-\delta^D\right)\nu\delta^A\gamma^A\left[\begin{array}{c}\left((1+\rho)\,\Pi\left(n\right)-\left(1-\delta^D\right)(1-\nu)\frac{\psi\left(\delta_1^B\right)}{\Pi(n)}\right)\\ -\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_1^B\right)\right)\left(1-\frac{1}{\gamma^A}\right)\end{array}\right]\end{array}\right]}{\left[\begin{array}{c}\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[(1-\nu)\left(1-\delta_1^B\right)+\nu\delta^A\gamma^A\right]\right)\left((1+\rho)\frac{\Pi\left(x_2^*\right)}{\Pi\left(x_1^*\right)}-\left(1-\delta^D\right)(1-\nu)\frac{\psi\left(\delta_2^B\right)}{\Pi\left(x_1^*\right)}\right)\\ +\left(1-\delta^D\right)\nu\delta^A\gamma^A\left[\begin{array}{c}\left(1+\rho-\left(1-\delta^D\right)(1-\nu)\frac{\psi\left(\delta_1^B\right)}{\Pi\left(x_1^*\right)}\right)\\ -\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_1^B\right)\right)\left(1-\frac{1}{\gamma^A}\frac{\Pi(y^*)}{\Pi\left(x_1^*\right)}\right)\end{array}\right]\end{array}\right]}.$$

Since $\Pi\left(n\right)<\Pi\left(x_1^*\right)$ and $\Pi\left(y^*\right)\geq\Pi\left(x_1^*\right)$ then $A>1$. In addition, $\Pi\left(x_2^*\right)\geq\Pi\left(x_1^*\right)$ and $\Pi\left(y^*\right)\geq\Pi\left(x_1^*\right)$ plus $\psi\left(\delta_2^B\right)/\Pi\left(x_1^*\right)<\psi\left(\delta_2^B\right)/\Pi\left(n\right)$ and $\psi\left(\delta_1^B\right)/\Pi\left(x_1^*\right)<\psi\left(\delta_2^B\right)/\Pi\left(n\right)$ imply that $B<1$, so that $A/B>1$ and

$$\frac{\delta_0^{A,Nash}}{\delta_{2,0}^{B,Nash}}>\frac{\delta_0^{A,coop}}{\delta_{2,0}^{B,coop}}.$$

If instead exogenous innovation is free, then the ψ terms disappear from B and we would get $B\leq1$ so that we still have $A/B>1$.

Using (B.99) together with (B.111) and (B.110)—these expressions are valid as they do not use (B.99) for δ_1^B—we obtain:

$$\psi'\left(\delta_{1,0}^{B,coop}\right) = \frac{\begin{bmatrix} \gamma^B\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[1-\nu+\nu\delta^A\gamma^A\right]\right)(1+\rho)\Pi\left(x_2^*\right) \\ -\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[1-\nu+\nu\delta^A\gamma^A\right]\right)\left(\gamma^B\psi\left(\delta_2^B\right)-\psi\left(\delta_1^B\right)\right)\left(1-\delta^D\right)(1-\nu) \\ -\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left(\nu\gamma^B\delta^A\gamma^A+(1-\nu)\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\right)\right)(1+\rho)\Pi\left(x_1^*\right) \\ -\left(1-\delta^D\right)\left(\gamma^B-1\right)\left((1-\nu)\delta_2^B+\nu\delta^A\gamma^A\right)\psi\left(\delta_1^B\right)\left(1-\delta^D\right)(1-\nu) \\ -\left(\gamma^B-1\right)\left(\Pi\left(x_1^*\right)-\frac{1}{\gamma^A}\Pi\left(y^*\right)\right)\left(1-\delta^D\right)\nu\delta^A\gamma^A\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_2^B\right)\right)\end{bmatrix}}{\begin{bmatrix}\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[(1-\nu)\left(1-\delta_1^B\right)+\nu\delta^A\gamma^A\right]\right)\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\right) \\ -(1-\nu)\delta_1^B\gamma^B\left(1-\delta^D\right)^2\nu\delta^A\gamma^A\end{bmatrix}},$$

and similarly

$$\psi'\left(\delta_{1,0}^{B,Nash}\right) = \frac{\begin{matrix} \gamma^B\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[1-\nu+\nu\delta^A\gamma^A\right]\right)(1+\rho)\Pi(n) \\ -\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[1-\nu+\nu\delta^A\gamma^A\right]\right)\left(\gamma^B\psi\left(\delta_2^B\right)-\psi\left(\delta_1^B\right)\right)\left(1-\delta^D\right)(1-\nu) \\ -\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left(\nu\gamma^B\delta^A\gamma^A+(1-\nu)\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\right)\right)(1+\rho)\Pi(n) \\ -\left(1-\delta^D\right)\left(\gamma^B-1\right)\left((1-\nu)\delta_2^B+\nu\delta^A\gamma^A\right)\psi\left(\delta_1^B\right)\left(1-\delta^D\right)(1-\nu) \\ -\left(\gamma^B-1\right)\left(1-\frac{1}{\gamma^A}\right)\Pi(n)\left(1-\delta^D\right)\nu\delta^A\gamma^A\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_2^B\right)\right)\end{matrix}}{\begin{bmatrix}\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[(1-\nu)\left(1-\delta_1^B\right)+\nu\delta^A\gamma^A\right]\right)\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\right) \\ -(1-\nu)\delta_1^B\gamma^B\left(1-\delta^D\right)^2\nu\delta^A\gamma^A\end{bmatrix}}.$$

Therefore, we get:

$$\left(\frac{\delta_0^{A,Nash}/\delta_{1,0}^{B,Nash}}{\delta_0^{A,coop}/\delta_{1,0}^{B,coop}}\right)^{\psi-1} = \frac{\psi'\left(\delta_0^{A,Nash}\right)/\psi'\left(\delta_{1,0}^{B,Nash}\right)}{\psi'\left(\delta_0^{A,coop}\right)/\psi'\left(\delta_{1,0}^{B,coop}\right)} = \frac{A}{C},$$

with

$$C \equiv \frac{\begin{matrix} \gamma^B\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[1-\nu+\nu\delta^A\gamma^A\right]\right)(1+\rho) \\ -\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left(\nu\gamma^B\delta^A\gamma^A+(1-\nu)\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\right)\right)(1+\rho) \\ -\left(\gamma^B-1\right)\left(1-\frac{1}{\gamma^A}\right)\left(1-\delta^D\right)\nu\delta^A\gamma^A\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_2^B\right)\right) \\ -\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[(1-\nu)+\nu\delta^A\gamma^A\right]\right)\frac{\left(\gamma^B\psi\left(\delta_2^B\right)-\psi\left(\delta_1^B\right)\right)}{\Pi(n)}\left(1-\delta^D\right)(1-\nu) \\ -\left(1-\delta^D\right)\left(\gamma^B-1\right)\left((1-\nu)\delta_2^B+\nu\delta^A\gamma^A\right)\frac{\psi\left(\delta_1^B\right)}{\Pi(n)}\left(1-\delta^D\right)(1-\nu)\end{matrix}}{\begin{matrix} \gamma^B\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[(1-\nu)+\nu\delta^A\gamma^A\right]\right)(1+\rho)\frac{\Pi\left(x_2^*\right)}{\Pi\left(x_1^*\right)} \\ -\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left(\nu\gamma^B\delta^A\gamma^A+(1-\nu)\left(1-\delta_2^B+\delta_2^B\gamma^B\right)\right)\right)(1+\rho) \\ -\left(\gamma^B-1\right)\left(1-\frac{1}{\gamma^A}\frac{\Pi\left(y^*\right)}{\Pi\left(x_1^*\right)}\right)\left(1-\delta^D\right)\nu\delta^A\gamma^A\left(1+\widetilde{\rho}-\left(1-\delta^D\right)(1-\nu)\left(1-\delta_2^B\right)\right) \\ -\left(1+\widetilde{\rho}-\left(1-\delta^D\right)\left[1-\nu+\nu\delta^A\gamma^A\right]\right)\frac{\gamma^B\psi\left(\delta_2^B\right)-\psi\left(\delta_1^B\right)}{\Pi\left(x_1^*\right)}\left(1-\delta^D\right)(1-\nu) \\ -\left(1-\delta^D\right)\left(\gamma^B-1\right)\left((1-\nu)\delta_2^B+\nu\delta^A\gamma^A\right)\frac{\psi\left(\delta_1^B\right)}{\Pi\left(x_1^*\right)}\left(1-\delta^D\right)(1-\nu)\end{matrix}}.$$

$\Pi\left(x_2^*\right) \geq \Pi\left(x_1^*\right)$ and $\Pi\left(y^*\right) \geq \Pi\left(x_1^*\right)$ both push towards $C \leq 1$, we then get that $C < 1$ provided that

$$\begin{aligned}&\left(1+\rho-\left(1-\delta^D\right)\left[(1-\nu)+\nu\left(1-\delta^A+\delta^A\gamma^A\right)\right]\right)\left(\gamma^B\psi\left(\delta_2^B\right)-\psi\left(\delta_1^B\right)\right)\\&+\left(1-\delta^D\right)\left(\gamma^B-1\right)\left((1-\nu)\delta_2^B+\nu\delta^A\gamma^A\right)\psi\left(\delta_1^B\right)\\&>0,\end{aligned}$$

a sufficient condition is then that $\gamma^B\psi\left(\delta_2^B\right) \geq \psi\left(\delta_1^B\right)$. We then have

$$\frac{\delta_0^{A,Nash}}{\delta_{1,0}^{B,Nash}} > \frac{\delta_0^{A,coop}}{\delta_{1,0}^{B,coop}}.$$

If innovation is free when exogenous then the ψ terms in C disappear and we directly have $C \leq 1$.

B.9. Slow diffusion of innovation

We consider a cooperative equilibrium where at the beginning of any relationship a good match cooperates as much as possible whether at the frontier or not, while there is no cooperation in bad matches. As explained in the text, the equilibrium is characterized by the levels of cooperation in frontier good matches (x^*) and in outdated good matches (y^*). It is direct to derive the IC constraints (10) and (29).

On equilibrium path, a producer switches between suppliers (favoring those with the frontier technology) until he finds a good match. Once he has found one, he optimally decides between switching to an innovator (when innovation occurs) or staying with the outdated good match supplier. If it is optimal to switch to the innovator and the innovator turns out to be a good match, he stays in a relationship with that innovator. If she turns out to be a bad match, then in the following period, the producer resumes his relationship with his old supplier if that supplier obtained the frontier technology and tries another frontier firm otherwise.

In this appendix, we first derive the condition under which a producer who knows a good match switches to the innovator in the cooperative case. Then, we look at the corresponding condition in the contractible or Nash cases. Afterward, we describe in more details the IC constraints and check for the existence of the equilibrium. Finally we derive the equations determining the innovation rates in the three cases.

B.9.1. Switching in the cooperative equilibrium. We focus on the cooperative equilibrium. As in the baseline model, Bertrand competition ensures that

$$V_1^s = V_1^T - V_0^{T,b}. \tag{B.116}$$

V_1^T is the joint value with a (cooperating) good match supplier. $V_0^{T,b}$ is the joint value of starting a relationship with a frontier firm without the option to fall back on a cooperating outdated good match, which we indicated through the superscript

b. Making this distinction is now helpful since a producer working with an outdated good match supplier may try to start a new relationship with a frontier supplier (even in periods without innovation), without necessarily being punished for doing so (as the outdated supplier would forgive if the frontier supplier turns out to be a bad match). In this case, the outside option of a producer working with a frontier supplier is to try another frontier supplier, but his previous good match would not forgive him for doing so.

Consider now a cooperating outdated good match. If the producer were to try a frontier firm, her expected value does not become 0 because she could get the producer back in the following period if the frontier firm turns out to be a bad match. As before, we denote by V_A^s, the expected value of an outdated good match when the producer tries out a frontier firm. Bertrand competition ensures that the value of an outdated good match W_1^s must satisfy $W_1^s \geq V_A^s$. Following the same reasoning as in Appendix A.1.1, we get that (A.2) holds. That is $W_1^p = V_I^{p,g}$, where $V_I^{p,g}$ is the value that a producer obtains in a relationship with the innovator when the producer already knows a cooperating outdated good match, and W_1^p is the value the producer captures with an outdated good match. This ensures that a producer (who knows a good match) switches to the innovator if (A.3) holds that is $V_I^{T,g} > W_1^T - V_A^s$, where $V_I^{T,g}$ is the joint value of a relationship with the innovator (when the producer knows a cooperating outdated good match). As a result, we get that (as in the baseline model):

$$W_1^s = V_A^s + \left(W_1^T - V_A^s - V_I^{T,g}\right)^+ . \tag{B.117}$$

Note that $V_I^{T,g}$ is the same as $V_0^{T,g}$, the joint value of starting a relationship with any frontier firm in a period without innovation when the producer also knows a cooperating outdated good match. Therefore in periods without innovation, a producer who knows a good match switches to a frontier firm under the same circumstances (that is whenever $V_I^{T,g} = V_0^{T,g} > W_1^T - V_A^s$). However, Bertrand competition ensures that $V_0^{p,g} = V_0^{T,g}$ since in periods without innovation there is more than one firm with the frontier technology, so that, generally $V_0^{p,g} \neq V_I^{T,g}$.

We obtain the law of motion

$$\begin{aligned} V_I^{T,g} &= V_0^{T,g} = (1-b)\, V_1^T \\ &+ b\left(\theta\Pi(n) + \frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)\left(\Delta V_1^p + (1-\Delta)\max\left(V_0^{T,g}, W_1^p\right)\right) + \delta^I \gamma W_1^p\right)\right). \end{aligned} \tag{B.118}$$

With probability $1-b$ the match is good, and the value is given by V_1^T. With probability b the match is bad, generating profits $\theta\Pi(n)$ this period; in the following period, there are three possibilities. i) No innovation occurred but the previous good match got access to the frontier technology, then the producer resumes his previous relationship and obtains V_1^p. ii) No innovation occurred and the previous good match did not inherit the technology, then the producer optimally decides between staying with his previous good match or trying another frontier supplier; since there are now several suppliers the producer would get the full value of a relationship with a frontier firm if that is higher than $W_1^T - V_A^s$; otherwise he stays with the outdated supplier and gets $W_1^p = W_1^T - \left(V_A^s + \left(W_1^T - V_A^s - V_I^{T,g}\right)\right) = V_0^{T,g}$ in that case. iii) An innovation occurs, in which case the producer can secure γW_1^p (as the frontier has moved one step). We can then rewrite more simply:

$$\begin{aligned} V_I^{T,g} &= V_0^{T,g} = (1-b)\, V_1^T \\ &+ b\left(\theta \Pi(n) + \frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)\left(\Delta V_1^p + (1-\Delta)\, V_0^{T,g}\right) + \delta^I \gamma W_1^p\right)\right). \end{aligned} \tag{B.119}$$

The expected value of an outdated supplier when the producer tries a good match, V_A^s, obeys the following law of motion:

$$V_A^s = b\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)\Delta V_1^s + \left(\left(1-\delta^I\right)(1-\Delta) + \delta^I \gamma\right) W_1^s\right). \tag{B.120}$$

With probability $1-b$, the producer met a good match and therefore the outdated supplier turns into a non-cooperating good match and her value becomes 0. Otherwise, she resumes her relationship with the producer in the following period and obtains V_1^s if she imitates the frontier technology and no innovation occurred. If she does not get access to the frontier technology (either because of a new innovation or because the previous one did not diffuse), the supplier's normalized value is W_1^s.

Combining the two, we get that

$$\begin{aligned} & V_0^{T,g} + V_A^s \\ = \; & (1-b)\, V_1^T + b\theta\Pi(n) + \frac{b\left(1-\delta^D\right)}{1+\rho} \\ & \times\left(\left(1-\delta^I\right)\left(\Delta V_1^T + (1-\Delta)\left(V_0^{T,g} + V_A^s + \left(W_1^T - V_A^s - V_0^{T,g}\right)^+\right)\right) + \delta^I \gamma W_1^T\right) \end{aligned} \tag{B.121}$$

The joint value of a relationship with the frontier supplier is still given by (4), which we reproduce here:

$$V_1^T = \Pi\left(x^*\right) + \frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right) V_1^T + \delta^I \gamma W_1^T\right). \tag{B.122}$$

The joint value of a relationship with an outdated producer obeys the following law of motion:

$$\begin{aligned} W_1^T &= \gamma^{-1}\Pi\left(y^*\right) + \frac{1-\delta^D}{1+\rho} \\ & \times\left(\left(1-\delta^I\right)\left(\Delta V_1^T + (1-\Delta)\left(W_1^T + \left(V_0^{T,g} + V_A^s - W_1^T\right)^+\right)\right) + \delta^I \gamma W_1^T\right). \end{aligned} \tag{B.123}$$

The current flow of profits is given by $\gamma^{-1}\Pi\left(y^*\right)$ since the producer does not have access to the frontier technology. In the following period, the relationship becomes a frontier one if the supplier gets access to the frontier technology (which occurs with probability $\left(1-\delta^I\right)\Delta$). If the technology does not diffuse then the producer should try a frontier supplier if $V_0^{T,g} > W_1^T - V_A^s$, in that case he obtains $V_0^{T,g}$ (since there are several frontier firms) and the supplier gets the expected value $W_1^s = V_A^s$; on the other hand if $V_0^{T,g} < W_1^T - V_A^s$, the producer stays with the outdated good match and they obtain W_1^T together. If another innovation occurs, then the innovator would be the only one with the frontier technology and would obtain any surplus of a relationship with her, hence the joint value of the producer and the (previous) supplier is W_1^T.

Combine (B.121), (B.122) and (B.123) to obtain:

$$\begin{aligned} & V_0^{T,g} + V_A^s - W_1^T \qquad \text{(B.124)} \\ = \; & (1-b)\,\Pi\left(x^*\right) + b\theta\Pi\left(n\right) - \gamma^{-1}\Pi\left(y^*\right) + \frac{1-\delta^D}{1+\rho}\left(1-\delta^I\right)(1-\Delta)\times \\ & \left[\begin{array}{c}(1-b)\,V_1^T + \\ b\left(V_0^{T,g} + V_A^s + \left(W_1^T - V_A^s - V_0^{T,g}\right)^+\right) - \left(W_1^T + \left(V_0^{T,g} + V_A^s - W_1^T\right)^+\right)\end{array}\right]. \end{aligned}$$

Next, using (B.122) and (B.123), we get:

$$\begin{aligned} V_1^T - W_1^T \; = \; & \Pi\left(x^*\right) - \gamma^{-1}\Pi\left(y^*\right) \\ & + \frac{1-\delta^D}{1+\rho}\left(1-\delta^I\right)(1-\Delta)\left(V_1^T - W_1^T - \left(V_0^{T,g} + V_A^s - W_1^T\right)^+\right). \end{aligned}$$

Hence

$$\begin{aligned} V_1^T - W_1^T \; = \; & \frac{(1+\rho)\left(\Pi\left(x^*\right) - \gamma^{-1}\Pi\left(y^*\right)\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I\right)(1-\Delta)} \qquad \text{(B.125)} \\ & - \frac{\left(1-\delta^D\right)\left(1-\delta^I\right)(1-\Delta)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I\right)(1-\Delta)}\left(V_0^{T,g} + V_A^s - W_1^T\right)^+. \end{aligned}$$

Plugging this expression in (B.124), we get

$$\begin{aligned} & V_0^{T,g} + V_A^s - W_1^T \\ = \; & (1-b)\,\Pi\left(x^*\right) + b\theta\Pi\left(n\right) - \gamma^{-1}\Pi\left(y^*\right) \\ & + \frac{\left(1-\delta^D\right)\left(1-\delta^I\right)(1-\Delta)(1-b)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I\right)(1-\Delta)}\left[\left(\Pi\left(x^*\right) - \gamma^{-1}\Pi\left(y^*\right)\right) - \left(V_0^{T,g} + V_A^s - W_1^T\right)^+\right] \end{aligned}$$

This implies that $V_0^{T,g} + V_A^s - W_1^T > 0$ if and only if (30) holds. Further

$$\begin{aligned} & \left(V_0^{T,g} + V_A^s - W_1^T\right)^+ \qquad \text{(B.126)} \\ = \; & \frac{\left(\begin{array}{c} b\left(1+\rho-\left(1-\delta^D\right)\left(1-\delta^I\right)(1-\Delta)\right)\left(\theta\Pi\left(n\right) - \gamma^{-1}\Pi\left(y^*\right)\right) \\ + (1+\rho)(1-b)\left(\Pi\left(x^*\right) - \gamma^{-1}\Pi\left(y^*\right)\right)\end{array}\right)^+}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I\right)(1-\Delta)} \end{aligned}$$

B.9.2. Switching in the contractible and Nash cases. In the contractible and Nash cases the same logic applies, since once a producer has found two good matches he can only remember the last one. Therefore, as in the equilibrium path described above, a producer would only return to the his last good match after having tried out the frontier firm (which implies that whether the technology diffuses to suppliers with whom he worked before does not matter). We can then directly copy the previous equations but replacing all investment levels by m in the contractible case and n in the Nash case. Therefore a producer switches if and only if (31) holds.

B.9.3. IC constraints in the cooperative case. To compare x^* and y^*, we need to compare the right-hand side of (10) and (29). To do that we first combine (B.117) and (B.120) to get

$$V_A^s = \frac{b\left(1-\delta^D\right)\left(\left(1-\delta^I\right)\Delta V_1^s + \left(\left(1-\delta^I\right)(1-\Delta)+\delta^I\gamma\right)\left(W_1^T - V_A^s - V_I^{T,g}\right)^+\right)}{1+\rho-b\left(1-\delta^D\right)\left(\left(1-\delta^I\right)(1-\Delta)+\delta^I\gamma\right)},$$

and

$$W_1^s = \frac{b\left(1-\delta^D\right)\left(1-\delta^I\right)\Delta V_1^s + (1+\rho)\left(W_1^T - V_A^s - V_I^{T,g}\right)^+}{1+\rho-b\left(1-\delta^D\right)\left(\left(1-\delta^I\right)(1-\Delta)+\delta^I\gamma\right)}. \tag{B.127}$$

Define $IC_y \equiv \gamma\left(1-\delta^I\right)\Delta V_1^s + \gamma\left(\left(1-\delta^I\right)(1-\Delta)+\delta^I\gamma\right)W_1^s$ and $IC_x \equiv \left(1-\delta^I\right)V_1^s + \delta^I\gamma W_1^s$, then we can rewrite (10) and (29) as

$$\varphi\left(x^*\right) \le \frac{1-\delta^D}{1+\rho} IC_x \text{ and } \varphi\left(y^*\right) \le \frac{1-\delta^D}{1+\rho} IC_y.$$

Then, using (B.127), we get:

$$\begin{aligned} & IC_y - IC_x \\ = \; & \frac{1}{1+\rho-b\left(1-\delta^D\right)\left(\left(1-\delta^I\right)(1-\Delta)+\delta^I\gamma\right)} \times \\ & \left(\begin{array}{c} \left((1+\rho)(\gamma\Delta-1) + b\left(1-\delta^D\right)(1-\Delta)\left(\left(1-\delta^I\right)+\delta^I\gamma\right)\right)\left(1-\delta^I\right)V_1^s \\ + \left(\left(1-\delta^I\right)(1-\Delta)+\delta^I(\gamma-1)\right)\gamma(1+\rho)\left(W_1^T - V_A^s - V_I^{T,g}\right)^+ \end{array}\right). \end{aligned}$$

In equilibrium $V_1^s > 0$, therefore a sufficient condition to ensure that $IC_y - IC_x \geq 0$ is that $(1+\rho)(\gamma\Delta-1) + b\left(1-\delta^D\right)(1-\Delta)\left(\left(1-\delta^I\right)+\delta^I\gamma\right) \geq 0$, which is satisfied for any δ^I as long as $\Delta > \left(1+\rho-b\left(1-\delta^D\right)\right)/\left(\gamma(1+\rho)-b\left(1-\delta^D\right)\right)$.

To ensure that an equilibrium exists, we must check that the IC constraints are not binding at n. This requires finding an expression for V_1^s. To do that, first note that the value of a relationship with a frontier firm for a supplier who does not know a cooperating outdated good match, $V_0^{T,b}$ follows:

$$V_0^{T,b} = (1-b)V_1^T + b\theta\Pi(n) + \frac{1-\delta^D}{1+\rho} b\left(\left(1-\delta^I\right)V_0^{T,b} + \delta^I\gamma W_0^T\right). \tag{B.128}$$

With probability $1-b$, the producer meets a good match. Otherwise, the situation is the same in the next period if there has been no innovation, while if an innovation occurs, the producer would try the innovator but would only capture his outside option, namely starting a relationship with an outdated supplier (as there is only one frontier firm then, $V_I^{p,b} = W_0^T$).

Similarly, the law of motion of W_0^T is:

$$W_0^T = (1-b)\,W_1^T + b\theta\gamma^{-1}\Pi(n) + b\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)V_0^{T,b} + \delta^I\gamma W_0^T\right). \tag{B.129}$$

With probability $1-b$, the producer meets an outdated good match, generating the joint value W_1^T. Otherwise, the producer gets current profit $\theta\gamma^{-1}\Pi(n)$ (with an outdated bad match), and in the following period, he tries one of the frontier suppliers and capture the full value if no innovation occurs, while he can only capture the value of a relationship with a new outdated supplier if an innovation occurs.

Combining (B.128) and (B.129), we get

$$V_0^{T,b} - W_0^T = (1-b)\left(V_1^T - W_1^T\right) + b\theta\left(1-\gamma^{-1}\right)\Pi(n). \tag{B.130}$$

Further, combining (B.122), (B.129) and using (B.130), we get

$$\begin{aligned} V_1^s &= V_1^T - V_0^{T,b} \\ &= b\begin{pmatrix} \Pi(x^*) - \theta\Pi(n) \\ +\frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I+\delta^I\gamma\right)V_1^s - b\delta^I\gamma\left(V_1^T - W_1^T\right) + \delta^I b\theta(\gamma-1)\Pi(n)\right) \end{pmatrix} \end{aligned}$$

Therefore, using (B.125), we get:

$$V_1^s = \frac{b(1+\rho)L(x^*,y^*)}{\left(1+\rho-\left(1-\delta^D\right)\left(1-\delta^I\right)(1-\Delta)\right)\left(1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I\gamma\right)\right)},$$

with

$$\begin{aligned} L(x^*,y^*) \equiv{}& \left(1+\rho-\left(1-\delta^D\right)\left(1-\delta^I\right)(1-\Delta) - b\delta^I\gamma\left(1-\delta^D\right)\right)\left(\Pi(x^*)-\theta\Pi(n)\right) \\ &+b\delta^I\left(1-\delta^D\right)\left(\Pi(y^*)-\theta\Pi(n)\right) \\ &-b\delta^I\gamma\frac{\left(1-\delta^D\right)^2}{1+\rho}\left(1-\delta^I\right)(1-\Delta)\left(\theta\left(1-\gamma^{-1}\right)\Pi(n) + \left(V_0^{T,g}+V_A^s-W_1^T\right)^+\right) \end{aligned}$$

Using (B.126), we get:

$$\begin{aligned}
& L\left(x^*, y^*\right) \\
\geq\ & \Pi\left(x^*\right)-\theta \Pi(n)-\frac{b \delta^I \gamma\left(1-\delta^D\right)\left(\Pi\left(x^*\right)-\gamma^{-1} \Pi\left(y^*\right)\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I\right)(1-\Delta)} \\
& +\frac{b \delta^I \gamma\left(1-\delta^D\right) \theta\left(1-\gamma^{-1}\right) \Pi(n)}{1+\rho} \\
& +\frac{b \delta^I \gamma}{1+\rho} \frac{\left(1-\delta^D\right)^2\left(1-\delta^I\right)(1-\Delta)}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I\right)(1-\Delta)} \\
& \times\left[b\left(\theta \Pi(n)-\gamma^{-1} \Pi\left(y^*\right)\right)+\frac{(1+\rho)(1-b)\left(\Pi\left(x^*\right)-\gamma^{-1} \Pi\left(y^*\right)\right)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I\right)(1-\Delta)}\right] .
\end{aligned}$$

Hence

$$\begin{aligned}
\frac{L(n, n)}{\Pi(n)} & \geq(1-\theta)\left(1-\frac{b \delta^I \gamma\left(1-\delta^D\right)}{1+\rho}\left(1-\gamma^{-1}+\frac{b\left(1-\delta^D\right)\left(1-\delta^I\right)(1-\Delta)}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I\right)(1-\Delta)}\right)\right) \\
& \geq(1-\theta)\left(\frac{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I+\delta^I \gamma\right)(1-\Delta)}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I\right)(1-\Delta)}+\frac{b \delta^I\left(1-\delta^D\right)}{1+\rho}\right)>0
\end{aligned}$$

The last inequality is obtained because we must have $\rho>\delta^I(\gamma-1)$ to ensure that utility is finite. Therefore the IC constraints do not bind at $x^*=y^*=n$, which ensures that there exist values such that $x^*, y^* \in(n, m]$ and either the IC constraints bind or the first best is achieved without violating the constraints.

Our equilibrium assumes that in periods with an innovation, the best offer that a producer receives comes from either the innovator or an outdated good match if he knows one. Further, the outdated good match is not willing to offer the full value of the relationship to the producer because even if the producer chooses the innovator, she still secures a postie expected value from the relationship. Therefore, in principle, we should check that the offer from a new outdated supplier cannot be better than that of the current outdated good match supplier. First assume that the producer would rather stay with her current supplier than switch to the innovator ($W_1^T-V_A^s>V_0^{T,g}$), then since the innovator dominates a new outdated supplier, it is clear that the good match supplier's offer is better than that of a new outdated supplier.

Let us then assume that $W_1^T-V_A^s<V_0^{T,g}$, we must then check that we have $W_0^T<W_1^T-V_A^s$. Combining (B.116), (B.120) with $W_1^s=V_A^s$, (B.123), and (B.129), we get:

$$\begin{aligned}
& \left(W_1^T-V_A^s-W_0^T\right)\left(1-b \frac{1-\delta^D}{1+\rho} \delta^I \gamma\right) \\
=\ & b \gamma^{-1}\left(\Pi\left(y^*\right)-\theta \Pi(n)\right)+b \frac{1-\delta^D}{1+\rho}\left(\left(1-\delta^I\right)(1-\Delta)\left(V_0^{T,g}-V_0^{T,b}\right)\right) .
\end{aligned}$$

Similarly using (B.118) and (B.128), we get:

$$\left(V_0^{T,g}-V_0^{T,b}\right)\left(1-b\frac{1-\delta^D}{1+\rho}\left(1-\delta^I\right)(1-\Delta)\right)=b\frac{1-\delta^D}{1+\rho}\delta^I\gamma\left(W_1^T-V_A^s-W_0^T\right).$$

Therefore both $V_0^{T,g}>V_0^{T,b}$ and $W_1^T-V_A^s-W_0^T>0$, which ensures that the two best options for a producer in a period with innovation are the innovator and a good match outdated supplier if he knows one.

B.9.4. Endogenous innovation. Finally, we want to determine the innovation rates, which requires to find the reward from innovation Z_K (for $K=cont, coop$ or $Nash$). Z_K is still defined as $Z_K=\omega V_{I,K}^{s,b}+(1-\omega)V_{I,K}^{s,g}$, and we still have that the expected mass of producers who do not know a good match supplier in steady-state is equal to $\omega=\delta^D/\left(1-b\left(1-\delta^D\right)\right)$.

In the cooperative case, we obtain $V_I^{s,g}=\left(V_I^{T,g}-\left(W_1^T-V_A^s\right)\right)^+$ which is given in (B.126). For producers who do not know an outdated good match, an innovator captures $V_I^{s,b}=V_0^{T,b}-W_0^{T,b}$, which using (B.130) and (B.125) is given by

$$\begin{aligned}V_I^{s,b} &= (1-b)\frac{(1+\rho)\left(\Pi(x^*)-\gamma^{-1}\Pi(y^*)\right)-\left(1-\delta^D\right)\left(1-\delta^I\right)(1-\Delta)\left(V_0^{T,g}+V_A^s-W_1^T\right)^+}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I\right)(1-\Delta)}\\ &\quad +b\theta\left(1-\gamma^{-1}\right)\Pi(n).\end{aligned}$$

We then get that

$$Z_{coop}=\frac{\Pi(x^*)}{1-b\left(1-\delta^D\right)}\times\left(\begin{array}{c}\frac{\delta^D(1-b)(1+\rho)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I\right)(1-\Delta)}\left(1-\gamma^{-1}\frac{\Pi(y^*)}{\Pi(x^*)}\right)+\delta^D b\theta\left(1-\gamma^{-1}\right)\frac{\Pi(n)}{\Pi(x^*)}\\ +\frac{(1-b)\left(1-\delta^D\right)\left(1+\rho-\left(1-\delta^I\right)(1-\Delta)\right)}{1+\rho-b\left(1-\delta^D\right)\left(1-\delta^I\right)(1-\Delta)}\left(\begin{array}{c}b\left(\theta\frac{\Pi(n)}{\Pi(x^*)}-\gamma^{-1}\frac{\Pi(y^*)}{\Pi(x^*)}\right)\\ +\frac{(1+\rho)(1-b)}{1+\rho-\left(1-\delta^D\right)\left(1-\delta^I\right)(1-\Delta)}\left(1-\gamma^{-1}\frac{\Pi(y^*)}{\Pi(x^*)}\right)\end{array}\right)^+\end{array}\right).$$

Therefore as in the baseline case Z_{coop} can be written as a function of $\Pi(x^*)$, $\Pi(y^*)/\Pi(x^*)$ and $\Pi(n)/\Pi(x^*)$, which is increasing in $\Pi(x^*)$, decreasing in $\Pi(y^*)/\Pi(x^*)$ and increasing in $\Pi(n)/\Pi(x^*)$. The same expression applies in the contractible and Nash cases if one replaces $\Pi(x^*)$ by $\Pi(m)$ or $\Pi(n)$ respectively and the profit ratios by 1. For ψ sufficiently convex, we can compare the innovation rates across the three cases by comparing the three Z_K.

Therefore, if $\Delta>\left(1+\rho-b\left(1-\delta^D\right)\right)/\left(\gamma(1+\rho)-b\left(1-\delta^D\right)\right)$, so that $y^*\geq x^*$, we must have $\delta^{cont}>\delta^{coop}$. Moreover, if relationships break in the Nash but not the cooperative case (so $V_0^{T,g}-V_A^s>W_1^T$ holds in the Nash but not the cooperative cases), then $\delta^{coop}<\delta^{Nash}$ for δ^D small enough (as then Z_{coop} is proportional to δ^D but Z_{Nash} is not).

B.10. Proof of Proposition A.4

As in the baseline model, the reward from cooperation for a good match supplier is independent of his current productivity, as her next period productivity is independent of the current one. Denoting IC, this reward from cooperation, we get that the incentive constraints for a good match suppliers are

$$\gamma^2 \varphi(x_2) \le IC, \gamma\varphi(x_1) \le IC \text{ and } \varphi(x_0) \le IC.$$

The higher is current productivity, the larger is the incentive to deviate, so that we must have $x_0 \ge x_1 \ge x_2$.

Using (1), we can write welfare as

$$U = \frac{1+\rho}{\rho}\left(1 + \int_0^1 \theta_{jk} A_k W(x)\right),$$

with $W(x) \equiv \sigma x^{\sigma-1}/(\sigma-1) - x$ the normalized social surplus when the supplier's normalized investment is x. Note that on (n, m), W is increasing in x.

First part: We prove that when $\lambda = 1$, welfare is necessarily higher in the cooperative case relative to the Nash case. In this case, producers face the same distribution of productivities for the alternative supplier in the cooperative and Nash cases.

First, consider a producer who draws suppliers whose productivities are such that he switches neither in the Nash nor in the cooperative case. Then, the social welfare from that line is $\gamma^i W(x_i)$ with $i \in \{0, 1, 2\}$ in the cooperative case and $\gamma^i W(n)$ in the Nash case, and we have $\gamma^i W(x_i) > \gamma^i W(n)$ since $x_i > n$.

Then suppose draws such that the producer switches in both cases. Then the social welfare in the cooperative case is $\gamma^i((1-b) W(x_i) + b\theta W(n))$ versus $\gamma^i(1 - b + b\theta) W(n)$ in the Nash case. Here as well, we have: $\gamma^i((1-b) W(x_i) + b\theta W(n)) > \gamma^i(1 - b + b\theta) W(n)$.

Finally, let us consider a situation where the producer switches in the cooperative case but not in the Nash case (the reverse being impossible). For instance, assume that the producer does not switch in the cooperative case when the previous good match's productivity is 1 and the alternative supplier's productivity is γ. This requires that $\gamma((1-b)\Pi(x_1) + b\theta\Pi(n)) < \Pi(x_0)$. At the same time, the switch occurs in the Nash case ($\gamma > \gamma^{Nash}$). Note that:

$$\begin{aligned}
&((1-b)\Pi(x_1) + b\theta\Pi(n)) W(x_0) - ((1-b) W(x_1) + b\theta W(n))\Pi(x_0) \\
= &\left((1-b)\left[x_1^{\frac{-1}{\sigma}} - x_0^{\frac{-1}{\sigma}}\right] x_1 + b\theta n\left[n^{\frac{-1}{\sigma}} - x_0^{\frac{-1}{\sigma}}\right]\right)\frac{x_0}{\sigma-1} > 0,
\end{aligned}$$

since $n < x_0$ and $x_1 \le x_0$. As a result we must have $((1-b)\Pi(x_1) + b\theta\Pi(n))/\Pi(x_0) > ((1-b) W(x_1) + b\theta W(n))/W(x_0)$. Hence we have that

$$W(x_0) > \gamma((1-b) W(x_1) + b\theta W(n)) > \gamma(1 - b + b\theta) W(n).$$

Therefore in that case too, the social surplus is larger in the cooperative case than in the Nash case. The same logic applies to the other cases with a switch in the cooperative but not the Nash case.

We can then conclude that welfare is strictly higher in the cooperative than in the Nash case.

Second part. . For simplicity, we consider parameters such that $x_0 = x_1 = x_2 = m$ (this occurs if b is high and ρ is low) and we focus on γ such that $\gamma^{Nash} = (1 - b + b\theta)^{-1} < \gamma < \gamma^{coop} = (1 - b + b\theta\Pi(n) / \Pi(m))^{-1} < \gamma^2$.

First let us look at the first round of producers in the Nash case. Since $\gamma > \gamma^{Nash}$, we get that the producer will pick the supplier with the highest productivity. Hence the chosen supplier's productivity is 1 only if both the original and the alternative suppliers' productivities are 1, which only happens with probability $1/9$. It is given by γ^2 if either supplier got a productivity γ^2, which occurs with probability $1/3 + (2/3)(1/3) = 5/9$ (either the first supplier got γ^2 or he did not but the second one did). Finally the chosen supplier's productivity is γ with probability $1 - 1/9 - 5/9 = 1/3$.

Let us then consider the second round of producers:

- With probability $1/3$, the previous supplier's productivity is γ^2. Therefore that supplier is chosen and the social surplus is given by $\gamma^2 W(n)$.
- With probability 1/3, the previous supplier's productivity is γ.
  - Further, with probability $5/9$, the alternative supplier's productivity is γ^2. In that case, the producer chooses the alternative supplier and the (expected) social surplus is given by $(1 - b + b\theta)\gamma^2 W(n)$.
  - Otherwise, the alternative supplier's productivity is weakly lower. In that case, the producer chooses the previous supplier and the social surplus is given by $\gamma W(n)$.
- With probability $1/3$, the previous supplier's productivity is 1.
  - Then, with probability $5/9$, the alternative supplier's productivity is γ^2, so that the social surplus is $(1 - b + b\theta)\gamma^2 W(n)$.
  - With probability $1/3$, the alternative supplier's productivity is γ, so that the social surplus is $(1 - b + b\theta)\gamma W(n)$.
  - Finally with probability $1/9$, the alternative supplier's productivity is 1, so that the producer keeps his previous supplier, leading to a social surplus $W(n)$.

Denote by L the set of lines for which the producer belongs to the second round. Then the expected social surplus for these lines in the Nash case is given by:

$$E\left(W_j^{Nash} | j \in L\right) = \frac{W(n)}{3}\left(\gamma^2\left(1 + \frac{10}{9}(1 - b + b\theta)\right) + \left(\frac{4}{9} + \frac{1}{3}(1 - b + b\theta)\right)\gamma + \frac{1}{9}\right). \tag{B.131}$$

Let us similarly consider the cooperative case. Since $\gamma < \gamma^{coop} < \gamma^2$, switching occurs only when the previous producer's technology is 1 while the alternative supplier's one is γ^2. This event occurs with probability 1/9. The chosen supplier's productivity is 1 if the original supplier's productivity is 1 and the alternative supplier did not draw γ^2: this happens with probability $(1/3)(1 - 1/3) = 2/9$. The chosen supplier's productivity is γ if the original supplier's productivity is γ (as then she is always picked), which happens with probability $1/3$. Finally the chosen supplier's productivity is γ^2 with probability $1 - 1/3 - 2/9 = 4/9$. Let us then consider the second round of producers:

- With probability $1/3$, the previous supplier's productivity is γ^2. Therefore that supplier is chosen and the social surplus is given by $\gamma^2 W(m)$.
- With probability $1/3$, the previous supplier's productivity is γ. Therefore that supplier is chosen and the social surplus is given by $\gamma W(m)$.
- With probability $1/3$, the previous supplier's productivity is 1.
  - Then, with probability $4/9$, the alternative supplier's productivity is γ^2, so that the social surplus is $((1-b) W(m) + b\theta W(n))\gamma^2$.
  - Otherwise, with probability $5/9$, the previous supplier is chosen and the social surplus is $W(m)$.

The expected social surplus for these lines in the cooperative case is given by:

$$E\left(W_j^{coop}|j \in L\right) = \frac{1}{3}\left(\gamma^2\left(W(m) + \frac{4}{9}(1-b) W(m) + b\theta W(n)\right) + \gamma W(m) + \frac{5}{9} W(m)\right). \tag{B.132}$$

Take the difference between (B.132) and (B.131):

$$\begin{aligned} & E\left(W_j^{coop}|j \in L\right) - E\left(W_j^{Nash}|j \in L\right) \\ = \; & \frac{W(m)}{3}\left[\begin{array}{c} \left(\left(1+\frac{4}{9}(1-b)\right)\gamma^2 + \gamma + \frac{5}{9}\right)\left(1 - \frac{W(n)}{W(m)}\right) - \frac{1}{9}(\gamma - 1)\frac{W(n)}{W(m)} \\ -\left(\frac{2}{3}\gamma + \frac{1}{3}\right)\left[(1-b+b\theta)\gamma - 1\right]\frac{W(n)}{W(m)} \end{array}\right]. \end{aligned}$$

Since $(1-b+b\theta)\gamma - 1 > 0$, this expression shows that for n sufficiently close to m (which is obtained by increasing β), then $E\left(W_j^{coop}|j \in L\right) < E\left(W_j^{Nash}|j \in L\right)$. When λ is close to 0, then nearly all lines belong to L therefore we also have $U^{Nash} > U^{coop}$.

C. Appendix: Data and regression

In this appendix, we describe our data and run some additional regressions involving other countries than the US and Japan.

C.1. Data

We use the patent data set of the OECD (OECD, 2015) which is built on data from the European Patent Office (EPO). The data set records all patent applications to the EPO. The year of a patent corresponds to the earliest year of application. Each patent is associated to a country depending on the address of its inventors (if a patent is associated with inventors from several countries, we weight each patent x country combination according to the share of inventors from each country). We restrict attention to patents granted by 2009 (from 2010, more than 10% of patent applications have been neither granted nor rejected yet, in addition, few patents have had the time to receive citations, which is problematic for the accuracy of our measure of generality). Furthermore we consider only the patents that have never been withdrawn. The generality measure is computed by the OECD and we use it directly (it is computing using citations made by other EPO patents). We drop patents for which the measure is not computed (either because the patents have not received any citations—indicating their low value—or because of missing information).

To compute our “Rauch index,” we first use a PATSTAT file which attributes 2, 3 or 4 digit NACE Rev 2 codes (depending on the sector) to each patent. Some patents might have multiple weighted associated NACE codes. Then, we use the liberal classification from Rauch (1999) which labels each 4 digit SITC 2 code as either “goods traded on organized exchange”, “reference priced” or “differentiated”. We attribute a “Rauch index" 1 to goods which are labeled as differentiated or reference-priced, and give an index of 0 to the other goods (method II), but also differentiated (I) as well as 1 to differentiated and 1/2 to reference-priced. Results in Table 2 are virtually unchanged regardless of what measure we use. We convert this into SITC 3 codes using the conversion table from http://econweb.ucsd.edu/~jrauch/rauch_classification.html. This is close to a one-to-one conversion. We use a conversion table from SITC 3 to NACE Rev 1 from World Integrated Trade Systems (http://wits.worldbank.org/product_concordance.html) to convert the SITC 3 to NACE Rev 1. If a NACE Rev 1 is associated with multiple SITC 3 codes we take the average value of the SITC 3 codes. Finally, we convert the NACE Rev 1 into NACE Rev 2 using a concordance table from Eurostat (http://ec.europa.eu/eurostat/web/nace-rev2/correspondence_tables). Again, if a NACE Rev 2 code corresponds to multiple NACE Rev 1 we use an unweighted average. We use the two digit NACE Revision 2 in our analysis, which leaves 27 distinct NACE categories with a Rauch index varying between 0.16 and 1. The Rauch index of each sector is given in Table C.1 below.

Data on trust (used for the additional regression below) come from the World Value Survey longitudinal data file 1981-2014. We focus on questions G007_33 which ask to respondents whether they trust people they know personally and G007_34 which ask them whether they trust people they meet for the first time. There are 4 levels of trust and we linearly transform each variable so that they are in [0, 1] with a high value corresponding to a high level of trust. These two questions where only asked from 2005 onward. For each country and wave, we average the answer given by individuals using the weights provided in the dataset (as recommended we censor weights below 0.33 and above 3).[10] For countries which are present in multiple waves, we further average across waves. Then, we take the difference between the trust level towards people met for the first time and trust towards people already known to define our “Dif Trust" variable: a low level indicates a relatively higher trust towards known people which favors the establishment of cooperation in long-term relationships. Since our trust measures are all from 2005, we restrict attention to patents filed after 1995. Furthermore, we focus on countries with more than 100 patents with generality data, though little depends on this choice. Table C.1b reports the full list of countries with their differential trust measure.

10. For the United Kingdom we use the value corresponding to Great Britain (which excludes Northern Ireland) and for Serbia we use results conducted for Serbia and Montenegro.

TABLE C.1. Country and Product category information used in the regressions

(A) Extend of differentiation for NACE Rev.2 product categories (details in text)

NACE, rev. 2	Label	Differentiation (I)	Differentiation (II)
10	Food Products	0.28	0.68
11	Beverages	0.16	0.96
12	Tobacco Products	0.17	1.00
13	Textiles	0.84	0.94
14	Wearing Apparel	1.00	1.00
15	Leather And Related Products	0.95	1.00
16	Wood Products	0.79	0.85
17	Paper And Paper Products	0.55	0.98
18	Printing And Reproduction Of Recorded Media	0.94	1.00
19	Coke And Refined Petroleum Products	0.25	0.93
20	Chemicals And Chemical Products	0.50	0.97
21	Basic Pharmaceutical Products	0.61	0.99
22	Rubber And Plastic Products	0.78	1.00
23	Other Non-Metallic Mineral Products	0.78	1.00
24	Basic Metals	0.23	0.78
25	Fabricated Metal Products	0.91	0.99
26	Computer, Electronic And Optical Products	0.88	1.00
27	Electrical Equipment	0.78	1.00
28	Machinery And Equipment N.E.C.	0.98	1.00
29	Motor Vehicles, Trailers And Semi-Trailers	0.85	1.00
30	Other Transport Equipment	0.97	1.00
31	Furniture	0.98	1.00
32	Other Manufacturing	0.93	0.94
43	Specialised Construction Activities	0.95	1.00
62	Computer Programming	1.00	1.00

Differentiation measure from Rauch (1999). (I) uses only n=1, (II) uses n=1 or r=1 (See text for details).

(B) Trust measures for countries used in regression (details in text)

Country	Trust Person: You know	Trust Person: You just met	Difference
Australia	0.79	0.45	-0.34
Brazil	0.54	0.21	-0.33
Canada	0.81	0.47	-0.34
Switzerland	0.77	0.48	-0.29
China	0.65	0.30	-0.35
Hong Kong	0.74	0.36	-0.38
Taiwan	0.71	0.38	-0.34
Germany	0.71	0.36	-0.34
Spain	0.75	0.39	-0.36
Finland	0.80	0.49	-0.31
France	0.87	0.44	-0.43
United Kingdom	0.83	0.46	-0.37
India	0.63	0.38	-0.26
Italy	0.57	0.31	-0.26
Japan	0.65	0.28	-0.37
South Korea	0.65	0.30	-0.35
Netherlands	0.71	0.36	-0.35
Norway	0.86	0.56	-0.31
Russian Federation	0.67	0.27	-0.40
Sweden	0.81	0.54	-0.28
Singapore	0.73	0.38	-0.35
Turkey	0.67	0.29	-0.38
United States	0.74	0.41	-0.33
South Africa	0.62	0.39	-0.24

C.2. Regression

We first run a regression analogous to that of Table 2 but at the lowest level of disaggregation available in the patent data (2 to 4 digit NACE level depending on the sector). The results are in Table C.2. There is little difference in the estimates.

TABLE C.2. Regression Results for 4 digit NACE.

	(I)	(II)	(III)
	Generality	Generality	Generality
US.	0.055***	0.025***	0.026
	(13.60)	(6.23)	(1.45)
US x Differentiated			0.029 ***
			(3.83)
Fixed Effects	No	Year, NACE	Year, NACE
Observations	339681	339681	339681

Standardized beta coefficients; t statistics in parentheses. Std. errors clustered at NACE x country level for (III). * $p < 0.10$, ** $p < 0.01$, ***$p < 0.001$.

We now report results on the larger set of countries which are consistent with our theory that cooperation in an environmental of weak contractability should deter general innovations. Direct evidence for whether countries are in the 'cooperative' or 'Nash' equilibrium is difficult to come by, we use the Dif Trust variable built above using World Values Survey data. A high value corresponds to a high relative trust in strangers, which reduces the scope for the establishment of cooperation in existing relationships. Japan gets the fifth lowest score and France the lowest, whereas the United States is slightly above the middle with South Africa receiving the highest score. We focus on countries with more than 100 patents with generality data, though little depends on this choice. This leaves us with 324,156 patents and a total of 24 countries. We then run the following regression:

$$gen_{i,t,s,c} = \beta_0 + \beta_1 Diff\ Trust_c \times Differentiated_s + \delta_c + \delta_s + \delta_t + \varepsilon_{i,t,s,c},$$

where $gen_{i,t,s,c}$ is the generality of patent i, which was filed in year t, corresponds to sector s and whose inventors were from country c, $Differentiated_s$ is the measure of the importance of contractibility issues in sector s and δ_c, δ_s and δ_t are country, sector and year fixed effects. shows this regression to be consistent with our predictions as a high relative trust in strangers (i.e. a lower scope for cooperation in existing relationships) is associated with relatively more general patents in more differentiated industries. The result is no longer statistically significant if we employ the NACE code at 4 digits.

Table C.3. Regression results for 24 countries

	(I)
	Generality
Trust x Differentiated	0.009**
	(1.99)
Fixed Effects	NACE, Country, Year
Observations	324,156

Standardized beta coefficients; t-statistics in parentheses. Column (I) uses 2 digit NACE (details in text).
* $p < 0.10$, ** $p < 0.05$

References

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