# Intellectual Property Rights in a Quality-Ladder Model with Persistent Leadership*

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**Abstract**

This article analyzes the effects of intellectual property rights in a quality-ladder model of endogenous growth in which incumbent firms preemptively innovate in order to keep their position of leadership. Unlike in models with leapfrogging, granting forward protection, and imposing a non-obviousness requirement reduces growth. In the main case where entrants and incumbents have free access to the same R&D technology, infinite protection against imitation, granted independently of the size of the lead, maximizes growth. If entrants have to engage in costly catch up before they can undertake frontier R&D, growth is maximal for a finite (expected) length of protection against imitation. (JEL L40, O31, O34)

**Keywords:** Intellectual property rights, cumulative innovation, persistent leadership, forward protection, non-obviousness requirement

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# 1 Introduction

This article studies the effects of different intellectual property right (IPR) policies in a quality-ladder model of endogenous growth. In contrast to most models of this type, where entrants undertake all R&D, so that there is leapfrogging, this article analyzes the case in which incumbent firms innovate preemptively in order to prevent being replaced by entrants, and in which there is persistence in leadership.

Empirical studies show that incumbent firms carry out a considerable number of innovations and remain the industry leaders for a substantial period of time. Camerani and Malerba (2007) study patents granted by the European patent office to firms from six countries (France, Italy, Germany, Sweden, USA and UK) in 13 sectors between 1990 and 2003. **Table 1** shows the fraction of patents that were granted to technological entrants who patented for the first time in a given sector.

**Table 1**
Technological entrants share per sector and period (patents)

| Sector | 1990-91 | 1992-93 | 1994-95 | 1996-97 | 1998-99 | 2000-01 | 2002-03 |
|---|---|---|---|---|---|---|---|
| Audiovisual technology | 0.16 | 0.18 | 0.16 | 0.23 | 0.19 | 0.21 | 0.17 |
| Biotechnologies | 0.14 | 0.15 | 0.15 | 0.13 | 0.16 | 0.16 | 0.19 |
| Information technology | 0.13 | 0.14 | 0.17 | 0.21 | 0.26 | 0.28 | 0.21 |
| Machine tools | 0.35 | 0.31 | 0.30 | 0.32 | 0.31 | 0.30 | 0.27 |
| Macromolecular chemistry | 0.07 | 0.09 | 0.08 | 0.10 | 0.10 | 0.11 | 0.13 |
| Materials; Metallurgy | 0.21 | 0.22 | 0.21 | 0.25 | 0.23 | 0.24 | 0.24 |
| Medical engineering | 0.27 | 0.22 | 0.21 | 0.22 | 0.23 | 0.22 | 0.20 |
| Optics | 0.10 | 0.09 | 0.11 | 0.14 | 0.16 | 0.15 | 0.18 |
| Pharmaceuticals; Cosmetics | 0.15 | 0.15 | 0.14 | 0.14 | 0.16 | 0.17 | 0.17 |
| Semiconductors | 0.11 | 0.10 | 0.10 | 0.12 | 0.13 | 0.15 | 0.15 |
| Space technology; Weapons | 0.17 | 0.18 | 0.18 | 0.22 | 0.25 | 0.23 | 0.22 |
| Control/Measures/Analysis | 0.23 | 0.22 | 0.23 | 0.24 | 0.25 | 0.24 | 0.24 |
| Telecommunications | 0.11 | 0.10 | 0.11 | 0.11 | 0.10 | 0.13 | 0.10 |

*Source:* Camerani and Malerba (2007), Table 6

As those fractions are relatively small in all sectors and periods, firms that innovate persistently within one sector obtain a disproportionate share of all patents.

Córcoles and Triguero (2013) study the persistence of innovation using firm level panel data on R&D expenditures and innovation of Spanish manufacturing firms for the period 1990 - 2008. They find that on average 72.6% of the innovators in one year also innovated in the subsequent year and that 58.1% also innovated five years later. Persistence was even higher in R&D decisions: on average 87.56% of the firms that undertook R&D in one year also did so in the following year and

78.55% also undertook R&D five years later. Numerous other empirical studies find high levels of firm innovation persistence when measured with patent, innovation, or R&D data.[1]

Garcia-Macia et al. (2015) study U.S. manufacturing data from 1992, 1997 and 2002. Using a growth model, they indirectly infer that most growth resulted from innovation by incumbents and from improvements of existing varieties, carried out mostly by the incumbents who had previously produced them. Given that intellectual property rights are used to stimulate innovation, it is therefore important to understand how they should be designed in order to encourage cumulative innovation in a context where a large fraction of innovations are carried out by incumbent firms.

The following model setup is used in order to generate persistent leadership: innovation is assumed to increase the quality of different intermediate goods that are used to produce a final consumption good. There is free entry into R&D in each intermediate good sector, and the R&D productivity of entrants and incumbents is the same. Contrary to most leapfrogging models, it is assumed that the R&D technology is characterized by decreasing returns at the industry level and that incumbents move first in the R&D game.[2] By increasing her own R&D effort, an incumbent can therefore decrease the profitability of R&D undertaken by entrants in her sector and preempt entry.

The analysis mainly focuses on the case of quasi-drastic innovations in which an incumbent who has undertaken two successive innovations (is two steps ahead) can charge the unconstrained monopoly price and earns larger profits than an entrant who has to compete with the previous incumbent in an intermediate good market. As incremental profits are lower for an incumbent than for entrants due to the Arrow replacement effect, incumbents have a lower incentive to innovate than entrants if they do not face any entry pressure. Under free entry, however, incumbents find it profitable to preempt entry as they value not being replaced and keeping their (two step) lead more than entrants value entry (which brings a

[1] Using an innovation panel data set on German manufacturing and service firms for the period 1994 – 2002, Peters (2009) finds that in manufacturing (services), about 89% (about 70%) of the innovators in one year also innovated in the next year. Jang et al. (2008) calculate transition probabilities using patent data for the period 1990-2003. They find that between 70 and 80% of firms in the U.S., Japan, South Korea, and Taiwan that applied for more than 6 patents in one year also applied for more than 6 patents two years later. Foster et al. (2013) find that there is high persistence among the top 200 R&D performing firms in the US. They document that the firms that were among the top 200 in 1976 still made up more than half of the total R&D expenditures of the top 200 firms in 2003 and more than 35% of total US R&D expenditures in 2003. Le Bas and Scellato (2014) provide an overview over 33 recent empirical papers on firm innovation persistence and classify 25 (2) of them as finding high (low) innovation persistence.

[2] Even without the latter assumption, a similar Walrasian equilibrium can be considered, as explained in footnote 19.

one step lead). Therefore, incumbents carry out all the R&D in equilibrium and there is persistent leadership.

The analysis builds on Denicolò (2001), who introduces the preemption mechanism of Gilbert and Newbery (1982) into a quality-ladder growth model, but does not study the role of IPRs. In equilibrium, the amount of R&D that an incumbent undertakes in order to prevent being replaced depends positively on the value of an innovation for an entrant expecting to become the new industry leader upon entry[3].

Within this setting of persistent leadership, the following IPR policies are analyzed: (i) forward protection (new innovations infringe the IPRs (patents) of previous innovators), (ii) a non-obviousness requirement (minimal inventive step), and (iii) protection against imitation (duration until rivals can copy a protected technology). The analysis generates three main results:

*First*, granting forward protection reduces growth. Under forward protection, entry is discouraged as incumbents can block entrants' innovations. Even though it is assumed that forward protection allows incumbents and entrants to collude in prices, entrants never obtain larger profits than without forward protection and without collusion. Therefore, granting forward protection unambiguously reduces entry pressure, the amount of R&D undertaken by incumbents, and growth. This result differs from that obtained by O'Donoghue and Zweimüller (2004), who study forward protection in the leapfrogging case in which patents for any two succeeding innovations are held by different firms. In their setting, forward protection also discourages innovation by leading to more backloaded profit flows. However, entrants can benefit from price collusion with infringed incumbents more than what they lose from profit backloading. As a result, the overall effect of forward protection on growth can be positive.

The *second* main result is that imposing a non-obviousness (patentability) requirement reduces growth. The reason for this is that such a requirement reduces entry pressure and therefore the amount of R&D that incumbents need to undertake in order to preempt entry and that, for given R&D spending, incumbents themselves find selecting the growth-maximizing step size profitable. This result differs from those obtained in the context of leapfrogging, where imposing a non-obviousness requirement can increase innovation by reducing the threat that innovators lose their monopoly power due to "creative destruction" by marginal follow-on innovations (see O'Donoghue (1998), O'Donoghue and Zweimüller (2004), and Hunt (2004)).

Finally, it is shown that full protection against imitation for any size of the

[3] The prediction that incumbents invest more in R&D if entry pressure increases is supported by an empirical study of Czarnitzki et al. (2014), who use a dataset from the German manufacturing sector.

lead maximizes growth. This is the *third* main result. In order to allow for less than full protection, it is assumed that with a certain hazard rate, competitors can copy a given invention and compete away all profits. As in Acemoglu and Akcigit (2012), the general case of state-dependent intellectual property (IP) protection is considered in which the rate at which protection against imitation expires can depend on the size of the lead of an IP holding firm. Unlike in Acemoglu and Akcigit (2012), however, growth is not maximal under state dependent IP protection, but rather in the case of full uniform protection, i.e. if protection never expires.

The last result, however, needs to be qualified: If there is *trade secrecy* in the sense that entrants have to incur some fixed catch-up costs before they can use the state-of-the-art R&D technology, it is shown that incumbents can preempt entry without doing any R&D. In this case, no innovation takes place in equilibrium under full IP protection against imitation[4]. If, however, IP protection expires with a positive probability, so that incumbents lose their lead from time to time, this regularly creates a neck and neck situation in which firms innovate and try to become the next leader in an industry. Because of that, the rate of growth is maximal for an intermediate probability of IP expiration (i.e. if patents are of finite expected duration). This result differs from that obtained in the case of leapfrogging, where, absent forward protection, full protection against imitation maximizes growth[5], even if there are catch-up costs due to trade secrecy.

In the following, the related literature is discussed in more detail. The question of how antitrust policies should be designed in innovative industries where entrants expect to become the next incumbents is analyzed by Segal and Whinston (2007), who find that entrants should be well protected against incumbents in most cases in order to guarantee that profit flows for successful innovators do not become too backloaded. They therefore identify the same effect as O'Donoghue and Zweimüller (2004), who show that innovation increases if profit flows become more frontloaded. Chu (2009) studies a generalized version of the model of O'Donoghue and Zweimüller (2004) and quantitatively estimates the effect of blocking patents on R&D using US data. O'Donoghue et al. (1998) analyze the role of forward protection in a quality-ladder model with leapfrogging and compare the case of short and broad to long and narrow patents, assuming that forward protection allows firms to collude and that investment opportunities arrive at an exogenous rate.

Bessen and Maskin (2009) and Llanes and Trento (2012) analyze models of sequential innovation in which patents grant blocking power over follow-on inno-

[4]In order to obtain this result, simplifying assumptions about the form of the R&D technology are made.

[5]Llobet and Suarez (2013), show that innovation and welfare can be maximal for an intermediate strength of protection if patents at the same time grant protection against imitation and (entry-discouraging) forward protection.

vations and in which licensing is inefficient due to asymmetric information. Assuming that firms can appropriate some surplus in the final goods markets even in the absence of IP protection, they show that innovation can be larger when there is no (forward) IP protection. In both articles, it is assumed that innovation does not lead to the replacement of the previous technology, so that issues like the Arrow replacement effect or the possibility of preemption that arise in a quality-ladder context are not considered. The main difference between the above-mentioned and the present article is the fact that the former do not study the case in which incumbents undertake R&D.[6] While several articles have analyzed the conditions under which (some) persistence in leadership can arise in quality-ladder models[7], the role of IP protection has only received little attention in settings in which incumbent firms also innovate.

Acemoglu and Akcigit (2012) analyze a model of step-by-step innovation in which there is a race between two firms in each sector and where the laggard first has to catch up through duplicative (but non-infringing) R&D before he can undertake frontier R&D. They argue that IP protection should be stronger for firms that have a larger technological lead over their rivals. In their model, IP protection affects innovation by affecting the incremental profits that the two incumbent firms obtain from moving one step ahead.[8] Reducing IP protection solely for firms with a smaller lead can therefore increase innovation incentives for laggards and the rate of growth by increasing incremental profits. In contrast, there is free entry into (frontier) R&D in the model analyzed here, and while reducing IP protection in a state-dependent way can induce firms with a lower lead to innovate at a faster rate, it comes at the cost of reducing the value of entering and the amount of R&D incumbents need to undertake in order to preempt entry. This entry-discouraging

[6] Segal and Whinston (2007) also study the case of innovation by leaders but do not analyze the role of IPRs in this context.

[7] In most quality-ladder growth models (like Aghion and Howitt (1992)), the case of leapfrogging is analyzed, although incumbents are actually indifferent about their share in total R&D if the R&D sector is competitive and markets are Walrasian, so that there might as well be some persistence in leadership (see Cozzi (2007)). In many continuous-time patent race models (like Reinganum (1983 and 1985)) where marginal R&D costs are increasing at the firm-, but not at the industry level, preemption is not possible and incumbents invest less in R&D than challengers in the standard case with simultaneous moves and drastic innovations. In a similar setup with fixed costs of entering the R&D sector, Etro (2004) finds that in the case where there is free entry and industry leaders move first in the R&D game (are Stackelberg leaders), they do more R&D than entrants so that there is some persistence in leadership. Denicolò (2001), on which the current article builds, analyzes the case where preemption is possible due to decreasing R&D productivity at the industry level and finds that there is persistent leadership if innovations are non-drastic and incumbents move first. Fudenberg et al. (1983) analyze conditions under which preemption is possible and when there can be competition in patent races.

[8] It is assumed that parameters are such that the laggard never stops innovating and never drops out of the market.

effect is so strong that, contrary to Acemoglu and Akcigit (2012), reducing IP protection in a state-dependent way always reduces growth compared to the case of full uniform IP protection. While this result accords with the findings of first-generation quality ladder models with free entry (based on Aghion and Howitt, 1992), it is derived in a setup where innovation rates can be lead dependent, giving rise to composition effects that otherwise only occur in step-by step models, but not in models with leapfrogging.

Denicolò and Zanchettin (2012) study a growth model with non-drastic innovations and constant returns to R&D in which incumbents and entrants simultaneously decide about the sizes of their R&D investments and in which there is no preemption. Unlike the present article, they assume that the R&D productivity of incumbents is larger than that of entrants so that incumbents might find it profitable to innovate, in spite of the Arrow replacement effect. They find that there can be stochastic leadership cycles, meaning that incumbents are not replaced immediately, but undertake some R&D and (on average) advance to a certain lead before they are replaced. The authors show that requiring successful outsiders to pay a licensing fee to the previous incumbent in this setting can increase the rate of growth, even if such forward protection does not facilitate collusion. Introducing state-dependent patent breadth (modeled as varying price caps) is shown to affect the share of R&D the incumbent undertakes, but it has no effect on the rate of growth.[9]

The article is structured as follows: in Section 2, the model is introduced and in Section 3, the equilibrium is derived. Section 4 studies the effects of different IPR policies like forward protection (Section 4.1), a non-obviousness requirement (Section 4.2) and the case of expiring IP protection (Sections 4.3 and 4.4). Section 5 concludes.

# 2 Model Setup and Equilibrium

This analysis is based on the quality-ladder growth model of Barro and Sala-i-Martin (2004). While Denicolò (2001) analyzes a one-sector version of this model, there is a continuum of sectors in the version of the model studied here. This allows for non-stochastic growth, but does not affect the qualitative results.

[9]Other papers in which (some) persistence in leadership results because incumbents are assumed to be more productive in doing R&D than entrants are Segerstrom and Zolnierek (1999) and Segerstrom (2007). In Ledezma (2013) there can be persistent leadership as incumbents can increase the R&D costs of entrants by biasing the direction of R&D. In Acemoglu and Cao (2015), only incumbents are capable of undertaking incremental innovations, while entrants engage in more radical innovations. Athey and Schmutzler (2001) analyze conditions under which dominance of an incumbent firm arises if there is ongoing firm-specific investment, but no entry into frontier R&D.

## 2.1 Preferences

There is a measure one of identical, infinitely lived individuals. Time is continuous and intertemporal preferences are given by

$$U(\tau) = \int_{t=\tau}^{\infty} c\,(t)\, e^{-r(t-\tau)} dt \tag{1}$$

with $c(t)$ denoting consumption at time $t$. As intertemporal preferences are linear, the equilibrium rate of interest is constant and equal to the rate of time preference $r$. Moreover, individuals are risk neutral, so that all assets must yield the same instantaneous expected net rate of return $r$ in equilibrium[10].

## 2.2 Goods Production

There is a final good $y$ which can be consumed, used for research, or used to produce intermediate goods, of which there is a continuum of measure one. The quality of each intermediate good $\omega$ can be increased step-wise through innovation. It is assumed that each innovation increases the quality by the factor $q > 1$. Normalizing the initial quality of all intermediate goods at time 0 to one, the highest available quality of an intermediate good $\omega$ at time $t$ is given by $q^{k(\omega,t)}$, with $k(\omega, t)$ indicating the number of innovations that have been achieved in industry $\omega$ by time $t$.

The final good is produced using labor $L$ and intermediate goods of vintages $v \in \{0, 1, ..., k(\omega, t)\}$ and quantities $x(v, \omega, t)$ according to the following constant returns production function:

$$y(t) = L^{1-\alpha} \int_{\omega=0}^{1} \left[ \sum_{v=0}^{k(\omega,t)} q^v x(v, \omega, t) \right]^{\alpha} d\omega, 0 < \alpha < 1$$

In order to produce one unit of any type of intermediate good, one unit of the final good is needed as an input. Both the final good and labor are supplied in perfectly competitive markets, while intermediate goods can be protected by IPRs. If more than one firm supplies intermediate goods in sector $\omega$, there is price competition. Since different vintages of intermediate goods are perfect substitutes within their sector, only the newest vintage $v = k(\omega, t)$ with the highest available quality is used in equilibrium. Each individual inelastically supplies one unit of labor at each point in time so that the total labor supply is given by $L = 1$. Therefore, the final good production function reduces to

[10] The qualitative results of the article are robust to the introduction of more general intertemporal preferences (see the last paragraph of Appendix A2).

$$y(t) = \int_{\omega=0}^{1} \left[q^{k(\omega,t)} x(k,\omega,t)\right]^{\alpha} d\omega \tag{2}$$

The final good is used as the numeraire.

### 2.3 The R&D Sector

Given $k$ is the newest vintage of the intermediate good in sector $\omega$, the next vintage $k+1$ can be invented if R&D is undertaken. The instantaneous arrival rate $\phi_\omega(k+1)$ with which the innovation occurs is given by

$$\phi_\omega(k+1) = min\left\{\left(\frac{n_\omega}{cg^k}\right)^{\frac{1}{1+\epsilon}}, \bar{\phi}\right\} \tag{3}$$

where $n_\omega$ denotes the total amount of the final good used for R&D in sector $\omega$ and $g \equiv q^{\frac{\alpha}{1-\alpha}} > 1$. $c > 0$ is an inverse measure of R&D productivity and $\epsilon > 0$ indicates the extent of decreasing returns to R&D at the industry level. $\bar{\phi} > 0$ is an upper bound that the innovation arrival rate cannot exceed due to technological reasons. This upper bound only becomes relevant in *Sections 4.3* and *4.4* of the analysis where, for reasons of tractability, the limit case of constant returns to R&D ($\epsilon \to 0$) is considered. The total R&D costs in terms of the final good are therefore given by

$$C(\phi_\omega(k+1)) = \begin{cases} cg^k\phi_\omega^{1+\epsilon} \text{ if } \phi_\omega \leq \bar{\phi} \\ \infty \text{ if } \phi_\omega > \bar{\phi} \end{cases} \tag{4}$$

In order to obtain a balanced growth path (BGP), these costs increase by the factor $g$ if the vintage (quality-level) $k$ of the good increases by one step due to an innovation[11]. There is free entry into frontier R&D, meaning that all firms have access to the same R&D technology and are capable of undertaking frontier R&D in all sectors without having to duplicate previous innovations of other firms first.

As $\epsilon > 0$, marginal and average R&D costs increase in $\phi_\omega$. If more than one firm does R&D, it is assumed that firm $i$ obtains the innovation with arrival rate $\phi_{\omega i} = \beta_{\omega i}\phi_\omega$ if its share in total R&D costs in sector $\omega$ is given by $\beta_{\omega i}$. The R&D costs of firm $i$ are then given by $C_i(\phi_{\omega i}(k+1)) = cg^k\phi_{\omega i}\phi_\omega^\epsilon$. A firm therefore increases the R&D costs of all other firms if it increases the (industry) arrival rate $\phi_\omega$ by increasing its own R&D spending. A reason for these decreasing returns might be that it is impossible to perfectly coordinate all R&D activities, implying that the probability of duplicative research (even within one firm) increases if more

[11] As every innovation turns out to increase profits by the factor $g$ (see equations 6 and 7), R&D costs have to increase by the same factor in order to obtain a BGP on which $\phi_\omega$ is constant.

R&D is undertaken in a given sector[12]. It is assumed that incumbent firms have a first-mover advantage in R&D due to their visibility, meaning that entrants at each instant of time first observe the R&D effort undertaken by an incumbent before they decide about their own R&D effort.

### 2.4 Intellectual Property (IP) Protection

It is assumed that an inventor of a new vintage of a good gets IP protection on it, allowing her to exclude other firms from producing her vintage. Initially, IP protection is assumed to be of infinite duration (e.g., patents are infinitely lived). The effects of forward protection, of a non-obviousness requirement and of expiring IP protection are discussed in *Section 4*.

### 2.5 Equilibrium Concept and Steady State

It is assumed that the perfectly competitive final good and labor markets clear at any point in time. In the imperfectly competitive intermediate goods markets, firms behave strategically in the races for new innovations and Markov Perfect Equilibria are analyzed in which strategies only depend on payoff-relevant state variables[13].

The analysis focuses on balanced growth paths. While productivity-enhancing innovations arrive at random intervals within a given intermediate good industry, the economy grows smoothly as there is a continuum of intermediate good industries, implying that there is no randomness in the production of the final good due to the law of large numbers. Along a BGP, the output of the final and of all intermediate goods, consumption, the wage rate, and total R&D expenditures all grow at a constant rate. In the case where IP protection expires, the fraction of industries in which the industry leader leads by a certain step size and the average innovation rate within an industry are constant.

## 3 General Equilibrium

### 3.1 Goods Market Equilibrium

Profit maximization of perfectly competitive firms in the final goods sectors implies that the demand for the latest vintage $k(\omega, t)$ of an intermediate good of type $\omega$

[12]In case of duplication, an increase in the R&D effort exerted by one firm increases the expected R&D costs per obtained IPR (patent) of rivals as either only one firm gets IP protection on the innovation or as the profits derived from the IPR have to be split among several innovators.

[13]Non-stationary endogenous cycle equilibria that can occur in quality-ladder models are not considered.

as a function of its price $p(k,\omega,t)$ is given by

$$x(k,\omega,t) = \alpha^{\frac{1}{1-\alpha}} g^k p(k,\omega,t)^{-\frac{1}{1-\alpha}} \tag{5}$$

The demand for intermediate good $\omega$ is independent of the prices of intermediate goods in other sectors. The demand function has the constant elasticity $\frac{1}{1-\alpha}$ and shifts up by the factor $g \equiv q^{\frac{\alpha}{1-\alpha}}$ if an innovation, which increases $k$ by one step, occurs. Producers of different vintages of intermediate goods within the same sector can be treated as if they were supplying the same good, measured in efficiency units, but at a different cost (in terms of the final good) per unit: the costs of vintage $k-l$ are therefore $q^l$ times larger than those of vintage $k$.

The firm that supplies the newest vintage $k$ of the intermediate good in industry $\omega$ maximizes profits under the constraint that the price has to be so low that no rival firm in the same industry finds it profitable to enter. If the industry leader has obtained $l$ successive IP protected innovations and therefore has a lead of $l$ steps, entry is deterred if $p(k,\omega,t) \leq q^l$ holds[14]. Profits, that are given by $\pi(k,\omega,t) = (p(k,\omega,t)-1)\,x(k,\omega,t)$, with $x(k,\omega,t)$ taken from equation 5, are therefore maximized for the unconstrained monopoly price $p_m = \frac{1}{\alpha}$ if $\frac{1}{\alpha} \leq q^l$ and for the limit price $p = q^l$ if $q^l < \frac{1}{\alpha}$. Let $s$ denote the minimal lead size which allows a leading firm to engage in (unconstrained) monopoly pricing. Formally, $s$ is an integer implicitly defined by the following condition:

$$q^{s-1} < \frac{1}{\alpha} \leq q^s$$

Equilibrium profits can be derived as:

$$\pi_m(k,\omega,t) = (1-\alpha)\,\alpha^{[(1+\alpha)/(1-\alpha)]} g^{k(\omega,t)} \equiv \pi_m g^{k(\omega,t)}, l \geq s \tag{6}$$

$$\pi_l(k,\omega,t) = \left(q^l - 1\right)\alpha^{1/(1-\alpha)} q^{-\frac{l}{1-\alpha}} g^{k(\omega,t)} \equiv \pi_l g^{k(\omega,t)}, l < s \tag{7}$$

Profits increase in the vintage $k$ of the good and increase in the lead $l$ relative to the closest competitor if $l < s$, but are independent of $l$ if $l \geq s$, i.e. if firms are unconstrained monopolists. In order to simplify the notation, the definitions $\pi_m \equiv (1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}}$ and $\pi_l \equiv \left(q^l-1\right)\alpha^{\frac{1}{1-\alpha}} q^{-\frac{l}{1-\alpha}}$ are used in the following and referred to as "normalized profits".

[14]The marginal costs of producing an efficiency-equivalent to one unit of a good of vintage $k$ (the marginal costs of which are one) are equal to $q^l$ for producers of vintage $k-l$, so that they cannot break even if the price lies below these marginal costs.

**Incremental Profits**

In order to understand a firm´s incentive to innovate, one needs to know how undertaking an innovation changes the firm´s profits depending on the size of the firm´s initial lead. Taking into account that an innovation increases the vintage and also the lead $l$ by one step, **incremental profits** are given by

$$\triangle\pi(k,l,\omega,t) = \pi(k+1,l+1,\omega,t) - \pi(k,l,\omega,t)$$

The following Lemma shows how, for a given vintage $k$ of an intermediate good, incremental profits depend on the initial lead of a firm in the industry.

**Lemma 1.** ***a****): If $q < \frac{1}{\alpha} \leq q^2$ (**Condition A**), incremental profits fall in the initial lead $l$ if $0 \leq l \leq 2$. For $l \geq 2$, firms are unconstrained monopolists and incremental profits are independent of $l$.*

***b****): Given that $q^s = \frac{1}{\alpha}$ $(s \geq 1)$, incremental profits decrease in $l$ if $0 \leq l \leq s$. For $l \geq s$, incremental profits are independent of $l$.*

***c****): For any step size $q > 1$, incremental profits are larger for entrants $(l = 0)$ than for unconstrained monopolists.*

*Proof.* See Appendix A1 □

Given that $q < \frac{1}{\alpha} \leq q^2$ (Condition A) holds, innovations are **quasi-drastic**. This means that each single innovation is non-drastic so that firms with a one step lead engage in limit pricing, while a firm with a two or more step lead is an unconstrained monopolist. In this case, incremental profits are largest for entrants $(l = 0)$, lower for incumbents with a one step lead $(l = 1)$, and even lower for unconstrained monopolists with an initial lead of two or more steps $(l \geq 2)$. Part b) of the Lemma (used in *Section 4.1)* shows that the result that undertaking an innovation increases profits less for firms with a larger initial lead $l$ also holds if the step size is such that firms become unconstrained monopolists with a lead of exactly $s$ steps. Part c) is needed in *Sections 4.1* and *4.2*.

The feature that incremental profits decrease in the size of the lead arises in many contexts and more general settings and is the source of the "Arrow replacement effect" (Arrow (1962)) and the "escape competition effect" (Aghion et al. (2001)), which are widely discussed in the literature[15]. In order to keep the derivations and the exposition simple, the following analysis focuses (unless otherwise stated) on the case of quasi-drastic innovations, which arises under Condition A.

[15] Denicolò (2001), on which this article builds, however, does not prove that incremental profits fall in the lead in the setting analyzed here.

## 3.2 Innovation Race

The expected value of being the leading firm that supplies vintage $k$ of the intermediate good in industry $\omega$ is denoted by $V_1(\omega, k, t)$ if the firm has a one step lead and by $V_2(\omega, k, t)$ if the firm has a lead of two or more steps. $V_1(\omega, k, t) < V_2(\omega, k, t)$ must hold as firms with a one step lead have lower profits in period $t$ ($\pi_1(\omega, k, t) = \pi_1 g^{k(\omega,t)} < \pi_2(\omega, k, t) = \pi_m(\omega, k, t) = \pi_m g^{k(\omega,t)}$) and can only catch up and obtain a more profitable two step lead if they undertake an innovation in the future. Along a BGP, the expected value of being the leading firm with an $l$ step lead increases by the factor $g$ if an innovation takes place, so that $V_l(\omega, k+1, t) = gV_l(\omega, k, t)$. In the following, the equilibrium innovation behavior in an intermediate good industry $\omega$ is studied. The indices $\omega$ and $t$ are dropped for convenience, as there are no interactions between industries and as time only affects payoffs in an indirect way by affecting $k$.

Suppose that $k$ is the newest vintage of an intermediate good in an industry. The expected value that an entrant obtains from introducing generation $k+1$ is then given by $V_1(k+1) = gV_1(k)$, as it brings a one step lead over the previous incumbent. An entrant $i$ chooses the innovation arrival rate $\phi_i$ in order to maximize expected discounted profits

$$V_0(k) = \phi_i V_1(k+1) - cg^k \phi_i \phi^\epsilon$$

It is therefore profitable for entrants to do R&D and to select $\phi_i > 0$ as long as the average (industry) R&D costs, that are given by $a(\phi) = cg^k\phi^\epsilon$ ($= \frac{C(\phi)}{\phi}$, see equation 4), do not exceed $V_1(k+1)$[16]. As there is free entry into R&D, $V_1(k+1) \leq cg^k\phi^\epsilon$ therefore has to hold as entrants would otherwise find it profitable to do more R&D and to increase $\phi$.

The case where $V_1(k+1) < cg^k\phi^\epsilon$ can only hold if the incumbent finds it profitable to do all of the R&D in the industry and to increase $\phi$ beyond the point where the average R&D costs start to exceed $V_1(k+1)$ and where entrants drop out of the innovation race. This would require that, absent any threat of entry, the incumbent values obtaining the next innovation more than entrants do. This is, however, not the case, as incremental profits are lower for the incumbent than for entrants, independently of whether the incumbent has an initial lead of one or two steps (see Lemma 1)[17]. Therefore $V_1(k+1) < cg^k\phi^\epsilon$ does not hold in the case

[16] The average and not the marginal costs in the corresponding industry $\omega$ matter as a single firm does not take into account that by increasing its own R&D effort it increases the R&D costs of all other firms, so that it still finds entry profitable if the marginal (industry) costs are higher than $V_1(k+1)$, as long as the average costs do not exceed $V_1(k+1)$.

[17] Without any threat of entry, the incumbent takes into account that by increasing the innovation rate, she not only increases the chance of discovering the next vintage of the good but also the chance of displacing her currently supplied vintage from the market, which makes her less willing to invest in R&D than entrants.

studied here[18] and the following free entry condition needs to hold with equality.

$$V_1(k+1) = cg^k(\phi^*)^\epsilon \tag{8}$$

Rearranging this condition gives $\phi^* = \left(\frac{V_1(k+1)}{cg^k}\right)^{\frac{1}{\epsilon}}$, implying that the equilibrium innovation rate $\phi^*$ in an industry is an increasing function of the value of an innovation for an entrant, $V_1(k+1)$. The fact that $V_1(k+1)$ determines $\phi^*$, however, does not imply that entrants actually carry out R&D in equilibrium.

**Persistent Leadership**

As it is assumed that incumbents move first (are Stackelberg leaders) in the innovation race, entrants adjust their R&D spending after observing the level of R&D that the incumbent undertakes. As equation 8 always holds, one unit of R&D undertaken by the incumbent crowds out one unit undertaken by entrants, leaving the industry innovation rate $\phi^*$ unchanged. An incumbent can therefore reduce the instantaneous probability of replacement from $\phi^*$ to $(1-\beta)\phi^*$ if she bears the costs $\beta cg^k(\phi^*)^{1+\epsilon}$ and if she innovates herself with arrival rate $\beta\phi^*$.

While the R&D costs are the same for entrants and the incumbent, the incumbent values not being replaced and obtaining the next innovation herself more than an entrant values entry: while successfully innovating entrants have to compete with the previous incumbent in the intermediate good market and only get the value $V_1(k+1)$ upon entry, the incumbent gets the larger value $V_2(k+1)$ in the case of a successful innovation, as it guarantees her unconstrained monopoly power.

With the total innovation rate $\phi^*$ pinned down by the free entry condition, incumbents lose their current value of incumbency, which is given by $V_2(k)$ in the case of a two step lead and by $V_1(k)$ in the case of a one step lead with the hazard rate $\phi^*$. As this replacement rate is independent of the share of R&D and of the innovation rate $\beta\phi^*$ of the incumbent, the incumbent has incentives to increase her R&D effort as long as $\beta < 1$ as she values obtaining the next innovation herself more than entrants do. Because of this "efficiency effect" (that was first analyzed by Gilbert and Newbery (1982) and introduced in the growth model presented here by Denicolò (2001)), the incumbent therefore fully preempts entry by doing as much R&D as needed to push the average R&D costs up to the value of an innovation for an entrant.

**Lemma 2.** *Given that the incumbent moves first in the innovation race, she undertakes exactly as much R&D as needed to discourage R&D of entrants in her*

[18]In *Section 4*, cases are discussed in which incumbents with a one step lead can have higher incentives to innovate than entrants.

*industry. Consequently, only the incumbent innovates and there is persistent leadership*

As the same reasoning applies when incumbents need a lead of more than two steps in order to obtain unconstrained monopoly power ($s > 2$), there is also persistent leadership in this more general case[19].

### 3.3 Equilibrium Innovation and Growth

In order to derive the equilibrium innovation rate $\phi^*$ from the free entry condition $\phi^* = \left(\frac{V_1(k+1)}{cg^k}\right)^{\frac{1}{\epsilon}}$, we need to first derive $V_1(k+1)$, the value of an innovation for an entrant. As successful entrants anticipate becoming the new incumbents and extending their lead to two and more steps, this value again depends on the value $V_2(k+2) = g^2 V_2(k)$ of having a lead of two or more steps. $V_2(k)$ can be derived from the following arbitrage condition:

$$rV_2(k) = \pi_m g^k - cg^k \left(\phi^*\right)^{1+\epsilon} - \phi^* V_2(k) + \phi^* V_2(k+1)$$

The first term on the right hand side indicates per period profits derived from supplying vintage $k$ of the intermediate good and the second term the R&D costs of the incumbent who does the entire R&D. The third and fourth terms on the right hand side indicate that, if an innovation occurs with the arrival rate $\phi^*$, the firm stops supplying vintage $k$ and loses the value $V_2(k)$, but at the same time starts supplying vintage $k+1$ and gains the value $V_2(k+1)$. Replacing $V_2(k+1) = gV_2(k)$ and solving for $V_2(k)$ gives

$$V_2(k) = \frac{\pi_m g^k - cg^k \left(\phi^*\right)^{1+\epsilon}}{r - \phi^* \left(g-1\right)} \tag{9}$$

In the following, it is assumed that $r > \phi^*(g-1)$ (**Condition B**) holds in order for $V_2(k)$ to be bounded. The value of being an incumbent with a one step lead supplying vintage $k$ of the good can be derived from the arbitrage condition

[19] A formal proof of Lemma 2 is provided in online Appendix B. The equilibrium analyzed here can either be interpreted as one where the incumbent is a Stackelberg leader in the R&D game or as a Walrasian equilibrium where the total demand for R&D inputs is equal to the supply and in which the auctioneer allocates all R&D to the incumbent who is willing to pay at least as much for it as the entrants. As the entrants earn zero profits in equilibrium, they are indifferent about the amount of R&D that they undertake and, if an incumbent would do less R&D than the equilibrium level, the Walrasian auctioneer would simply assign a larger amount of R&D to entrants in order to obtain the equilibrium (see Cozzi (2007) for a more detailed discussion). A similar equilibrium might also be obtained as the outcome of an auction in which incumbents and entrants simultaneously bid for R&D inputs.

$$rV_1(k) = \pi_1 g^k - cg^k \left(\phi^*\right)^{1+\epsilon} - \phi^* V_1(k) + \phi^* V_2(k+1)$$

which takes into account that this incumbent initially earns the lower profits $\pi_1 g^k$ and then advances to a two step lead with the next innovation. This arbitrage condition can be solved for

$$V_1(k) = \frac{\pi_1 g^k - cg^k \left(\phi^*\right)^{1+\epsilon} + \phi^* V_2(k+1)}{r + \phi^*}$$

Inserting $V_2(k+1) = gV_2(k)$ from equation 9 into this condition, plugging $V_1(k+1) = gV_1(k)$ into Equation 8 and dividing by $g^k$, the free entry condition can be written as:

$$\frac{g\left(\pi_1 - c\left(\phi^*\right)^{1+\epsilon}\right)}{r + \phi^*} + \frac{\phi^*}{r + \phi^*} \frac{g^2\left(\pi_m - c\left(\phi^*\right)^{1+\epsilon}\right)}{r - \phi^*\left(g-1\right)} = c\left(\phi^*\right)^{\epsilon} \tag{10}$$

This equation implicitly defines the equilibrium innovation rate $\phi^*$ in a given sector.

**Proposition 1.** *The equilibrium innovation rate $\phi^*$ is the same in all sectors and the equilibrium rate of growth $G^*$ of the economy is given by $G^* = (g-1)\,\phi^*$.*

*Both $\phi^*$ and $G^*$ increase in $\pi_1$, $\pi_m$ and $g$ and decrease in $r$ and $c$. In the case where incumbents need a lead of $s > 2$ steps in order to obtain unconstrained monopoly power, $\phi^*$ and $G^*$, moreover, increase in the (normalized) profits $\pi_l$ that incumbents obtain for lead sizes $1 < l < s$. In order for an equilibrium to exist, $r > G^*$ (Condition B) must hold, which is always satisfied if $r$ is sufficiently large.*

*Proof.* See Appendix A2 □

The intuition for these results is straightforward: Without the threat of entry, incumbents have a lower incentive to innovate than entrants due to the replacement effect and therefore only conduct as much R&D as needed to preempt entry when entry is possible. This preemptive R&D level depends positively on entry pressure, i.e. on the incentives of entrants to undertake R&D in order to enter and to become the next incumbent. Entry pressure decreases in the R&D costs $c$ and increases in the value of an innovation for an entrant which depends positively on (normalized) profits $\pi_1$ (or more generally $\pi_l$) and $\pi_m$ and negatively on the rate of interest $r$ due to a discounting effect. When $g$ increases, entry pressure increases as each innovation then leads to a larger increase in profits. Taking into account that $g \equiv q^{\frac{\alpha}{1-\alpha}}$ and that $\pi_1$ (and also $\pi_l$ for $l < s$) depends positively on $q$, $\phi^*$ therefore increases in the innovation step size $q$. As the equilibrium rate of growth of output is given by $G^* = (g-1)\phi^*$, all parameters have the same qualitative effect on $G^*$ that they have on $\phi^*$.

# 4 The Effects of different IPR Policies

## 4.1 Forward Protection

It is now assumed that an entrant´s IPR that covers vintage $k+1$ of a certain intermediate good infringes on the IPR of the previous vintage $k$ of the same good (but not on vintage $k-1$), so that the incumbent can block the production of vintage $k+1$ of the good by an entrant. An IPR therefore grants protection against future superior innovations, labeled "forward protection". It is assumed that there is still a research exemption so that entrants are permitted to undertake frontier R&D as long as they do not produce the invented goods (for an analysis of the case where there is no research exemption, see *Section 4.4.2*). Moreover, it is assumed that the antitrust authority allows entrants and incumbents to collude in prices if they negotiate a licensing agreement in the case of blocking IPRs but that collusion is prohibited if there is no forward protection. Incumbents are assumed to always collude with entrants if they find it profitable to do so once entry has occurred. Therefore, they cannot ex ante commit not to collude ex post.

**Proposition 2.** *Forward protection reduces growth. This result generalizes to the case where $s > 2$ innovation steps are needed in order to obtain unconstrained monopoly power and to the case where $q^s = \frac{1}{\alpha}$, $s \geq 2$, and where forward protection allows the blockage of $R > 1$ future innovations.*

*Proof.* See Appendix A3 □

This is the first main result of the article and the intuition (for the simple case) is the following: while absent forward protection entrants earn the profits $\pi_1 g^{k+1}$ if they obtain a one step lead, they maximally earn the incremental profits $\pi_m(g^{k+1} - g^k)$ by which their innovation increases monopoly profits in the case of forward protection. This is because the incumbent has the possibility to keep producing her old vintage and earning profits $\pi_m g^k$ (until the entrant obtains a two step lead) if she blocks the entrant from producing vintage $k+1$ and is therefore only willing to accept a collusive licensing agreement with the entrant if it gives her at least the same, leaving less than $\pi_m(g^{k+1} - g^k)$ per period for the entrant. As $\pi_m(g^{k+1} - g^k) < \pi_1 g^{k+1}$ holds due to declining incremental profits (Lemma 1), forward protection therefore reduces profits of entrants in the period where they have a one step lead. Once they have obtained a two step lead, profits are again given by $\pi_m g^{k+1}$, as in the case without forward protection. Therefore, forward protection reduces the value of an innovation for an entrant, entry pressure, and growth.

This result differs from those obtained in models where there is leapfrogging, like O'Donoghue and Zweimüller (2004) and O'Donoghue et al. (1998). In these models, firms never lead by more than one step so that, absent forward protection, profits for entrants are given by $\pi_1 g^{k+1}$ until they are replaced by the next innovation, $k+2$. Forward protection then has two effects: by giving blocking power to the incumbent, it forces the entrant to pay licensing fees to the previous incumbent, but allows him to recoup licensing fees from the next entrant. For constant markups, blocking power then leads to a more backloaded (and therefore more heavily discounted) profit flow for an entrant and can be shown to reduce the incentives to innovate. At the same time, forward protection allows firms to collude and to increase the joint profits that accrue to entrants and replaced incumbents who still get licensing revenues. If the backloading effect is not too large (because the interest rate is low or because entrants are strong bargainers), forward protection can therefore increase the value of an innovation for an entrant and the equilibrium innovation rate. In the case of persistent leadership, this cannot happen for the following reasons: forward protection then also has a blocking effect, as entrants have to pay licensing fees to previous incumbents, but do not expect to obtain any licensing fees in the future, as they plan to become the new industry leaders and to preempt all entry in the future[20]. Moreover, collusion only allows to increase joint profits in the case where an entrant has obtained a one step lead, but not anymore when he has advanced to a two step lead, as he then obtains unconstrained monopoly profits even absent forward protection. As an incumbent with blocking power never leaves a share of the collusive profits that exceeds $\pi_1 g^{k+1}$ to the entrant when $l = 1$, entry is unambiguously discouraged under forward protection as the entrant does not reap any of the collusive gains.

It should be noted that simply permitting entrants and incumbents to collude in prices (or to sell their IPRs or exclusive licenses to each other) without granting forward protection unambiguously increases growth in both the case of persistent leadership and in that of leapfrogging[21]. The reason for this is that by granting blocking power to incumbents, forward protection greatly discourages entry by increasing the incumbents' bargaining power.

The analysis is restrictive in the sense that it only focuses on cumulative in-

[20]Forward protection, however, also brings delayed rewards in the form of reduced R&D costs as it reduces the entry-deterring innovation rate $\phi^*$. The blocking effect of forward protection is similar to the effect that imposing entry fees would have: It is simple to show that such fees that entrants have to pay upon entry into a product market lead to reduced entry pressure and growth.

[21]In the case of persistent leadership, collusion allows avoiding the phase of competition when entrants have a one step lead and to increase their profits above the level $\pi_1 g^{k+1}$ that they can obtain absent collusion. This increases entry pressure and growth. As entry no longer reduces joint profits in the case of collusion, the incumbent values not being replaced and obtaining the next innovation equally much as the entrants value entering, as she still gets a share of the surplus in the case of entry. Therefore, the incumbent is indifferent about her share in total R&D and there need not be persistent leadership anymore.

novations along given quality ladders $\omega$ without allowing for product innovations that increase the set of intermediate goods. If the inventions of the first vintages ($k = 1$) in each intermediate good industry $\omega$ were also endogenized, forward protection would encourage these product innovations, as it increases profits for first innovators by giving them blocking power over follow-on innovations. Then, a trade-off would arise as forward protection would encourage product innovation while discouraging quality-improving follow-on innovations (see Chu et al. (2012) and also Denicolò (2002)). If growth is mainly driven by improvements of existing varieties, as Garcia-Macia et al. (2015) argue, the effects that intellectual property policy might have on growth through the channel of product innovations might, however, be relatively small.

## 4.2 Non-Obviousness Requirement

So far it has been assumed that firms cannot influence the size of the quality improvement $q$ that an innovation brings and that $q$ is such that a two (or $s$) step lead allows charging the unconstrained monopoly price $\frac{1}{\alpha}$. This section looks at the case where firms are capable of targeting different innovation step sizes $q > 1$ at different costs. Unconstrained monopoly power (a quality difference larger than $\frac{1}{\alpha}$ relative to the closest competitor) can therefore be attained by either undertaking many little steps (small $q$) or a few large steps (large $q$). Incremental profits resulting from an innovation are now endogenous and increase in the step size $q$. In this setting, there is an additional instrument that intellectual property policy can use: a non-obviousness requirement that sets a lower bound $\underline{q}$ on the inventive step below which an inventor cannot obtain IP protection.

It is assumed that the R&D technology is the same in all intermediate good sectors $\omega$. The R&D costs in terms of the final good for a firm $i = j$ that wants to improve upon the currently highest quality $Q_\omega(t)$ of an intermediate good by the factor $q_j$ ($> 1$) and that targets the arrival rate $\phi_j$ are given by

$$C_j(\omega, t) = c\left(Q_\omega(t)\right)^{\left(\frac{\alpha}{1-\alpha}\right)} \phi_j \lambda(q_j) \left(\sum_i \phi_i \lambda\left(q_i\right)\right)^{\epsilon}$$

with $\frac{\partial \lambda(q)}{\partial q} > 0$ and with $\sum \phi_i \lambda\left(q_i\right)$ indicating the overall R&D effort in sector $\omega$. For given overall R&D spending, targeting a larger inventive step $q_j$ therefore implies a lower hazard rate $\phi_j$. Given that $\epsilon > 0$, R&D costs for firm $j$ increase in the overall R&D effort in the industry. Due to this assumption, an incumbent can again preempt entry by undertaking a large enough amount of R&D in order to increase the entrants' R&D costs[22]. In order to allow for balanced growth,

[22] It is therefore assumed that only the total R&D effort of the incumbent matters for the

the growth factor $(Q_\omega(t))^{\left(\frac{\alpha}{1-\alpha}\right)}$ is included due to which R&D costs increase in line with profits when the quality $Q_\omega(t)$ of the newest vintage of an intermediate good $\omega$ increases. It is assumed that it is prohibitively costly to target a drastic innovation ($q > \frac{1}{\alpha}$) which gives an entrant unconstrained monopoly power in a single step.

In this more general setting, the efficiency effect is present again: as for any size of the lead, an incumbent earns larger profits due to larger monopoly power, she again values not being replaced and obtaining the next innovation herself more than an entrant values entry. While entrants and incumbents might find it optimal to select different step sizes for their innovations, it is never more costly for incumbents to innovate than for entrants, as they always have the option of selecting the same step size as entrants. Therefore, incumbents again find it profitable to preempt entry and there is persistent leadership (as in Denicolò (2001)). Due to the fact that for any step size, incremental profits are lower for unconstrained monopolists than for entrants (see Lemma 1, Part c), incumbents who have advanced to a lead that is large enough to earn unconstrained monopoly profits never want to do more R&D than needed to preempt entry (as, whatever step size is most profitable for incumbents, incremental profits for entrants are larger if they select the same step size).

In order to simplify the notation, the indices $\omega$ and $t$ are omitted in the following. Expected profits of an entrant who targets step size $q_e$ and innovation rate $\phi_e$ in order to obtain a one step lead over the initial quality level $Q$ are given by

$$V_0(Q) = \phi_e V_1(Q, q_e) - cQ^{\left(\frac{\alpha}{1-\alpha}\right)}\phi_e\lambda(q_e)\left(\phi_e\lambda(q_e) + \sum_{i\neq e}\phi_i\lambda(q_i)\right)^{\epsilon}$$

with $V_1(Q, q_e)$ denoting the maximum value that the entrant can derive from obtaining a one step lead. $V_1(Q, q_e)$ not only depends on the size $q_e$ of the first inventive step, but on the whole path of inventive steps and innovation rates that the entrant finds optimal to choose in the future in order to advance his lead and to preempt entry of others. The free entry condition ($V_0(Q) = 0$) that must be satisfied in an industry where the incumbent is an unconstrained monopolist and undertakes all the R&D is therefore given by

$$V_1(Q, q_e) = cQ^{\left(\frac{\alpha}{1-\alpha}\right)}\lambda(q_e)\left(\phi_m\lambda(q_m)\right)^{\epsilon} \tag{11}$$

entrants' R&D costs, but not the combination of step size and innovation rate that the incumbent chooses. This assumption seems plausible in settings where the price of (uniform) R&D inputs increases if demand for them increases and where it is possible to preempt entry by simply buying enough of these inputs in order to increase their price. If, however, the incumbent´s R&D activity affected the entrants' R&D profitability by increasing the risk of duplication, the combination of step size and innovation rate would also matter.

This condition pins down the R&D effort $\phi_m \lambda(q_m)$ that the incumbent undertakes in order to preempt entry as an increasing function of the maximized and normalized value that an innovation has for an entrant, $\tilde{V}_1 \equiv \frac{V_1(Q,q_e)}{Q^{\frac{\alpha}{1-\alpha}}\lambda(q_e)}$.

Without any non-obviousness requirement, entrants choose the sequence and sizes of inventive steps (including the first one, $q_e$) that maximize the present discounted value of their R&D activity (and therefore also $\tilde{V}_1$). Therefore, any binding non-obviousness requirement that restricts the R&D decisions of entrants and of incumbents who have not yet reached a lead large enough to be unconstrained monopolists decreases the maximized value $\tilde{V}_1$ that entrants derive from undertaking R&D, and therefore reduces entry pressure and the total R&D effort $\phi_m \lambda(q_m)$ of incumbents who are unconstrained monopolists. Moreover, the following can be shown:

**Lemma 3.** *For a given R&D effort $\phi_m \lambda(q_m)$, unconstrained monopolists find it optimal to select the growth-maximizing innovation step size $q_m^* = q^*$.*

*Proof.* See Appendix A4 □

Lemma 3 therefore implies that imposing a non-obviousness requirement cannot increase growth by increasing the innovation step size that unconstrained monopolists choose. As such a requirement at the same time reduces the R&D effort of industry leaders, we can state the following:

**Proposition 3.** *Imposing a non-obviousness requirement reduces equilibrium growth.*

This result differs from the previous literature. O'Donoghue and Zweimüller (2004) and Hunt (2004) show that, in the case of leapfrogging, imposing a patentability (non-obviousness) requirement can increase the rate of innovation, growth, and welfare. O'Donoghue (1998) reaches the same conclusion in a model where even incumbent firms do some R&D in equilibrium, but in which preemption is not possible, so that there is no persistent leadership. The following mechanism is at work in these models: when an entrant selects the profit-maximizing combination of step size and innovation arrival rate for a given R&D spending, he does not take into account that increasing the latter (but not the former) reduces the expected profits of the previous incumbent by leading to a faster rate of "creative destruction". Imposing a non-obviousness requirement can therefore reduce the threat that innovators will lose their monopoly power due to marginal (low step size) follow-on innovations and can, ceteris paribus, prolong the duration of monopoly power. This effect turns out to be stronger than the direct innovation-discouraging effect stemming from increased R&D costs (which is partially offset by larger markups and profits resulting from a larger step size), so that imposing a (weak) non-obviousness requirement can increase innovation in the case of leapfrogging.

In the case of persistent leadership, allowing entrants to target small innovation step sizes and thereby increasing the risk of creative destruction for the incumbent has different effects: as the incumbent wants to keep her unconstrained monopoly power, she responds to increased entry pressure by increasing her own R&D spending. As entry does not occur in equilibrium and as incumbents find it profitable to target the growth-maximizing step sizes, marginal innovations on which firms could obtain IP protection are never carried out and industry profits are never reduced below the monopoly level[23].

## Expiring Intellectual Property Protection

In order to make the model more tractable, the limit case in which there are constant returns to R&D ($\epsilon = 0$) is studied in the following sections. As for $\epsilon = 0$, the current innovation rate is undetermined even though the free entry condition $V_1(k+1) = cg^k$ pins down the expected future innovation rates, it is assumed that the equilibrium is selected that results as the limit if $\epsilon \to 0$. Given that the incumbent has unconstrained monopoly power, the current and the expected future innovation rates coincide in this preemption equilibrium and are given by $\phi_2^*$.

With constant returns to R&D the innovation rate might not always be pinned down by the free entry condition and some firms might find it profitable to increase their R&D effort infinitely for certain lead sizes. Because of this, it is assumed that there is an upper bound $\bar{\phi}$ (as specified in equations 3 and 4) that the innovation rate cannot surpass and that is selected in these cases. In order to avoid the uninteresting cases where, absent the threat of future entry, not even entrants find it profitable to undertake R&D or where even incumbents with a two step lead want to achieve the maximal innovation rate $\bar{\phi}$, the following condition is assumed

[23] Chu and Pan (2013) study the effects of forward protection in a model with leapfrogging and endogenous innovation step sizes. They show that (marginally) increasing the blocking power of incumbent firms can push entrants to pursue larger innovation step sizes and can thereby increase innovation. In the case of persistent leadership, increasing the blocking power of incumbents cannot have such a positive effect, as it either reduces entry pressure by reducing profits for entrants that infringe on the incumbent's IPRs (see Section 4.1) or by pushing entrants to target larger step sizes than they would find optimal absent forward protection.

to be satisfied[24] (**Condition C**):

$$\frac{\pi_m\left(g^{k+1}-g^k\right)}{r} < cg^k < \frac{\pi_1 g^{k+1}}{r}$$

## 4.3 State-dependent Intellectual Property Protection

This section analyzes the effects of IP expiration in the basic model in which IPRs protect against imitation, but in which there is no forward protection and no collusion and in which the step size is constant and exogenous. Like in Acemoglu and Akcigit (2012), the general case of "state-dependent intellectual property protection" is analyzed in which the probability of IP expiration can depend on whether a firm has a one step or a two (or more) step lead over its rivals. The IPRs of a firm with a one step lead are assumed to expire with the instantaneous Poisson arrival rate $\gamma_1$, and those of a firm with a two step lead with the instantaneous arrival rate $\gamma_2$. When IP protection expires in an industry, the newest vintage of the intermediate good falls in the public domain, allowing competitors to copy it and to fully catch up[25]. Profits therefore fall to zero in the case of IP expiration.

In the case of persistent leadership, $V_1(k)$ (that means the value of being one step ahead) and the value $V_2(k)$ of being two (or more) steps ahead can be derived from the following arbitrage conditions, where $\phi_l$ denotes the innovation rate chosen by an incumbent with an $l$ step lead:

$$rV_2(k) = \pi_m g^k - cg^k\phi_2 - \gamma_2 V_2(k) + \phi_2\left(V_2(k+1) - V_2(k)\right)$$

$$rV_1(k) = \pi_1 g^k - cg^k\phi_1 - \gamma_1 V_1(k) - \phi_1 V_1(k) + \phi_1 V_2(k+1)$$

Taking into account that $V_l(k+1) = gV_l(k)$ along a BGP, we can solve for:

$$V_1(k+1) = g^{k+1}\frac{\pi_1 - c\phi_1}{r+\gamma_1+\phi_1} + \frac{\phi_1}{r+\gamma_1+\phi_1}g^{k+2}\frac{\pi_m - c\phi_2}{r+\gamma_2-\phi_2\,(g-1)} \tag{12}$$

A firm with a one step lead sets its innovation rate $\phi_1$ as a function of the IP expiration rates $\gamma_1$ and $\gamma_2$ which determine the relative profitability of a two step lead compared to a one step lead.

[24]Absent the threat of future entry and with infinite IP protection, the value of an innovation for an entrant is given by $\triangle V = V_1(k+1) = \frac{\pi_1 g^{k+1}}{r}$ and that of an incumbent with a two step lead by $\triangle V = V_2(k+1) - V_2(k) = \frac{\pi_m\left(g^{k+1}-g^k\right)}{r}$. As the marginal (and average) R&D costs are given by $cg^k$, firms that maximize expected profits $\phi(\triangle V - cg^k)$ therefore find it optimal to increase $\phi$ if $\triangle V > cg^k$ and to not undertake any R&D if $\triangle V < cg^k$.

[25]This specification implicitly assumes that IP protection on second-newest vintages never expires. The case in which firms with a two step lead can lose this lead for a one step lead (because their IPR on the second-newest vintage expires) is not considered.

**Lemma 4.** *Given that* $\frac{\pi_m(g-1)}{r} < c < \frac{\pi_1 g}{r}$ *(Condition C) and that* $\bar{\phi}$ *is sufficiently large, the following cases arise:*

***i):*** *If* $\gamma_1 > \frac{g\pi_1}{c} - r$ *and if* $\gamma_2 < \frac{g^2\pi_m}{(g+1)c} - r$*, a firm with a one step lead selects innovation rate* $\phi_1^* = \bar{\phi} > \phi_2^*$

***ii):*** *If* $\gamma_1 \leq \frac{g\pi_1}{c} - r$ *and if* $\gamma_2 \leq \gamma_1 + \frac{g(\pi_m - \pi_1)}{c}$*, a firm with a one step lead selects the preemptive innovation rate* $\phi_1^* = \phi_2^* < \bar{\phi}$

***iii):*** *If* $\gamma_1 \leq \frac{g\pi_1}{c} - r$ *and if* $\gamma_2 > \gamma_1 + \frac{g(\pi_m - \pi_1)}{c}$*, a firm with a one step lead does not do any R&D (*$\phi_1^* = 0$*), so that there is leapfrogging.*

*Proof.* See Appendix A5 □

As $\pi_1 g > \pi_2 g - \pi_1$ (Lemma 1), $\gamma_1 > \gamma_2$ needs to hold in Regime (i), meaning that IP protection needs to expire sufficiently more quickly in the case of a one step lead than in the case of a two step lead in this regime. This makes a two step lead relatively more profitable, so that firms with a one step lead do the maximal amount of R&D, $\bar{\phi}$, in order to reach a two step lead as quickly as possible.

In Regime (ii), firms with a one step lead value not being replaced and reaching a two step lead more than entrants value entry, but they do not want to do more R&D than necessary to preempt entry so that $\phi_1^* = \phi_2^*$. In Regime (iii), IPRs expire so much faster in the case of a two step lead than in the case of a one step lead that firms with a one step lead do not find reaching a two step lead worthwhile and stop innovating. In Regimes (i) and (ii), the innovation rate in the case where the IPR on the currently newest vintage of an intermediate good has expired ("zero step lead") is given by $\phi_0^* = \phi_2^*$. While entrants might undertake R&D in this case, the innovation rate is the same as the preemptive level $\phi_2^*$ that incumbents select in order to prevent entry, as it is pinned down by the same free entry condition $V_1(k+1) = cg^k$. In Regime (ii), the innovation rate is therefore independent of the size of the lead, while it is higher in industries with a one step lead than in industries with a zero or a two step lead in Regime (i). In Regime (iii), entrants undertake all R&D, and the innovation rate is given by $\phi^*$, independently of whether IP protection on the currently newest vintage of the good in a given sector has expired or not. We can now analyze the effect of IP expiration on equilibrium growth $G^*$ and derive the third main result of the article.

**Proposition 4.** *Suppose that Condition C holds and that* $\bar{\phi}$ *is sufficiently large. Given a change in* $\gamma_i$ *does not lead to a switch between regimes, the equilibrium rate of growth* $G^*$ *decreases in* $\gamma_1$ *in all Regimes and decreases in* $\gamma_2$ *in Regimes (i) and (ii). If a marginal shift in parameters leads to a switch from Regime (ii) to Regime (i), there is a discontinuous increase in* $G^*$*, while there is no discontinuity in the case of a switch between Regimes (ii) and (iii).* $G^*$ *is maximal if* $\gamma_1 = \gamma_2 = 0$*, that means under full uniform IP protection.*

*Proof.* See Appendix A6 □

Reducing IP protection by increasing $\gamma_1$ or $\gamma_2$ decreases the value of an innovation for entrants and entry pressure in all regimes (with the exception that $\gamma_2$ does not affect outcomes in Regime iii)). This directly implies a reduction in the equilibrium innovation and growth rates in Regimes iii) and ii), and also a decrease in the innovation rates $\phi_2^* = \phi_0^*$ in the cases of a two and a zero step lead in Regime (i). In Regime (i), there is, however, also a "composition effect" which goes in the opposite direction: increasing $\gamma_2$ can now increase the fraction of industries in which there is a one step lead and in which the innovation rate is maximal.[26] However, this composition effect is not strong enough to overcompensate the negative effect that an increase in $\gamma_2$ has on $\phi_2^* = \phi_0^*$, so that an increase in $\gamma_2$ still leads to a reduction in the equilibrium rate of growth $G_i^*$ in Regime i). $G_i^*$ is therefore maximal if $\gamma_2 = 0$, in which case the composition effect disappears as leaders never lose a two step lead. But given that $\gamma_2 = 0$, the growth rate decreases in $\gamma_1$, as increasing the rate with which IP protection for firms with a one step lead expires merely reduces the value of an innovation for entrants and therefore the amount of R&D that incumbents need to undertake in order to preempt entry. Consequently, growth is maximal under full uniform IP protection and, unlike in Acemoglu and Akcigit (2012), cannot be increased by reducing IP protection in a state-dependent way (i.e. by increasing $\gamma_1$ more than $\gamma_2$).

Given that $\gamma_1$ and $\gamma_2$ are already positive and at a certain threshold value, a small increase in $\gamma_1$ can, however, lead to an increase in growth by inducing a switch from Regime (ii) to Regime (i). The reason for this is that such a decrease in IP protection for firms with a one step lead can induce them to switch their strategy from only undertaking the amount of R&D that is necessary to preempt entry to undertaking the maximal amount of R&D in order to reach a two step lead as quickly as possible. There is therefore a nonlinear relation between $\gamma_1$ and the rate of growth so that the "incentive effect" identified by Acemoglu and Akcigit (2012) is at work at a specific threshold value of $\gamma_1$.

## 4.4 Potential Perils of strong Protection against Imitation

In the previous section the assumption was made that entrants and incumbents have free access to the same R&D technology. It was shown that in this case reducing the strength with which IPRs protect against imitation (i.e. increasing $\gamma_1$ or $\gamma_2$) cannot increase the rate of growth through a composition effect, i.e. by increasing the fraction of industries in which R&D incentives are higher due to a lower lead. In the following subsections, cases are analyzed in which this is different and where reducing IP protection can increase growth through such a composition

[26]Increasing $\gamma_1$ always decreases this fraction (see Appendix A6).

effect. It is again assumed that there is no forward protection and that the step size is constant and exogenous.

#### 4.4.1 Trade Secrecy

So far it has been assumed that there is free access to the R&D sectors and that entrants have complete knowledge about the currently newest technologies and can directly start doing frontier R&D. While, ideally, a firm is only granted IP (patent) protection when it discloses the functioning of its innovation, firms often succeed in keeping part of their knowledge secret so that competing firms first have to engage in some duplicative catch-up R&D before they can start conducting frontier R&D. It is therefore assumed that entrants first have to spend the fixed catch-up costs $Fg^k$ before they can undertake frontier R&D directed at inventing vintage $k+1$ in a given industry[27]. It is assumed that the incumbent can observe an entrant's catch-up and thus can adjust her R&D spending after observing entry into the R&D sector. IP protection expires with the constant hazard rate $\gamma \geq 0$ and, in the case of expiration in a given industry, the newest available vintage of the intermediate good falls in the public domain and is supplied at the marginal cost of one[28]. In this case, profits fall to zero, and it is assumed for simplicity that all knowledge about the currently newest vintage of the good falls in the public domain.

The equilibrium can be derived through backward induction. Given the incumbent in a certain industry has a lead of one or two steps, and entry into the corresponding R&D sector has occurred, the preemption equilibrium results in which the incumbent does enough R&D to completely discourage R&D by the entrant and in which the entrant earns zero profits. Expecting this, no entrant finds it profitable to pay the catch-up costs $Fg^k$, so that there is no entry into the R&D sector in an industry in which the currently newest vintage of the good is protected by IPRs. This, however, implies that there is no entry pressure and that incumbents do not need to undertake any R&D in order to preempt entry. Due to Condition C, incumbents thus stop innovating after having obtained a two step lead and might even find it profitable to stop innovating after having obtained a one step lead.

If IPRs have expired and if the currently newest vintage of an intermediate

[27] In the online Appendix B, the somewhat related problem where two R&D stages must be completed in order to obtain a marketable final innovation is analyzed. It is shown that in such a case, growth is maximal if entrants are allowed to obtain IP protection on the intermediate R&D inputs generated in the first R&D stage while incumbents are not.

[28] This specification is chosen for reasons of simplicity, and neglects the case where firms with a two step lead lose this lead for a one step lead because their IPR on the second-newest vintage expires. It is therefore implicitly assumed that IPRs on second-newest vintages never expire.

good is in the public domain, firms might, however, find it profitable to undertake R&D in order to become the next leader, given that IP protection does not expire too quickly. Denoting the equilibrium innovation rates in the case of an $l$ step lead by $\phi_l^*$ and the innovation rate in the case where IPRs have expired by $\phi_0^*$, the following proposition holds:

**Proposition 5.** *Given that Condition C holds, three regimes arise:*
***Regime A****): if* $\gamma \leq \frac{\pi_m g - \pi_1}{c} - r$, $\phi_0^* = \phi_1^* = \bar{\phi}$ *and* $\phi_2^* = 0$
***Regime B****): if* $\frac{\pi_m g - \pi_1}{c} - r < \gamma \leq \frac{\pi_1 g}{c} - r$, $\phi_0^* = \bar{\phi}$ *and* $\phi_1^* = \phi_2^* = 0$
***Regime C****): if* $\gamma > \frac{\pi_1 g}{c} - r$, $\phi_0^* = \phi_1^* = \phi_2^* = 0$
*The equilibrium rate of growth is given by:*

$$G^* = \begin{cases} (g-1)\,\frac{\bar{\phi}\gamma\left(\gamma+2\bar{\phi}\right)}{(\bar{\phi}+\gamma)^2} \text{ in Regime A} \\ (g-1)\,\frac{\bar{\phi}\gamma}{\bar{\phi}+\gamma} \text{ in Regime B} \\ 0 \text{ in Regime C} \end{cases}$$

Given that $\frac{\pi_m g - \pi_1}{c} - r > 0$ and $\bar{\phi}(\frac{2g\pi_m - (2+g)\pi_1}{c} - r) + \left(\frac{\pi_m g - \pi_1}{c} - r\right)^2 > (<) 0$, $G^*$ is maximal if $\gamma = \frac{\pi_m g - \pi_1}{c} - r$ $\quad(\gamma = \frac{\pi_1 g}{c} - r)$. If $\frac{\pi_m g - \pi_1}{c} - r < 0$, $G^*$ is maximal if $\gamma = \frac{\pi_1 g}{c} - r > 0$.

*Proof.* See Appendix A7 □

There is therefore an inverted-U relation between the rate of growth and the strength of IP protection[29]. If IPRs are fully protected (the sub-case of Regime A where $\gamma = 0$), the growth rate is zero, as there is no entry pressure and as monopolists rest on their laurels. Reducing the strength of IP protection then increases the fraction of industries in which no firm has a lead over its rivals due to expired IPRs and in which innovation incentives are maximal, but also the fraction of industries in which the incumbent only has a one step lead and might (in Regime A) find it profitable to do R&D in order to obtain a two step lead. If, however, IP protection is too weak (Regime C), the growth rate is again zero, as appropriability is so low that firms do not find it profitable to do R&D, even if the currently newest vintage of an intermediate good is in the public domain.

[29] It should be noted that this result holds for any size of $F$, as long as incumbents can observe catch-up and entry and can readjust their R&D effort ex post. There is therefore a discontinuity in the effect that IP protection has on growth when $F$ falls from $\epsilon > 0$ to 0. This discontinuity, however, disappears if incumbents are not capable of observing the catch-up (which might be plausible if catch-up costs $Fg^k$ are low): then, they actually have to undertake R&D in order to preempt entry, so that the analysis is again very similar to that in the previous sections. The fixed catch-up costs then only decrease entry pressure while stronger IP protection (a lower $\gamma$) again encourages innovation.

It should be noted that this result only arises in the present setup where incumbents can (and want to) preempt entry, but not in models with leapfrogging in which only entrants innovate. In the case of leapfrogging, full protection against imitation ($\gamma = 0$) always maximizes innovation and growth, as it encourages firms to be the first to overcome the fixed catch-up costs and to then win the innovation race in which they cannot be preempted by an incumbent anymore.

### 4.4.2 Lack of Research Exemption and different Forms of Preemption

Even if entrants are capable of undertaking frontier R&D without having to catch up first, broad intellectual property protection granted to previous innovators and the lack of a research exemption might make it illegal for entrants to do frontier R&D without first negotiating licensing contracts with previous innovators. If these contracts specify a positive up front licensing fee and if the incumbent can observe licensing, such a fee, however small it might be, plays the same role as the catch-up costs in the section above and leads to a complete blockage of entry. Therefore, imposing a research exemption for entrants seems to be particularly important to encourage cumulative innovation when incumbents can innovate preemptively.

Proposition 5 holds more generally in any situation where an incumbent (or several incumbents colluding on R&D decisions) does not face any entry pressure[30] and it is in line with the findings of Horowitz and Lai (1996) and Cadot and Lippman (1995) who assume that only one incumbent firm is capable of doing R&D[31]. It was assumed in the analysis above that incumbents can only discourage entrants' R&D by doing R&D themselves. There might, however, be cases in which incumbents can increase the R&D costs of potential entrants, or decrease their expected benefits of innovating, without doing R&D themselves. If there is an upward-sloping supply curve for R&D labor in an industry, incumbents might, for example, hire a certain amount of R&D labor in order to increase the wages that rivals need to pay, but employ the researchers in other areas than R&D. In the case where knowledge is to some extent tacit and where only researchers who were involved in past R&D are capable of doing frontier R&D, incumbents can simply offer them long term contracts or make them sign non-compete clauses in order to prevent them from doing research for entrants. While preemption is not costless

[30]Given the constant-return R&D technology with the upper bound $\bar{\phi}$, $\phi_0$ is the same in the cases where one or several firms capable of doing R&D.

[31]If only a single firm is capable of doing R&D in a given industry, the result even extends to the case where innovations are drastic (i.e. where $q > \frac{1}{\alpha}$). If $\frac{\pi_m(g-1)}{r} < c < \frac{\pi_m g}{r}$, such a firm only has incentives to innovate when IP protection has expired. Then, growth is maximal for the expiration rate $\gamma = \frac{\pi_m g}{c} - r$ at which firms still find it profitable to set $\phi_0^* = \bar{\phi}$ and that maximizes the fraction of industries in which IP protection has expired and in which innovation takes place.

in these cases, the result that incumbents stop innovating once they have obtained a sufficient lead is therefore the same. Because of that, the rate or growth is again zero if IPRs are fully protected and might increase if IP protection is decreased.

# 5 Conclusions

While most of the literature analyzing the role of intellectual property rights in models of cumulative innovation focuses on the case of leapfrogging, this article studies the case of persistent leadership. This is relevant from an empirical perspective, since a considerable amount of R&D is undertaken by firms that innovate on a permanent basis. Three main results come out of this analysis: *first*, that forward protection reduces growth and *second*, that imposing a non-obviousness requirement reduces growth. Both of these results differ from those obtained in models with leapfrogging. The *third* main result is that full uniform protection against imitation, and not state-dependent IP protection, maximizes growth. The main driving force behind these results is that the corresponding policies reduce entry pressure and the amount of R&D that incumbents need to undertake in order to preempt entry.

A limitation of the analysis is that it focuses on growth and not on the welfare effects of different policies, which are not straightforward in quality-ladder models in which growth can be excessive from a welfare point of view (for a detailed analysis, see Denicolò and Zanchettin (2014)). Given that growth is excessive, it is, however, not recommendable to reduce it by introducing forward protection or a non-obviousness requirement if it is instead possible to reduce the maximal price that unconstrained incumbents can charge[32] and to diminish growth in this way. The reason for this is that only the latter policy reduces monopoly distortions while the other policies mainly reduce entry pressure without affecting the price setting power of unconstrained incumbents, who supply all intermediate goods in equilibrium[33].

The model generates persistent leadership by assuming that innovations are non-drastic and that incumbents are capable of preempting entry, but – unlike other models – it does not rely on the assumption that incumbents are more productive in undertaking R&D than entrants. Within this setting, entry can occur if IP protection has expired or if entrants and incumbents can collude or sell their IPRs. Therefore, the model is also consistent with cases in which there is less than perfect persistence. If, however, entry occurs because innovations are drastic,

[32]This can for example be done by imposing price regulations or by reducing the breadth of IP protection, forcing incumbents to engage in limit pricing in order to prevent entry of competitors supplying non-infringing imitates of inferior quality.

[33]A formal welfare analysis is provided in online Appendix B.

because entrants' R&D productivity is larger than that of incumbents, or because preemption is not possible (or not an optimal strategy for incumbents) due to technological or informational reasons, the mechanisms that arise in leapfrogging models become more relevant. As the effects of IPR policies differ in models with leapfrogging and persistent leadership, it might be beneficial to use different policies in sectors that differ substantially in the persistence of innovative activities. In order to study such questions in more detail, it would be interesting to analyze a more general model in which entry plays a more prominent and realistic role. This extension is left for future research.

# Appendix

## A1: Proof of Lemma 1

*Proof.* Part b) is presented first (in three sub-steps) as Part a) builds on it. "$k$" instead of "$k(\omega, t)$" is used to save on notation.

**b1)**: For $l \geq s$, $\triangle\pi(k,l) = \pi_m(k)\,(g-1)$ can be derived from equation 6 and is independent of $l$.

**b2)**: For $0 \leq l \leq s-1$, $\triangle\pi(k,l) = \alpha^{\frac{1}{1-\alpha}} g^k q^{-\frac{l}{1-\alpha}} \left[\left(q^{l+1}-1\right) g q^{-\frac{1}{1-\alpha}} - q^l + 1\right]$ can be derived from equation 7 and falls in $l$ if $\triangle\pi(k,l) < \triangle\pi(k,l-1)$ (starting from $l \geq 1$). The latter inequality holds if

$$\left(q^{l+1}-1\right) g q^{-\frac{1}{1-\alpha}} - q^l + 1 < q^{\frac{1}{1-\alpha}} \left[\left(q^l - 1\right) g q^{-\frac{1}{1-\alpha}} - q^{l-1} + 1\right]$$

Replacing $g = q^{\frac{\alpha}{1-\alpha}}$, this implies the inequality $q - 1 < q^{\frac{1}{1-\alpha}}\,(q-1)$, which is satisfied as $q > 1$ and $\alpha < 1$.

**b3)**: For $l = s$, $\triangle\pi(k, l = s) = \pi_m(k)\,(g-1)$, and for $l = s-1$, $\triangle\pi(k, l = s-1) = \pi_m(k) g - \pi(k, s-1)$, with $\pi(k, s-1) = \left(q^{s-1}-1\right) \alpha^{\frac{1}{1-\alpha}} g^k q^{-\frac{s-1}{1-\alpha}}$ (see equation 7). As $\pi(k, s-1) < \pi_m(k)$, $\triangle\pi(k, l = s-1) > \triangle\pi(k, l \geq s)$ holds, so that $\Delta\pi$ falls if $l$ increases from $l = s-1$ to $l = s$.

**a)**: $\Delta\pi$ falls in $l$ for $l \geq 1$ (see sub-steps *b3* and *b1*). If $l$ rises from 0 to 1, $\Delta\pi$ falls if (**Condition (Co.) 1**):

$$\triangle\pi(k,0) = \pi(k+1,1) > \triangle\pi(k,1) = \pi_m(k+1) - \pi(k,1)$$

which holds if

$$M \equiv \left(\left(q-1\right) \alpha^{\frac{1}{1-\alpha}} q^{-\frac{1}{1-\alpha}}\right) \frac{(g+1)}{g} > \alpha^{\frac{1+\alpha}{1-\alpha}}\,(1-\alpha)$$

Co. 1 holds for $q = \frac{1}{\alpha}$ (see *b3*) and also for $q = \frac{1}{\sqrt{\alpha}}$ (see *b2*), so that it also holds for all values $\frac{1}{\sqrt{\alpha}} < q < \frac{1}{\alpha}$ (i.e. under Co. A) if $M$ lies above its boundary values ($M(q = \frac{1}{\alpha})$ and $M\left(q = \frac{1}{\alpha^2}\right)$) in this range of $q$. This holds as $M$ is a continuous and concave function in this range: Replacing $g = q^{\frac{\alpha}{1-\alpha}}$, we get

$$\frac{\partial^2 M}{\partial q^2} = \frac{\alpha^{\frac{1}{1-\alpha}}}{(1-\alpha)^2} q^{-2-\frac{1}{1-\alpha}} \left[\alpha\,(q+1) - 2 + 2q^{-\frac{\alpha}{1-\alpha}}\,(q\alpha\,(1+\alpha) - 1 - \alpha)\right] < 0$$

**c)**: If $q \geq \frac{1}{\alpha}$, $\triangle\pi(k, l = 0) = \pi_m(k) g > \triangle\pi(k, l \geq 1) = \pi_m(k)\,(g-1)$ always holds. If $1 < q < \frac{1}{\alpha}$, it can be shown that incremental profits for entrants are larger than those of unconstrained monopolists if (**Co. 2**):

$$\frac{q-1}{q\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)} > \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)$$

As this inequality holds if $q = \frac{1}{\alpha}$, it also holds if $1 < q < \frac{1}{\alpha}$ if the left hand side (LHS) of Co. 2 decreases in $q$ in this range. $\frac{\partial LHS}{\partial q} < 0$ holds if $1 - \alpha > q^{\frac{\alpha}{1-\alpha}} - \alpha q^{\frac{1}{1-\alpha}} \equiv N$ (**Co. 3**). As $\frac{\partial N}{\partial q} < 0$, Co. 3 always holds as it is satisfied with equality for the smallest feasible value of $q$, i.e. for $q = 1$. □

## A2: Proof of Proposition 1

*Proof.* For $l < s$, the arbitrage condition

$$V_l(k) = \frac{1}{r + \phi_l} \left[ \pi_l g^k - c g^k (\phi_l)^{1+\epsilon} + \phi_l V_{l+1}(k+1) \right] \tag{A1}$$

must hold, with either $\phi_l = \phi^*$ if $\left.\frac{\partial V_l(k)}{\partial \phi_l}\right|_{\phi_l = \phi^*} \leq 0$ or $\phi_l > \phi^*$ if $\left.\frac{\partial V_l(k)}{\partial \phi_l}\right|_{\phi_l = \phi^*} > 0$ (the latter can only arise if $\Delta\pi$ locally rises in $l$, as in Sections 4.1 and 4.3). Iterating equation A1 and replacing $V_l(k) = V_m(k)$ from equation 9 for $l \geq s$, we obtain

$$V_1(k+1) = g^{k+1} \frac{\pi_1 - c\phi_1^{1+\epsilon}}{r + \phi_1} + \sum_{i=1}^{s-2} \frac{\prod_{j=1}^{i} \phi_j}{\prod_{k=1}^{i+1} (r + \phi_k)} \left[ \pi_{i+1} - c\phi_{i+1}^{1+\epsilon} \right] g^{k+1+i}$$

$$+ \frac{\prod_{j=1}^{s-1} \phi_j}{\prod_{k=1}^{s-1} (r + \phi_k)} \left[ \frac{\pi_m - c\left(\phi^*\right)^{1+\epsilon}}{r - \phi^* (g-1)} \right] g^{k+s} \tag{A2}$$

Due to persistent leadership, $l > s$ and $\phi = \phi^*$ hold in all industries along a BGP. $\phi^*$ is determined by equation 8, with $V_1(k+1)$ taken from equation A2. If $\phi_l = \phi^*$ for all $l$, $V_1(k+1)$ increases in $\pi_l$, $\pi_m$ and $g$ and decreases in $r$, $c$ and $\phi^*$. These results are unchanged if $\phi_l > \phi^*$ holds for some $l$ (with $\phi_l$ determined by the condition $\frac{\partial V_l}{\partial \phi_l} = 0$ because $V_l$ is concave in $\phi_l$) as the indirect effects that a variable $Z$ has on $V_1(k+1)$ through affecting $\phi_l$ are of second order. The latter holds as $\frac{\partial V_l}{\partial \phi_l} = 0$ implies that $\frac{\partial V_1}{\partial \phi_l} = 0$ (see A1), so that $\frac{\partial V_1}{\partial \phi_l}\frac{\partial \phi_l}{\partial Z} = 0$. The result that $V_1(k+1)$ decreases in $\pi_l$ extends the case of a non-marginal change of $\pi_l$. As the right hand side of equation 8 increases in $c$ and in $\phi^*$, $\phi^*$ increases in $\pi_l$, $\pi_m$ and $g$ and decreases in $r$ and $c$. $\phi^* > 0$ always holds if $\epsilon > 0$ and if $r > \phi^* (g-1)$ (Condition B ("Co. B")) as the right hand side of equation 8 continuously increases from zero to infinity if $\phi^*$ increases from zero to infinity, while the left hand side (equation A2) is initially positive and then decreases continuously. As $\phi^*$ decreases in $r$, Co. B holds if $r$ is sufficiently large.

Along a BGP, $p = p_m = \frac{1}{\alpha}$ holds in all industries. Using equations 5 and 2 and $g = q^{\frac{\alpha}{1-\alpha}}$, we obtain

$$y(t) = \alpha^{\frac{2\alpha}{1-\alpha}} \int_{\omega=0}^{1} g^{k(\omega,t)} d\omega$$

Consequently,

$$G^* = \frac{\dot{y}(t)}{y(t)} = \frac{\int_{\omega=0}^{1} \left(g^{k(\omega,t)+1} - g^{k(\omega,t)}\right)\phi^* d\omega}{\int_{\omega=0}^{1} g^{k(\omega,t)} d\omega} = (g-1)\,\phi^*$$

As $\frac{\dot{c}(t)}{c(t)} = G^*$ along a BGP, $r > G^*$ (Co. B) needs to hold in order for $U(\tau)$ (equation 1) to be bounded.

For more general intertemporal preferences, $r$ is endogenous and depends positively on $G$ through the Euler equation. $G^*$ is then determined by equation 8 (a negative relation between $G$ and $r$) and the Euler equation and Proposition 1 still holds. □

## A3: Proof of Proposition 2

*Proof.* While it is not proven that incumbents strictly prefer to do all the R&D under forward protection, the case of persistent leadership is analyzed as, due to equation 8, the effects of forward protection on $G^*$ are the same in this case and in the (potentially possible, see online Appendix B) case where incumbents are indifferent about staying the industry leaders.

$R = 1$ and $s > 1$: Forward protection only affects $V_1(k+1)$ through the profits $\tilde{\pi}_1 g^{k+1}$ when $l = 1$. As the previous incumbent can earn $\pi_m g^k$ by blocking entry and as licensing allows to increase joint profits to $\pi_m g^{k+1}$ (collusion is allowed over the whole jointly owned product range), she is maximally willing to give $\tilde{\pi}_1^{max} g^{k+1} = \pi_m(g^{k+1} - g^k) < \pi_1 g^{k+1}$ to the entrant when $l = 1$. The inequality holds due to Lemma 1 c), implying that forward protection reduces $G^*$ by reducing $\pi_1$ (see Proposition 1).

$R > 1$, $q^s = \frac{1}{\alpha}$ and $s > 1$: Comparing profits under blocked entry and licensing, the previous incumbent is maximally willing to leave $\tilde{\pi}_l^{max} g^{k+1} = g^k\left(g^l - 1\right)\pi_m$ to the entrant when $l \leq R$. This is less than the profits $g^{k+l}\pi_l$ the entrant earns absent forward protection:

$$g^k\left(g^l - 1\right)\pi_m = g^k\left[(g-1)\,\pi_m + \left(g^2 - g\right)\pi_m + ... + \left(g^l - g^{l-1}\right)\pi_m\right]$$

$$< g^k\left[(\pi_1 g) + \left(\pi_2 g^2 - \pi_1 g\right) + ... + \left(\pi_l g^l - \pi_{l-1} g^{l-1}\right)\right] = g^{k+l}\pi_l$$

The inequality holds as Lemma 1 b) implies that $(g-1)\,\pi_m < \pi_1 g$ and that $\left(g^l - g^{l-1}\right)\pi_m \leq \pi_l g^l - \pi_{l-1} g^{l-1}$ (with strict inequality when $l \leq s$). Proposition 1 therefore implies that forward protection reduces $G^*$. If entrants are weak bargainers or if $R$ is sufficiently large, $V_1(k+1) < V_m(k+1) - V_m(k)$ might hold, in which case forward protection eliminates all entry pressure, leading to a maximal reduction in $G^*$. □

## A4: Proof of Lemma 3

*Proof.* The value $V_m(Q)$ of being an unconstrained monopolist supplying quality $Q$ can be derived from the arbitrage condition

$$rV_m(Q) = \pi_m Q^{\left(\frac{\alpha}{1-\alpha}\right)} - cQ^{\left(\frac{\alpha}{1-\alpha}\right)} \left(\phi_m \lambda(q_m)\right)^{1+\epsilon} - \phi_m V_m(Q) + \phi_m V_m(Q + q_m)$$

Inserting $(\phi_m \lambda(q_m))^{1+\epsilon} = \left(\frac{\tilde{V}_1}{c}\right)^{\frac{1+\epsilon}{\epsilon}}$ and $\phi_m = \frac{\left(\frac{\tilde{V}_1}{c}\right)^{\frac{1}{\epsilon}}}{\lambda(q_m)}$ from equation 11 ($\tilde{V}_1 \equiv \frac{V_1(Q,q_e)}{Q^{\frac{\alpha}{1-\alpha}}\lambda(q_e)}$ is constant along a BGP) and taking into account that $V_m(Q+q_m) = V_m(Q)\,(q_m)^{\frac{\alpha}{1-\alpha}}$ along a BGP, we obtain:

$$V_m(Q) = Q^{\frac{\alpha}{1-\alpha}} \left( \frac{\pi_m - c\left(\frac{\tilde{V}_1}{c}\right)^{\frac{1+\epsilon}{\epsilon}}}{r - \left(\frac{\tilde{V}_1}{c}\right)^{\frac{1}{\epsilon}} \left(\frac{(q_m)^{\frac{\alpha}{1-\alpha}}-1}{\lambda(q_m)}\right)} \right)$$

Given an interior solution $1 < q_m^* < \frac{1}{\alpha}$ (with $r - \left(\frac{\tilde{V}_1}{c}\right)^{\frac{1}{\epsilon}} \left(\frac{(q_m^*)^{\frac{\alpha}{1-\alpha}}-1}{\lambda(q_m)}\right) > 0$) exists, the step size $q_m^*$ that maximizes the term $\frac{(q_m)^{\frac{\alpha}{1-\alpha}}-1}{\lambda(q_m)}$ maximizes $V_m(Q)$. We obtain $\phi_m^* = \frac{\left(\frac{\tilde{V}_1}{c}\right)^{\frac{1}{\epsilon}}}{\lambda(q_m^*)}$ from equation 11 and

$$G^* = \left((q_m^*)^{\frac{\alpha}{1-\alpha}} - 1\right)\phi_m^* = \left(\frac{\tilde{V}_1}{c}\right)^{\frac{1}{\epsilon}} \left(\frac{(q_m^*)^{\frac{\alpha}{1-\alpha}} - 1}{\lambda\left(q_m^*\right)}\right)$$

As $q_m^*$ maximizes the term in the second bracket on the right hand side, it maximizes $G^*$, so that $q_m^* = q^*$ holds. □

## A5: Proof of Lemma 4

*Proof.* **Regime i)**: We obtain $\phi_1 = \bar{\phi}$ if $\frac{\partial V_1(k)}{\partial \phi_1} > 0$, which holds if (**Co. 4**):

$$\phi_2^* < \frac{(r+\gamma_1)\left(g\pi_m - c\left(r+\gamma_2\right)\right) - \pi_1(r+\gamma_2)}{(r+\gamma_1)\,c - (g-1)\,\pi_1}$$

Given that $\phi_1 = \bar{\phi}$, we can derive $\phi_2^*$ from the free entry condition ($V_1(k+1) = cg^k$) and obtain $\lim_{\bar{\phi}\to\infty} \phi_2^* = \frac{g^2\pi_m}{c} - (r+\gamma_2)\,(g+1)$. Plugging this into Co. 4 (note that $(r+\gamma_1)\,c - (g-1)\,\pi_1 > rc - (g-1)\,\pi_m > 0$ under Co. C), we get

$$\pi_m\,(g-1)\left(c\left(r+\gamma_1\right) - g\pi_1\right) < c(r+\gamma_2)(c\left(r+\gamma_1\right) - g\pi_1)$$

As $\pi_m (g-1) < cr < c(r+\gamma_2)$ due to Co. C, this condition holds if $c(r+\gamma_1) - g\pi_1 > 0$, i.e. if $\gamma_1 > \frac{g\pi_1}{c} - r$ (**Co. 5**). If $\bar{\phi}$ is sufficiently large, Co. 5 therefore implies that $\phi_1 = \bar{\phi}$ holds and $\phi_2^*$ is interior if $\bar{\phi} > \lim_{\bar{\phi} \to \infty} \phi_2^* > 0$, which holds if $\gamma_2 < \frac{g^2\pi_m}{(g+1)c} - r$ (**Co. 6**).

**Regimes ii)** and **iii)**: If $\gamma_1 \leq \frac{g\pi_1}{c} - r$ (**Co. 7**, i.e. Co. 5 does not hold), we either have $\phi_1 = \phi_2^* = \phi_0^* = \phi^*$ (Regime (ii)) or $\phi_1 = \phi_2 = 0$ and $\phi_0^* = \phi^*$ (Regime (iii); leapfrogging). Comparing $V_1(k+1)$ in both regimes ($V_1^{iii}(k+1) = g^{k+1}\frac{\pi_1}{r+\gamma_1+\phi^*}$; $V_1^{ii}(k+1)$ is given by equation 12 with $\phi_1 = \phi_2 = \phi^*$), Regime ii) arises if $V_1^{ii}(k+1) > V_1^{iii}(k+1)$, which holds if $\phi^* < \frac{g\pi_m}{c} - r - \gamma_2$ (**Co. 8**). At the regime boundary where $V_1^{ii}(k+1) = V_1^{iii}(k+1)$, $\phi^* = \frac{g\pi_1}{c} - r - \gamma_1$ (**Co. 9**) holds due to the free entry condition $V_1^{iii}(k+1) = cg^k$. Inserting Co. 9 into Co. 8, we find that Regime (ii) arises if $\gamma_2 \leq \gamma_1 + \frac{g(\pi_m - \pi_1)}{c}$ and Regime (iii) if $\gamma_2 > \gamma_1 + \frac{g(\pi_m-\pi_1)}{c}$. Under C 7, $\phi^*$ is positive in both regimes (due to Co. 9 and the fact that $V_1^{ii} > V_1^{iii}$ in Regime (ii)). □

## A6: Proof of Proposition 4

*Proof.* **Regime i):** The fraction $\sigma_l$ of industries in which the leading firm has an $l$-step lead (is in "state $l$") are constant along a BGP, implying that the following three equations that equate expected inflows into a state (left hand sides) to expected outflows (right hand sides) have to hold:

$l = 0$: $\gamma_1\sigma_1 + \gamma_2\sigma_2 = \sigma_o\phi_2^*$
$l = 1$: $\sigma_0\phi_2^* = \sigma_1(\bar{\phi} + \gamma_1)$
$l = 2$: $\sigma_1\bar{\phi} = \sigma_2\gamma_2$

These equations, with $\sigma_0 + \sigma_1 + \sigma_2 = 1$, can be solved for the BGP industry shares:

$$\sigma_0^* = \frac{\left(\bar{\phi} + \gamma_1\right)\gamma_2}{\left(\bar{\phi} + \gamma_1\right)\gamma_2 + \phi_2^*\left(\gamma_2 + \bar{\phi}\right)}$$

$$\sigma_1^* = \frac{\phi_2^*\gamma_2}{\left(\bar{\phi} + \gamma_1\right)\gamma_2 + \phi_2^*\left(\gamma_2 + \bar{\phi}\right)}$$

$$\sigma_2^* = \frac{\phi_2^*\bar{\phi}\left(\bar{\phi} + \gamma_1\right)\gamma_2}{\left(\bar{\phi} + \gamma_1\right)\gamma_2\left(\left(\bar{\phi} + \gamma_1\right)\gamma_2 + \phi_2^*\left(\gamma_2 + \bar{\phi}\right)\right)}$$

Equilibrium growth in Regime i) is given by

$$G_i^* = (g-1)\left(\bar{\phi}\sigma_1^* + \phi_2^*(1-\sigma_1^*)\right) = (g-1)\frac{\phi_2^*\left(\gamma_2\left(\bar{\phi} + \gamma_1\right) + \bar{\phi}(\phi_2^* + \gamma_2)\right)}{\left(\bar{\phi} + \gamma_1\right)\gamma_2 + \phi_2^*\left(\gamma_2 + \bar{\phi}\right)}$$

Solving the free entry condition $V_1(k+1) = cg^k$ for $\phi_2^*$, we can derive

$$sign\frac{\partial \phi_2^*}{\partial \gamma_1} = sign\left[\frac{\pi_m\,(g-1)}{r+\gamma_2} - c\right] < 0$$

(the inequality holds due to Condition C) and

$$\frac{\partial \phi_2^*}{\partial \gamma_2} = -\frac{\left(\bar{\phi}c\,(g+1) + c\,(r+\gamma_1) - g\pi_1\right)}{\bar{\phi}c - (g-1)\,(c\,(r+\gamma_1) - g\pi_1)} < 0$$

The last inequality holds as $c(r+\gamma_1) - g\pi_1 > 0$ (Co. 5 from *Appendix* A5) and as the denominator is positive given that $\bar{\phi}$ is sufficiently large. As

$$sign\frac{\partial \sigma_1^*}{\partial \gamma_1} = sign\left\{\frac{\partial \phi_2^*}{\partial \gamma_1}\left(\bar{\phi}+\gamma_1\right)\gamma_2 - \phi_2^*\gamma_2\right\} < 0,$$

$$\frac{\partial G_i^*}{\partial \gamma_1} = (g-1)\left(\frac{\partial \sigma_1^*}{\partial \gamma_1}(\bar{\phi} - \phi_2^*) + \frac{\partial \phi_2^*}{\partial \gamma_1}(1-\sigma_1^*)\right) < 0$$

holds. Moreover,

$$sign\frac{\partial G_i^*}{\partial \gamma_2} =$$

$$sign\left\{\frac{\partial \phi_2^*}{\partial \gamma_2}\left[\left(\gamma_2(\bar{\phi}+\gamma_1) + \bar{\phi}(\phi_2^*+\gamma_2)\right)\gamma_2(\bar{\phi}+\gamma_1) \;\; + \phi_2^*\bar{\phi}\left(\gamma_2(\bar{\phi}+\gamma_1) + \phi_2^*(\gamma_2+\bar{\phi})\right)\right]\right.$$

$$\left. + \bar{\phi}(\phi_2^*)^2(\bar{\phi} - \phi_2^*)\right\} < 0$$

This derivative is negative as $\frac{\partial \phi_2^*}{\partial \gamma_2} < -g-1$ under Co. 5 and for $\bar{\phi}$ sufficiently large, implying that the positive term $\bar{\phi}^2(\phi_2^*)^2$ in the lowest line is dominated by the negative term $\frac{\partial \phi_2^*}{\partial \gamma_2}\bar{\phi}^2(\phi_2^*)^2$ that is part of the middle line. Therefore, $G_i^*$ is maximal for $\gamma_2 = 0$. If $\gamma_2 = 0$, $G_i^* = \phi_2^*$ decreases in $\gamma_1$. Therefore, $G_i^*$ is maximal if $\gamma_1 = \gamma_2 = 0$.

**Regime ii):** $\phi_2^*$ and therefore $G_{ii}^* = (g-1)\,\phi_2^*$ decrease in $\gamma_i$. This can be inferred from the free entry condition $V_1^{ii}(k+1) = cg^k$ as $V_1^{ii}(k+1)$ (equation 12 with $\phi_1 = \phi_2$) decreases in $\gamma_i$ and also in $\phi_2$.

**Regime iii)**: Using Condition 9, we obtain:

$$G_{iii}^* = (g-1)\,\phi^* = (g-1)\left(\frac{g\pi_1}{c} - r - \gamma_1\right)$$

Consequently, $G_{iii}^*$ decreases in $\gamma_1$.

Across all Regimes, $G^*$ is therefore maximal if $\gamma_1 = \gamma_2 = 0$. At the switching point between Regimes ii) and iii), $V_1^{ii}(k+1) = V_1^{iii}(k+1)$ holds and there is no discontinuous jump in $\phi^*$ or $G^*$. At the switching point between Regimes i) and ii), $V_1^{i}(k+1) = V_1^{ii}(k+1)$ holds, so that there is no discontinuity in $\phi_0^*$ and $\phi_2^*$. However, $\phi_1$ and also $G^*$ increase in a discontinuous way when a slight shift in parameters leads so a switch from Regime ii) to Regime i). □

## A7: Proof of Proposition 5

*Proof.* With $\phi_2^* = 0$, we obtain $V_2(k+1) = \frac{\pi_m g^{k+1}}{r+\gamma}$ and

$$V_1(k) = \frac{\pi_1 g^k - \phi_1 c g^k + \phi_1 V_2(k+1)}{r + \phi_1 + \gamma}$$

We get $\phi_1^* = \bar{\phi}$ if $\frac{\partial V_1(k)}{\partial \phi_1} > 0$, which holds if $\gamma < \frac{\pi_m g - \pi_1}{c} - r$ (Regime A) and $\phi_1^* = 0$ if $\gamma > \frac{\pi_m g - \pi_1}{c} - r$ (Regime B). When IP protection has expired, $\phi_0^* = \bar{\phi}$ holds if for any $\phi < \bar{\phi}$ the marginal benefits of doing R&D exceed the average costs, i. e. if $V_1(k+1) > cg^k$. This condition is satisfied (even in Regime B) if $\frac{\pi_1 g^{k+1}}{r+\gamma} > cg^k$, i.e. if $\gamma < \frac{\pi_1 g}{c} - r$ (note that $\frac{\pi_1 g}{c} - r > \frac{\pi_m g - \pi_1}{c} - r$ due to Lemma 1). We therefore get $\phi_0^* = \phi_1^* = \bar{\phi}$ if $\gamma \leq \frac{\pi_m g - \pi_1}{c} - r$ (Regime A), $\phi_0^* = \bar{\phi}$ and $\phi_1^* = 0$ if $\frac{\pi_m g - \pi_1}{c} - r < \gamma \leq \frac{\pi_1 g}{c} - r$ (Regime B), and $\phi_0^* = \phi_1^* = 0$ if $\gamma > \frac{\pi_1 g}{c} - r$ (Regime C).

Denoting the fraction of industries in which the leading firm has an $l$-step lead (is in "state $l$") by $\sigma_l$ ($l = 0$ stands for expired IP protection), we obtain:

$$G = (g-1)\left(\sigma_0\phi_0 + \sigma_1\phi_1 + \sigma_2\phi_2\right)$$

Along a BGP, $\sigma_l$ is constant and the following equations that equate expected inflows into a state (left hand sides) to expected outflows (right hand sides) have to hold:

$l = 0$: $\gamma(\sigma_1 + \sigma_2) = \sigma_0\phi_0^*$
$l = 1$: $\sigma_0\phi_0^* = \sigma_1\phi_1^* + \sigma_1\gamma$
$l = 2$: $\sigma_1\phi_1^* = \sigma_2\gamma = (1 - \sigma_0 - \sigma_1)\gamma$

Solving these equations for the equilibrium values $\sigma_l^*$ we can derive $G_A^* = (g-1)\,\frac{\bar{\phi}\gamma\left(\gamma+2\bar{\phi}\right)}{(\bar{\phi}+\gamma)^2}$ in Regime A and $G_B^* = (g-1)\,\frac{\bar{\phi}\gamma}{\bar{\phi}+\gamma}$ in Regime B. In both Regimes, $G^*$ increases in $\gamma$, and it therefore maximal at the boundary values $\gamma_a = \frac{\pi_m g - \pi_1}{c} - r$ in Regime A and $\gamma_b = \frac{\pi_1 g}{c} - r > \gamma_a$ in Regime B. If $\frac{\pi_m g - \pi_1}{c} - r > 0$, $G_A^*(\gamma_a) > (<)\, G_B^*(\gamma_b)$ holds if $\phi_m(2\gamma_a - \gamma_b) + \gamma_a^2 > (<)\, 0$, that means if

$$\bar{\phi}\left(\frac{2g\pi_m - (2+g)\pi_1}{c} - r\right) + \left(\frac{\pi_m g - \pi_1}{c} - r\right)^2 > (<) 0$$

If $\frac{\pi_m g - \pi_1}{c} - r < 0$, $\phi_1^* = 0$, so that $G^*$ is maximal if $\gamma = \gamma_b = \frac{\pi_1 g}{c} - r > 0$ ($\gamma_B > 0$ holds due to Condition C). □


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