International Trade and Consumer Heterogeneity: The Role of Continuous Income Distributions

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Abstract

This thesis develops a static general equilibrium model that discusses the impact of within-country inequality on international trade. The endowments are distributed uniformly over a continuous range of households, which have non-homothetic preferences. In autarky, more inequality leads to a higher product variety, but the majority of households consumes less goods and the utilitarian welfare of the economy decreases. In the open economy, the more unequally distributed country hosts the firms with a rich critical client, whereas the other country hosts the firms selling to most households. If the countries are sufficiently similar in their within-country inequalities, both feature mass producers and more similarity leads to a higher volume of trade. Otherwise, the unequal country hosts all mass producers and even some firms that have a bigger home market in the equal country.

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1 Introduction

There is a wide range of literature on the impact of trade on income inequality.[1] This work instead goes the other way around, trying to explain the impact of consumer heterogeneity on international trade, an idea that has been widely neglected in theory. On one hand, most models use homothetic preferences, which, because of their mathematical properties, do not allow for income inequality to have an effect on the outcomes, as the only thing that matters in such models are aggregated quantities. On the other hand, the few papers that use non-homothetic preferences often examine the impact of across-country inequality on trade (e.g. Foellmi et al., 2010). This work in contrast focuses on within-country inequality.

To emphasize the difference between those two ideas, an illustrative example, such as the one stated by Foellmi et al., should be given. In their paper, they compare two countries, Austria and Nigeria, with the same PPP-adjusted national income, but Nigeria being large and poor and Austria being small and rich, and ask the question about the effect on trade patterns. In contrast to this, I compare two countries that have the same national income and population (and therefore also per capita national income), but one country has more unequally distributed labor endowments than the other. Consider the case of Sweden and Switzerland, which had in the year 2000 the same PPP-adjusted national income of 246 bill I\$[2] and 245 bill I\$ respectively and similar populations of 8.9 mill and 7.2 mill. The dimension in which these countries differed is their income inequality, with Sweden having a Gini coefficient of 0.25, whereas Switzerland's was 0.34. This indicates that Switzerland had a higher share of very rich and probably also very poor[3] households compared to Sweden. The aim of this thesis is to show in what ways such differences affect the market structure and trade patterns.

Due to the production technology, which features fixed and variable costs, a Chamberlinian monopolistic competition market structure, such as the one derived in Dixit and Stiglitz (1977), emerges in equilibrium. The model built here contributes to the New Trade Theory going back to the elementary work of Krugman (1979, 1980), where the gains from trade come from increasing returns to scale. But instead of CES-preferences, non-homothetic 0/1-preferences are used to allow for income inequality to have an impact on the outcomes. Such preferences are of the kind that only the first consumed unit of a good increases the utility of a household and to my knowledge, were first used in Murphy et al. (1989). This assumption moves the focus away from the intensive to the extensive margin of trade, as a change in

[1] A good review of this literature is given by Harrison et al. (2011).
[2] I\$ stands for International Dollar. The data used in this example is from data.worldbank.org.
[3] The term "very poor" has to be understood relative rather than absolute.

the endowment leads to a different number of goods rather than quantities consumed by a household. The distribution of endowments, i.e. the within-country inequality, is not modeled as a discrete two groups (one rich group, one poor group) distribution, but instead a continuous uniform endowment distribution is assumed. Continuous endowment distributions are rarely used,[4] but can lead to interesting additional information, because they allow to show what happens to the households that lie between the richest and the poorest, something that is not possible with a two groups distribution.

Summarized, this work sheds light on two barely explored aspects, in modeling the impact of within-country inequality on trade by using a continuous endowment distribution. The goal of this thesis is to give useful insights on the mechanisms and channels through which within-country income inequality affects outcomes in trade. This may hopefully be the fundament that later works can build on. The remainder is structured in the following manner: Section 2 states the environment of the model. Section 3 solves the autarky equilibrium, which is then opened up for trade and discussed in Section 4. The last Section 5 concludes the results.

[4] One example is e.g. Matsuyama (2000), who uses a continuous endowment distribution in a Ricardian trade model.

2 Environment

This section states the environment of the model, which is later derived in Section 3 for an autarky and extended in Section 4 to an open economy with two countries. The assumptions described in this section are fundamental and valid throughout this whole work, unless otherwise noted. I first introduce the supply side, i.e. the production technology of the firms and then go over to the demand side of the model, showing the households' endowments and utility function.

2.1 Firms and the Supply Side

As the main point of this work is to stress the impact of consumer heterogeneity on international trade, the supply side of this model is the same as in the fundamental New Trade Theory models of Krugman (1979, 1980).

A continuum of firms produce each a differentiated good indexed by j, where labor is the only input in the production of those consumer goods. A fixed amount of labor F is needed to set up a firm. Thereafter, every produced good needs an additional labor input of $1/a$.[5] This economies of scale production technology is the same for all firms and induces the familiar monopolistic competition market structure in the equilibrium.

2.2 Households and the Demand Side

In contrast to the supply side, the demand side crucially differs from the standard model in two points. First, instead of just having one representative agent, the endowments are distributed over a continuous range, and second, a non-homothetic utility function is introduced.

The economy is populated by a continuum of L households. The labor endowments θ are uniformly[6] distributed over the range $[\mu - r, \mu + r]$, where μ is the mean labor endowment and r is the distance of the lowest as well as the highest labor endowment to the mean. Accordingly, it follows that $r \leq \mu$ has to hold, as there would be negative labor endowments otherwise. The total labor endowment in this economy is therefore μL. As it is assumed that labor is perfectly mobile within a country and that the labor market is competitive, the wage rate W is the same for all households in an economy.

[5] Notice that an additional labor input of $1/a$ for every produced good implies that one unit of labor input produces a goods, i.e. a is the productivity of labor.

[6] Working with this specific function allows for closed-form expressions and simplifies the understanding of the mechanisms in a model with a more general continuous endowment distribution.

The span width of the labor endowments is $2r$. Therefore, the probability density function (PDF) is given by

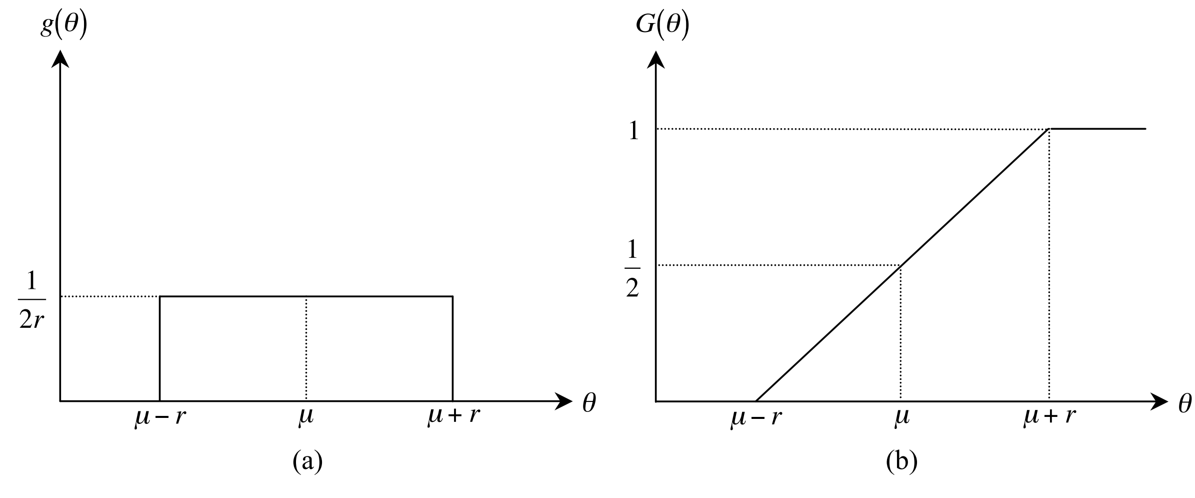
$$g(\theta) = \begin{cases} \frac{1}{2r} & \text{for} \quad \theta \in [\mu - r, \mu + r] \\ 0 & \text{otherwise} \end{cases}$$

and the cumulative density function (CDF) is given by

$$G(\theta) = \begin{cases} 1 & \text{for} \quad \theta \in [\mu + r, \infty) \\ \frac{\theta - (\mu - r)}{2r} & \text{for} \quad \theta \in [\mu - r, \mu + r) \\ 0 & \text{for} \quad \theta \in [0, \mu - r). \end{cases}$$

Figure 1 shows the PDF (a) and CDF (b) of this economy. The uniform endowment distribution has the nice attribute that, given the mean labor endowment μ, there is just one parameter, r, which determines the inequality in this economy.[7] To follow the comparative statics performed in Section 3 and 4, it is important to understand the impact of a change in r on the PDF and especially the CDF. More inequality, i.e. a higher r, means a wider endowment distribution and therefore a lower density of households over this range, which is equivalent to a flatter CDF. This is an important mechanism in the following sections and should always be kept in mind.

Figure 1: PDF and CDF of the uniform endowment distribution respectively



Having described the distribution of the endowments, the utility function, which is the same for all households, is given by

$$U_i = \int_0^\infty c_i(j) dj,$$

[7]This also holds true by looking at the Gini coefficient of the uniform endowment distribution, which is $r/(3\mu)$. Appendix A shows the derivation of the Lorenz curve and the Gini coefficient.

where $c_i(j)$ is an indicator function with $c_i(j) = 1$ if good j is consumed by household i and $c_i(j) = 0$ if not.[8] Therefore, the number of goods consumed determines the utility of a household and it is not possible to generate a higher utility by increasing the consumption of a specific good that is already consumed. It follows that every household wants to consume as many goods as possible, i.e. increase the variety consumed, but is restricted in doing so by the labor income. The budget restriction of a household i with income θ_i is given by

$$\theta_i W = \int_0^\infty p(j) c_i(j) dj,$$

where $p(j)$ is the price for good j.

Notice the difference between a non-homothetic utility function, as the one used in this model, and a homothetic one, such as CES-preferences used in the standard Krugman model. Here, as described above, a household wants to consume as many goods as possible, taking into account his budget restriction. Therefore, poorer households are not able to consume the same number of goods as richer households. As shown in Section 3, this will lead in the equilibrium to some firms excluding poorer households by setting a higher price in order to take advantage of the higher willingness to pay of richer households. The intensive margin of consumption does per definition not play any role, as a household only gains utility from the first unit of consumption. This is a contrast to CES-preferences, where the intensive margin plays a role as a household, taking into account his budget restriction, wants to consume as much of a good as possible. The relative consumption shares of the different goods are then pinned down by the elasticity of substitution, which is constant by assumption and therefore does not change with a variation in the households' income. The extensive margin of consumption does not play a role, due to the fact that a household's willingness to pay goes to infinity as the consumption approaches zero and therefore every household always consumes all available goods. Hence, the non-homothetic 0/1-preferences used in this model are crucial in letting consumer heterogeneity have an impact on aggregate demand, which would not be possible by using homothetic CES-preferences.

[8] This is the reason, why this utility function is also sometimes called 0/1-preferences.

3 Closed Economy Equilibrium

I start by looking at a closed economy, facing the environment stated in Section 2. The wage rate is set as numéraire, so $W = 1$. Therefore, "endowment" and "income" are used as synonyms. First, the maximization problems of the different individuals, i.e. the households and the firms, is solved. In a second step, two constraints that need to hold in equilibrium are introduced. The free entry condition, which leads to zero profits, and the fact that all households exhaust their budgets. Those optimality conditions allow to solve for the equilibrium and to discuss the impact of inequality on the outcomes in a last section. The derivation of the autarky equilibrium is closely based on Bertola et al. (2006, Exercise 43), who shortly discuss this case for a general continuous endowment distribution.

3.1 Maximization Problems

Imagine a household facing the decision whether to consume a good j or not. Although the utility function is not differentiable, the standard mechanisms can still be used to solve this problem. A household compares his marginal utility of good j, which is 1, with the marginal cost of the consumption, which can be expressed as the price of the good $p(j)$ multiplied with the shadow price of income, i.e. the marginal cost of strengthening the budget constraint by one unit. Therefore, the first order condition of a household i looks as follows

$$c_i(\theta_i, j) = \begin{cases} 1, & \text{if} \quad 1 - \lambda(\theta_i)p(j) \geq 0 \Leftrightarrow 1/\lambda(\theta_i) \geq p(j) \\ 0, & \text{otherwise} \end{cases}$$

where $\lambda(\theta_i)$ is the shadow price of household i with endowment θ_i or the inverse of his willingness to pay. This maximization problem therefore adds up to the intuitive decision rule that a household i with income θ_i wants to consume good j, if its willingness to pay exceeds the price. As a richer household is willing to pay a higher price than a poorer household, this will be interesting to explore for some firms in equilibrium.

Given this simple consumption rule of the households, it is now possible to describe the firms profit maximization. Each firm faces a trade-off between the price and the quantity. The higher the price of the good is set, the less households are willing to buy this goods, i.e. the smaller is the market. As it is not possible to use price discrimination within a country, a firm obviously sets the price equal the willingness to pay of its poorest client $\tilde{\theta}$[9]

$$p(\tilde{\theta}, j) = p(\tilde{\theta}) = 1/\lambda(\tilde{\theta}),$$

which is identical for all firms with the same critical client,[10] and sell a total quantity of

$$x(\tilde{\theta}, j) = x(\tilde{\theta}) = (1 - G(\tilde{\theta}))L = \frac{\mu + r - \tilde{\theta}}{2r} L$$

to all individuals with an equal or higher reservation price than the critical client $\tilde{\theta}$. Notice that graphically, the market of a firm corresponds to the vertical distance above the CDF, multiplied by the labor force. In equilibrium, every household must be the critical client for at least one firm, as otherwise its willingness to pay would be infinity, which would induce some firms to explore this opportunity.

3.2 Additional Equilibrium Constraints

After showing how the households make their consumption decisions and the firms set their prices, I now impose two equilibrium constraints in order to solve the model. The first is the free entry condition, which assures that all firms make zero profits in equilibrium. The intuition is that new firms enter the market for goods with a critical client $\tilde{\theta}$, as long as there is a positive profit. This leads to more goods consumed by households $\tilde{\theta}$ and therefore a lower willingness to pay. As the firms set the price equal the willingness to pay of their poorest client, this goes on until the price is such that the firms make zero profits. Hence, the price for a good with the poorest client $\tilde{\theta}$ derived from the zero profit condition is

$$\begin{aligned} F &= \left(p(\tilde{\theta}) - \frac{1}{a} \right) x(\tilde{\theta}) \\ \Leftrightarrow \quad p(\tilde{\theta}) &= \frac{aF + \left[(\mu + r - \tilde{\theta})/(2r) \right] L}{\left[(\mu + r - \tilde{\theta})/(2r) \right] L} \frac{1}{a}. \end{aligned} \tag{1}$$

Notice that in equilibrium the firms are not willing to set a price lower than their average costs, as otherwise they would make losses. It follows that $p(\tilde{\theta})$ increases in $\tilde{\theta}$ as the market a firm caters to gets smaller the richer its critical client is and the consuming households then have to cover a bigger share of the fix costs.

[9]To simplify the reading, I will talk throughout this work of "the poorest client $\tilde{\theta}$" instead of "the poorest client with a labor endowment $\tilde{\theta}$". Additionally will I use "critical client" as a synonym for "poorest client".

[10]Due to symmetry, all variables concerning the same critical client $\tilde{\theta}$ are equal. That is why the goods' index j is dropped to ease the notation.

The second equilibrium constraint is that all households have to exhaust their budget. Together with the derived prices of Equation (1), this determines the number of firms for each critical client. Starting with the budget constraint of the poorest client with an income $\mu - r$, the number of mass producers $N(\mu - r)$ can be calculated as

$$\begin{aligned} \mu - r &= p(\mu - r)N(\mu - r) \\ \Leftrightarrow \quad N(\mu - r) &= \frac{aL}{aF + L}(\mu - r). \end{aligned} \tag{2}$$

The number of mass producers is pinned down by the income of the poorest households divided by the price for those goods. This relation makes sure that the poorest households exhaust their budgets.

Now look at the budget constraint of a household with an income $\in (\mu - r, \mu + r]$. Such a household consumes all goods with a poorer critical client plus those goods, he is the poorest client for. The budget constraint can therefore be written as

$$\tilde{\theta} = \int_{\mu - r}^{\tilde{\theta}} p(\theta)N'(\theta)d\theta + p(\mu - r)N(\mu - r).$$

As $N(\tilde{\theta})$ denotes the number of goods consumed by the households $\tilde{\theta}$, the derivative $N'(\tilde{\theta})$ stands for the number of firms producing with a critical client $\tilde{\theta}$. Hence, this equation states that the household's income $\tilde{\theta}$ needs to be equal to all goods it consumes multiplied with the respective prices. The number of firms with the critical client $\tilde{\theta}$ can be calculated by taking the derivative with respect to $\tilde{\theta}$ on both sides of the budget constraint and rearranging to

$$N'(\tilde{\theta}) = \frac{a\left[(\mu + r - \tilde{\theta})/(2r)\right]L}{aF + \left[(\mu + r - \tilde{\theta})/(2r)\right]L}, \tag{3}$$

which is the inverse of the according price. This makes intuitively sense, as the budget constraint assures that the additional number of goods consumed by any household (compared to the next poorer household) has to equal the additional infinitesimal unit of income divided by the price for those goods, which is equal to the inverse of the price, as the wage rate is set as numéraire.

From this, it is now possible to calculate the market structure

$$N(\tilde{\theta}) = \int_{\mu-r}^{\tilde{\theta}} N'(\theta)d\theta + N(\mu - r)$$

$$= 2ar\left[1 - \frac{\mu + r - \tilde{\theta}}{2r} - \frac{aF}{L}\log\left(\frac{aF + L}{aF + \left[(\mu + r - \tilde{\theta})/(2r)\right] L}\right)\right] + \frac{aL}{aF + L}(\mu - r) \quad (4)$$

as well as the overall product variety of this economy

$$N(\mu + r) = \int_{\mu-r}^{\mu+r} N'(\theta)d\theta + N(\mu - r)$$

$$= 2ar\left[1 - \frac{aF}{L}\log\left(\frac{aF + L}{aF}\right)\right] + \frac{aL}{aF + L}(\mu - r). \quad (5)$$

The closed-form expressions for the market structure and the product variety can be derived with a little bit of algebra, as can be seen in Appendix B.

The number of goods consumed by a household with income $\tilde{\theta}$, i.e. $N(\tilde{\theta})$, is also equal to its utility. Therefore, the overall product variety $N(\mu+r)$ is the utility of the richest households. It follows that the utilitarian or Benthamite welfare of this economy can be stated as

$$B = \int_{\mu-r}^{\mu+r} g(\theta)LN(\theta)d\theta$$

$$= a\left[L\mu - FN(\mu + r)\right]. \quad (6)$$

Equation (6) has a very nice intuition. As the number of goods consumed determines the utility of a household, the utilitarian welfare is equal to the total number of all goods produced in this economy. Look at the term in brackets on the RHS of the equation. $L\mu$ stands for the whole labor supply, whereas $FN(\mu + r)$ states the labor demand which is used to set up firms, as $N(\mu + r)$ is the overall number of firms and all need to bring up the fixed costs F. It follows that the expression in the brackets is equal to the labor allocated in the production sector. This needs to be multiplied by a to determine the number of goods produced in this economy, as a stands for, as mentioned earlier, the labor productivity. The detailed derivation of Equation (6) can be found in Appendix C.

Notice that given this market structure in the general equilibrium, the labor market clearing condition also holds due to Walras' Law

$$\mu L = \int_{\mu-r}^{\mu+r} \left(F + \frac{x(\theta)}{a}\right) N'(\theta) d\theta + \left(F + \frac{L}{a}\right) N(\mu - r).$$

Geometrically, the labor market demand can be interpreted as the area above the CDF, multiplied by the labor force. This is something that will be of more interest in the open economy equilibrium. As, given the mean labor endowment, the size of this area is not affected by a change in income inequality, the labor demand is always equal to the labor supply.

3.3 Impact of Consumer Heterogeneity in Autarky

This section shows the impact of consumer heterogeneity on the market structure in autarky. By referring to an increase or decrease in inequality, it is always assumed that this happens for a given mean labor endowment μ. Therefore, a rise in inequality can also be described as a mean preserving spread or simply as an increase in r. It is very important to understand the inequality's effect on the market sizes. The mechanism always works through this channel and the quantity a firm can sell is a driving force in this model. Figure 2 (a) shows that the CDF "turns clockwise", when inequality increases. Controlling for the mean income, more inequality, i.e. a higher r, therefore leads to shrinking and growing market sizes for firms with the poorest client in the ranges $\tilde{\theta} \in (\mu - r, \mu)$ and $\tilde{\theta} \in (\mu, \mu + r)$ respectively.[11]

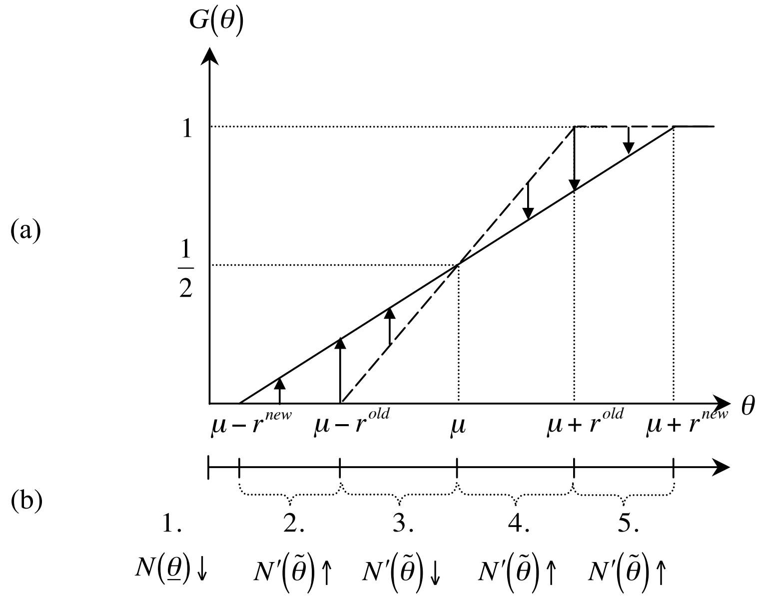
The changes in the quantities that these firms can sell, then have an effect on the prices set by them. The shrinking (growing) market sizes for firms with the critical client below (above) the mean, lead to higher (lower) prices for those goods. The reason for this is that firms need to set the price equal their average costs and a smaller (bigger) customer base means that each household has to bear a higher (lower) share of the fix costs.

Finally, the adjustment in the prices has an influence on the number of firms selling to different critical clients, which is the inverse of the respective price, as mentioned above. More inequality therefore leads to five effects, shown in Figure 2 (b), affecting the overall product variety. First, the higher income dispersion means a lower income for the poorest households. As the price for mass consumption goods does not depend on the inequality,[12] this implies that there are less mass producers in equilibrium (negative effect on the product variety).

[11] Remember that the market of a firm corresponds to the vertical distance above the CDF, multiplied by the labor force.

[12] Mass consumption goods are per definition consumed by all households. Its price is therefore equal to the variable cost of one unit plus the associated share of the fix costs, which is equal to the set-up costs divided by the whole population. None of these factors depend on the inequality.

Figure 2: Impact of an increase in inequality on the CDF and the number of goods consumed



Annotation. $N(\underline{\theta})$ stands for the number of mass producers.

Second, there is now a positive number of firms that have a critical client in the income range $\tilde{\theta} \in (\mu - r^{new}, \mu - r^{old})$, which did not exist before the rise in inequality (positive effect on product variety). Third, the shrinking market sizes in the income range $\tilde{\theta} \in (\mu - r^{old}, \mu)$ lead to higher prices for those goods (negative effect on the product variety). The fourth effect is the opposite of the third, i.e. the growing market sizes in the income range $\tilde{\theta} \in (\mu, \mu + r^{old})$ lead to lower prices for those goods (positive effect on product variety). And fifth, there are now a positive amount of firms that have a critical client in the income range $\tilde{\theta} \in (\mu + r^{old}, \mu + r^{new}]$, which did not exist before the rise in inequality (positive effect on product variety). Before some interesting propositions are stated, I first proof Lemma 1.

Lemma 1. *Effect 1 in Figure 2 (b) dominates effect 2, i.e. a household with income $\mu - r^{old}$ consumes less goods, if inequality increases.*

Proof. This perhaps seems a little bit counterintuitive, as this household belongs to the group of the poorest households before the increase in inequality, whereas afterwards it does not anymore. The reason for this being true lies in the fact that the average price of its consumed goods rises. Consider the consumption basket of a household with income $\mu - r^{old}$ before and after the increase in inequality. Before the increase in inequality, such a household consumes only mass products that cost the cheap mass product price, whereas after the increase it consumes also some exclusive goods that are more expensive. As the household's labor income stays the same, it is obvious that the number of consumed goods has to decline. See Appendix D.1 for a mathematical proof. □

Proposition 1. *The majority of households loses utility from a mean preserving spread.*

Proof. As the firms with a critical client above the mean are able to set lower prices, it would be intuitive to think that all households with an income in this range benefit from higher inequality. Nevertheless, this neglects the fact that households with an income above the mean also consume all the goods with a poorer critical client. Consider the households with income μ in Figure 2. As Lemma 1 shows that effect 1 dominates effect 2 and because effect 3 is negative, the mean households clearly consume less goods if inequality increases. The households infinitesimal richer than the mean households additionally consume the increased number of goods they are the critical client for, which clearly can't offset the loss of utility coming from the other goods. This is the reason, why the majority of households loses utility from a mean preserving spread. See Appendix D.2 for a mathematical proof. □

Proposition 2. *A rise in inequality, i.e. a higher r, implies more overall variety.*

Proof. Considering the labor market, the intuition for this is straight forward. The labor supply is unaffected by a mean preserving spread in the endowment distribution, whereas the labor demand depends on the market structure, which is affected by a rise in inequality. As shown in Figure 2, the "clockwise turn" of the CDF leads to shrinking (growing) market sizes for firms with the poorest client below (above) the mean. This leads to less (more) firms with the critical client in this range, because the households can afford less (more) additional goods, due to the higher (lower) prices. Therefore, there exist less firms selling to a broad base of households and more of them selling exclusively to richer ones. As the labor demand of a firm is $F + x(\tilde{\theta})/a$, this means that one firm, which has a big market is replaced by more than one firm with a smaller market. That's why the overall number of firms and income inequality are positively related to each other. It is important to notice that a higher product variety directly benefits uniquely the richest households as they are the only households who consume all goods. See Appendix D.3 for a mathematical proof. □

Propositions 1 and 2 show that the richest households gain utility, whereas the majority of households loses. This raises the question, whether the utility gains of the minority of favored households are big enough to compensate for the losses of the other households.

Proposition 3. *The utilitarian welfare in a closed economy falls, if inequality increases.*

Proof. Having shown that the Proposition 2 holds true, this proof is straight forward. Remember from Section 3.2 that the utilitarian welfare is equal to the labor, allocated in the production sector, multiplied by the productivity. The increase in the number of firms (product variety), due to a mean preserving spread, is therefore the only impact on the welfare, as neither the productivity, nor the labor supply changes. More labor is needed to set up the firms and less is available for the production of goods, which clearly reduces the utilitarian welfare. Utilitarian welfare is maximized in an economy without inequality, where only mass producers exist. Those firms have the highest labor demand, which allows only for a few firms to emerge and therefore only a small share of labor is bound to the set-up sector. Notice that this is simply due to the existence of fixed costs. In a world without such costs, the whole labor supply is allocated in the production sector. This would imply that income inequality has no impact on the utilitarian welfare at all. See Appendix D.4 for a mathematical proof. □

Those three proofs lead to the conclusion that although the overall product variety increases due to a rise in inequality, the majority of households consumes less goods, which induces a lower overall welfare in the closed economy.

4 Open Economy Equilibrium

After deriving the closed economy equilibrium, I now assume that there are two countries of the kind described in Section 2 with equal labor force L and mean labor endowment μ, but with one country, called *Unequal*, having a more unequal endowment distribution than the other, called *Equal*[13]. Therefore, $r_U > r_E$ must be the case. In contrast to the standard Krugman (1979, 1980) trade framework, income inequality does have a very important impact on the market and trade structure in this model.[14] To illustrate this, I first treat the two economies just like one big economy, to derive the overall market structure and the prices. In a second step, I describe the location of those firms and the trade structure. As discussed later, this is indeterminate without trade costs and therefore based on crucial assumptions. Lastly, the impact of consumer heterogeneity on the market structure and trade patterns is described.

4.1 Overall Market Structure

Notice first that due to the continuous endowment distribution, there is always a marginal critical client that can be served by firms located in both countries. As those firms face the same revenues as well as production technology and because there are no trade costs, wages have to equalize for the zero profit condition to hold. The wage rate is therefore again set as numéraire, $W^E = W^U = 1$ and "endowment" as well as "income" are used as synonyms.

The decision rule of a household i, whether to consume a good or not, does not change qualitatively in an open economy.

$$c_i(\theta_i, j) = \begin{cases} 1, & \text{if} \quad 1 - \lambda(\theta_i)p(j) \geq 0 \Leftrightarrow 1/\lambda(\theta_i) \geq p(j) \\ 0, & \text{otherwise} \end{cases}$$

A household i with income θ_i still wants to consume good j, if its willingness to pay exceeds the price. As the price is the only thing that matters, the origin of the good does not directly influence the decision.

The firms profit maximization problem is not directly affected either. The firm still sets the price equal the willingness to pay of its poorest client.

$$p(\tilde{\theta}, j) = p(\tilde{\theta}) = 1/\lambda(\tilde{\theta})$$

[13] The description *Equal* does not imply that there is full equality in this country. This just states the fact that it has a more equally distributed endowment distribution than the other country.

[14] Appendix E shows that a continuous endowment distribution, i.e. within-country inequality, does not change the outcome in a Krugman trade model with CES-preferences.

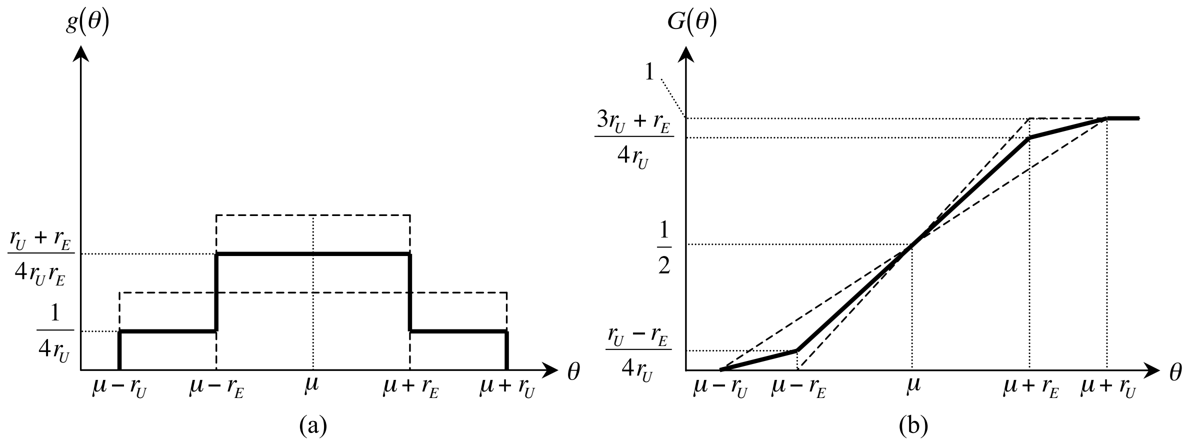
Because there are no transportation costs, it is not possible to use price discrimination across countries, as this would trigger off parallel trades. This implies that the price a firm can set for a good with the poorest client $\tilde{\theta}_E$ in *Equal* is the same it has to set in *Unequal* with a critical client $\tilde{\theta}_U$. Due to the fact that the price equals the willingness to pay of a household, it follows that the firm has the same critical client in both countries $\tilde{\theta}_E = \tilde{\theta}_U = \tilde{\theta}$.

The quantity a firm can sell is therefore given by

$$x(\tilde{\theta}, j) = x(\tilde{\theta}) = (1 - G_U(\tilde{\theta}))L + (1 - G_E(\tilde{\theta}))L.$$

I.e. a firm sells to all individuals with higher or equal reservation price in both countries. Obviously, the market sizes are bigger than in autarky for firms located in either country with the poorest client in the range $[\mu - r_U, \mu + r_E)$ and bigger or the same for firms with the critical client in the range $(\mu + r_E, \mu + r_U]$ located in *Equal* or *Unequal* respectively. Figure 3 shows the PDF (a) and CDF (b) for the overall world economy. Notice that the CDF for the overall world economy is not differentiable at $\mu - r_E$ and $\mu + r_E$. The kinks at those incomes are the reason, why the explicit formula for the quantity is different, dependent on the income of the poorest client of a firm. Table 1 gives an overview.[15]

Figure 3: PDF and CDF of the overall endowment distribution respectively



Annotation. The bold line depicts the PDF and CDF for the overall economy, whereas the dashed lines are for the separate countries.

[15] As there is not just one single quantity formula over the whole endowment range, solving for the closed-form expressions in the open economy case is especially complicated. Nevertheless, the valuable understanding of the mechanisms in autarky will turn out to be sufficient to discuss the impact of within-country inequality in an intuitive way.

Table 1: Quantity a firm can sell depending on its critical client

$\tilde{\theta}$	$= \mu - r_U$	$\in (\mu - r_U, \mu - r_E]$	$\in (\mu - r_E, \mu + r_E]$	$\in (\mu + r_E, \mu + r_U]$
$x(\tilde{\theta})$	$2L$	$\frac{\mu + 3r_U - \tilde{\theta}}{2r_U} L$	$\frac{(\mu - \tilde{\theta})(r_U + r_E) + 2r_U r_E}{2r_U r_E} L$	$\frac{\mu + r_U - \tilde{\theta}}{2r_U} L$

Using the zero profit condition, the price can be determined

$$F = \left(p(\tilde{\theta}) - \frac{1}{a} \right) x(\tilde{\theta})$$
$$\Leftrightarrow p(\tilde{\theta}) = \frac{aF + x(\tilde{\theta})}{x(\tilde{\theta})} \frac{1}{a}. \tag{7}$$

Notice that Equation (7) again states as Equation (1) did that the price equals the average costs, i.e. the variable costs for one unit plus the corresponding share of the set-up costs. In other words, the price is equal to the whole labor requirement for the production of the good, $F + x(\tilde{\theta})/a$, divided by the corresponding market, $x(\tilde{\theta})$. It follows that most prices for a given critical client are lower than in autarky. More precisely, all firms that can export some goods, set a lower price in the open economy, as the foreign customers also pay a share of the set-up costs.

Given the prices, the mechanism to derive the market structure is exactly the same as in the closed economy. From the budget constraint of the poorest client overall, i.e. the households in *Unequal* with an income of $\mu - r_U$, the number of mass producers can be calculated

$$\mu - r_U = p(\mu - r_U) N(\mu - r_U)$$
$$\Leftrightarrow N(\mu - r_U) = \frac{a2L}{aF + 2L} (\mu - r_U). \tag{8}$$

In equilibrium there are more mass producers in the open economy, as the market for mass consumption goods is bigger and therefore the mass consumption good price is lower.

The budget constraints of all households with an income $\tilde{\theta} \in (\mu - r_U, \mu + r_U]$

$$\tilde{\theta} = \int_{\mu - r_U}^{\tilde{\theta}} p(\theta) N'(\theta) d\theta + p(\mu - r_U) N(\mu - r_U)$$

determine the amount of firms for each critical client

$$N'(\tilde{\theta}) = \frac{ax(\tilde{\theta})}{aF + x(\tilde{\theta})}. \tag{9}$$

Qualitatively nothing changes compared to autarky, i.e. the number of firms for which the poorest client has an income of $\tilde{\theta}$ is still the inverse of the price, which is equal to the corresponding market divided by whole labor requirement for the production of the good. Therefore, quantitatively $N'(\tilde{\theta})$ is higher than in autarky for all goods with a lower price.

From this, the market structure

$$N(\tilde{\theta}) = \int_{\mu - r_U}^{\tilde{\theta}} \frac{ax(\tilde{\theta})}{aF + x(\tilde{\theta})} d\theta + \frac{a2L}{aF + 2L}(\mu - r_U) \tag{10}$$

and the overall product variety

$$N(\mu + r_U) = \int_{\mu - r_U}^{\mu + r_U} \frac{ax(\tilde{\theta})}{aF + x(\tilde{\theta})} d\theta + \frac{a2L}{aF + 2L}(\mu - r_U) \tag{11}$$

can be stated.[16]

Proposition 4. *Every household consumes more goods in an open economy compared to autarky, i.e. has a higher utility. It follows that the overall product variety rises.*

Proof. The reason for this is straight forward as all firms with a critical client $\in [\mu - r_U, \mu + r_E)$ are able to set a lower price in an open economy, due to the fact that they can pass on the fixed costs to a bigger market, whereas the remaining firms with a critical client $\in [\mu + r_E, \mu + r_U]$ don't change their prices. Therefore, at least a part of the goods consumed by a household are cheaper, allowing it to buy more goods. This is simply the familiar gains from trade result, as a consequence of the economies of scale production function. It follows that the overall product variety rises, because the richest household also consumes more goods in an open economy. □

[16]For the sake of completeness, the closed-form expression of the product variety is stated in Appendix F. As mentioned earlier, the calculations are especially cumbersome, due to the quantity formula $x(\tilde{\theta})$, which depends on the critical client and therefore has two kinks.

The utilitarian or Benthamite welfare in *Equal* and *Unequal* can be stated as

$$B_E = \int_{\mu - r_E}^{\mu + r_E} g_E(\theta) L N(\theta) d\theta \tag{12}$$

$$B_U = \int_{\mu - r_U}^{\mu + r_U} g_U(\theta) L N(\theta) d\theta. \tag{13}$$

Compared to the autarky, both countries gain, due to the fact that all households consume more goods, as shown in Proposition 4. Appendix F shows the closed-form expressions for Equations (12) and (13) as well as the fact that the welfare in both countries together are, as in autarky, equal to total production, i.e. the labor allocated in the production sector multiplied by the productivity.

4.2 Location Decision and Trade Structure

After the derivation of the overall market structure, I look at the location decision of those firms. As already mentioned, this is actually indeterminate in a setup without transportation costs. The introduction of such costs is not as straight forward though, because of their effects on the price setting behavior of the firms and the wage rates, in addition to the location decision. To simplify the analysis, I just introduce a home market bias assumption, to determine the location of the firms, without explicitly introducing trade costs. With this assumption the location decision is determinate without any of the additional effects arising from such an introduction.

Assumption 1. *There exists a sort of home market bias, which makes sure that a firm preferably locates in the country where it has the bigger home market.*

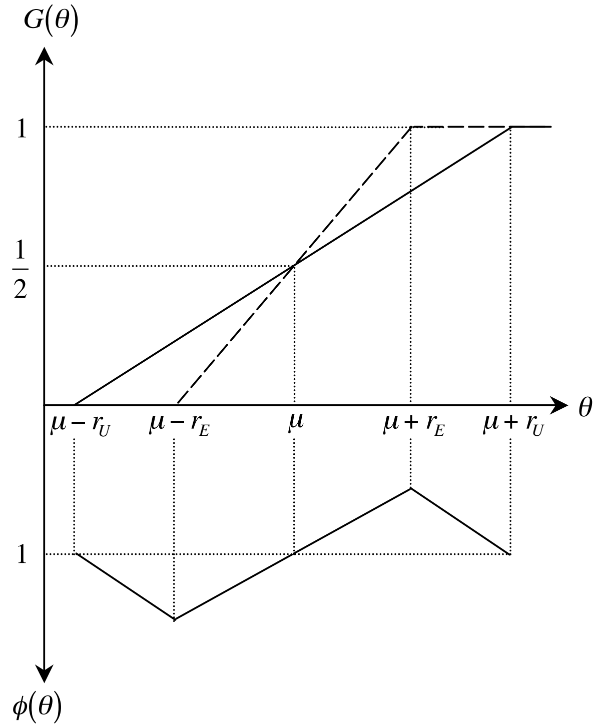
I justify this assumption, as it is intuitive that a firm will settle in the country with the bigger home market. Nevertheless, it always has to be kept in mind that this is a crucial assumption, which in the end determines the trade structure.

As soon as one country's labor force is too small to produce all the goods it has the bigger home market for, the other country will start producing some of those goods. In order to find out which goods, there needs to be a ranking. I assume there exists a function $\phi(\tilde{\theta})$, which states the preferability for a good with a critical client $\tilde{\theta}$ to be produced in *Equal*. I call it home market preferability function (HMPF).[17] If the preferability for goods, which

[17] Notice that it is also possible to describe a function, which states the preferability for a good to be produced in *Unequal*. This would simply give the opposite ranking.

have the same home market in both countries, is set to unity, then $\phi(\tilde{\theta}) > 1$ for all goods with a bigger home market in *Equal* and $\phi(\tilde{\theta}) < 1$ for all goods with a bigger home market in *Unequal*. In accordance with the home market bias assumption, it is intuitive to assume that the preferability is higher, the bigger the absolute and/or relative difference in the home markets of *Equal* and *Unequal*. Figure 4 shows such a function $\phi(\tilde{\theta})$ together with the CDF's of both countries. Remember that the home market is $x_C(\tilde{\theta}) = (1 - G_C(\tilde{\theta}))L$ for $C \in \{E, U\}$.

Figure 4: CDF's of both countries and the HMPF



Annotation. The function $\phi(\theta)$ has not to be linear. The solid and the dashed line in the upper graph depict the CDF of the unequal and equal country respectively.

By looking at the HMPF, it can be seen that it starts at 1, as the home markets for firms with the poorest client overall (with an income of $\mu - r_U$) are the same size in both countries. As the income of the critical client increases, the home market sizes will diverge and as *Equal* has the bigger home market, $\phi(\tilde{\theta})$ will continuously increase. This goes on until the poorest client has an income of $\mu - r_E$. Thereafter, a further increase in the income of the poorest client will lead to a convergence of the home market sizes and therefore a continuous decrease of $\phi(\tilde{\theta})$ until at the mean income the home markets will be the same size and $\phi(\tilde{\theta}) = 1$ again. A further increase in the income of the poorest client lets the HMPF drop below 1 and continuously decrease until an income of $\mu + r_E$ and then again increase until it reaches unity at the top income.

To determine the location of the firms, *Equal* will host those firms with the highest values of the HMPF, i.e. the highest ϕ's. Therefore, there needs to be a value $\hat{\phi}$, which clears the labor market in *Equal*[18] and all firms with a critical client $\tilde{\theta}$ for which $\phi(\tilde{\theta}) > \hat{\phi}$ is true, will settle in *Equal* and all others in *Unequal*.

I first look at a case for which $\hat{\phi} = 1$, i.e. both countries host the firms, for whose products they have the bigger markets and the mass producers are shared such that both labor markets clear. In a second step, the case, where only the unequal country has mass producers and therefore also hosts some firms, which *Equal* has the bigger market for, i.e. $\hat{\phi} > 1$, is discussed. As shown later, with the assumption that both countries have a uniform endowment distribution, a case where $\hat{\phi} < 1$ will never emerge.

4.2.1 Mass Producers in Both Countries

For the case $\hat{\phi} = 1$, *Equal* features the firms with the bigger markets ($\phi(\tilde{\theta}) > 1$), i.e. the firms whose poorest clients have a low labor endowment, and the unequal country will feature the firms with the smaller markets ($\phi(\tilde{\theta}) < 1$), i.e. the firms whose poorest clients have a high labor endowment. More precisely, firms with the critical client in the income range $(\mu - r_U, \mu)$ locate in *Equal* and firms with the critical client in the income range $(\mu, \mu + r_U)$ locate in *Unequal*. The firms that produce goods for everyone, i.e. with a critical client $\mu - r_U$, are located in both countries.

Therefore, the firms split-up is as follows

$$N_E = \int_{\mu - r_U}^{\mu} \frac{ax(\tilde{\theta})}{aF + x(\tilde{\theta})} d\theta + m \frac{a2L}{aF + 2L} (\mu - r_U) \tag{14}$$

$$N_U = \int_{\mu}^{\mu + r_U} \frac{ax(\tilde{\theta})}{aF + x(\tilde{\theta})} d\theta + (1 - m) \frac{a2L}{aF + 2L} (\mu - r_U), \tag{15}$$

where m is the share of the mass producers located in *Equal* and $\in (0, 1)$.

The split-up of the mass producers m needs to be such that the labor market clearing condition in *Equal* holds:

$$\mu L = \int_{\mu - r_U}^{\mu} \left(F + \frac{x(\theta)}{a} \right) N'(\theta) d\theta + m \left(F + \frac{2L}{a} \right) N(\mu - r_U)$$

$$\mu L = \int_{\mu - r_U}^{\mu} x(\theta) d\theta + m2L(\mu - r_U) \tag{16}$$

[18] If the labor market in *Equal* clears, *Unequal*'s does as well, because of Walras' Law.

$$\Rightarrow m = \frac{4\mu + r_E - 7r_U}{8(\mu - r_U)} \tag{17}$$

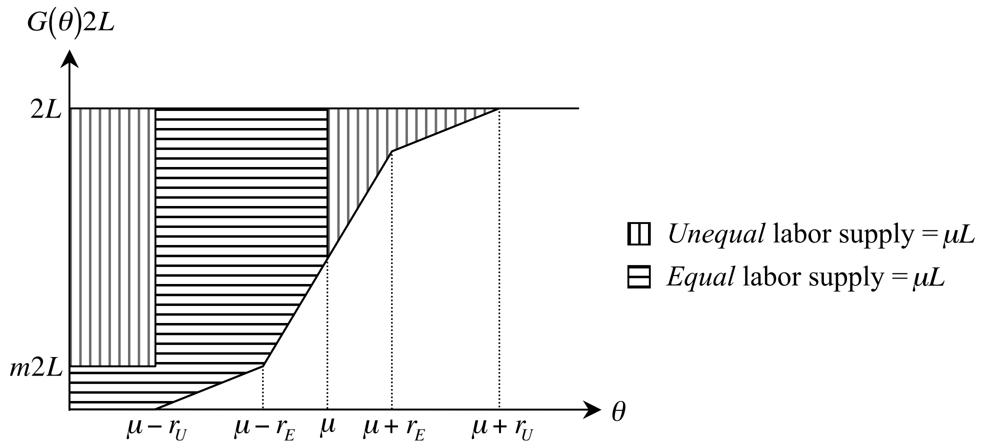
The calculations can be found in Appendix G. Due to Walras' Law, m could also be calculated using the labor market clearing condition of *Unequal* or the balanced payments condition. Similar to Equation (16), the labor market clearing condition in *Unequal* can be simplified to

$$\mu L = \int_{\mu}^{\mu+r_U} \left(F + \frac{x(\theta)}{a}\right) N'(\theta)d\theta + (1-m)\left(F + \frac{2L}{a}\right) N(\mu - r_U)$$

$$\mu L = \int_{\mu}^{\mu+r_U} x(\theta)d\theta + (1-m)2L(\mu - r_U). \tag{18}$$

Equations (16) and (18) are shown graphically in Figure 5. Notice that the share of mass producers m located in *Equal*, or correspondingly the share of mass producers $(1-m)$ in *Unequal*, is the variable making sure that both labor markets clear.

Figure 5: Firms split-up when both countries have mass producers



Annotation. The y-axis is scaled up by $2L$ such that the area above the CDF corresponds the market sizes.

As long as $m \in (0,1)$, both countries have mass producers. Considering Equation (17), $m < 1$ is always the case, as $4\mu > r_E + r_U$ holds (both variables, r_E and r_U, have to be smaller than μ). Therefore, the unequal country always features mass producers, i.e. $\hat{\phi} < 1$ is not possible. By contrast, $m > 0$ only holds for $4\mu > 7r_U - r_E$. If this is not the case, only *Unequal* has mass producers and there will be some firms in *Unequal* that produce goods, which *Equal* has the bigger home market for, i.e. $\hat{\phi} > 1$.

Lastly, it is possible to write down the balanced payments condition, which states that

Equal's exports to *Unequal*, have to match *Equal*'s imports from *Unequal* in terms of money

$$\int_{\mu-r_U}^{\mu} (1-G_U(\theta))Lp(\theta)N'(\theta)d\theta + mLp(\mu-r_U)N(\mu-r_U) =$$
$$\int_{\mu}^{\mu+r_E} (1-G_E(\theta))Lp(\theta)N'(\theta)d\theta + (1-m)Lp(\mu-r_U)N(\mu-r_U).$$

This can be simplified to

$$\int_{\mu-r_U}^{\mu} (1-G_U(\theta))Ld\theta + mL(\mu-r_U) = \int_{\mu}^{\mu+r_E} (1-G_E(\theta))Ld\theta + (1-m)L(\mu-r_U). \tag{19}$$

Notice that trade is balanced in terms of money, but not in terms of goods. Besides the mass products, *Equal* exports relatively many goods for a low price and imports relatively few goods for a high price. Of course it is vice versa for *Unequal*.

4.2.2 Mass Producers Only in the Unequal Country

In the case of $\hat{\phi} > 1$, the unequal country features some firms that produce goods, which *Equal* has the bigger home market for. Notice first that, not regarding the top income $\mu + r_U$ where anyway both market sizes are zero, every income has exactly one "twin" income with the same ϕ-value. Denoting the lower income for a specific value $\tilde{\phi}$ as $\theta_1(\tilde{\phi})$ and the higher as $\theta_2(\tilde{\phi})$, the firms split-up can be stated as

$$N_E = \int_{\theta_1(\hat{\phi})}^{\theta_2(\hat{\phi})} \frac{ax(\tilde{\theta})}{aF + x(\tilde{\theta})} d\theta \tag{20}$$

$$N_U = \int_{\mu-r_U}^{\theta_1(\hat{\phi})} \frac{ax(\tilde{\theta})}{aF + x(\tilde{\theta})} d\theta + \int_{\theta_2(\hat{\phi})}^{\mu+r_U} \frac{ax(\tilde{\theta})}{aF + x(\tilde{\theta})} d\theta + \frac{a2L}{aF+2L}(\mu - r_U). \tag{21}$$

Equation (20) shows that the more equally distributed country hosts all firms with a critical client in the range $(\theta_1(\hat{\phi}), \theta_2(\hat{\phi}))$, i.e. all those markets for which the HMPF is bigger than $\hat{\phi}$, whereas Equation (21) shows that the unequal country features all firms for which the HMPF is below $\hat{\phi}$, including all the mass producers.

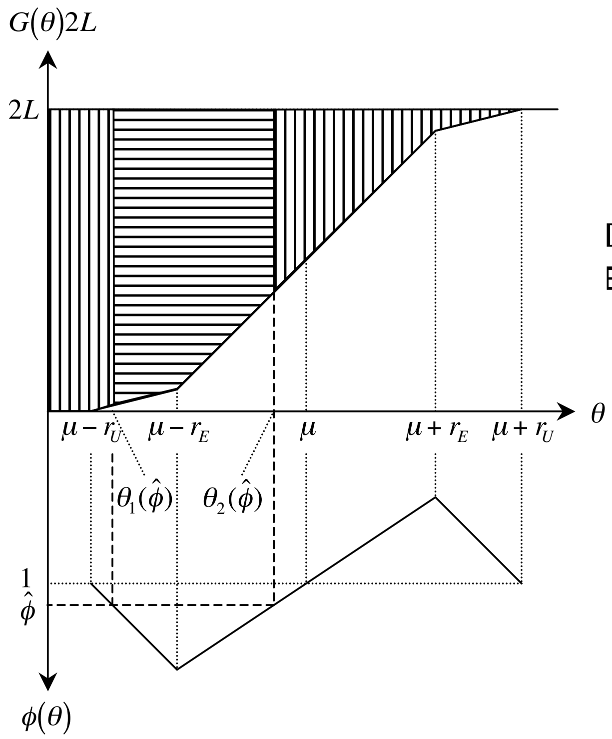
The labor market clearing conditions can again be simplified to

$$\mu L = \int_{\theta_1(\hat{\phi})}^{\theta_2(\hat{\phi})} x(\theta) d\theta \tag{22}$$

$$\mu L = \int_{\mu - r_U}^{\theta_1(\hat{\phi})} x(\theta) d\theta + \int_{\theta_2(\hat{\phi})}^{\mu + r_U} x(\theta) d\theta + \frac{a2L}{aF + 2L}(\mu - r_U) \tag{23}$$

and are illustrated in Figure 6. Now that all mass producers are located in *Unequal*, i.e. $m = 0$, $\hat{\phi}$ is the variable making sure that both labor markets clear.

Figure 6: Firms split-up when only the unequal country has mass producers



Annotation. The y-axis is scaled up by $2L$ such that the area above the CDF corresponds to the market sizes.

The already simplified balanced payments condition in the case of $\hat{\phi} > 1$ looks as follows:

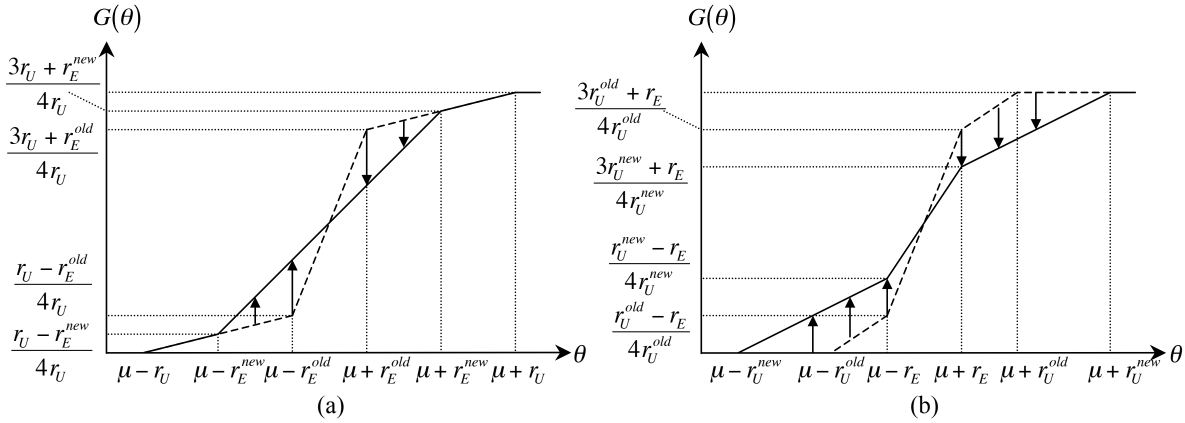
$$\int_{\theta_1(\hat{\phi})}^{\theta_2(\hat{\phi})} (1 - G_U(\theta)) L d\theta = \int_{\mu - r_U}^{\theta_1(\hat{\phi})} (1 - G_E(\theta)) L d\theta + \int_{\theta_2(\hat{\phi})}^{\mu + r_E} (1 - G_E(\theta)) L d\theta + L(\mu - r_U). \tag{24}$$

Equation 24 states that the exports of *Equal* (LHS) need to be the same as its imports (RHS) or vice versa for *Unequal*.

4.3 Impact of Consumer Heterogeneity in the Open Economy

This section shows the impact of consumer heterogeneity on the market and trade structure in the open economy. As always, an increase or decrease in inequality is assumed to be for a given mean labor endowment μ. Like in autarky, inequality affects the equilibrium through the channel of the market sizes. Therefore, it is important to understand its effect on the CDF of the overall economy. Figure 7 shows that the CDF "turns clockwise", when the inequality increases in *Equal* (a) and *Unequal* (b). Notice that in (a) this is only the case in the endowment range $[\mu - r_E^{new}, \mu + r_E^{new}]$, as obviously nothing changes in both pole regions of the endowment distribution, where only households in *Unequal* are located. Hence, it can be concluded that a rise in inequality leads to shrinking (growing) market sizes for firms with the critical client below (above) the mean within the income range that is affected by the changing endowment distribution.

Figure 7: Impact of an increase in inequality in *Equal* and *Unequal* on the CDF respectively



As the impact on the markets is the same as in autarky, the effects of a rise in inequality on the market structure are straight forward. All firms selling to a smaller (bigger) market, set a higher (lower) price, which enables the particular critical client to buy less (more) of those goods, i.e. less (more) of such firms enter those markets. Notice that Lemma 1 also holds in an open economy in the case where r_U increases.[19] The argumentation is exactly the same. A household with an income of $\mu - r_U^{old}$ consumes less goods after an increase in inequality, because the average price of his consumption basket rises.

Proposition 5. *Because a rise in inequality has the same effects as in autarky, Propositions 1 and 2 do also hold in an open economy.*

a) The majority of households loses utility from a mean preserving spread.

[19]In the case where r_E increases, the poorest households are not affected and therefore Lemma 1 is needless.

b) *A rise in inequality, i.e. a higher r, implies more overall variety.*

Proof. This is true for the same reasons as in autarky.

a) As a household above the mean also consumes all the more expensive goods, which decline in number, a rise in inequality leads also some households that have an income above the mean to be worse off.

b) Firms with a poor critical client, of which there are less in the new equilibrium, use more labor, as they have a higher labor demand in the production sector, whereas firms with a rich critical client, of which there are more in the new equilibrium, have a lower labor demand. Therefore, there are more goods available, as the labor markets clear in equilibrium.

It follows that indeed also in the open economy it is true that although a mean preserving spread increases the overall variety, the majority of households loses utility. □

Besides those effects on the market structure, which are the same as in autarky, a rise in inequality in an open economy has also an effect on the trade structure, as it affects the location decision of the firms. First of all, the inequality parameter r in both countries determines, which is more unequal than the other and therefore hosts the firms that preferably produce for the richer households, whereas the other country has the firms that preferably have a poorer critical client. Second, r_E and r_U are the deciding factors, whether both countries have mass producers or just the more unequal one does. Section 4.2.1 states that both, *Unequal* and *Equal*, host mass producers if

$$4\mu > 7r_U - r_E.$$

For a given mean labor endowment μ this holds true whenever the inequality in the more equally distributed country is high, i.e. a high r_E that is still lower than r_U, or the inequality in the more unequally distributed country is low, i.e. a low r_U that is still higher than r_E. It follows that both countries have mass producers, if they have a similar endowment distribution, whereas only the unequal country has mass producers, if the endowment distributions are very different. Additionally, the inequalities in both countries should not be too high for both of them having mass producers.

What do those effects of inequality on the market structure and firms location decision mean for the volume of trade in terms of money, i.e. does the hypothesis of Linder (1961) hold that similar countries trade more?

Proposition 6. *More similar endowment distributions, i.e. an increase (decrease) in inequality in the equal (unequal) country, lead to a higher volume of trade in terms of money, if both countries host mass producers.*

Proof. This can be shown easily, using Equation (19) and Equation (17).

$$\int_{\mu-r_U}^{\mu} (1 - G_U(\theta))Ld\theta + mL(\mu - r_U) = \int_{\mu}^{\mu+r_E} (1 - G_E(\theta))Ld\theta + (1 - m)L(\mu - r_U)$$
$$\frac{3r_U}{4}L + mL(\mu - r_U) = \frac{r_E}{4}L + (1 - m)L(\mu - r_U)$$
$$\frac{4\mu + r_E - r_U}{8}L = \frac{4\mu + r_E - r_U}{8}L$$

Therefore, a higher r_E or a lower r_U lead to a higher volume of trade in terms of money. □

As long as $4\mu \leq 7r_U - r_E$, it can not be shown unambiguously that more similar endowment distributions lead to a higher volume of trade, without further assumptions about the HMPF, as there is always more than one effect working in opposite directions. Nevertheless, as the endowment distributions eventually become sufficiently similar, both countries host mass producers, which is the case discussed in Proposition 6.

5 Conclusions

This thesis develops a model that discusses the impact of within-country inequality on international trade in an increasing returns to scale framework, using non-homothetic 0/1-preferences and continuous endowment distributions. Those two assumptions make sure that richer households consume more goods in equilibrium than poorer households, letting within-country income inequality have an important effect on the market structure and the trade patterns, which is not the case in a New Trade Theory model with CES-preferences.

In a first step, the autarky equilibrium is discussed. Firms set the prices equal the willingness to pay of the poorest household to whom they sell. The critical client of a firm determines its market, which is the driving force in this model. The prices equal the average costs of a firm, which are increasing in the critical client's income, due to the fact that the fixed costs have to be born by less households. Every household consumes all goods with a poorer critical client plus those he is the poorest client for, and his utility is equal to the number of goods consumed. The utilitarian welfare in this economy is therefore the overall number of goods consumed, which is equal to the labor working in the production sector times the productivity.

A rise in within-country inequality leads to shrinking markets for firms with a critical client below the mean income and to growing markets above. This leads to less firms with a critical client that have a high labor demand, and more firms that sell only to the richer households, which have a low labor demand. It follows that more inequality enables an economy to have a higher product variety, which directly benefits the richest households that consume all goods. Nevertheless, richer households also consume all the goods with a poorer critical client, which decline in number. This is the reason why even some households with an income above the mean are worse off, when the endowment distribution becomes more unequal. Hence, although the richest households benefit, the majority of households loses from higher income inequality. This can also be seen in the fact that the utilitarian welfare falls, as the higher product variety, leads to more labor needed in the set-up sector, which is why less goods can be produced.

In a second step, the open economy equilibrium is described for two identical economies except that one has a more unequal endowment distribution than the other. By just opening up this economy, the price setting behavior and the mechanisms for the determination of the market structure does not change in a world without trade costs. Nevertheless, the bigger world population means bigger markets for most firms, which are therefore able to set lower prices. This leads to more firms in equilibrium and gains from trade due to the increasing returns to scale, known from all models of the New Trade Theory.

A rise in inequality in an open economy has the same effects on the market structure as in a closed one. More inequality leads to a higher product variety benefiting only the richest households, whereas the majority of households is worse off. In an open economy, there is even an additional feature. Each firm has to decide, in which country it locates, therefore affecting the trade patterns. Assuming a home market bias, the unequal country hosts the firms producing only for the richer households with the smaller markets, whereas the more equally distributed country primarily hosts the firms that exclude only a few households and therefore have big markets. If the endowment distributions of both countries are very similar, either country hosts mass producers, selling to all households. The respective share of mass producers is determined such that both labor markets clear. In the case where the endowment distributions are sufficiently different, the labor force of the equal country is too small to produce all the goods, which it has the bigger home market for and therefore the unequal country hosts all mass producers and also some firms that would preferably locate in the equal country. The income inequality therefore not only has an impact on the market structure, but also on the location decision of the firms, which affects the trade structure. In the first case, where both countries have a similar endowment distribution and host mass producers, it can be shown that even more similarity in the endowments leads to a higher trade volume in terms of money. In the second case this can not be unambiguously determined, but more similarity in their endowment distributions will eventually lead to the first case.

A caveat against this model is that the location decision is based on the assumption of the existence of a home market bias, although no trade costs are modeled. Therefore, the next reasonable extension to the model should be the implementation of such costs. In a general case, this probably leads to different wages in both countries and therefore make the costs of a firm dependent on its location, which then affects the price setting behavior. A second difficulty that would make this extension not as straight forward is the fact that the the households utility maximization decision depends on the income, which is not anymore only given by the known endowment distribution, but also by the wage rate that is unknown in the first place. Another possible extension to the model could be to ease the assumption of a uniform endowment distribution and implement instead a Pareto or a totally general continuous distribution, although this would make it impossible to solve for closed-form expressions. Additionally, I assume that the mechanics and results of such a model, would be very similar to this one and therefore do not generate a lot of value added.

As stated in Section 1, the goal of this thesis was to give useful insights on the channels through which within-country inequality affects outcomes in trade. I hope that this work could inspire the readers to further investigate those ideas and can be used as a basis for future works.

References

Bertola, Giuseppe, Reto Foellmi, and Josef Zweimüller, *Income Distribution in Macroeconomic Models*, Princeton and Oxford: Princeton University Press, 2006.

Dixit, Avinash and Joseph Stiglitz, "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 1977, *67* (3), 297–308.

Foellmi, Reto, Christian Hepenstrick, and Josef Zweimüller, "Non-Homothetic Preferences, Parallel Imports and the Extensive Margin of International Trade," *CEPR Discussion Paper Series*, August 2010, (7939).

Harrison, Ann, Margaret McMillan, and John McLaren, "Recent Perspectives on Trade and Inequality," *Annual Review of Economics*, 2011, *3* (1), 261–289.

Krugman, Paul, "Increasing Returns, Monopolistic Competition, and International Trade," *Journal of International Economics*, 1979, *9* (4), 469–479.

_ , "Scale Economies, Product Differentiation, and the Pattern of Trade," *American Economic Review*, 1980, *70* (5), 950–959.

Linder, Staffan, *An Essay on Trade and Transformation*, Uppsala: Almqvist and Wiksells, 1961.

Matsuyama, Kiminori, "A Ricardian Model with a Continuum of Goods under Nonhomothetic Preferences: Demand Complementarities, Income Distribution, and North-South Trade," *Journal of Political Economy*, 2000, *108* (6), 1093–1120.

Murphy, Kevin M., Andrei Shleifer, and Robert Vishny, "Income Distribution, Market Size, and Industrialization," *Quarterly Journal of Economics*, 1989, *104* (3), 537–564.

A Lorenz Curve and the Gini Coefficient

The Lorenz curve shows the cumulative share of income earned as a function of the cumulative share of households from the lowest to the highest income. In terms of the PDF and the CDF, this can be written as

$$L(G) = \frac{\int_{\mu-r}^{\theta(G)} \theta W g(\theta) L d\theta}{\int_{\mu-r}^{\mu+r} \theta W g(\theta) L d\theta,}$$

where $\theta(G) = 2rG + (\mu - r)$ stands for the inverse of the CDF

$$\begin{aligned} &= \frac{\left[\frac{\theta^2}{4r}\right]_{\mu-r}^{2rG+(\mu-r)}}{\mu} \\ L(G) &= \frac{rG^2 + G(\mu - r)}{\mu}. \end{aligned}$$

From the Lorenz curve the derivation of the Gini coefficient is straight forward:

$$\begin{aligned} Gini &= \frac{\frac{1}{2} - \int_0^1 \frac{rG^2+G(\mu-r)}{\mu} dG}{\frac{1}{2}} \\ &= 1 - 2\left[\frac{rG^3}{3\mu} + \frac{G^2(\mu - r)}{2\mu}\right]_0^1 \\ &= \frac{r}{3\mu} \end{aligned}$$

The Gini coefficient of the uniform distribution is in the range $[0, 1/3]$ as $r \in [0, \mu]$ and therefore can never be higher than $1/3$. This is simply because of the uniform distribution's definition that the density is the same over the whole support and hence, a positive skew is not possible. Notice that the mean labor endowment and the Gini coefficient are negatively correlated. The reason is that for a given endowment range $2r$ the relative difference between the richest and the poorest household is bigger, the lower μ.

B Closed-Form Expression of the Autarky Market Structure

After some algebra it is possible to derive a closed-form expression for the market structure integral:

$$\int_{\mu-r}^{\tilde{\theta}} \frac{a\left[(\mu + r - \theta)/(2r)\right] L}{aF + \left[(\mu + r - \theta)/(2r)\right] L} d\theta$$

substitute $(\mu + r - \theta)/(2r) = t$ and $d\theta = -2rdt$

$$\Leftrightarrow -2ar \int_{1}^{(\mu+r-\tilde{\theta})/(2r)} \frac{Lt}{aF + Lt} dt$$

$$\Leftrightarrow -2ar \int_{1}^{(\mu+r-\tilde{\theta})/(2r)} \frac{-aF + aF + Lt}{aF + Lt} dt$$

$$\Leftrightarrow 2a^2 Fr \int_{1}^{(\mu+r-\tilde{\theta})/(2r)} \frac{1}{aF + Lt} dt - 2ar \int_{1}^{(\mu+r-\tilde{\theta})/(2r)} dt$$

substitute $aF + Lt = u$ and $dt = 1/Ldu$

$$\Leftrightarrow \frac{2a^2 Fr}{L} \int_{aF+L}^{aF+L(\mu+r-\tilde{\theta})/(2r)} \frac{1}{u} du - 2ar \int_{1}^{(\mu+r-\tilde{\theta})/(2r)} dt$$

$$\Leftrightarrow \left[\frac{2a^2 Fr \log(u)}{L} \right]_{aF+L}^{aF+L(\mu+r-\tilde{\theta})/(2r)} - [2art]_{1}^{(\mu+r-\tilde{\theta})/(2r)}$$

substitute back $u = aF + Lt$

$$\Leftrightarrow \left[\frac{2a^2 Fr \log(aF + Lt)}{L} \right]_{1}^{(\mu+r-\tilde{\theta})/(2r)} - [2art]_{1}^{(\mu+r-\tilde{\theta})/(2r)}$$

substitute back $t = (\mu + r - \theta)/(2r)$

$$\Leftrightarrow \left[\frac{2a^2 Fr \log(aF + L\left[(\mu + r - \theta)/(2r)\right])}{L} \right]_{\mu-r}^{\tilde{\theta}} - [a(\mu + r - \theta)]_{\mu-r}^{\tilde{\theta}}$$

$$\Leftrightarrow -\frac{2a^2 Fr}{L} \log\left(\frac{aF + L}{aF + \left[(\mu + r - \tilde{\theta})/(2r)\right] L} \right) - a(\mu + r - \tilde{\theta}) + 2ar$$

$$\Leftrightarrow 2ar \left[1 - \frac{\mu + r - \tilde{\theta}}{2r} - \frac{aF}{L} \log\left(\frac{aF + L}{aF + \left[(\mu + r - \tilde{\theta})/(2r)\right] L} \right) \right]$$

It follows that the market structure can be expressed as

$$N(\tilde{\theta}) = 2ar \left[1 - \frac{\mu + r - \tilde{\theta}}{2r} - \frac{aF}{L} \log\left(\frac{aF + L}{aF + \left[(\mu + r - \tilde{\theta})/(2r)\right] L} \right) \right] + \frac{aL}{aF + L}(\mu - r)$$

and the overall product variety as

$$N(\mu + r) = 2ar \left[1 - \frac{aF}{L} \log \left(\frac{aF + L}{aF}\right)\right] + \frac{aL}{aF + L}(\mu - r),$$

by setting $\tilde{\theta} = \mu + r$.

C Utilitarian Welfare in Autarky

The overall welfare in the closed economy is defined as

$$\begin{aligned} B &= \int_{\mu-r}^{\mu+r} g(\theta) L N(\theta) d\theta \\ &= aL \int_{\mu-r}^{\mu+r} 1 - \frac{\mu + r - \theta}{2r} - \frac{aF}{L} \log \left(\frac{aF + L}{aF + [(\mu + r - \theta)/(2r)]\, L}\right) + \frac{L(\mu - r)}{2r(aF + L)} d\theta \end{aligned}$$

First calculate the integral

$$\int_{\mu-r}^{\mu+r} \log \left(\frac{aF + L}{aF + [(\mu + r - \theta)/(2r)]\, L}\right) d\theta$$

substitute $aF + [(\mu + r - \theta)/(2r)]L = t$ and $d\theta = -2r/Ldt$

$$\Leftrightarrow -\frac{2r}{L} \int_{aF+L}^{aF} \log \left(\frac{aF + L}{t}\right) dt$$

substitute $1/t = u$ and $dt = -t^2 du = -1/u^2 du$

$$\begin{aligned} &\Leftrightarrow \frac{2r}{L} \int_{1/(aF+L)}^{1/(aF)} \frac{\log [u(aF + L)]}{u^2} du \\ &\Leftrightarrow -\left[\frac{2r \log [u(aF + L)]}{Lu}\right]_{1/(aF+L)}^{1/(aF)} + \frac{2r}{L} \int_{1/(aF+L)}^{1/(aF)} \frac{1}{u^2} du \\ &\Leftrightarrow -\left[\frac{2r(\log [u(aF + L)] + 1)}{Lu}\right]_{1/(aF+L)}^{1/(aF)} \end{aligned}$$

substitute back $u = 1/t$

$$\Leftrightarrow -\left[\frac{2rt(\log\left[(aF+L)/t\right]+1)}{L}\right]_{aF+L}^{aF}$$

substitute back $t = aF + [(\mu + r - \theta)/(2r)]L$

$$\Leftrightarrow -\left[\frac{2r(aF+[(\mu+r-\theta)/(2r)]L)(\log\left[(aF+L)/(aF+[(\mu+r-\theta)/(2r)]L)\right]+1)}{L}\right]_{\mu-r}^{\mu+r}$$
$$\Leftrightarrow -\frac{2aFr\log\left[(aF+L)/(aF)\right]}{L}+2r$$

Now it is possible to calculate the overall welfare

$$\begin{aligned}
B &= aL\int_{\mu-r}^{\mu+r} 1-\frac{\mu+r-\theta}{2r}-\frac{aF}{L}\log\left(\frac{aF+L}{aF+\left[(\mu+r-\theta)/(2r)\right]L}\right)+\frac{L(\mu-r)}{2r(aF+L)}d\theta \\
&= aL\left[\frac{\theta^2}{4r}-\frac{aF(\mu-r)\theta}{2r(aF+L)}\right]_{\mu-r}^{\mu+r}+\frac{2a^3F^2r\log\left[(aF+L)/(aF)\right]}{L}-2a^2Fr \\
&= aL\left[\mu-\frac{aF(\mu-r)}{aF+L}\right]+\frac{2a^3F^2r\log\left[(aF+L)/(aF)\right]}{L}-2a^2Fr \\
&= aL\mu - aF\left(2ar\left[1-\frac{aF}{L}\log\left(1+\frac{L}{aF}\right)\right]+\frac{aL}{aF+L}(\mu-r)\right) \\
&= a\left[L\mu - FN(\mu+r)\right]
\end{aligned}$$

D Proofs for Impact of Consumer Heterogeneity in Autarky

D.1 Proof of Lemma 1

Lemma 1. *Effect 1 in Figure 2 (b) dominates effect 2, i.e. a household with income $\mu - r^{old}$ consumes less goods, if inequality increases.*

Proof. To show formally that effect 1 in Figure 2 dominates effect 2, one can show that a household with income $\mu - r^{old}$ consumes less goods, if inequality increases.

$$N(\mu - r^{old})^{new} - N(\mu - r^{old})^{old} < 0$$
$$2ar^{new}\left[\frac{r^{new}-r^{old}}{2r^{new}}-\frac{aF}{L}\log\left(\frac{aF+L}{aF+\left[(r^{new}+r^{old})/(2r^{new})\right]L}\right)\right]-\frac{aL}{aF+L}(r^{new}-r^{old})<0$$
$$2ar^{new}\left[\frac{aF(r^{new}-r^{old})}{2r^{new}(aF+L)}-\frac{aF}{L}\log\left(\frac{aF+L}{aF+\left[(r^{new}+r^{old})/(2r^{new})\right]L}\right)\right]<0$$

Substitute $x = L/(aF)$ and show that the inequation holds true for all $x \in (0, \infty)$.

$$\frac{r^{new} - r^{old}}{2r^{new}(1+x)} - \frac{1}{x}\log\left(\frac{1+x}{1+\left[(r^{new}+r^{old})/(2r^{new})\right]x}\right) < 0$$

$$\frac{(r^{new} - r^{old})x}{2r^{new}(1+x)} < \log\left(\frac{1+x}{1+\left[(r^{new}+r^{old})/(2r^{new})\right]x}\right)$$

Now define

$$f(x) \equiv \frac{(r^{new} - r^{old})x}{2r^{new}(1+x)} \qquad \text{and} \qquad g(x) \equiv \log\left(\frac{1+x}{1+\left[(r^{new}+r^{old})/(2r^{new})\right]x}\right)$$

Show that both functions are monotonically increasing for all $x \in (0, \infty)$

$$f'(x) = \frac{r^{new} - r^{old}}{2r^{new}(1+x)^2} > 0, \qquad \forall x \in (0, \infty)$$

$$g'(x) = \frac{r^{new} - r^{old}}{2r^{new}(1+x)(1+\left[(r^{new}+r^{old})/(2r^{new})\right]x)} > 0, \qquad \forall x \in (0, \infty)$$

and that $g(x)$ has a steeper slope than $f(x)$

$$\frac{r^{new} - r^{old}}{2r^{new}(1+x)^2} < \frac{r^{new} - r^{old}}{2r^{new}(1+x)(1+\left[(r^{new}+r^{old})/(2r^{new})\right]x)}$$

$$\frac{r^{new} + r^{old}}{2r^{new}} < 1.$$

As

$$\lim_{x \to 0} \frac{(r^{new} - r^{old})x}{2r^{new}(1+x)} = 0$$

and

$$\lim_{x \to 0} \log\left(\frac{1+x}{1+\left[(r^{new}+r^{old})/(2r^{new})\right]x}\right) = 0$$

both functions start at the origin and $f(x)$ is always smaller than $g(x)$. It follows that $N(\mu - r^{old})^{new} - N(\mu - r^{old})^{old} < 0$ for all $L/(aF) \in (0, \infty)$. A household with income $\mu - r^{old}$ consumes less goods, if inequality increases. □

D.2 Proof of Proposition 1

Proposition 1. *The majority of households loses utility from a mean preserving spread.*

Proof. Differentiate $N(\mu)$ with respect to r and demonstrate that the derivative is negative, to show that the majority of households loses utility from a mean preserving spread.

$$N(\mu) = 2ar\left[\frac{1}{2} - \frac{aF}{L}\log\left(\frac{aF+L}{aF+L/2}\right)\right] + \frac{aL}{aF+L}(\mu - r)$$
$$\frac{\mathrm{d}N(\mu)}{\mathrm{d}r} = 2a\left[\frac{1}{2} - \frac{aF}{L}\log\left(\frac{aF+L}{aF+L/2}\right)\right] - \frac{aL}{aF+L}$$

Substitute $x = L/(aF)$ and show that the derivative is negative for all $x \in (0, \infty)$.

$$1 - \frac{2}{x}\log\left(\frac{2x+2}{x+2}\right) - \frac{x}{x+1} < 0$$
$$\frac{1}{x+1} - \frac{2}{x}\log\left(\frac{2x+2}{x+2}\right) < 0$$
$$\frac{x}{2x+2} < \log\left(\frac{2x+2}{x+2}\right)$$

Now define

$$f(x) \equiv \frac{x}{2x+2} \qquad \text{and} \qquad g(x) \equiv \log\left(\frac{2x+2}{x+2}\right)$$

Show that both functions are monotonically increasing for all $x \in (0, \infty)$

$$f'(x) = \frac{2}{4x^2+8x+4} = \frac{1}{2x^2+4x+2} > 0, \qquad \forall x \in (0,\infty)$$
$$g'(x) = \frac{x+2}{2(x+1)}\frac{2x+4-(2x+2)}{(x+2)^2} = \frac{2}{2(x+1)(x+2)} = \frac{1}{x^2+3x+2} > 0, \qquad \forall x \in (0,\infty)$$

and that $g(x)$ has a steeper slope than $f(x)$ for all $x \in (0, \infty)$

$$\frac{1}{2x^2+4x+2} < \frac{1}{x^2+3x+2}, \qquad \forall x \in (0,\infty).$$

As

$$\lim_{x\to 0}\frac{x}{2x+2} = 0$$

and

$$\lim_{x\to 0}\log\left(\frac{2x+2}{x+2}\right) = 0$$

it can be concluded that both functions start at the origin and $f(x)$ lies always below $g(x)$. It follows that $\mathrm{d}N(\mu)/\mathrm{d}r < 0$ for all $L/(aF) \in (0, \infty)$. The indifferent household which is not affected by a rise in inequality lies between the mean and the richest household. □

D.3 Proof of Proposition 2

Proposition 2. *A rise in inequality, i.e. a higher r, implies more overall variety.*

Proof. To show formally that a rise in inequality, i.e. a higher r, implies more overall variety, one can proceed as in Appendix D.2 by showing that the derivative $\mathrm{d}N(\mu+r)/\mathrm{d}r > 0$ for all $L/(aF) \in (0, \infty)$.

$$N(\mu+r) = 2ar\left[1 - \frac{aF}{L}\log\left(1+\frac{L}{aF}\right)\right] + \frac{aL}{aF+L}(\mu - r)$$
$$\frac{\mathrm{d}N(\mu+r)}{\mathrm{d}r} = 2a\left[1 - \frac{aF}{L}\log\left(1+\frac{L}{aF}\right)\right] - \frac{aL}{aF+L}$$

Substitute $x = L/(aF)$ and show that the derivative is positive for all $x \in (0, \infty)$.

$$2 - \frac{2}{x}\log(x+1) - \frac{x}{x+1} > 0$$
$$\frac{x+2}{x+1} - \frac{2}{x}\log(x+1) > 0$$
$$\frac{x^2+2x}{x+1} > 2\log(x+1)$$

Now define

$$f(x) \equiv \frac{x^2+2x}{x+1} \qquad \text{and} \qquad g(x) \equiv 2\log(x+1)$$

Show that both functions are monotonically increasing for all $x \in (0, \infty)$

$$f'(x) = \frac{x^2+2x+2}{(x+1)^2} > 0, \qquad \forall x \in (0, \infty)$$
$$g'(x) = \frac{2}{(x+1)} > 0, \qquad \forall x \in (0, \infty)$$

and that $f(x)$ has a steeper slope than $g(x)$ for all $x \in (0, \infty)$

$$\frac{x^2+2x+2}{(x+1)^2} > \frac{2}{(x+1)}$$
$$x^2 > 0, \qquad \forall x \in (0, \infty).$$

As

$$\lim_{x\to 0}\frac{x^2+2x}{x+1} = 0$$

and

$$\lim_{x \to 0} 2 \log (x + 1) = 0$$

it can be concluded that both functions start at the origin and $f(x)$ lies always above $g(x)$. It follows that $\mathrm{d}N(\mu + r)/\mathrm{d}r > 0$ for all $L/(aF) \in (0, \infty)$. More inequality implies more product variety. □

D.4 Proof of Proposition 3

Proposition 3. *The utilitarian welfare in a closed economy falls, if inequality increases.*

Proof. Differentiate the overall welfare U in the economy with respect to r and demonstrate that the derivative is negative, to show that more inequality is unfavorable for a closed economy.

$$B = a[L\mu - FN(\mu + r)]$$
$$\frac{\mathrm{d}B}{\mathrm{d}r} = -aF\frac{\mathrm{d}N(\mu + r)}{\mathrm{d}r}$$

Show that the derivative is negative.

$$-\frac{\mathrm{d}N(\mu + r)}{\mathrm{d}r} < 0$$
$$\frac{\mathrm{d}N(\mu + r)}{\mathrm{d}r} > 0$$

Which was already proofed in Appendix D.3. □

E Consumer Heterogeneity in a Krugman Model

To see what happens in a Krugman (1979, 1980) framework including consumer heterogeneity, assume two countries, called *Home* and *Foreign*, of the kind described in Section 2, except that the households have a CES-utility function

$$U_i = \int_0^\infty \frac{c_i(j)^{1-\alpha}}{1-\alpha} dj,$$

where $1/\alpha$ is the elasticity of substitution and the distribution of labor endowments $\in [\underline{\theta}_C, \overline{\theta}_C]$ with $C \in \{H, F\}$ is given by a general continuous distribution. From the first order condition of a household i's utility maximization problem, it is possible to derive the individual consumption

$$c_i(\theta_i, j)^{-\alpha} - \lambda(\theta_i)p(j) = 0$$
$$c_i(\theta_i, p(j)) = (\lambda(\theta_i)p(j))^{-\frac{1}{\alpha}},$$

which leads to the overall demand function

$$x(p(j)) = \left[L \int_{\underline{\theta}_H}^{\overline{\theta}_H} \lambda(\theta)^{-\frac{1}{\alpha}} g_H(\theta) d\theta + L \int_{\underline{\theta}_F}^{\overline{\theta}_F} \lambda(\theta)^{-\frac{1}{\alpha}} g_F(\theta) d\theta \right] \cdot p(j)^{-\frac{1}{\alpha}} \equiv \psi p(j)^{-\frac{1}{\alpha}},$$

assuming the same population size.

From the firms maximization problem, it is possible to derive the price, using the first order condition.

$$\frac{\alpha - 1}{\alpha} \psi p^C(j)^{-\frac{1}{\alpha}} + \frac{1}{\alpha} \frac{W^C}{a} \psi p^C(j)^{-\frac{1+\alpha}{\alpha}} = 0$$
$$p^C(j) = p^C = \frac{1}{1-\alpha} \frac{W^C}{a}, \qquad C \in \{H, F\}.$$

The elasticity of substitution does not depend on the income, because all households have a CES-utility function. As the mark-up only depends on the elasticity, the price doesn't change when introducing income inequality.

The quantity each firm can sell is given by the zero profit condition

$$W^C F = \left(p^C - \frac{W^C}{a} \right) x^C(j)$$
$$x^C(j) = x = \frac{aF(1-\alpha)}{\alpha}$$

and also does not depend on the income distribution.

The number of goods produced can be determined from the labor market constraint

$$L^C = L = N^C \left(F + \frac{x}{a} \right)$$
$$N^C = N = \frac{\alpha L}{F},$$

which is also the same as in the standard model, where both countries have only one income group.

Finally, it can be shown that the wage rates equalize in both countries. Using the budget constraint of the households, it is possible to calculate the amount of money a household uses for each good

$$p^F c_H^F(\theta,j) = \theta \frac{FW_H}{\alpha L\left(\omega^{\frac{1-\alpha}{\alpha}}+1\right)} \qquad \text{and} \qquad p^H c_F^H(\theta,j) = \theta \frac{FW_F}{\alpha L\left(\left(\frac{1}{\omega}\right)^{\frac{1-\alpha}{\alpha}}+1\right)}$$

for $\omega = W^F/W^H$. This can be plugged into the balanced payments condition, yielding

$$\int_{\underline{\theta}_F}^{\overline{\theta}_F} g_F(\theta) p^H c_F^H(\theta,j) d\theta \cdot LN = \int_{\underline{\theta}_H}^{\overline{\theta}_H} g_H(\theta) p^F c_H^F(\theta,j) d\theta \cdot LN$$

$$\omega^{\frac{1}{\alpha}} + \omega = \left(\frac{1}{\omega}\right)^{\frac{1-\alpha}{\alpha}} + 1.$$

Obviously only $\omega = 1$ makes sure that this equation holds. Hence it can be concluded that the endowment distribution has no impact in this framework at all.

F Open Economy Product Variety and Utilitarian Welfare

The overall product variety in an open economy with two countries can be calculated as

$$N(\mu + r_U) = \int_{\mu - r_U}^{\mu + r_U} \frac{ax(\tilde{\theta})}{aF + x(\tilde{\theta})} d\theta + \frac{a2L}{aF + 2L}(\mu - r_U).$$

As the function for the market $x(\tilde{\theta})$ has two kinks at $\mu - r_E$ and $\mu + r_E$, this leads to the equation

$$\begin{aligned} N(\mu + r_U) = & \int_{\mu - r_U}^{\mu - r_E} \frac{a\left[(\mu + 3r_U - \theta)/(2r_U)\right]L}{aF + \left[(\mu + 3r_U - \theta)/(2r_U)\right]L} d\theta \\ & + \int_{\mu - r_E}^{\mu + r_E} \frac{a\left[\left([\mu - \tilde{\theta}][r_U + r_E] + 2r_U r_E\right)/(2r_U r_E)\right]L}{aF + \left[\left([\mu - \tilde{\theta}][r_U + r_E] + 2r_U r_E\right)/(2r_U r_E)\right]L} d\theta \\ & + \int_{\mu + r_E}^{\mu + r_U} \frac{a\left[(\mu + r_U - \theta)/(2r_U)\right]L}{aF + \left[(\mu + r_U - \theta)/(2r_U)\right]L} d\theta + \frac{a2L}{aF + 2L}(\mu - r_U), \end{aligned}$$

which yields after some algebra

$$\begin{aligned} N(\mu + r_U) = 2ar_U & \left[1 - \frac{aF}{L}\log\left(\frac{aF + 2L}{aF}\right)\right. \\ & \left. + \frac{aFr_U}{L(r_U + r_E)}\log\left(\frac{aF + \left[(3r_U + r_E)/(2r_U)\right]L}{aF + \left[(r_U - r_E)/(2r_U)\right]L}\right)\right] + \frac{a2L}{aF + 2L}(\mu - r_U). \end{aligned}$$

Additionally, the utilitarian welfare of *Equal*

$$U_E = \int_{\mu - r_E}^{\mu + r_E} g_E(\theta) L N(\theta) d\theta$$

and *Unequal*

$$U_U = \int_{\mu - r_U}^{\mu + r_U} g_U(\theta) L N(\theta) d\theta$$

can be calculated as

$$U_E = aL\mu - aF\left(2ar_U\left[\frac{r_E}{r_E + r_U} + \log\left(\frac{aF + 2L}{aF + \left[(3r_U + r_E)/(2r_U)\right]L}\right)\right.\right.$$
$$\left.\left.- \frac{r_U r_E}{L(r_U + r_E)^2}\left(aF + \frac{r_U - r_E}{2r_U}L\right)\log\left(\frac{aF + \left[(3r_U + r_E)/(2r_U)\right]L}{aF + \left[(r_U - r_E)/(2r_U)\right]L}\right)\right] + \frac{aL}{aF + 2L}(\mu - r_U)\right)$$

and

$$U_U = aL\mu - aF\left(2ar_U\left[\frac{r_U}{r_E + r_U} - \left(\frac{aF}{L} + 1\right)\log\left(\frac{aF + 2L}{aF + \left[(3r_U + r_E)/(2r_U)\right]L}\right)\right.\right.$$
$$- \frac{r_U r_E}{L(r_U + r_E)^2}\left(\frac{aFr_E}{r_U} - \frac{r_U - r_E}{2r_U}L\right)\log\left(\frac{aF + \left[(3r_U + r_E)/(2r_U)\right]L}{aF + \left[(r_U - r_E)/(2r_U)\right]L}\right)$$
$$\left.\left.- \frac{aF}{L}\log\left(\frac{aF + \left[(r_U - r_E)/(2r_U)\right]L}{aF}\right)\right] + \frac{aL}{aF + 2L}(\mu - r_U)\right)$$

respectively. Adding the utilitarian welfare of those two countries leads to the following simple formula

$$U_E + U_U = a\left[2L\mu - FN(\mu + r)\right],$$

which actually has the same intuition as in autarky. The overall utility, i.e. all consumed goods, is equal to the labor allocated in the production sector multiplied with the productivity.

G Split-up of the mass producers

Starting from the labor market clearing condition in *Equal*, it is possible to calculate the share of mass producers m. To decompose the integral, the quantity a firm can sell $x(\tilde{\theta})$ from

Table 1 is needed.

$$
\begin{aligned}
\mu L &= \int\limits_{\mu - r_U}^{\mu} \left(F + \frac{x(\theta)}{a}\right) N'(\theta) d\theta + m \left(F + \frac{2L}{a}\right) N(\mu - r_U) \\
\mu L &= \int\limits_{\mu - r_U}^{\mu} x(\theta) d\theta + m2L(\mu - r_U) \\
\mu &= \int\limits_{\mu - r_U}^{\mu - r_E} \left(\frac{\mu + 3r_U - \theta}{2r_U}\right) d\theta + \int\limits_{\mu - r_E}^{\mu} \left(\frac{(\mu - \theta)(r_U + r_E) + 2r_U r_E}{2r_U r_E}\right) d\theta + m2(\mu - r_U) \\
\mu &= \left[\frac{(\mu + 3r_U)\theta}{2r_U} - \frac{\theta^2}{4r_U}\right]_{\mu - r_U}^{\mu - r_E} + \left[\frac{(\mu r_U + \mu r_E)\theta}{2r_U r_E} - \frac{(r_U + r_E)\theta^2}{4r_U r_E} + \theta\right]_{\mu - r_E}^{\mu} + m2(\mu - r_U) \\
\mu &= \frac{-r_U r_E + 7r_U^2}{4r_U} + m2(\mu - r_U) \\
\mu &= \frac{7r_U - r_E}{4} + m2(\mu - r_U) \\
\Rightarrow m &= \frac{4\mu + r_E - 7r_U}{8(\mu - r_U)}
\end{aligned}
$$

An equilibrium where both countries have mass producers, emerges as long as $m \in (0, 1)$. *Unequal* has all mass producers, if

$$
\begin{aligned}
m &\leq 0 \\
4\mu &\leq 7r_U - r_E.
\end{aligned}
$$

Equal has never all mass producers, as $\mu \geq r_U \geq r_E$ and therefore

$$
\begin{aligned}
m &\geq 1 \\
r_E + r_U &\geq 4\mu \qquad \lightning
\end{aligned}
$$

is not possible.