MMP II
Exercise Sheet 11

FS 19
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https://www.physik.uzh.ch/en/teaching/PHY322

Issued: 7.5.2019
Due: 14.5.2019

Exercise 1 [Oscillations of a Triangle (9 points)]
In the lecture a qualitative analysis of the small oscillations of CH_4 was given. Work out the corresponding steps for a system of three equal point masses moving in a plane. The equilibrium positions form a regular triangle (symmetry group C_{3v}). The potential energy is invariant with respect to orthogonal transformations

$$V(R\vec{x}_1, R\vec{x}_2, R\vec{x}_3) = V(\vec{x}_1, \vec{x}_2, \vec{x}_3) \qquad \forall R \in O(2) \tag{1}$$

and permutations of the masses

$$V(\vec{x}_{\pi 1}, \vec{x}_{\pi 2}, \vec{x}_{\pi 3}) = V(\vec{x}_1, \vec{x}_2, \vec{x}_3) \qquad \forall \pi \in S_3 \ . \tag{2}$$

Let the representation ρ of C_{3v} be defined as in the lecture.

a) For each class K of conjugate elements write explicitly down $\rho(g)$ for an arbitrary $g \in K$.

b) Decompose ρ into a direct sum of irreducible representations.

c) The rigid rotations and the translations are "oscillations with frequency zero". One can show that the rigid rotations (translations resp.) form a 1-dimensional (2-dimensional resp.) subspace and that ρ restricted to this subspace is equivalent to the A_2-representation (E-representation resp.). How many non-trivial oscillations does the system have? More precisely, determine the number of non-vanishing eigenvalues and the number of corresponding linearly independent eigenvectors.

Exercise 2 [Matrix Exponential (6 points)]
Compute $\exp(tA)$ ($t \in \mathbb{R}$) for the following matrices A:

$$\text{a.)} \quad \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{b.)} \quad \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} .$$