
Exercise 1 *Chemical potential in two dimensions*

(a) Show that the chemical potential of a Fermi gas in two dimensions is given by:

$$\mu(T) = k_{\mathrm{B}}T \ln \left[\exp\left(\frac{\pi n \hbar^2}{m k_{\mathrm{B}} T} \right) - 1 \right], \tag{1}$$

where $n = N/L^2$ is the area density of particles. Hint: Use $N = \int_0^\infty D(\epsilon) f(\epsilon) d\epsilon$ and remember that $D(\epsilon)$ is a constant in two dimensions.

(b) In two dimensions, derive the Fermi energy E_{F} and express $\mu(0)$ in terms of the Fermi energy E_{F}.

(c) Make a plot of equation (1).

Exercise 2 *Fermi energy E_{F} and Fermi temperature T_{F}*

The atom ^{3}He has spin ½ and is a fermion. The density of liquid ^{3}He is $0.081\,\mathrm{g/cm^3}$ near $T = 0\,\mathrm{K}$. Calculate the Fermi energy E_{F} and Fermi temperature T_{F}.

Exercise 3 *Kinetic energy of an electron gas*

Show that the kinetic energy of a three-dimensional gas of N free electrons at 0 K is $U = \frac{3}{5} N E_{\mathrm{F}}$.

Exercise 4 *Occupation of states*

Plot $D(E) \cdot f(E,T)$ against the energy E in units of the chemical potential μ, where $D(E)$ is the electronic density of states in three dimensions and $f(E,T)$ is the Fermi-Dirac distribution. You may neglect all the pre-factors for your plot. Use the temperatures (in units of μ) $k_{\mathrm{B}}T = 10^{-4}$, 10^{-3}, and 10^{-2}. In the legend of your plot, write the corresponding temperatures in Kelvin. If you need any material parameters for your plot, take the ones for copper.