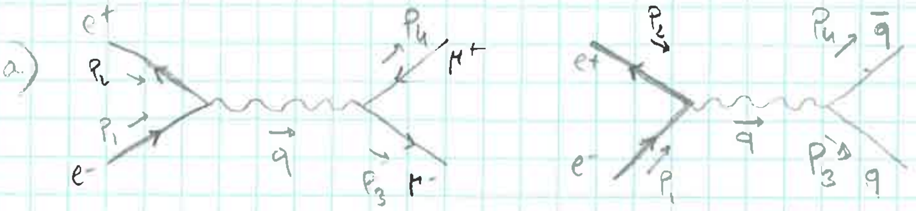
EXERCISE SHEET 4 - KT II

1) ELECTRON - POSITRON ANNIHILATION

$e^+e^- \to \mu^+\mu^-$, $e^+e^- \to q\bar{q}$

a)



b) $e^+e^- \to \mu^+\mu^-$

$$-iM = (2\pi)^4 \int dq^4\, \delta^4(p_1+p_2-q)\, \delta^4(p_3+p_4-q)$$
$$\times\, \bar{v}(2)\, i g_e \gamma^\mu u(1)\, \frac{i g_{\mu\nu}}{q^2}\, \bar{u}(3)\, i g_e \gamma^\nu v(4)$$

$$-iM = \frac{g_e^2}{(p_1+p_2)} [\bar{v}(2)\gamma^\mu u(1)][\bar{u}(3)\gamma_\mu v(4)]$$

$e^+e^- \to \bar{q}q$ $\to$ the matrix element is the same as for $e^+e^- \to \mu^+\mu^-$ but considering that the charge carried out by quarks is only a fraction of that carried by an electron, so

$$-iM = \frac{Q_q g_e^2}{p_1+p_2} [\bar{v}(2)\gamma^\mu u(1)][\bar{u}(3)\gamma_\mu v(4)] \times 3$$

where Q_q indicates the charge of the quark.

So $M(e^+e^- \to q\bar{q}) = Q_q\, M(e^+e^- \to q\bar{q})$

c) the total cross section of $e^+e^- \to q\bar{q}$ is equal to

$$\sigma(e^+e^- \to q\bar{q}) = 3Q_q^2\, \sigma(e^+e^- \to \mu^+\mu^-)$$

where the factor 3 takes into account the fact that we have a diagram per each colour of the quark.

Summing up over the different flavour quarks we get

$$\sigma(e^+e^- \to \text{hadrons}) = \sum_q \sigma(e^+e^- \to q\bar{q}) = 3 \sum_q Q_q^2 \, \sigma(e^+e^- \to \mu^+\mu^-)$$

where $\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3q^2}$

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3 \sum Q_q^2$$

d) $R_s = R(1 + \alpha_s/\pi)$

At $E_{cm} = 2.8$ GeV $\to$ u, d, s quarks can be produced
$E_{cm} = 5$ GeV $\to$ u, d, s, c
$E_{cm} = 15$ GeV $\to$ u, d, s, c, b

For $E_{cm} = 2.8$ GeV $\quad R = 3\left[\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2\right] = 2 \quad$ for u, d, s

For $E_{cm} = 5$ GeV $\quad R = 2 + 3\left(\frac{2}{3}\right)^2 = \frac{10}{3} \quad$ for u, d, s, c

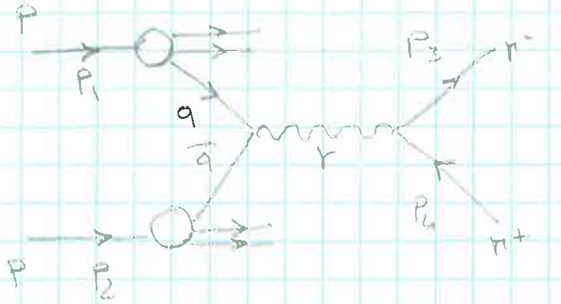
For $E_{cm} = 15$ GeV $\quad R = \frac{10}{3} + 3\left(\frac{1}{3}\right)^2 = \frac{11}{3} \quad$ for u, d, s, c, b

$\alpha_s(\mu^2 = E_{cm}^2) = \quad 0.27 \times E_{cm} = 2.8$ GeV
$\quad 0.2 \times E_{cm} = 5$ GeV $\qquad R_s = R(1 + \alpha_s/\pi)$
$\quad 0.18 \times E_{cm} = 15$ GeV

$R_s(E_{cm} = 2.8\,\text{GeV}) \simeq 2,17$
$R_s(E_{cm} = 5\,\text{GeV}) \simeq 3,54$
$R_s(E_{cm} = 15\,\text{GeV}) \simeq 3,88$

$$R = \frac{11}{3} + 3\left(\frac{2}{3}\right)^2 = \frac{15}{3} = 5$$

DRELL YAN PROCESS



$pp \longrightarrow \mu^+\mu^- X$

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3\hat{s}}$$

$$\sigma(q\bar{q} \to \mu^+\mu^-) = \frac{1}{N_c} Q_q^2 \frac{4\pi\alpha^2}{3\hat{s}} \longrightarrow \frac{1}{N_c} = \frac{1}{3}$$

this factor accounts for the fact that of the 9 possible combinations of $q\bar{q}$, the annihilation process can take place only for 3 $\to r\bar{r}$, $g\bar{g}$ and $b\bar{b}$

Quark and anti-quark bring only a fraction of the proton, namely x_1 and x_2 => $x_1 + \delta x_1$ and $x_2 + \delta x_2$

$$d^2\sigma = Q_u^2 \frac{4\pi\alpha^2}{3\cdot 3\hat{s}} u^p(x_1)\,dx_1\, \bar{u}^p(x_2)\,dx_2$$

$\hookrightarrow$ this comes from the definition of parton distribution function, PDF

$\bar{u}^p$ is the PDF for the anti-up-quark in the proton. the PDF of quarks in the proton are not the same as PDF of anti-quarks in proton

the centre of mass energy of the $q\bar{q}$ system can be written in terms of initial momenta of the protons:

$$\hat{s} = (x_1 P_1 + x_2 P_2)^2 = x_1^2 P_1^2 + x_2^2 P_2^2 + 2x_1 x_2 P_1 P_2$$

In high energy limit $P_1^2 = P_2^2 = m_p^2 \simeq 0 \implies \hat{s} \simeq 2x_1 x_2 P_1 P_2$

$\hat{s} \approx x_1 x_2 \, (2 P_1 \cdot P_2) = x_1 x_2 S \rightarrow S$ is the centre of mass of the colliding

pp system

$$d^2\sigma = \frac{4}{9} \cdot \frac{4\pi\alpha^2}{9 x_1 x_2 S} \, u^p(x_1)\, \bar{u}^p(x_2)\, dx_1 dx_2$$

We also have a contribution from $d\bar{d}$ annihilation

$$d^2\sigma = \frac{4\pi\alpha^2}{9x_1x_2S}\left[\frac{4}{9}\left\{u^p(x_1)\bar{u}^p(x_2)\right\} + \frac{1}{9}\left\{d^p(x_1)\bar{d}^p(x_2)\right\} + \frac{4}{9}\left\{\bar{u}^p(x_1)u^p(x_2)\right\} + \frac{1}{9}\left\{\bar{d}^p(x_1)d^p(x_2)\right\}\right]dx_1 dx_2$$

$\rightarrow$ We do not have just uud in the proton but rather gluon exchange that can fluctuate into virtual $q\bar{q}$ pairs. These sea and quark gluons are suppressed by a factor $1/q^2$, so they are produced mainly at low x.

$\Rightarrow$ This means we have $u(x) = u_v(x) + u_s(x)$ $\rightarrow$ accounting for the contribution from valence and sea up quarks

Concerning antiquarks $\rightarrow$ there are only sea quark contributions

$\bar{u}(x) = \bar{u}_s(x)$ and $\bar{d}(x) = \bar{d}_s(x)$

Given that the proton consists of 2 valence quarks and one valence down quark the valence quark PDFs are normalized differently

$$\int_0^1 u_v(x)\,dx = 2 \qquad \int_0^1 d_v(x)\,dx = 1$$

Few assumptions $q_s(x) = \bar{q}_s(x)$ because sea quarks and antiquarks are produced in couple

Moreover $m_u \sim m_d \Rightarrow$ sea PDFs can be assumed to be approximately the same

$$u_s(x) = \bar{u}_s(x) = d_s(x) = \bar{d}_s(x) \simeq S(x)$$

In the proton the normalisation factor for valence up quarks is 2

$\Rightarrow u_v(x) = 2 d_v(x)$

But, let's write the cross section in terms of valence and sea quarks.

$$d^2\sigma_{DY}^{pp} = \frac{4\pi\alpha^2}{9x_1x_2s}\frac{1}{9}\Big\{4\cdot u_v(x_1)\,\bar{u}_s(x_2) + d_v(x_1)\,\bar{d}_s(x_2) + 4\,\bar{u}_s(x_1)\,u_v(x_2)$$

$$+ \bar{d}_s(x_1)\,d_v(x_2) + \text{the same with } u_s(x) \text{ and } d_s(x)$$

$$= \frac{4\pi\alpha^2}{81x_1x_2s}\Big\{8d_v(x_1)S(x_2) + d_v(x_1)S(x_2) + 8S(x_1)d_v(x_2)$$

$$+ S(x_1)d_v(x_2) + 10S(x_1)S(x_2)\Big\}$$

$$= \frac{4\pi\alpha^2}{81x_1x_2s}\Big\{9d_v(x_1)S(x_2) + 9S(x_1)d_v(x_2) + 10S(x_1)S(x_2)\Big\}$$

In the case of $p\bar{p}$ collisions both $\bar{u}$ and $\bar{d}$ can be of valence, and we have

$$d^2\sigma_{DY}^{p\bar{p}} = \frac{4\pi\alpha^2}{81x_1x_2s}\Big\{17\,d_v(x_1)\,d_v(x_2) + 9\,d_v(x_1)S(x_2) + 9S(x_1)d_v(x_2) +$$

$$+ 10\,S(x_1)\,S(x_2)\Big\}dx_1dx_2$$

c) $$K = \frac{\sigma(\pi^+ C \to \mu^+\mu^- X)}{\sigma(\pi^- C \to \mu^+\mu^- X)}$$

$K \to 1$ for small Q^2/s

$K \to 1/4$ for $Q^2/s \to 1$

$\pi^+ \to u\bar{d}$ / $\pi^- \to \bar{u}d$

the PDFs for the 2 pions can be written in terms of valence and sea quarks:

$u^{\pi^+}(x) = u_v^{\pi^+}(x) + S^{\pi^+}(x) = u_v^{\pi}(x) + S^{\pi}(x)$

$\bar{d}^{\pi^+}(x) = \bar{d}_v^{\pi^+}(x) + S^{\pi^+}(x) = \bar{d}_v^{\pi}(x) + S^{\pi}(x)$

$\bar{u}^{\pi^+}(x) = S^{\pi^+}(x) = S^{\pi}(x)$

$d^{\pi^+}(x) = S^{\pi^+}(x) = S^{\pi}(x)$

Assuming isospin symmetry the down-quark PDF in π^- is identical to the up-quark PDF in π^+

$u^{\pi^-}(x) = S^{\pi^-}(x)$

$\bar{d}^{\pi^-}(x) = S^{\pi^-}(x)$

$\bar{u}^{\pi^-}(x) = \bar{u}_v^{\pi^-}(x) + S^{\pi}(x)$

$d^{\pi^-}(x) = d_v^{\pi^-}(x) + S^{\pi}(x)$

When $Q^2/s \to 0 \Rightarrow x_1 \simeq x_2 \simeq 0 \Rightarrow$ sea quarks dominate and there is no difference between π^+C and π^-C

When $Q^2/s \simeq 1$ $x_1 \simeq x_2 \simeq 1$ and the valence quarks dominate.

Putomming isospin symmetry $[d^{\pi}(x) = u^{\pi^+}(x)]$ and considering that C has the same number of up and down quarks $\Rightarrow$

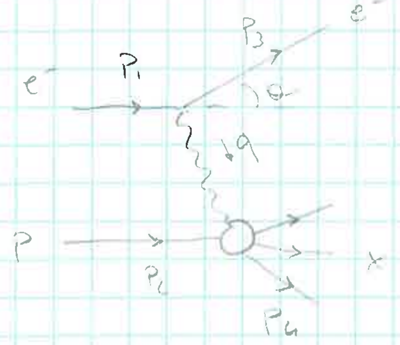
$$\frac{\sigma(\pi^+ C \to \mu^+\mu^- X)}{\sigma(\pi^- C \to \mu^+\mu^- X)} = \frac{e_d^2\, f_d^{\pi^+}\, f_d^C}{e_u^2\, f_u^{\pi^-}\, f_u^C} = \frac{e_d^2}{e_u^2} = \frac{1}{4}$$

$\Rightarrow$ d and u are valence quarks in C, but $\bar{u}$ and $\bar{d}$ are sea quarks

ELECTRON - PROTON INELASTIC SCATTERING

$$X = \frac{Q^2}{2P_2 \cdot q} \qquad \nu = \frac{P_2 \cdot q}{m_p}$$

$$\Rightarrow 2m_p \nu = W^2 + Q^2 - m_p^2$$



$Q^2 = -q^2 \Rightarrow$ negative four-momentum squared of the virtual photon

W is the four-momentum of the hadronic system (X)

$$W^2 = P_4^2 = (q + P_2)^2 = q^2 + 2P_2 \cdot q + P_2^2 = -Q^2 + 2P_2 \cdot q + m_p^2$$

but $\nu = \frac{P_2 \cdot q}{m_p} \Rightarrow W^2 + Q^2 - m_p^2 = 2\nu m_p$

In the rest frame of the proton $P_2 = (m_p, 0, 0, 0)$, the momenta of the initial state e^-, the final state e^- and the virtual photon can be written as:

$$P_1 = (E_1, 0, 0, E_1), \quad P_3 = (E_3, E_3 \sin\theta, 0, E_3 \cos\theta)$$
and $q = (E_1 - E_3, \vec{P}_1 - \vec{P}_3)$

$$\nu = \frac{P_2 \cdot q}{m_p} = \frac{1}{m_p} m_p (E_1 - E_3) = E_1 - E_3$$ $\Rightarrow$ this is simply the energy lost by the electron.