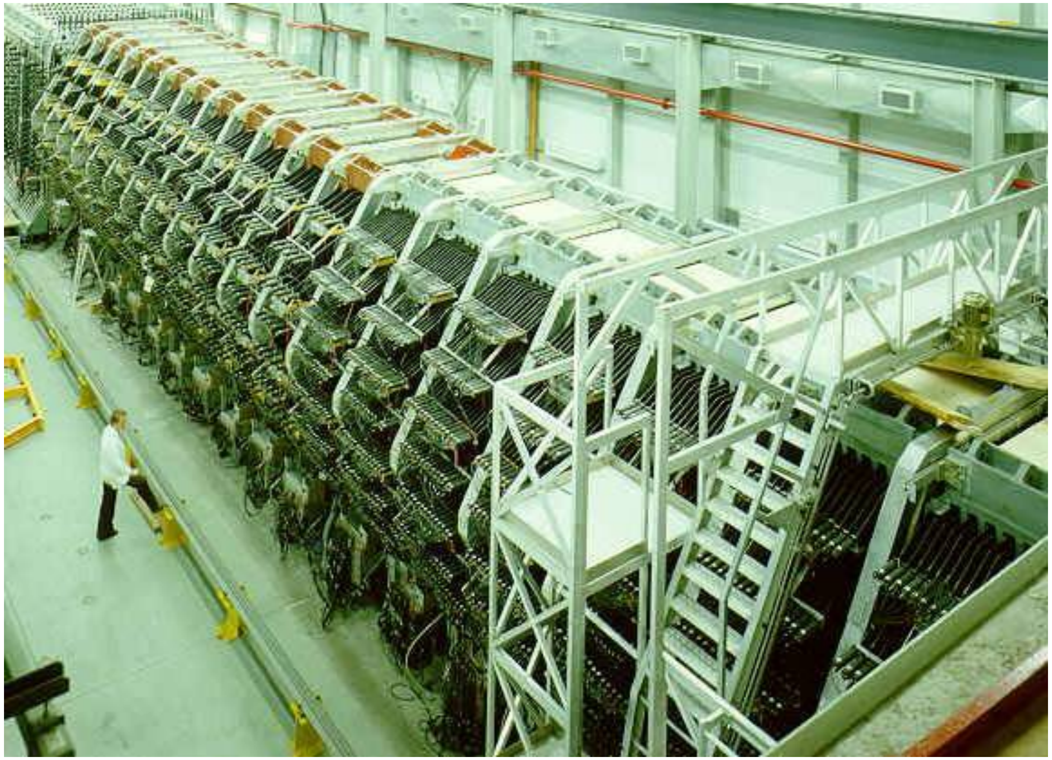
Particle Physics

Handout from Prof. Mark Thomson's lectures
Adapted to UZH by Prof. Canelli and Prof. Serra

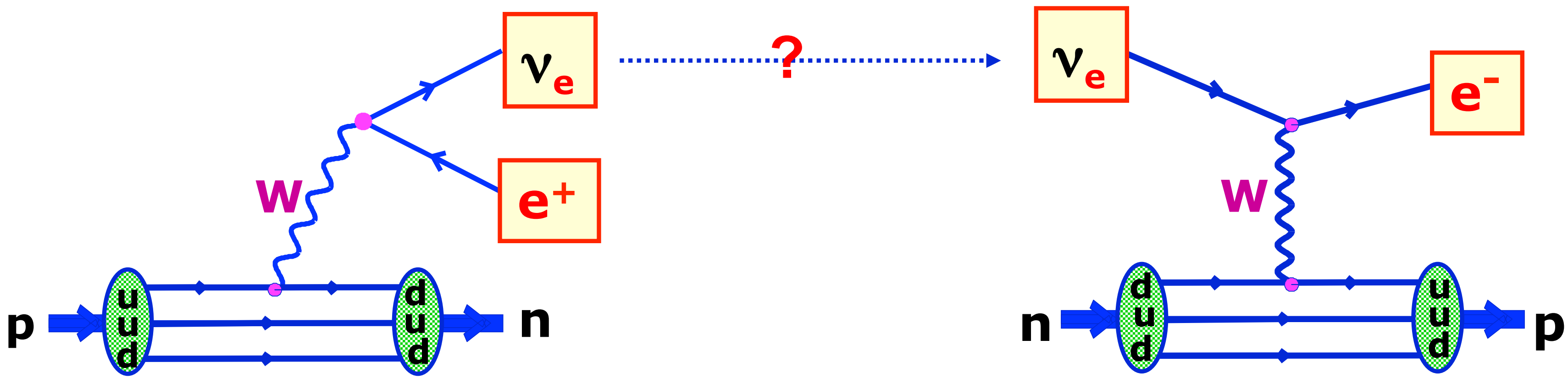


Handout 10 : Leptonic Weak Interactions and Neutrino Deep Inelastic Scattering

Aside : Neutrino Flavours

- ★ Recent experiments (see Handout 11) ➡ neutrinos have mass (albeit very small)
- ★ The textbook neutrino states, ν_e, ν_μ, ν_τ, are not the fundamental particles; these are ν_1, ν_2, ν_3
- ★ Concepts like "electron number" conservation are now known **not** to hold.
- ★ So what are ν_e, ν_μ, ν_τ ?
- ★ Never **directly** observe neutrinos – can only detect them by their weak interactions. Hence by **definition** ν_e is the neutrino state produced along with an electron. Similarly, charged current weak interactions of the state ν_e produce an electron

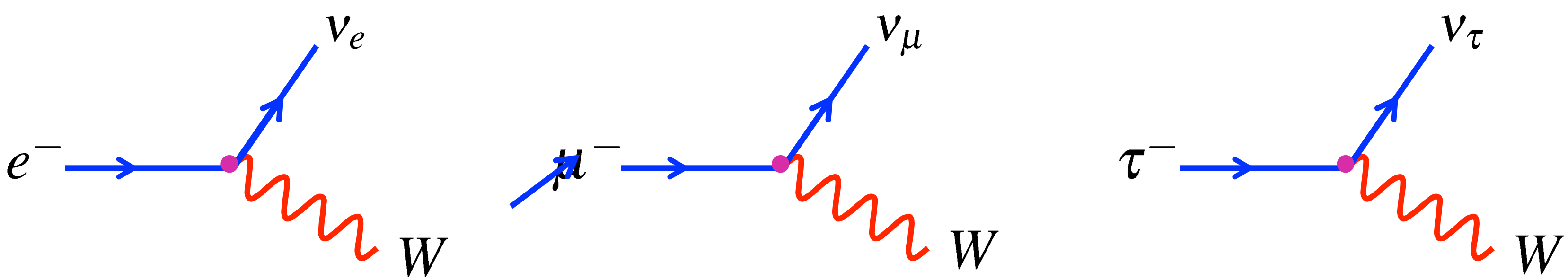
➡ ν_e, ν_μ, ν_τ = weak eigenstates



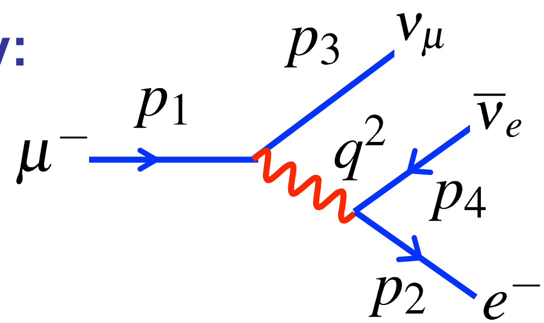
- ★ Unless dealing with very large distances: the neutrino produced with a positron will interact to produce an electron. For the discussion of the **weak interaction** continue to use ν_e, ν_μ, ν_τ as if they were the fundamental particle states.

Muon Decay and Lepton Universality

★The leptonic **charged current** ($W^\pm$) interaction vertices are:



★Consider muon decay:



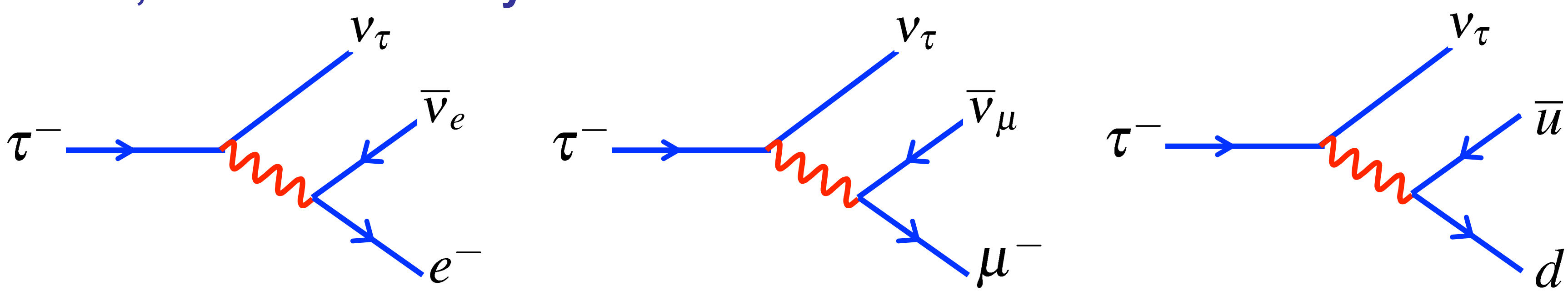
•It is straight-forward to write down the matrix element

$$M_{fi} = \frac{g_W^{(e)} g_W^{(\mu)}}{8m_W^2}[\overline{u}(p_3)\gamma^\mu(1-\gamma^5)u(p_1)]g_{\mu\nu}[\overline{u}(p_2)\gamma^\nu(1-\gamma^5)v(p_4)]$$

Note: for lepton decay $q^2 \ll m_W^2$ so propagator is a constant $1/m_W^2$
i.e. in limit of Fermi theory

•Its evaluation and subsequent treatment of a three-body decay is rather tricky (and not particularly interesting). Here will simply quote the result

- The muon to electron rate $\Gamma(\mu \to e\nu\nu) = \frac{G_F^e G_F^\mu m_\mu^5}{192\pi^3} = \frac{1}{\tau_\mu}$ with $G_F = \frac{g_W^2}{4\sqrt{2}m_W^2}$
- Similarly for tau to electron $\Gamma(\tau \to e\nu\nu) = \frac{G_F^e G_F^\tau m_\tau^5}{192\pi^3}$
- However, the tau can decay to a number of final states:



$Br(\tau \to e\nu\nu) = 0.1784(5)$ $\quad Br(\tau \to \mu\nu\nu) = 0.1736(5)$

- Recall total width (total transition rate) is the sum of the partial widths

$$\Gamma = \sum_i \Gamma_i = \frac{1}{\tau}$$

- Can relate partial decay width to total decay width and therefore lifetime:

$$\Gamma(\tau \to e\nu\nu) = \Gamma_\tau Br(\tau \to e\nu\nu) = Br(\tau \to e\nu\nu)/\tau_\tau$$

- Therefore predict

$$\tau_\mu = \frac{192\pi^3}{G_F^e G_F^\mu m_\mu^5} \qquad \tau_\tau = \frac{192\pi^3}{G_F^e G_F^\tau m_\tau^5} Br(\tau \to e\nu\nu)$$

• All these quantities are precisely measured:

$$m_\mu = 0.1056583692(94)\,\mathrm{GeV} \qquad \tau_\mu = 2.19703(4)\times 10^{-6}\,\mathrm{s}$$

$$m_\tau = 1.77699(28)\,\mathrm{GeV} \qquad \tau_\tau = 0.2906(10)\times 10^{-12}\,\mathrm{s}$$

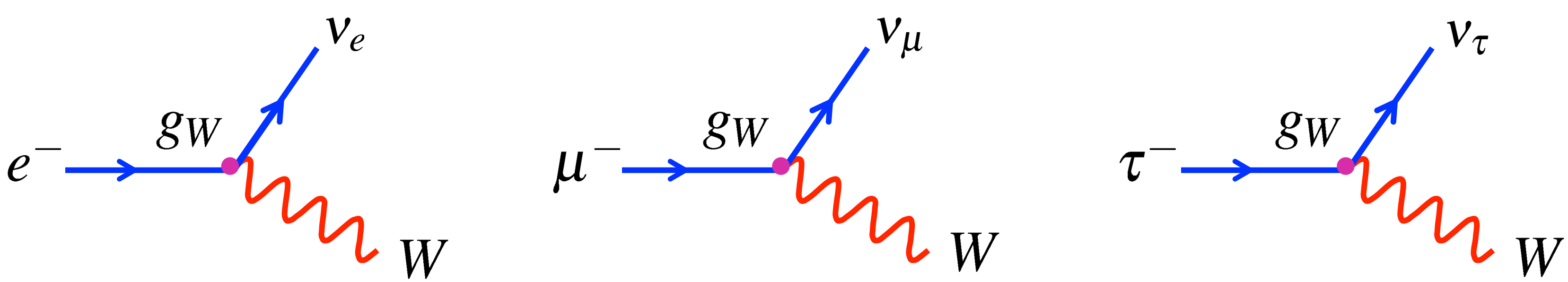
$$Br(\tau \to e\nu\nu) = 0.1784(5)$$

$$\Rightarrow \quad \frac{G_F^\tau}{G_F^\mu} = \frac{m_\mu^5 \tau_\mu}{m_\tau^5 \tau_\tau} Br(\tau \to e\nu\nu) = 1.0024 \pm 0.0033$$

• Similarly by comparing $Br(\tau \to e\nu\nu)$ and $Br(\tau \to \mu\nu\nu)$

$$\frac{G_F^e}{G_F^\mu} = 1.000 \pm 0.004$$

★ Demonstrates the weak charged current is the same for all leptonic vertices



$\Rightarrow$ **Charged Current Lepton Universality**

Neutrino Scattering

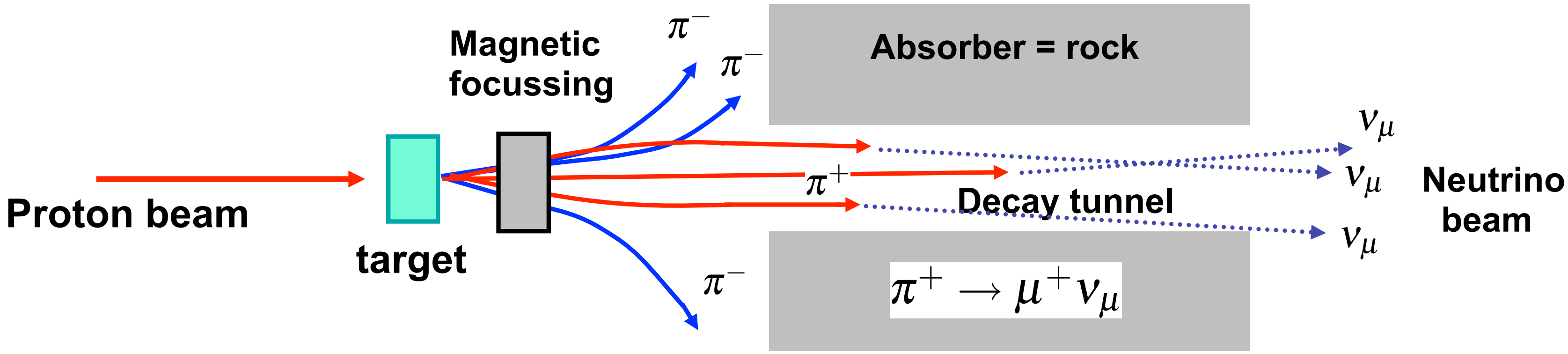
- In handout 6 considered electron-proton **Deep Inelastic Scattering** where a virtual photon is used to probe nucleon structure
- Can also consider the weak interaction equivalent: **Neutrino Deep Inelastic Scattering** where a virtual W-boson probes the structure of nucleons

  ➡ additional information about parton structure functions

  \+ provides a good example of calculations of weak interaction cross sections.

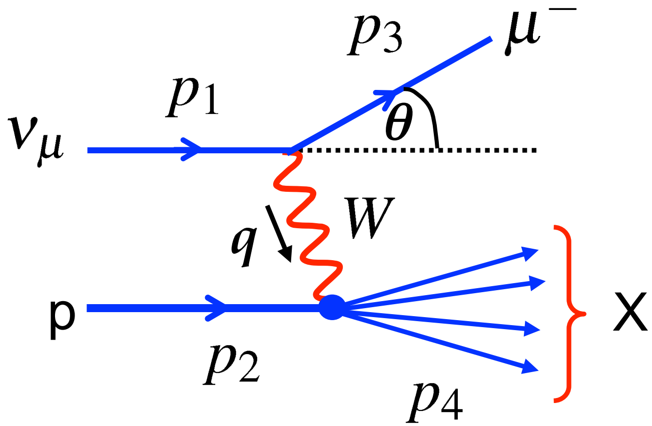
★ Neutrino Beams:

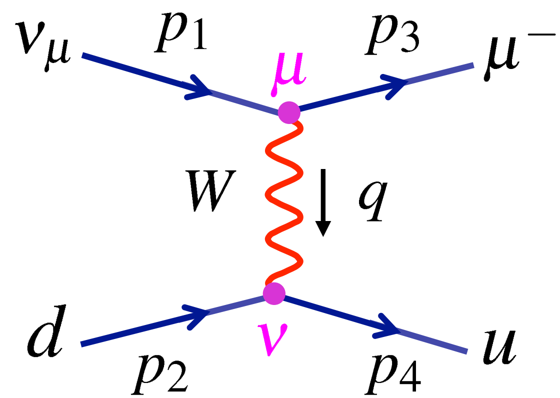
- Smash high energy protons into a fixed target ➡ hadrons
- Focus positive pions/kaons
- Allow them to decay $\pi^+ \rightarrow \mu^+ \nu_\mu$ + $K^+ \rightarrow \mu^+ \nu_\mu$ $(BR \approx 64\%)$
- Gives a beam of "collimated" ν_μ
- Focus negative pions/kaons to give beam of $\overline{\nu}_\mu$



Neutrino-Quark Scattering

★For ν_μ -proton Deep Inelastic Scattering the underlying process is $\nu_\mu d \rightarrow \mu^- u$





★In the limit $q^2 \ll m_W^2$ the W-boson propagator is $\approx ig_{\mu\nu}/m_W^2$

•The Feynman rules give:

$$-iM_{fi} = \left[-i\frac{g_W}{\sqrt{2}}\overline{u}(p_3)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u(p_1)\right]\frac{ig_{\mu\nu}}{m_W^2}\left[-i\frac{g_W}{\sqrt{2}}\overline{u}(p_4)\tfrac{1}{2}\gamma^\nu(1-\gamma^5)u(p_2)\right]$$

$$M_{fi} = \frac{g_W^2}{2m_W^2}g_{\mu\nu}\left[\overline{u}(p_3)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u(p_1)\right]\left[\overline{u}(p_4)\tfrac{1}{2}\gamma^\nu(1-\gamma^5)u(p_2)\right]$$

•Evaluate the matrix element in the extreme relativistic limit where the muon and quark masses can be neglected

•In this limit the helicity states are equivalent to the chiral states and

$$\tfrac{1}{2}(1-\gamma^5)u_\uparrow(p_1)=0 \qquad \tfrac{1}{2}(1-\gamma^5)u_\downarrow(p_1)=u_\downarrow(p_1)$$

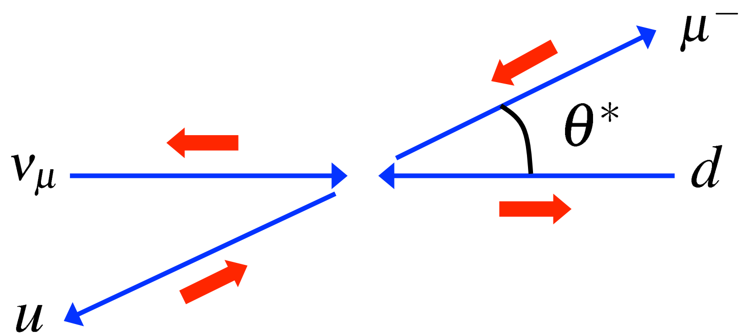
➡ $M_{fi}=0$ **for** $u_\uparrow(p_1)$ **and** $u_\uparrow(p_2)$

•Since the weak interaction "conserves the helicity", the only helicity combination where the matrix element is non-zero is

$$M_{fi}=\frac{g_W^2}{2m_W^2}g_{\mu\nu}\left[\overline{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1)\right]\left[\overline{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2)\right]$$

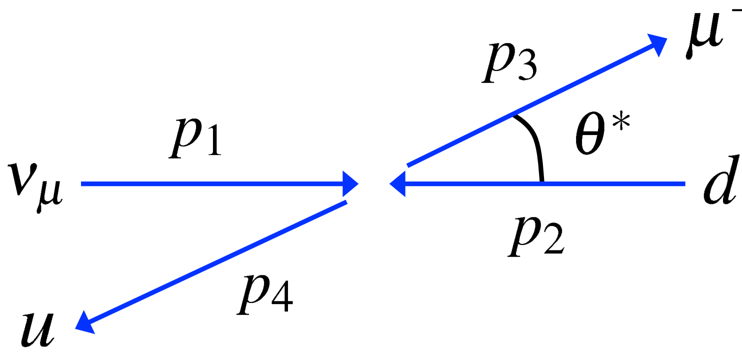
NOTE: we could have written this down straight away as in the ultra-relativistic limit only LH helicity particle states participate in the weak interaction.

★Consider the scattering in the C.o.M frame



Evaluation of Neutrino-Quark Scattering ME

- Go through the calculation in gory detail (fortunately only one helicity combination)
- In the $\nu_\mu d$ CMS frame, neglecting particle masses:



$$p_1 = (E, 0, 0, E),$$
$$p_2 = (E, 0, 0, -E)$$
$$p_3 = (E, E\sin\theta^*, 0, E\cos\theta^*)$$
$$p_4 = (E, -E\sin\theta^*, 0, -E\cos\theta^*)$$

- Dealing with LH helicity particle spinors. From handout 3 (p.80), for a massless particle travelling in direction (θ, ϕ):

$$u_\downarrow = \sqrt{E}\begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix} \qquad c = \cos\frac{\theta}{2}; \quad s = \sin\frac{\theta}{2}$$

- Here $(\theta_1, \phi_1) = (0, 0)$; $(\theta_2, \phi_2) = (\pi, 0)$; $(\theta_3, \phi_3) = (\theta^*, 0)$; $(\theta_4, \phi_4) = (\pi - \theta^*, \pi)$

giving:

$$u_\downarrow(p_1) = \sqrt{E}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix};\; u_\downarrow(p_2) = \sqrt{E}\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix};\; u_\downarrow(p_3) = \sqrt{E}\begin{pmatrix} -s \\ c \\ s \\ -c \end{pmatrix};\; u_\downarrow(p_4) = \sqrt{E}\begin{pmatrix} -c \\ -s \\ c \\ s \end{pmatrix}$$

- To calculate

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\overline{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1)\right] \left[\overline{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2)\right]$$

need to evaluate two terms of form

$$\begin{aligned}
\overline{\psi}\gamma^0\phi &= \psi^\dagger\gamma^0\gamma^0\phi = \psi_1^*\phi_1 + \psi_2^*\phi_2 + \psi_3^*\phi_3 + \psi_4^*\phi_4 \\
\overline{\psi}\gamma^1\phi &= \psi^\dagger\gamma^0\gamma^1\phi = \psi_1^*\phi_4 + \psi_2^*\phi_3 + \psi_3^*\phi_2 + \psi_4^*\phi_1 \\
\overline{\psi}\gamma^2\phi &= \psi^\dagger\gamma^0\gamma^2\phi = -i(\psi_1^*\phi_4 - \psi_2^*\phi_3 + \psi_3^*\phi_2 - \psi_4^*\phi_1) \\
\overline{\psi}\gamma^3\phi &= \psi^\dagger\gamma^0\gamma^3\phi = \psi_1^*\phi_3 - \psi_2^*\phi_4 + \psi_3^*\phi_1 - \psi_4^*\phi_2
\end{aligned}$$

- Using

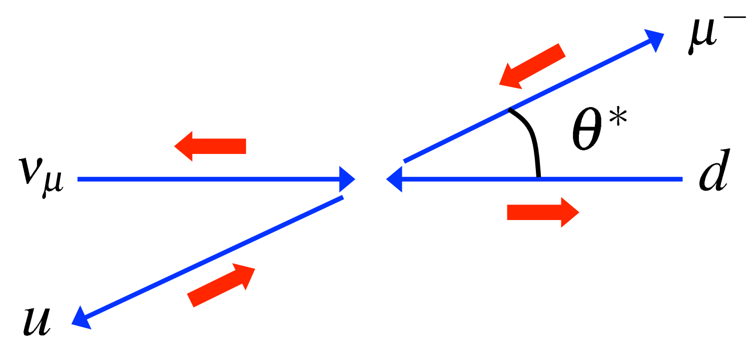
$$u_\downarrow(p_1) = \sqrt{E}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix};\; u_\downarrow(p_2) = \sqrt{E}\begin{pmatrix}-1\\0\\1\\0\end{pmatrix};\; u_\downarrow(p_3) = \sqrt{E}\begin{pmatrix}-s\\c\\s\\-c\end{pmatrix};\; u_\downarrow(p_4) = \sqrt{E}\begin{pmatrix}-c\\-s\\c\\s\end{pmatrix}$$

$$\Rightarrow \quad \begin{aligned}
\overline{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1) &= 2E(c, s, -is, c) \\
\overline{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2) &= 2E(c, -s, -is, -c)
\end{aligned}$$

$$\Rightarrow \quad M_{fi} = \frac{g_W^2}{2m_W^2} 4E^2(c^2 + s^2 + s^2 + c^2) = \frac{g_W^2 \hat{s}}{m_W^2} \qquad \hat{s} = (2E)^2$$

★ **Note the Matrix Element is isotropic**

$$M_{fi} = \frac{g_W^2}{m_W^2}\hat{s}$$



we could have anticipated this since the helicity combination (spins anti-parallel) has $S_z = 0$ ➡ no preferred polar angle

★ **As before need to sum over all possible spin states and average over all possible initial state spin states. Here only one possible spin combination (LL→LL) and only 2 possible initial state combinations (the neutrino is always produced in a LH helicity state)**

$$\langle |M_{fi}|^2 \rangle = \frac{1}{2} \cdot \left| \frac{g_W^2}{m_W^2}\hat{s} \right|^2$$

The factor of a half arises because half of the time the quark will be in a RH states and won't participate in the charged current Weak interaction

★ **From handout 1, in the extreme relativistic limit, the cross section for any 2→2 body scattering process is**

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 \hat{s}} \langle |M_{fi}|^2 \rangle$$

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2\hat{s}}\langle|M_{fi}|^2\rangle = \frac{1}{64\pi\hat{s}}\frac{1}{2}\left(\frac{g_W^2\hat{s}}{m_W^2}\right)^2 = \left(\frac{g_W^2}{8\sqrt{2}\pi m_W^2}\right)^2\hat{s}$$

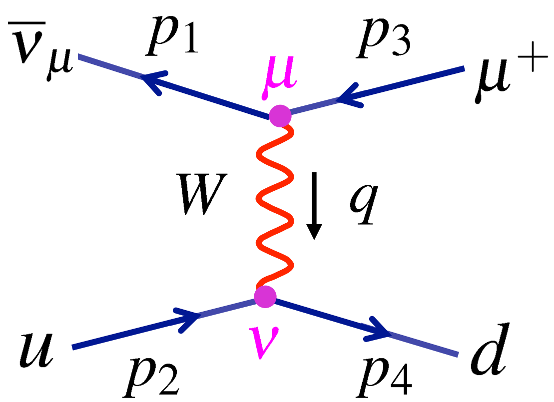
using $\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}$ → $$\boxed{\frac{d\sigma}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}}$$

★Integrating this isotropic distribution over $d\Omega^*$

→ $$\boxed{\sigma_{\nu q} = \frac{G_F^2\hat{s}}{\pi}} \qquad \textbf{(1)}$$

•cross section is a Lorentz invariant quantity so this is valid in any frame

Antineutrino-Quark Scattering



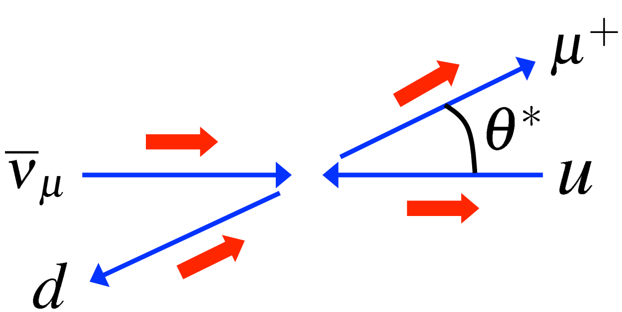
- In the ultra-relativistic limit, the charged-current interaction matrix element is:

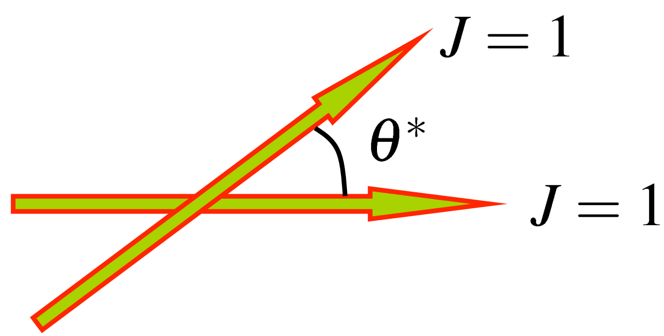
$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\overline{v}(p_1)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)v(p_3)\right] \left[\overline{u}(p_4)\gamma^\nu \tfrac{1}{2}(1-\gamma^5)u(p_2)\right]$$

★ In the extreme relativistic limit only **LH Helicity particles** and **RH Helicity anti-particles** participate in the charged current weak interaction:

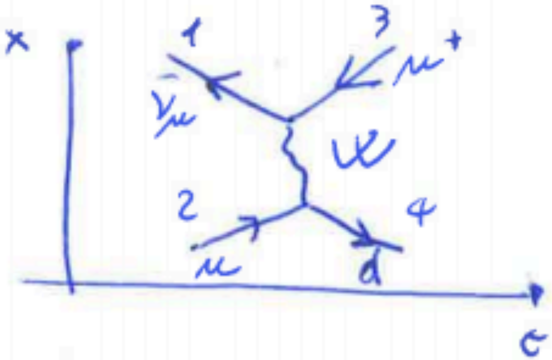
$$\Rightarrow \quad M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\overline{v}_\uparrow(p_1)\gamma^\mu v_\uparrow(p_3)\right] \left[\overline{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2)\right]$$

★ In terms of the particle spins it can be seen that the interaction occurs in a total angular momentum 1 state





Antineutrino-Quark Scattering

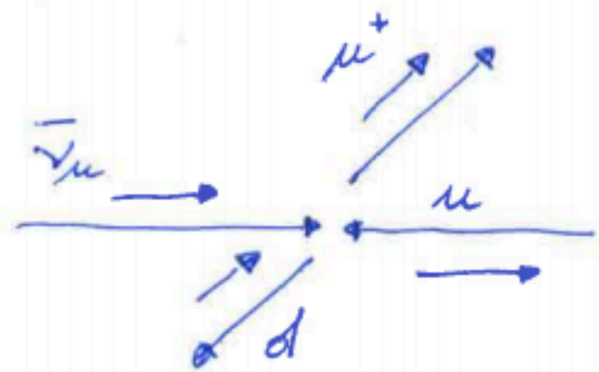


$$M \propto \left\{ \bar{v}_{\uparrow}(1)\, \gamma^{\mu}\, v_{\uparrow}(3) \right\} \left\{ \bar{u}_{\downarrow}(4)\, \gamma_{\mu}\, u_{\downarrow}(2) \right\}$$

$$\bar{u}_{\downarrow}(4)\, \gamma^{\nu}\, u_{\downarrow}(2) = 2E\,(c, -s, -is, -c)$$

quark is LH in the massless limit

$$v_{\uparrow} = \sqrt{E} \begin{pmatrix} s \\ -c\,e^{i\phi} \\ -s \\ c\,e^{i\phi} \end{pmatrix}$$



$J = 1$

$J = 1$

Antineutrino-Quark Scattering

$$
\begin{aligned}
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\overline{\psi}\gamma^1\phi &= \psi^\dagger\gamma^0\gamma^1\phi = \psi_1^*\phi_4 + \psi_2^*\phi_3 + \psi_3^*\phi_2 + \psi_4^*\phi_1 \\
\overline{\psi}\gamma^2\phi &= \psi^\dagger\gamma^0\gamma^2\phi = -i(\psi_1^*\phi_4 - \psi_2^*\phi_3 + \psi_3^*\phi_2 - \psi_4^*\phi_1) \\
\overline{\psi}\gamma^3\phi &= \psi^\dagger\gamma^0\gamma^3\phi = \psi_1^*\phi_3 - \psi_2^*\phi_4 + \psi_3^*\phi_1 - \psi_4^*\phi_2
\end{aligned}
$$

$$v_\uparrow(1) = \begin{pmatrix}0\\-1\\0\\1\end{pmatrix}\sqrt{E} \qquad v_\uparrow(3) = \begin{pmatrix}s\\-c\\-s\\c\end{pmatrix}$$

$$\bar{v}(1)_\uparrow\gamma^\mu v_\uparrow(3) = 2E(c, s, is, c)$$

$$\bar{u}_\downarrow(4)\gamma^\nu u_\downarrow(2) = 2E(c, -s, -is, -c)$$

$$M_{fi} = \frac{g_W^2}{2m_W^2}4E^2(c^2+s^2-s^2+c^2) =$$

$$= \frac{g_W^2}{2m_W^2}\hat{s}\left(\cos^2\frac{\theta^*}{2}\right) = \frac{g_W^2\hat{s}}{8m_W^2}\frac{1}{2}(1+\cos\theta^*)$$

Anti-neutrino scattering to protons

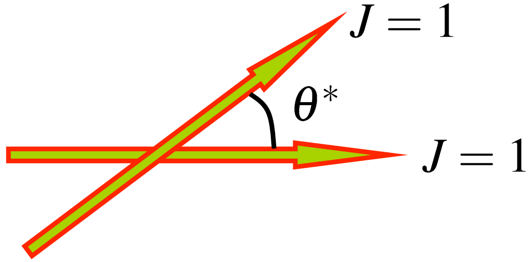
$$M_{fi} = \frac{g_W^2}{2m_W^2}4E^2(c^2+s^2+s^2+c^2) = \frac{g_W^2\hat{s}}{m_W^2}$$

neutrino scattering to protons

★In a similar manner to the neutrino-quark scattering calculation obtain:

$$\frac{d\sigma_{\overline{\nu}q}}{d\Omega^*} = \frac{d\sigma_{\nu q}}{d\Omega^*}\frac{1}{4}(1+\cos\theta^*)^2$$

•The factor $\frac{1}{4}(1+\cos\theta^*)^2$ can be understood in terms of the overlap of the initial and final angular momentum wave-functions



★Similarly to the neutrino-quark scattering calculation obtain:

$$\frac{d\sigma_{\overline{\nu}q}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$$

$$d\Omega = d\phi\sin\theta d\theta \rightarrow d\phi d(\cos\theta)$$

★Integrating over solid angle:

$$\int(1+\cos\theta^*)^2 d\Omega^* = \int(1+\cos\theta^*)^2 d(\cos\theta^*)d\phi = 2\pi\int_{-1}^{+1}(1+\cos\theta^*)^2 d(\cos\theta^*) = \frac{16\pi}{3}$$

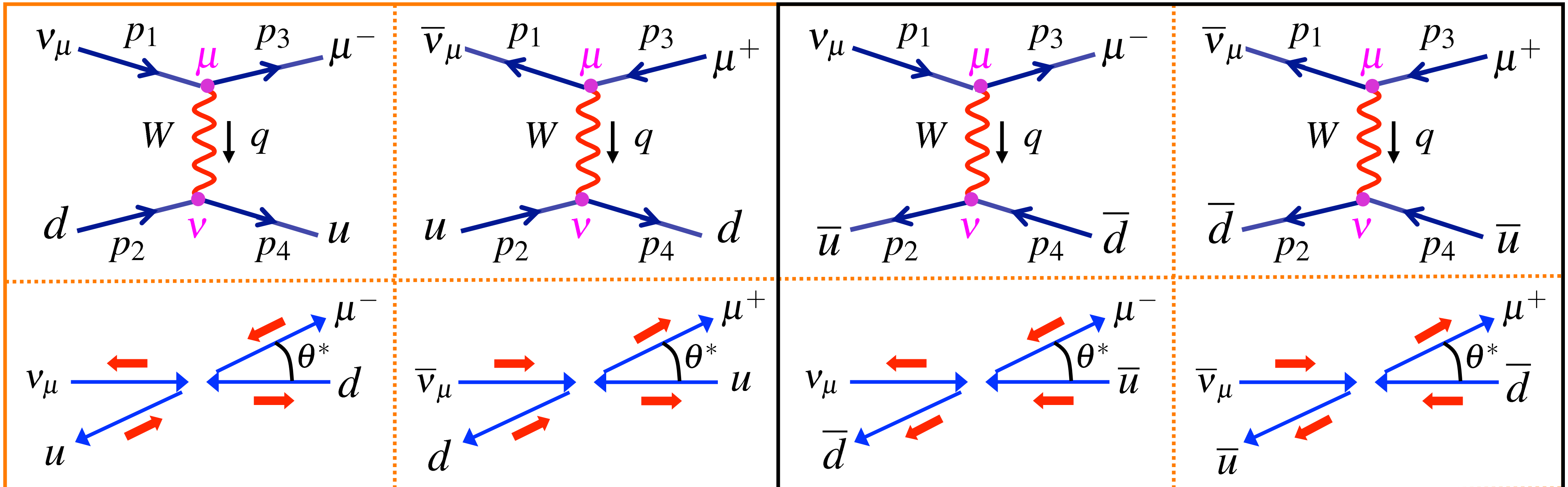
$$\Rightarrow \boxed{\sigma_{\overline{\nu}q} = \frac{G_F^2\hat{s}}{3\pi}}$$

★This is a factor three smaller than the neutrino quark cross-section

$$\boxed{\frac{\sigma_{\overline{\nu}q}}{\sigma_{\nu q}} = \frac{1}{3}}$$

(Anti)neutrino-(Anti)quark Scattering

- Non-zero anti-quark component to the nucleon ⇒ also consider scattering from $\overline{q}$
- Cross-sections can be obtained immediately by comparing with quark scattering and remembering to only include **LH particles and RH anti-particles**

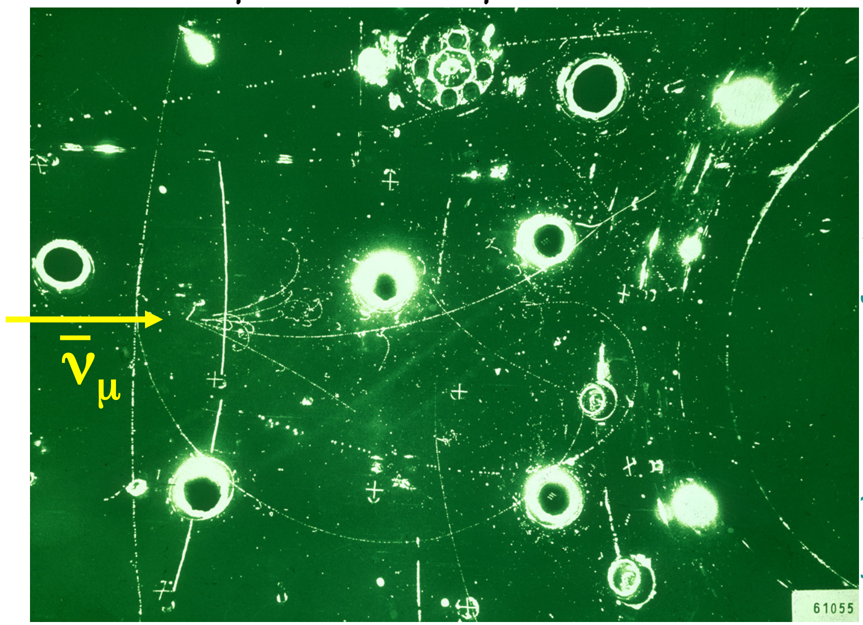


$S_z = 0$	$S_z = +1$	$S_z = -1$	$S_z = 0$
$\frac{d\sigma_{\nu q}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$	$\frac{d\sigma_{\overline{\nu} q}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$	$\frac{d\sigma_{\nu \overline{q}}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$	$\frac{d\sigma_{\overline{\nu}\overline{q}}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$
$\sigma_{\nu q} = \frac{G_F^2\hat{s}}{\pi}$	$\sigma_{\overline{\nu} q} = \frac{G_F^2\hat{s}}{3\pi}$	$\sigma_{\nu \overline{q}} = \frac{G_F^2\hat{s}}{3\pi}$	$\sigma_{\overline{\nu}\overline{q}} = \frac{G_F^2\hat{s}}{\pi}$

Weak Neutral Current

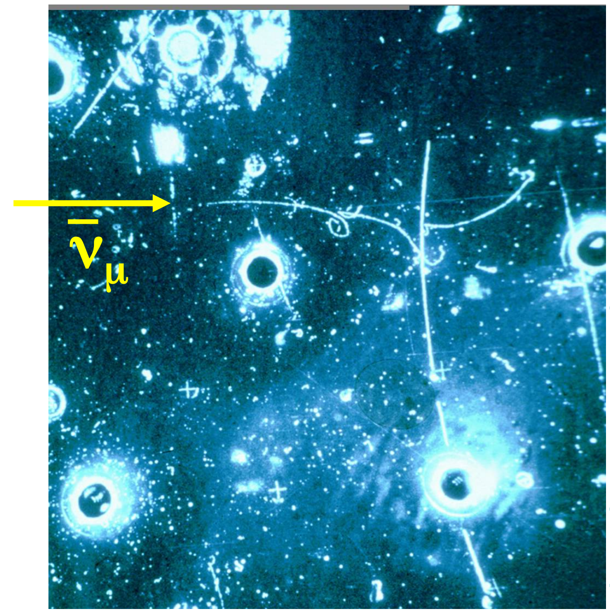
★ Neutrinos also interact via the Neutral Current. First observed in the Gargamelle bubble chamber in 1973. Interaction of muon neutrinos produce a final state muon

$$\overline{\nu}_\mu + N \rightarrow \overline{\nu}_\mu + \text{hadrons}$$



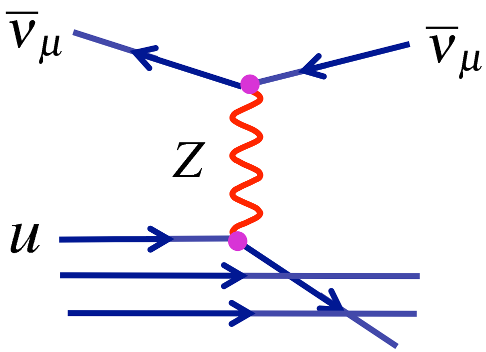
F.J. Hasert et al., Phys. Lett. 46B (1973) 138

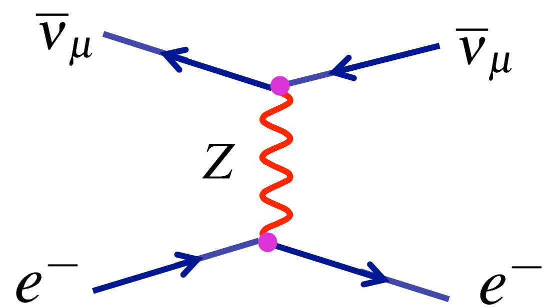
$$\overline{\nu}_\mu + e^- \rightarrow \overline{\nu}_\mu + e^-$$



F.J. Hasert et al., Phys. Lett. 46B (1973) 121

★ Cannot be due to W exchange - first evidence for Z boson





Summary

★ Derived neutrino/anti-neutrino – quark/anti-quark weak charged current (CC) interaction cross sections

★ Neutrino – nucleon scattering yields extra information about parton distributions functions:

- ν couples to d and $\overline{u}$; $\overline{\nu}$ couples to u and $\overline{d}$
  ➡ investigate flavour content of nucleon
- can measure anti-quark content of nucleon

★ Finally observe that neutrinos interact via weak neutral currents (NC)