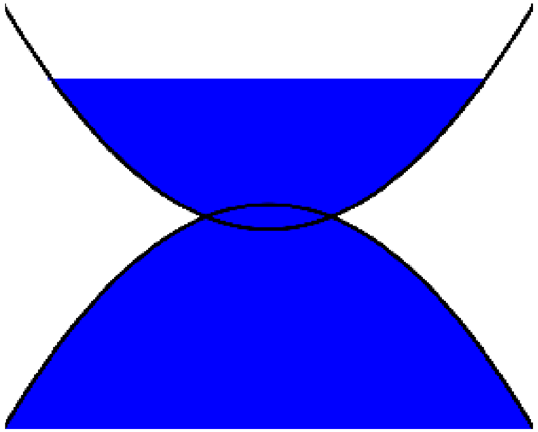
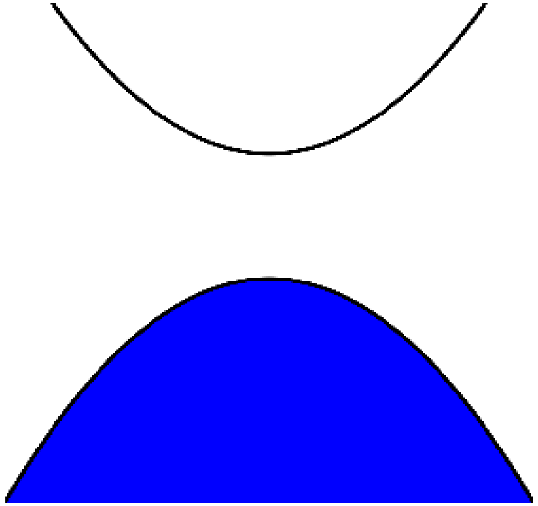
Semiconductors

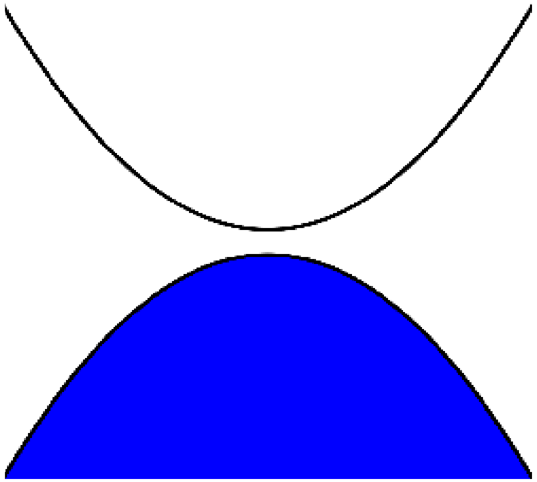
Kittel Chapter 8
Simon (oxford) - chapter 17



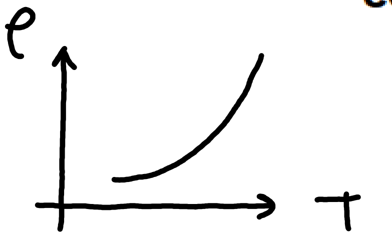
conductor



insulator



semiconductor

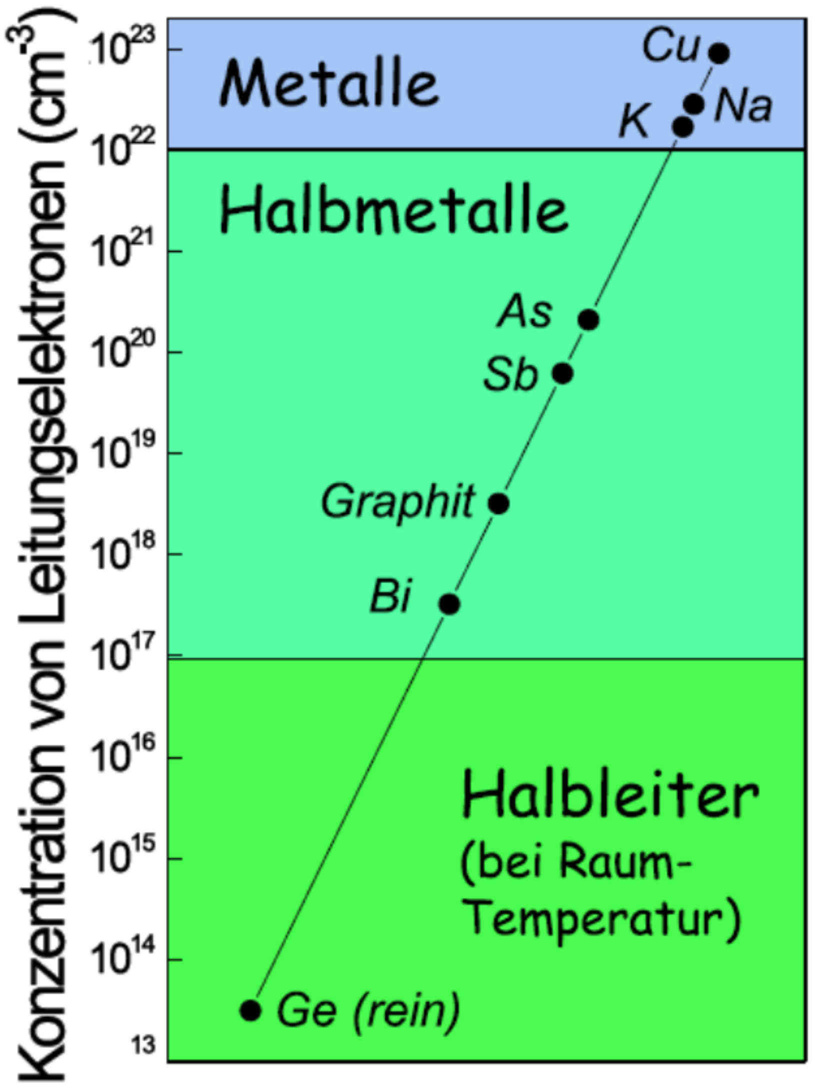


Resistivity: $\frac{d\rho}{dT} < 0$

pure metal "$\rho \sim 10^{-10}\,\Omega\text{cm}$"

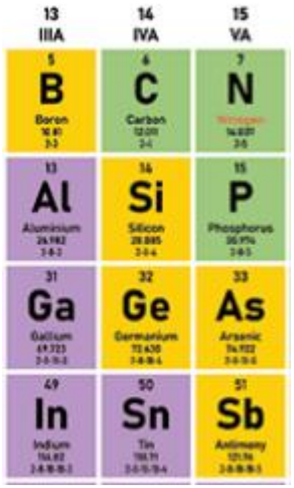
"$\rho > 10^{14}\,\Omega\text{cm}$"

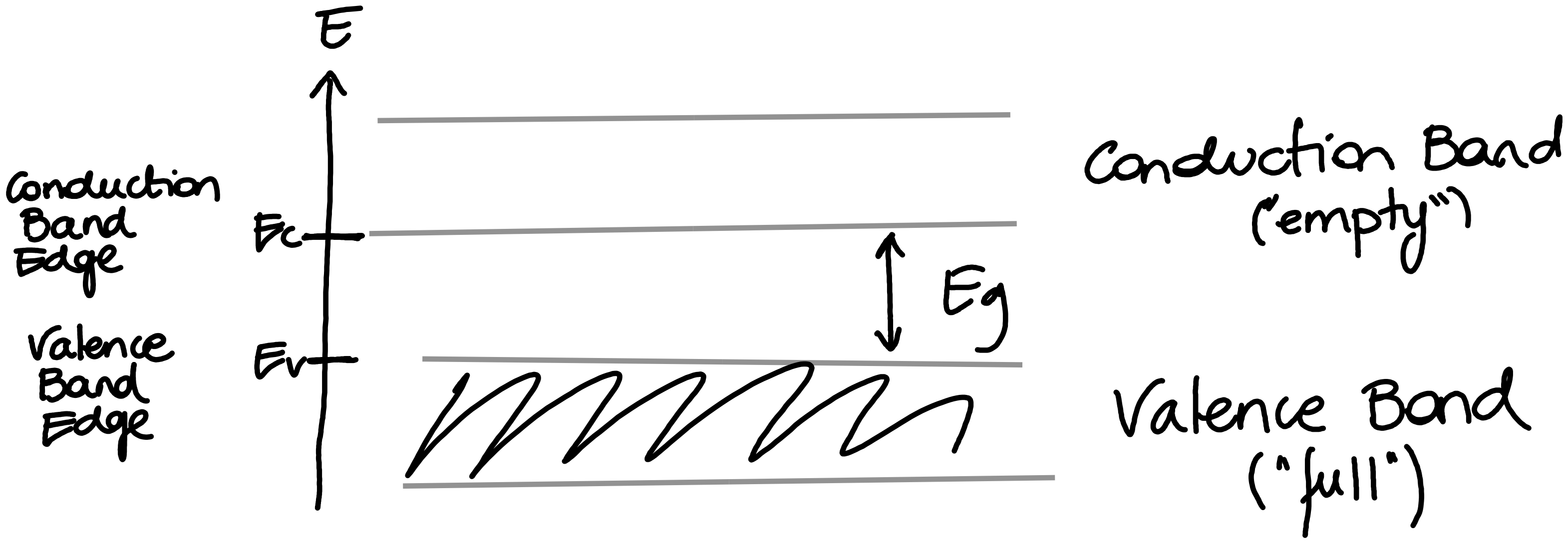
$\rho \sim 10^{-2} - 10^{9}\,\Omega\text{cm}$ @ RT



Ladungsträgerkonzentration:
Halbleiter, Halbmetalle, Metalle

Semiconductors – useful nomenclature





AB (chemical formula)

III-V GaAs, InSb

II-VI ZnS, CdS

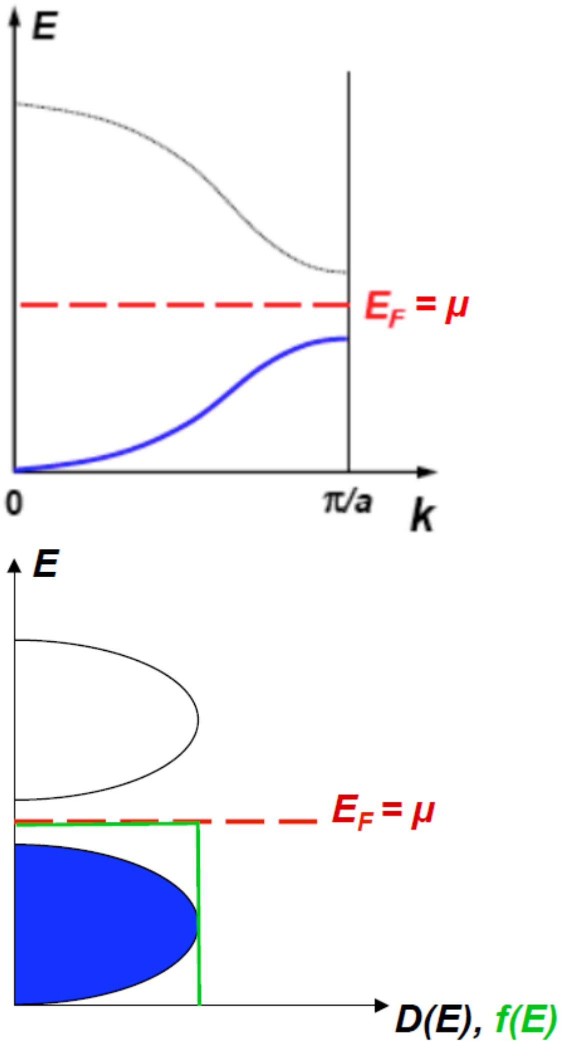
IV-IV SiC

type Diamond Ge, Si

{ Intrinsic Conductivity. (in pur crystals)
Extrinsic conductivity (related to impurities]

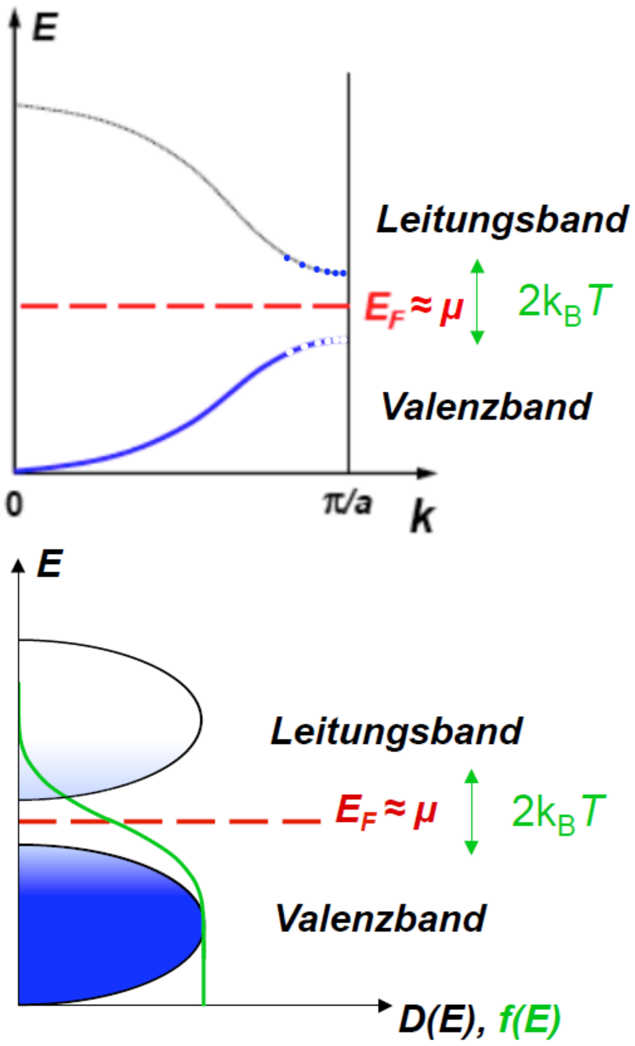
$T = 0$

Isolator/Halbleiter:



$T > 0$

As T increases, ≠ processes might allow e^- to be excited to the conduction band and empty states to be formed at the valence band.



Parameter E_g/k_BT (which compares energies) is important

at RT,
$k_BT \approx 26\,meV$

Band structure and optical properties

Semiconductors	$E_g(0\,\mathrm{K})$ (eV)	$E_g(300\,\mathrm{K})$ (eV)
Si	1.17	1.12
Ge	0.75	0.67
GaAs	1.52	1.43
InSb	0.24	0.18
InAs	0.43	0.35
InP	1.42	1.35
ZnO	3.44	3.2
ZnS	3.91	3.6
CdS	2.58	2.42
CdTe	1.61	1.45

Colors corresponding to photon energies

Color	$\hbar\omega$
Infrared	< 1.65 eV
Red	~ 1.8 eV
Orange	~ 2.05 eV
Yellow	~ 2.15 eV
Green	~ 2.3 eV
Blue	~ 2.7 eV
Violet	~ 3.1 eV
Ultraviolet	~ 3.2 eV

Optical absorption of several semiconductors

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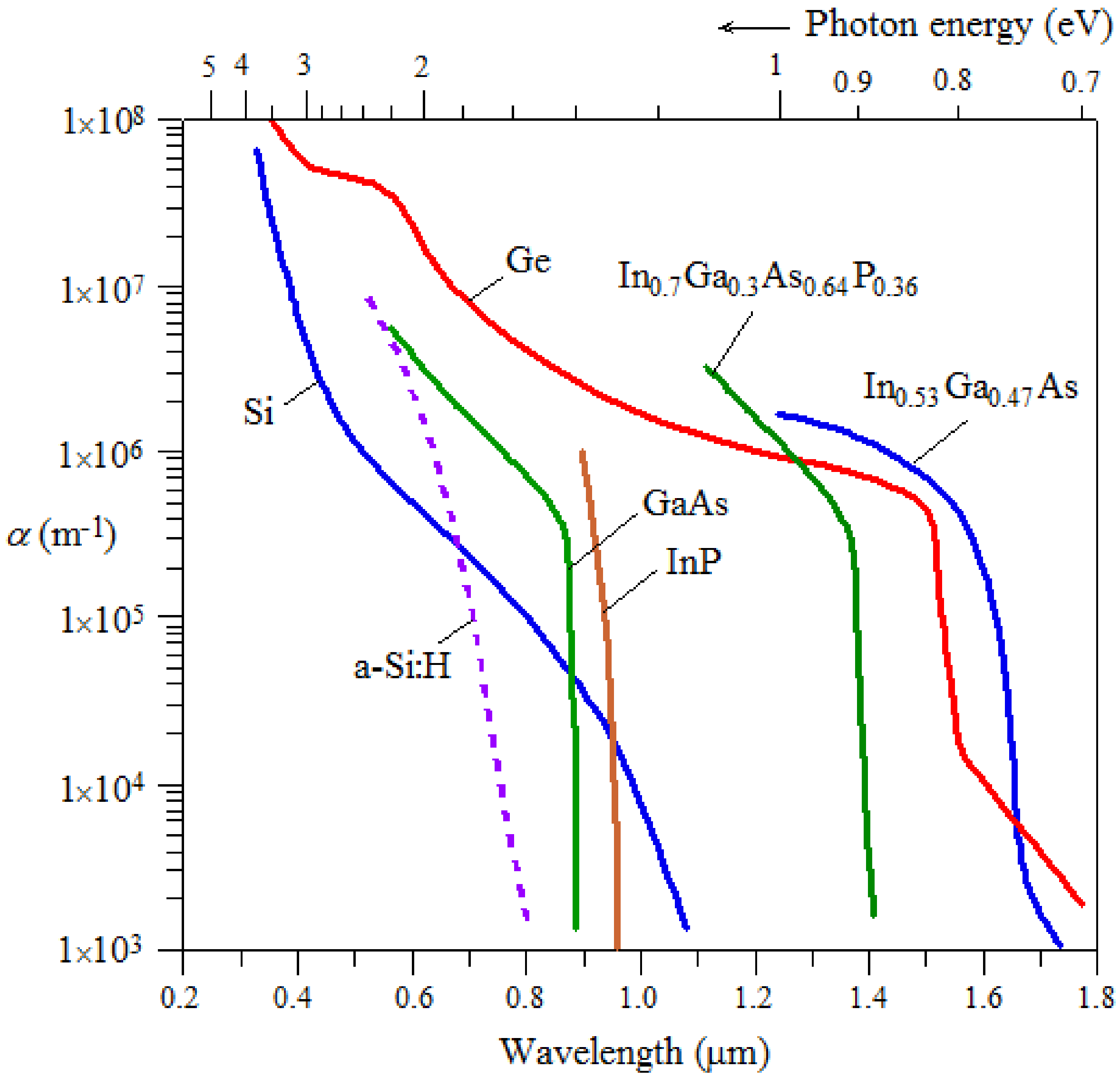
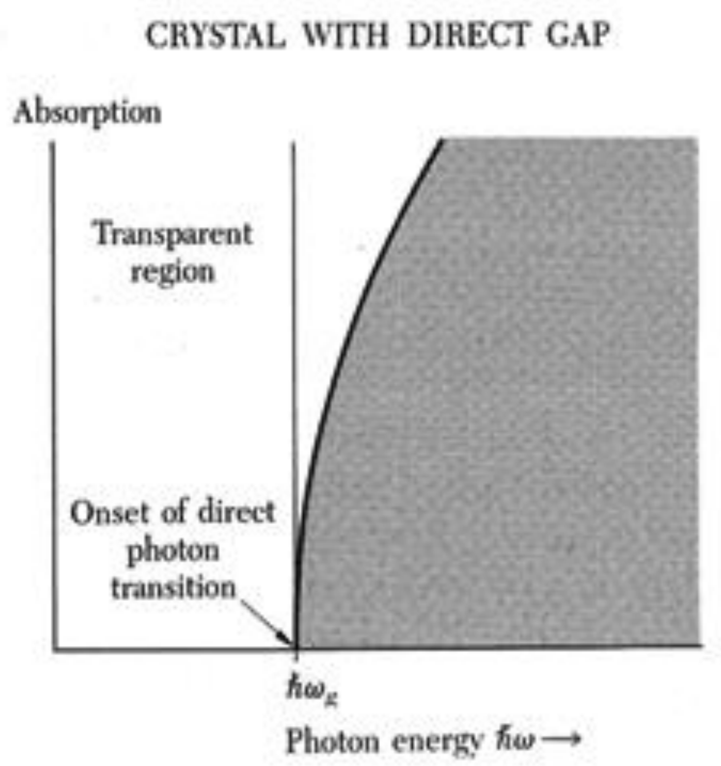


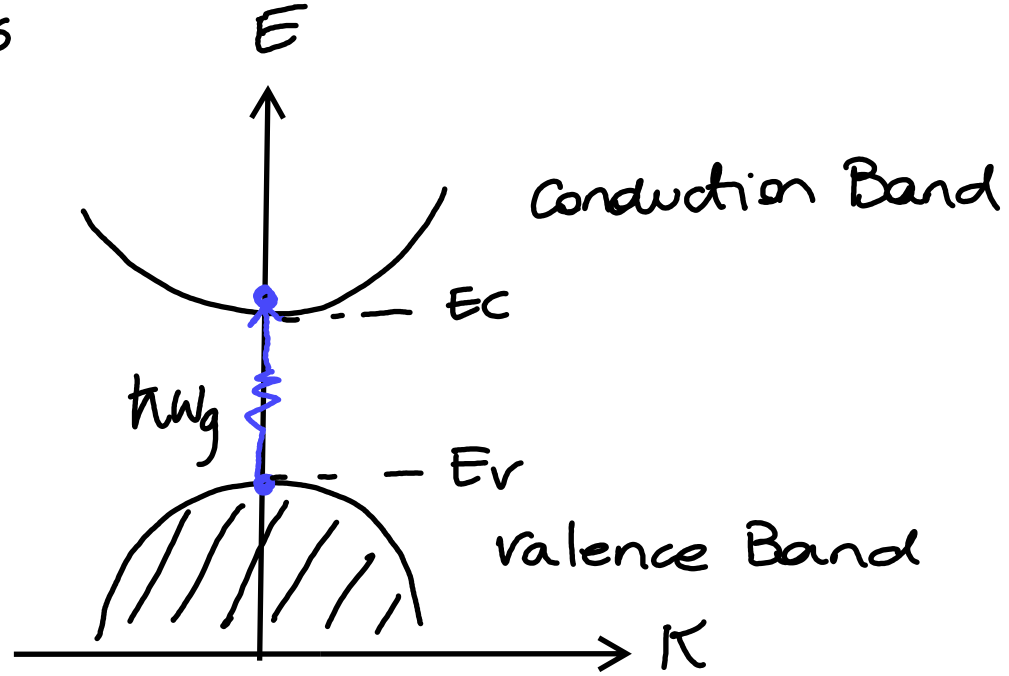
https://hub.hamamatsu.com/jp/en/technical-note/sipm-physics-operation/index.html

How to measure the band gap?



DIRECT OPTICAL TRANSITION

transition between two electronic states with no significant change of K

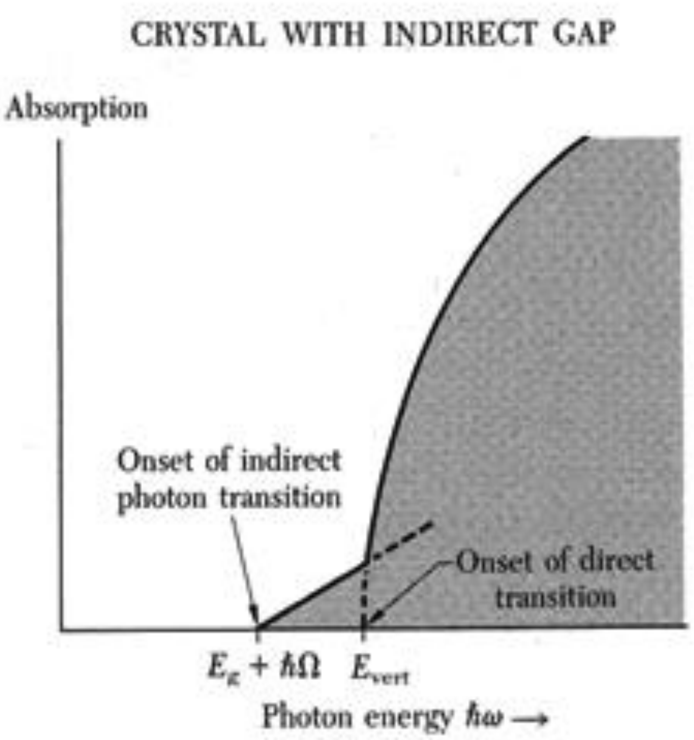


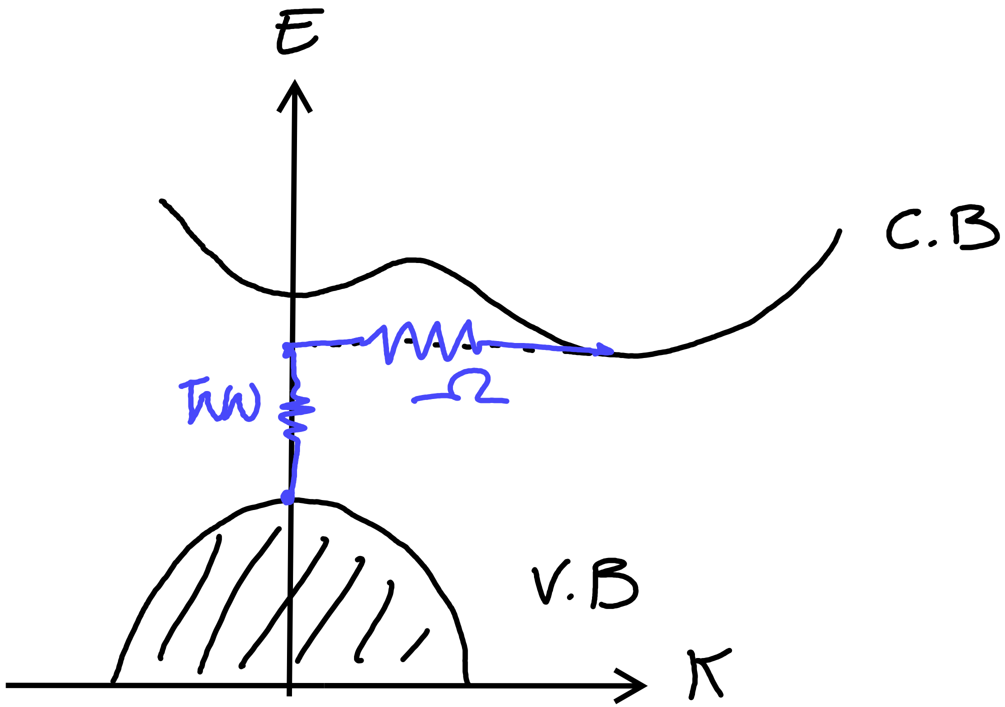
$K_{photon} \ll K_{electr.}$

$$\hbar|K| = \hbar\frac{\omega}{c} = \frac{energy}{c}$$

Absorption of photons leads to "vertical" transitions

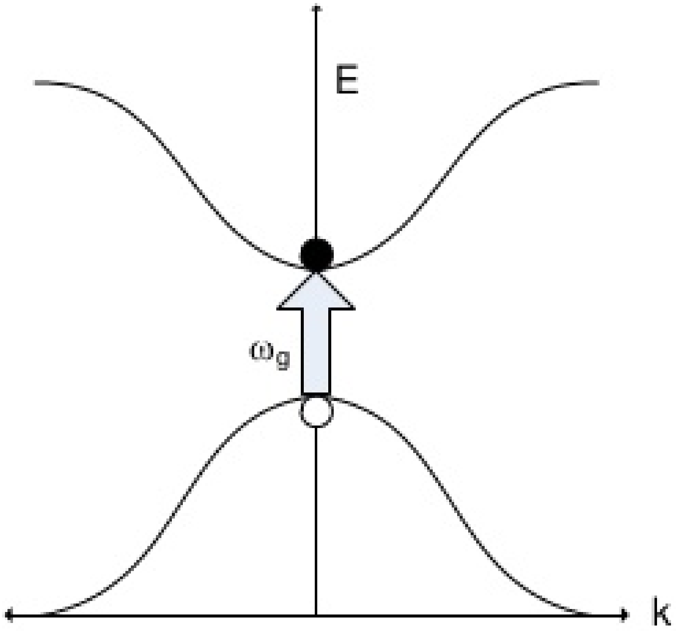
INDIRECT ABSORPTION PROCESS



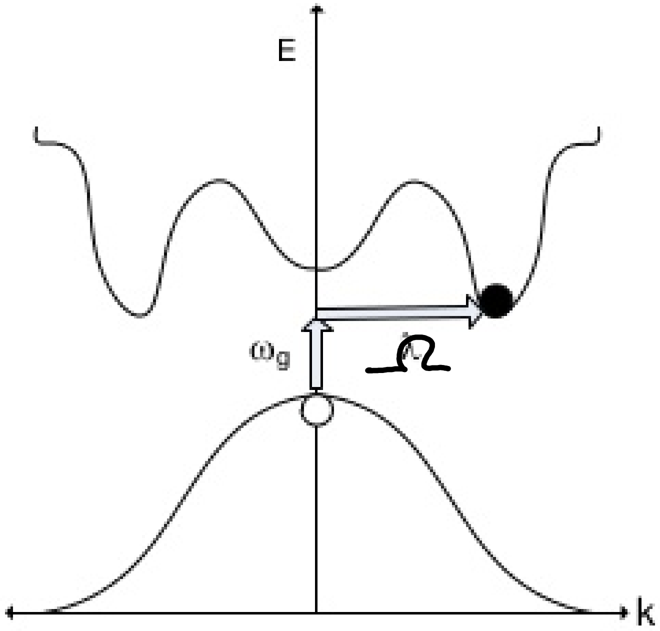


$$\begin{cases} \hbar\omega = E_g + \hbar\underbrace{\Omega}_{\text{freq. phonon}} \\ \vec{K}(\text{photon}) \approx 0 = \bar{K}_e + \bar{K}_{phonon} \end{cases}$$

Direct and indirect absorption

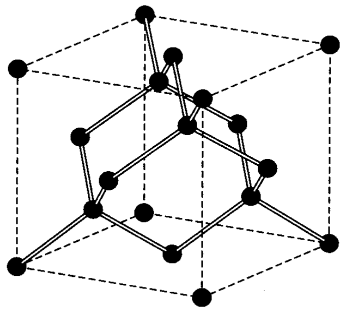


Direct absorption process



Indirect absorption process

Germanium



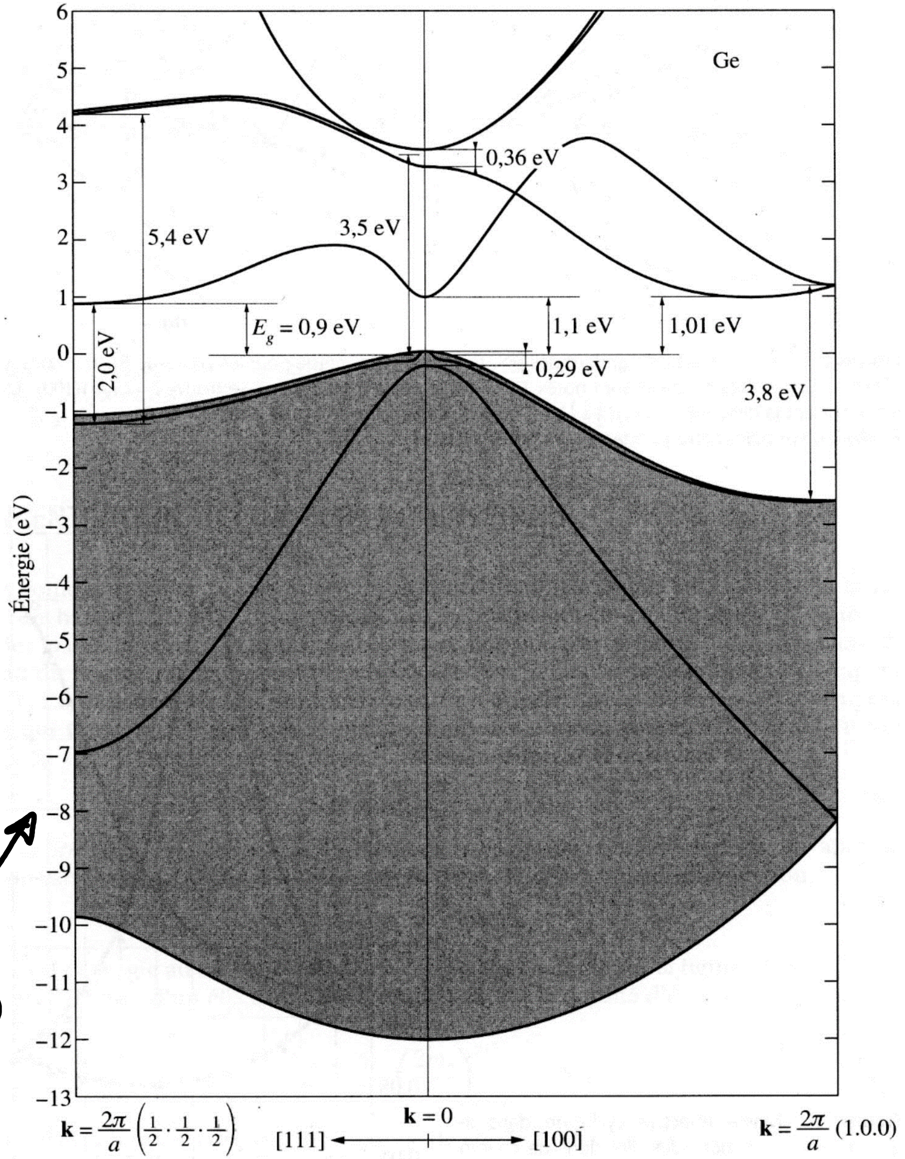
primitive u.c. : 2 atoms

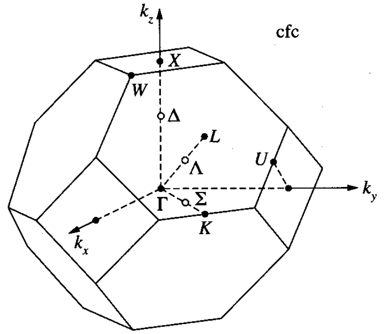
Each Ge atom ⇒ 4e- valence

8e- per primitive cell

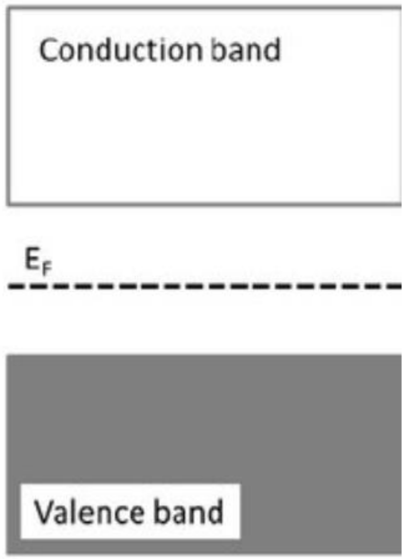
⇒ 8N e- in the crystal

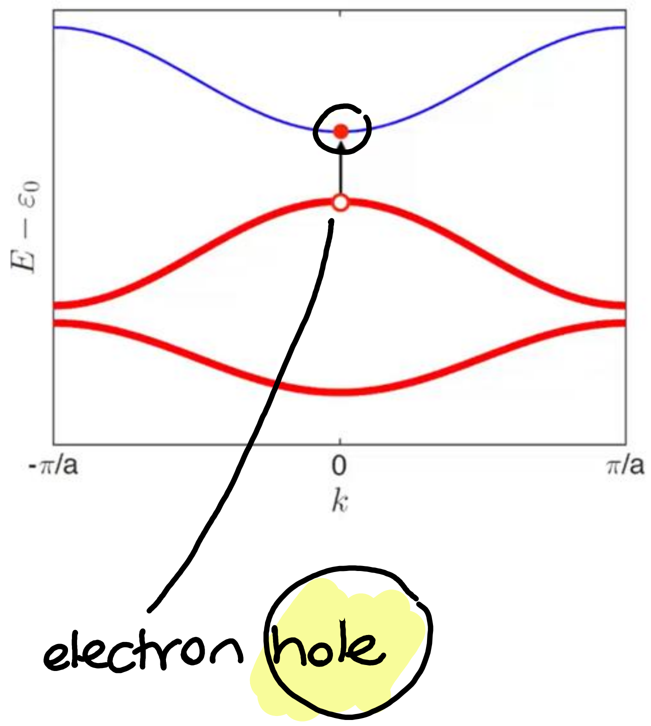
⇒ 8Ne- will occupy exactly (0K)
4 bands till the gap



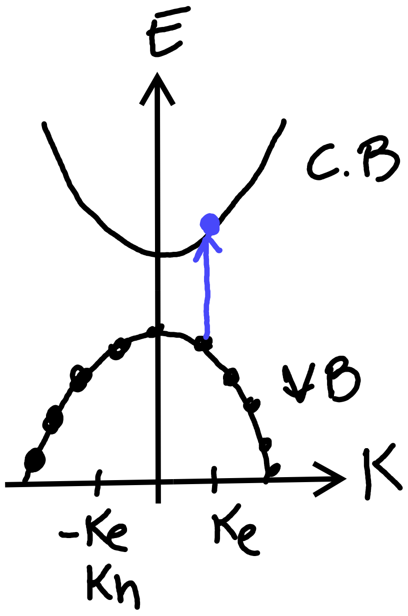


the properties of empty states in an otherwise filled bands are important in semiconductors!





Holes



- For a full band: $\sum_j \bar{K}_j = 0$

- If we remove an e- to create an excitation $\longrightarrow$ HOLE
  the band gets a net wavector

$$\bar{K}_h = \sum_{j \neq e} \vec{K}_j = -\bar{K}_e \longrightarrow \boxed{\bar{K}_h = -\bar{K}_e}$$

- Energy $\boxed{E_h(K_h) = -E_e(K_e)}$

- Group velocity $\boxed{v_h = \frac{1}{\hbar}\nabla_{K_h} E_h = \frac{1}{\hbar}\nabla_{-K_e}\left[-E(K_e)\right] = v_e}$

- Effective mass $\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{d^2E}{dK^2}$

$$\boxed{\frac{1}{m_h^*} = \frac{1}{\hbar^2}\frac{d}{d(-K_e)}\frac{d}{d(-K_e)}(-E_e) = -\frac{1}{\hbar^2}\frac{d^2E}{dK^2} = -\frac{1}{m_e^*}}$$

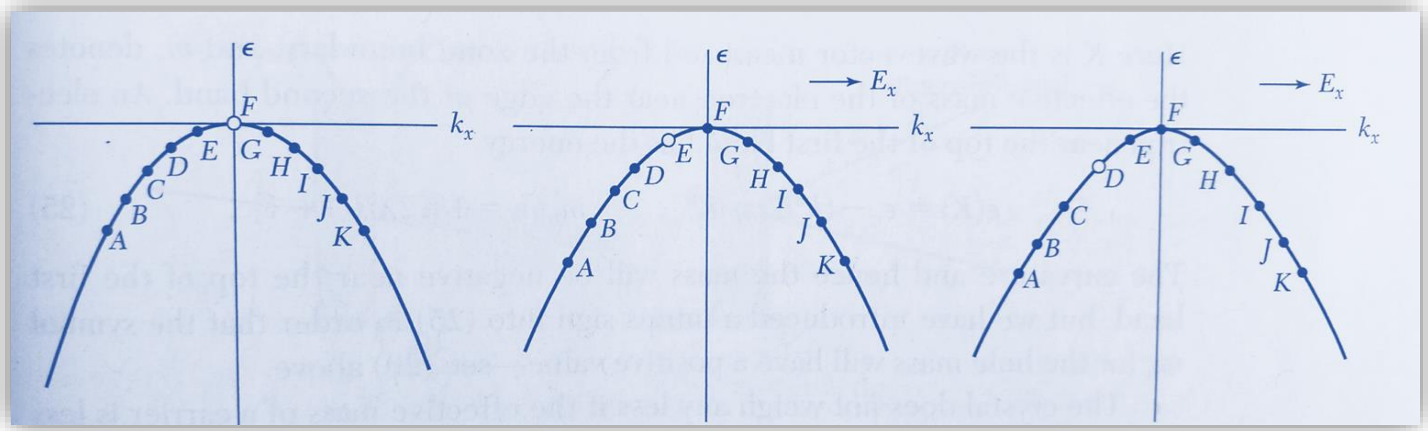
- A filled band does not transport current:

$$\vec{j} = \sum_i (-e)\vec{v}_i = 0$$

if we remove an e^-,

$$\sum_{i \neq l} (-e)\vec{v}_i = \sum_i (-e)\vec{v}_i - (-e)\vec{v}_e = 0 + e v_e = +e v_h$$

electric charge of a hole $\boxed{q_{hole} = +e} > 0$

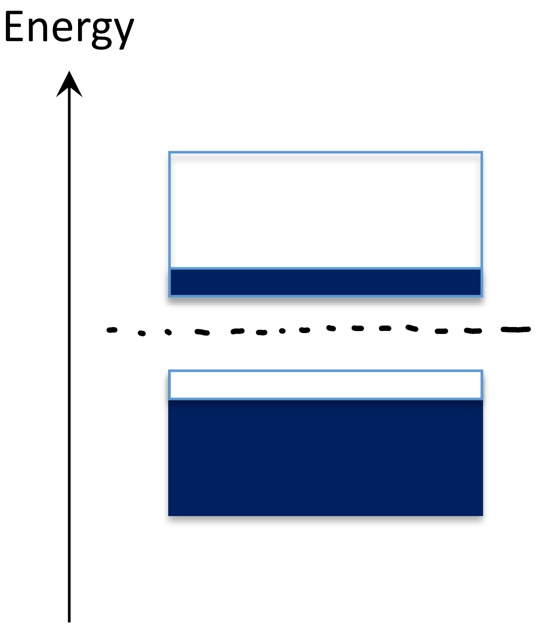


Intrinsic carrier concentration

We want to know the number of e- and holes in the conduction and valence band, respectively.

Let's consider the case:

- Fermi level (μ) is in the gap
- sufficiently low-T so the thermal occupation of the conduction band is low



$E-\mu(T) \gg K_B T$, the Fermi Dirac distribution

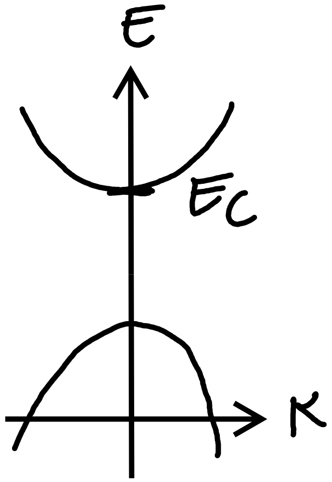
$$f_{FD}(E,T) = \frac{1}{e^{\left(\frac{E-\mu}{K_B T}\right)}+1} \sim e^{\frac{\mu(T)-E}{K_B T}}$$

semiconductor $E_g \sim 1\,eV$

$1\,[eV] \simeq K_B\ 11600\ [K]$

$26\,[meV] \simeq K_B\ 300\ [K]$

Concentration of electrons in the conduction band



$$E(K) = E_c + \frac{\hbar^2 K^2}{2m_e^*}$$

$$N(E) = \frac{2^{1/2} m^{3/2}}{\pi^2 \hbar^3} E^{1/2} \longrightarrow N_e(E) = \frac{2^{1/2} m_e^{*\,3/2}}{\pi^2 \hbar^3} (E - E_c)^{1/2}$$

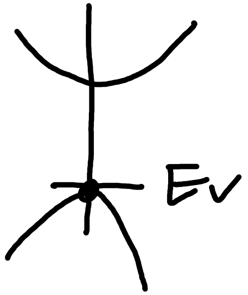
$$n(E) = \int_{E_c}^{\infty} N_e(E) \cdot f_{FD}(E,T)\, dE = \frac{2^{1/2} m_e^{*\,3/2}}{\pi^2 \hbar^3} e^{\frac{\mu(T)}{K_B T}} \int_{E_c}^{\infty} (E - E_c)^{1/2} e^{-E/K_B T}\, dE$$

$$\Rightarrow \cdots \Rightarrow \boxed{n_e = 2 \left(\frac{m_e^* K_B T}{2\pi \hbar^2} \right)^{3/2} e^{\frac{\mu(T) - E_c}{K_B T}} = n}$$

Concentration of holes in the valence band

$$f_{FD,e}(E,T) + f_{FD,h}(E,T) = 1 \longrightarrow f_h = 1 - f_e$$

$$f_{FD,h} = 1 - \frac{1}{e^{\frac{E-\mu}{K_B T}} + 1} \sim e^{\frac{E-\mu}{K_B T}}$$



$$\mu(T) - E > 0 \gg K_B T$$

$$N_h(E) = \frac{2^{1/2} (m_h^*)^{3/2}}{\pi^2 \hbar^3} (E_v - E)^{1/2}$$

$$n_h = \int_{-\infty}^{E_v} N_h(E) \, f_{FD,h}(E,T) \, dE$$

$$\boxed{n_h = 2 \left(\frac{m_h^* K_B T}{2\pi \hbar^2} \right)^{3/2} e^{\frac{(E_v - \mu(T))}{K_B T}} \equiv p}$$

Law of mass action

$$n \cdot p = 4\left(\frac{K_B T}{2\pi\hbar^2}\right)^3 (m_e^* m_h)^{3/2} e^{-E_g/K_B T}$$

$E_g = E_c - E_v$

Ex $np =$ $6.55 \times 10^{12}\ cm^{-6}$ GaAS
$2.1 \times 10^{19}\ cm^{-6}$ Si
$2.89 \times 10^{26}\ cm^{-6}$ Ge

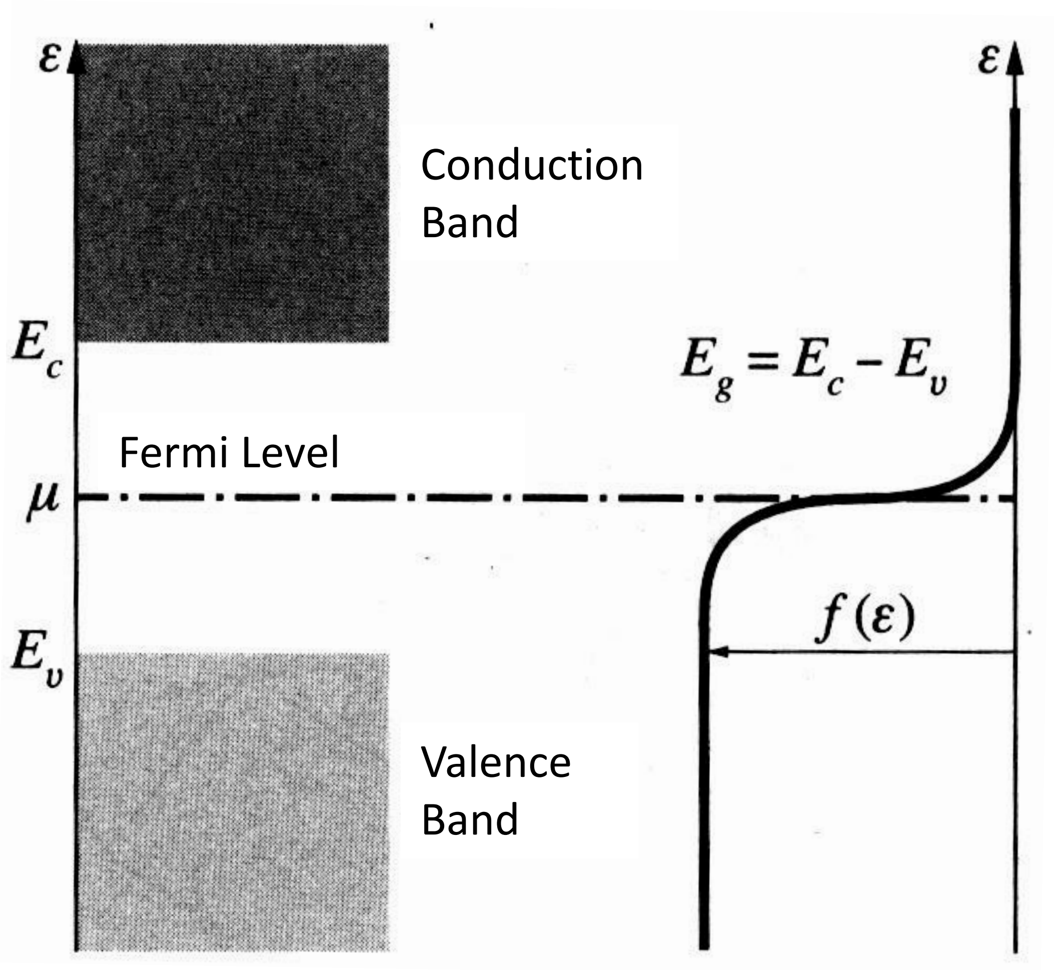
For an intrinsic semiconductor $n_i = p_i$

$$n = p = \sqrt{np} = 2\left(\frac{K_B T}{2\pi\hbar^2}\right)^{3/2} (m_e^* m_h^*)^{3/4} e^{-E_g/2K_B T}$$

$$\frac{p}{n} = \left(\frac{m_h^*}{m_e^*}\right)^{3/2} e^{E_v - \mu - \mu + E_c / K_B T} = 1$$

$$\frac{3}{2}\ln\left(\frac{m_h^*}{m_e^*}\right) + \frac{E_v + E_c}{K_B T} - \frac{2\mu(T)}{K_B T} = 0 \Rightarrow \mu(T) = \frac{E_v + E_c}{2} + \frac{3}{4} K_B T \ln \frac{m_h^*}{m_e^*}$$

Chemical potential is close to the middle of the gap


ε
ε
Conduction Band
E_c
$E_g = E_c - E_v$
Fermi Level
μ
$f(\varepsilon)$
E_v
Valence Band