University of Zurich[UZH]

Solid State Physics
Exercise Sheet 12
Band structure

HS19
Prof. Dr. Marta Gibert

Assistant: Gabriele De Luca

Received on 6th December
Discussed on 13th December

Exercise 1 *Band structure of a semi-metal*
The band structure of semi-metals (As, Sb, Bi) is characterized by the fact that the valence and conduction bands overlap in such a manner that the corresponding dispersion relations are described by:

$$E(V) = -(\hbar^2/2m_h^*)k^2 + E_0 \qquad E(C) = (\hbar^2/2m_e^*)k^2 \tag{1}$$

where $E_0 > 0$, m_e^* and m_h^* are the effective masses of electrons and holes such that $m_e^* < m_h^*$.

1. Draw the band diagram for a semi-metal and compare it with that of an alkali metal and a semiconductor.
2. Starting from the density of states for a free electron gas, find the expression for the density of conduction electrons n and the density of holes p.
3. Knowing that in semi-metals $n = p$, express the Fermi energy E_F as a function of E_0, m_e^* and m_h^*.

Exercise 2 *Semiconductor band structure with multiple valence bands*
Let's consider the case of an intrinsic semiconductor with two valence bands E_v with effective masses m_{h1}^* and m_{h2}^* and a conduction band E_c with effective mass m_e^*. The valence bands and the conduction band are separated by a bandgap E_g.

1. Express the chemical potential $\mu(T)$ as a function of the given parameters.
2. Compare the obtained result with the one described in the lecture for a single valence band: Where is located the chemical potential in this case for $m_h^* = m_e^*$?
3. The presence of the second valence band will change the value of the potential. Does this kind of displacement seem logical to you? Explain.

Exercise 3 *Quantum oscillations on quasi two-dimensional systems*
In $Tl_2Ba_2CuO_{6+\delta}$, quantum oscillations with a frequency of $F = 18.1\,\mathrm{kT}$ are observed (B. Vignolle et al., Nature **455**, 952-955 (2008)).

1. Use the Onsager relation ($S = 2\pi\frac{eF}{\hbar}$) to calculate the Fermi surface area.
2. If we assume a circular Fermi surface shape, what is the Fermi momentum?

Exercise 4 *Quantum oscillations in gold*
Estimate the Fermi energy of gold (in eV) based on the oscillations of the spin susceptibility in a magnetic field, see figure 1. Which of the two superimposed oscillations corresponds to the largest orbit on the Fermi-sphere? Compare the result with the literature value $\epsilon_F = 5.51\,\mathrm{eV}$. Where is the other oscillation originating from?

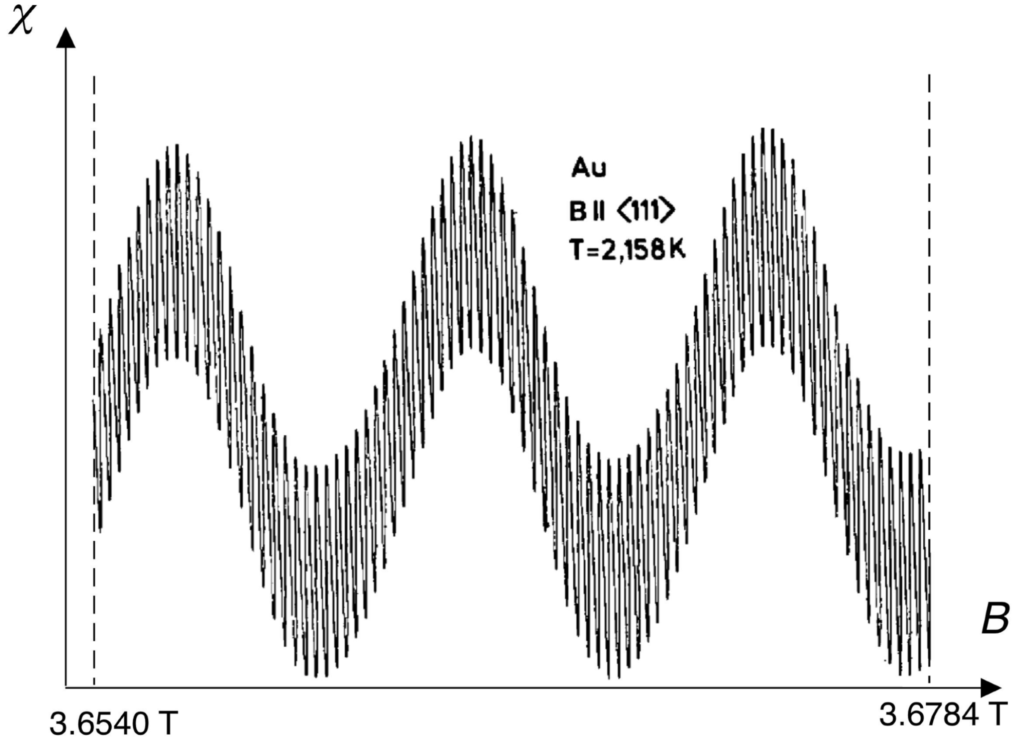

Figure 1: The spin susceptibility of gold in a magnetic field.