
Exercise 1 *Quantum oscillations on quasi two-dimensional systems*
In $Tl_2Ba_2CuO_{6+\delta}$, quantum oscillations with a frequency of $F = 18.1\,\mathrm{kT}$ are observed (B. Vignolle et al., Nature **455**, 952-955 (2008)).

(a) Use the Onsager relation ($S = 2\pi\frac{eF}{\hbar}$) to calculate the Fermi surface area.

(b) If we assume a circular Fermi surface shape, what is the Fermi momentum?

Exercise 2 *Quantum oscillations in gold*
Estimate the Fermi energy of gold (in eV) based on the oscillations of the spin susceptibility in a magnetic field, see figure 1. Which of the two superimposed oscillations corresponds to the largest orbit on the Fermi-sphere? Compare the result with the literature value $\epsilon_{\mathrm{F}} = 5.51\,\mathrm{eV}$. Where is the other oscillation originating from?

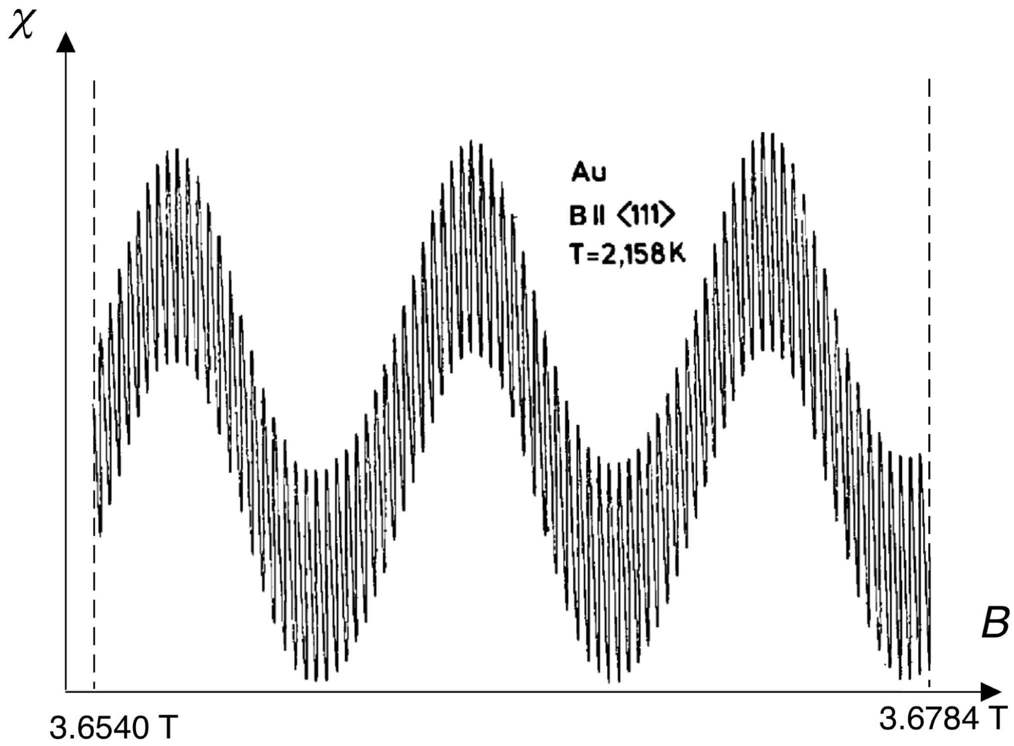

Figure 1: The spin susceptibility of gold in a magnetic field.

Exercise 3 *Tight binding model*
In the lecture, we derived the tight-binding expression for a two-dimensional square lattice:

$$\epsilon_k = -\epsilon_0 - 2t[\cos(k_x a) + \cos(k_y a)] \tag{1}$$

(a) Plot, using your favourite computer program, (1) the full three-dimensional band structure ϵ_k versus k_x and k_y as a surface plot, (2) the band structure along the zone diagonal $k_x = k_y$, and (3) the Fermi surface ($\epsilon_k = \epsilon_F$) for systems with $\mu = \epsilon_F$ (metals), having the values $\epsilon_F = -\epsilon_0$ and $\epsilon_F = -\epsilon_0 \pm 2t$. [Hint: Set $t = 1$ meaning that ϵ_k is plotted in units of t, set $\epsilon_0 = 0$, plot k_x, k_y in units of π/a]

(b) In the lecture, we developed the tight binding only to first order. Let's include second order terms. We define t' as the integral over next-nearest neighbours that are given by $\vec{\rho}_m = (\pm a, \pm a)$ and $(\pm a, \mp a)$. Show that the tight-binding dispersion becomes:

$$\epsilon_k = -\epsilon_0 - 2t[\cos(k_x a) + \cos(k_y a)] - 4t'[\cos(k_x a)\cos(k_y a)] \tag{2}$$

(c) Let's say that $\mu = -\epsilon_0 - 0.87t$. Compare the Fermi surfaces for $t' = 0$ and $t' = -0.2t$.

(d) Figure 2 displays a Fermi surface of a two-dimensional electron gas (2DEG) produced by depositing potassium on an insulating substrate (Ca_2RuO_4). Extract the Fermi momentum k_F.

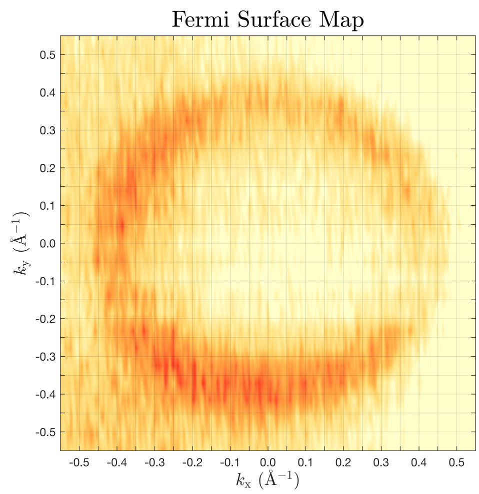

Figure 2: Fermi surface of a 2DEG on Ca_2RuO_4. Dark colours correspond to high intensities.

(e) Show that in two dimensions, the electron density is given by $n = k_F^2/(2\pi)$. Compare this with the results for three-dimensions.

(f) The lattice constant is 3.89 Å. What is the area of the Fermi surface? What is the Brillouin zone area and what is the ratio between the two? How does it relate to the electron density?

(g) Calculate the electronic density of states in two-dimensions. Show that it is independent of ϵ_k. Compare with the three-dimensional result.