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# Global symmetries and gravity

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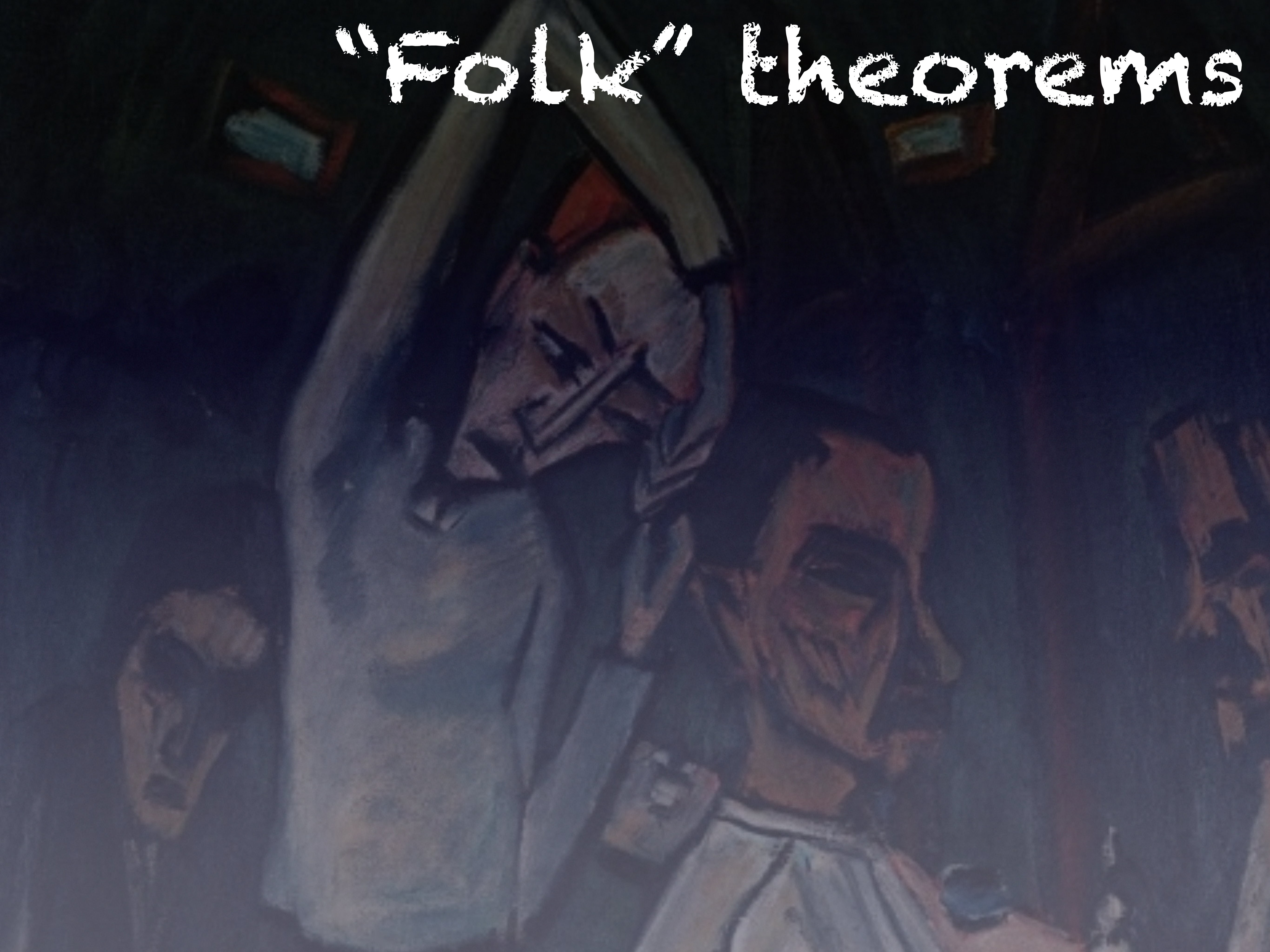
ETH, Zurich, 31 Oct 2017

Based on arXiv:1706.07415
with Rodrigo Alonso

# "Folk" theorems



# "Folk" theorems

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Q ???

BH

# Scalar field in Schwarzschild BG

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$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{16\pi}\mathcal{R} + g^{\mu\nu}(\partial_\mu\Phi)^*(\partial_\nu\Phi)\right.$$

$$\Phi \longrightarrow e^{i\alpha}$$

# Scalar field in Schwarzschild BG

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

$$\Box\Phi = \mu^2\Phi$$

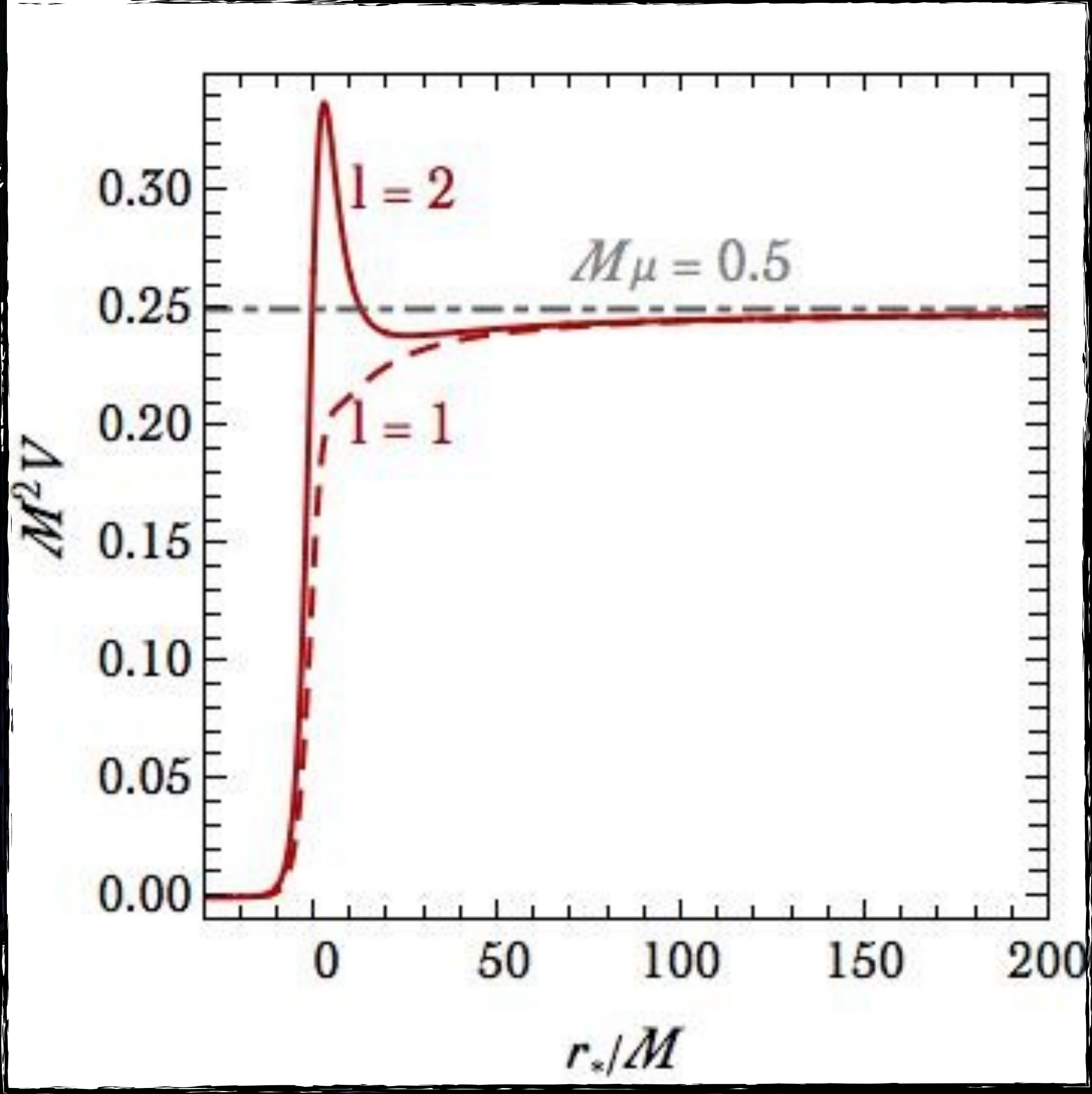
$$\Phi(t, r, \theta, \phi) = \sum_{l,m} Y_l^m(\theta, \phi) e^{-i\omega t} \frac{R(r)}{r}$$

# Scalar field in Schwarzschild BG

$$\left[-\frac{d^2}{dr_*^2} + V_{\text{eff}}(r)\right] R(r) = \omega^2 R(r)$$

$$V_{\text{eff}}(r) = \left(1 - \frac{2M}{r}\right)\left[\frac{2M}{r^3} + \frac{l(l+1)}{r^2} + \mu^2\right]$$

# Scalar fi... Schwarzsc...




$$\left[-\frac{d^2}{dr_*^2}+V_{\rm eff}(r)\right]R(r)=\omega^2 R(r)$$

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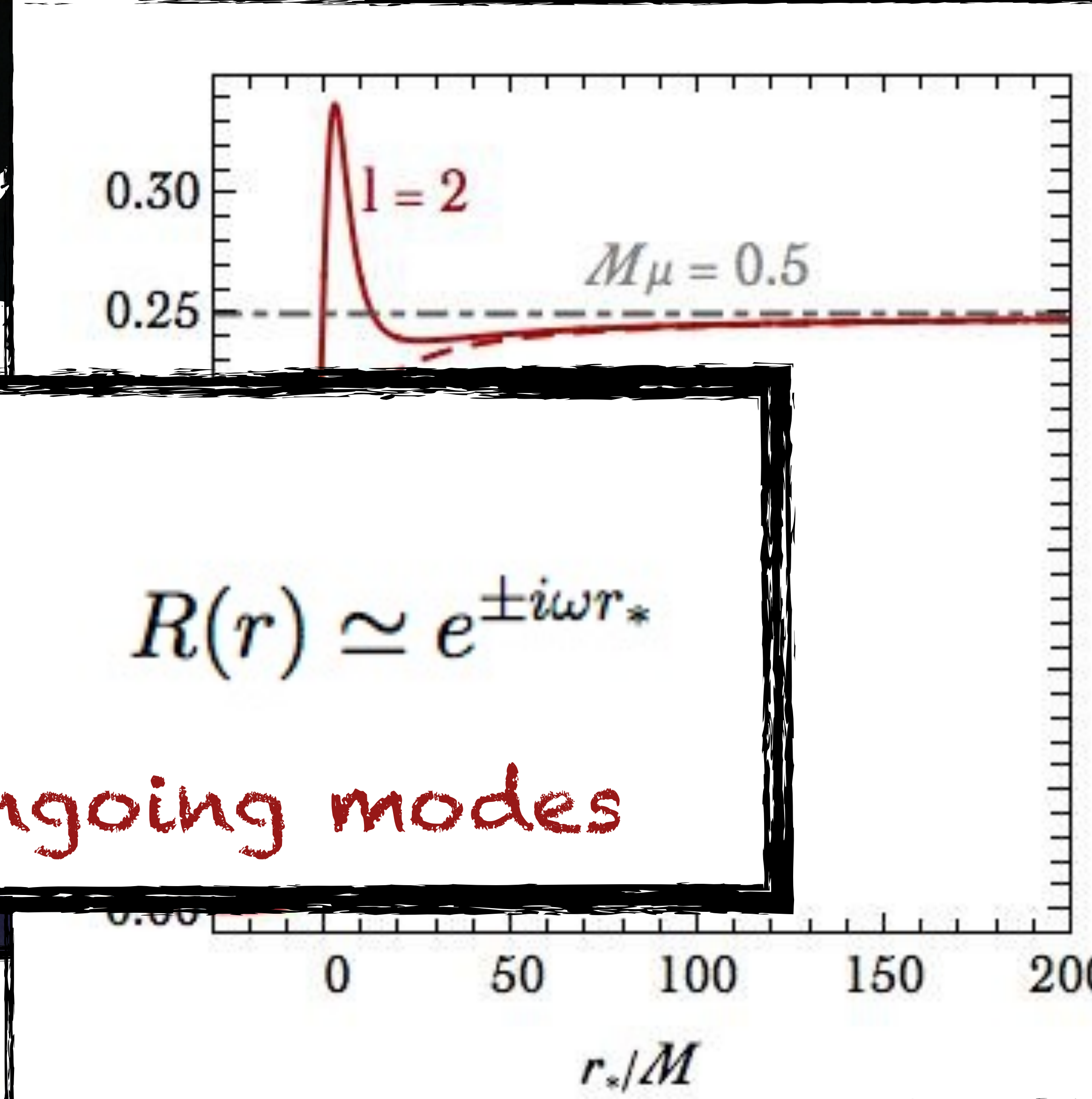
# Scalar fi




$$-\frac{d^2R(r)}{dr_*^2} \simeq \omega^2 R(r) \qquad \Longrightarrow \qquad R(r) \simeq e^{\pm i\omega r_*}$$

At the horizon we impose ingoing modes

$$\left[-\frac{d^2}{dr_*^2} + V_{\text{eff}}(r)\right] R(r) = \omega^2 R(r)$$

$$V_{\text{eff}}(r) = \left(1 - \frac{2M}{r}\right)\left[\frac{2M}{r^3} + \frac{l(l+1)}{r^2} + \mu^2\right]$$

in BG

$$\Phi(t,r,\theta,\phi) = \sum_{l,m} Y_l^m(\theta,\phi) e^{-i\omega t} \frac{R(r)}{r}$$

$$e^{-i\omega t} = e^{-i(\omega_{\mathrm{R}} + i\omega_{\mathrm{I}})t} = e^{-i\omega_{\mathrm{R}} t} e^{\omega_{\mathrm{I}} t}$$

$\ell = 1$

| $\mu$ | $\omega$ |
|---|---|
| 0.1 | $0.09987 - 1.5182 \times 10^{-11} i$ |
| 0.2 | $0.19895 - 4.0586 \times 10^{-8} i$ |
| 0.3 | $0.29619 - 9.4556 \times 10^{-6} i$ |
| 0.4 | $0.38955 - 5.6274 \times 10^{-4} i$ |
| 0.5 | $0.47759 - 5.5441 \times 10^{-3} i$ |

$\ell = 2$

| $\mu$ | $\omega$ |
|---|---|
| 0.1 | $0.09994 - 8.6220 \times 10^{-17} i$ |
| 0.2 | $0.19954 - 5.9249 \times 10^{-14} i$ |
| 0.3 | $0.29844 - 4.9002 \times 10^{-11} i$ |
| 0.4 | $0.39619 - 1.1703 \times 10^{-8} i$ |
| 0.5 | $0.49219 - 1.2271 \times 10^{-6} i$ |
| 0.6 | $0.58541 - 6.9974 \times 10^{-5} i$ |
| 0.7 | $0.67385 - 1.4987 \times 10^{-3} i$ |
| 0.8 | $0.75788 - 8.1511 \times 10^{-3} i$ |

# U(1) Goldstone boson

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$$\mathcal{L} = (\partial_\mu \Phi)^*(\partial^\mu \Phi) - \lambda \left( |\Phi|^2 \right.$$

$$\Phi \to e^{i\alpha}\Phi$$

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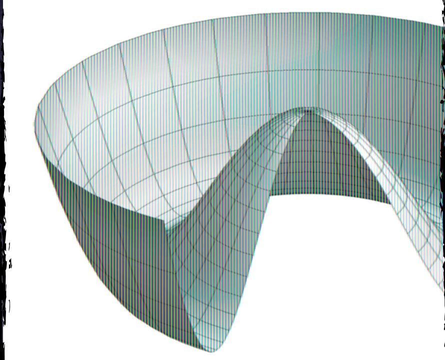
$$\Phi \to e^{i\alpha}$$

$$\Phi = \frac{1}{\sqrt{2}}(f_a + \rho)e^{i\phi/f_a} \qquad \langle|\Phi|$$



# U(1) Goldstone boson

$$\mathcal{L} = (\partial_\mu \Phi)^* (\partial^\mu \Phi) - \lambda \left( |\Phi|^2 \right.$$

$$\Phi \rightarrow e^{i\alpha}\Phi$$

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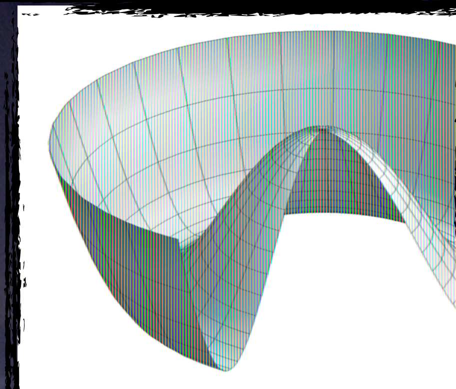


$$\mathcal{L} = \frac{1}{2}(\partial_\mu \rho)(\partial^\mu \rho) + \frac{1}{2}\left(\frac{f_a + \rho}{f_a}\right)(\partial_\mu \phi)(\partial^\mu \phi) - \lambda(f_a^2 \rho$$

$$\begin{aligned} \rho &\rightarrow \rho \\ \phi &\rightarrow \phi + \alpha \end{aligned}$$

# U(1) Goldstone boson

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi)$$

$$\phi \longrightarrow \phi + \alpha\, f_a$$

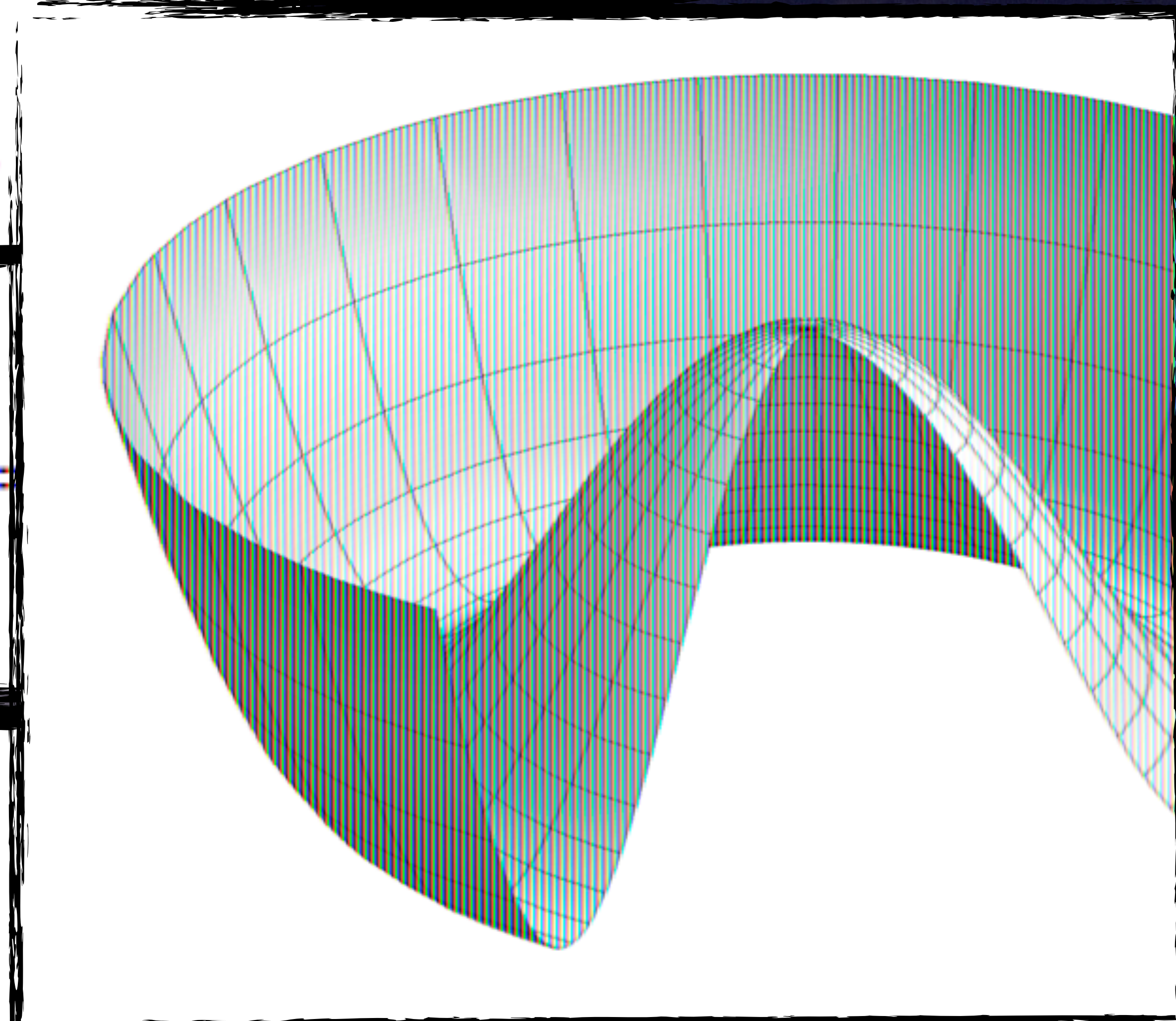
$$j^\mu = f_a(\partial^\mu \phi)\ , \qquad Q = \int d^3x$$

# U(1) Goldstone boson

$$\mathcal{L} \quad = \quad \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi)$$

$$\phi \quad \longrightarrow \quad \phi + \alpha\, f_a$$

$$j^\mu =$$



$$\phi \longrightarrow \phi + 2k\pi\, f_a,$$

# U(1) Goldstone boson

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu$$

$$V_{\rm QCD}(\phi) = \Lambda^4_{\rm QCD}\,\mathrm{cc}$$

$$\phi \rightarrow \phi + \alpha\, f_a$$

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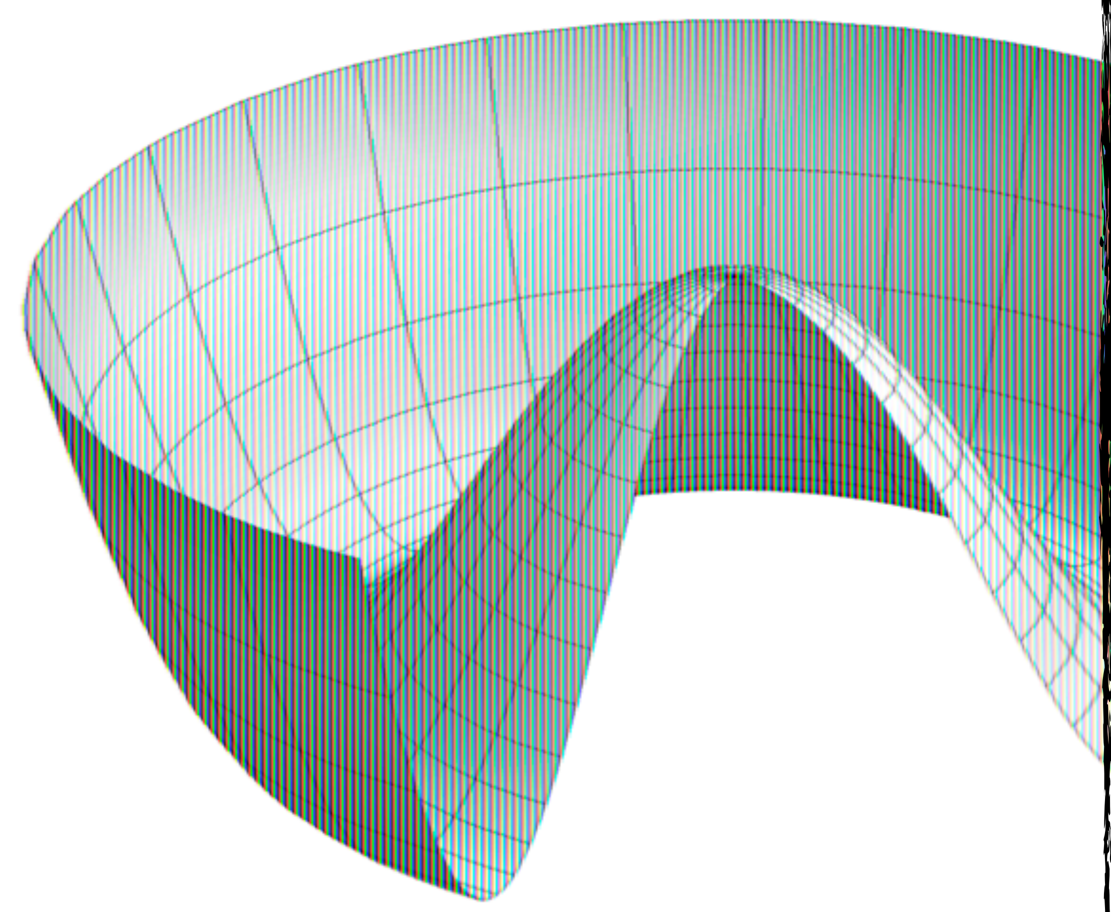

$$\phi \rightarrow \phi + 2k\pi\, f_a,$$

# Perturbative breaking

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$$V_{\rm gravity}(\Phi) = g_{2m+n} \frac{|\Phi|^{2m}\Phi^{n}}{M_{\rm Pl}^{2m+n-}}$$

M. Kamionkowski, J. March-Russell,
Phys. Lett. B282 (1992) 137-141

# Perturbative breaking

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{\rm Pl}^2}{16\pi} \mathcal{R} + \frac{f_a^2}{2} (\partial \right.$$

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$$V_{\rm gravity}(\Phi) \ldots {}^{2m+n} \frac{|\Phi|^{2m} \Phi^n \ldots}{M_{\rm Pl}^{2m+n-\ldots}}$$

# Non-perturbative solutions

$$S = \int d^4x\sqrt{-g}\,\left[\frac{M_{\rm Pl}^2}{16\pi}\,\mathcal{R} + \frac{f_a^2}{2}(\partial$$

Non-trivial stationary points of the Euclidean action

# Non-perturbative solutions

Solution of Einstein's equations + EOM scalar

Non-trivial stationary points of the Euclidean action

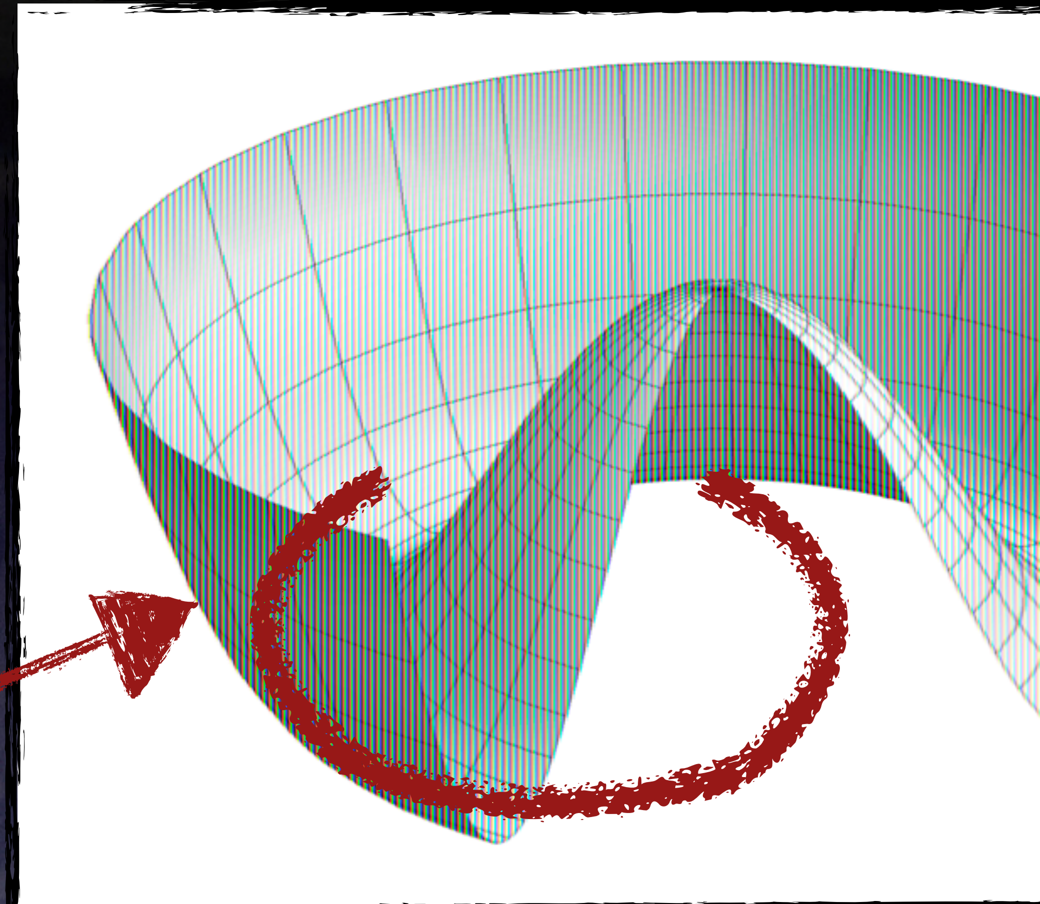
# Non-perturbative solutions

The action is non-zero if you plug in back the solution

Non-trivial stationary points of the Euclidean action

# Non-perturbative solutions

Trivial solutions with zero action are the infinite degenerate vacua with flat space and constant axion field



## Non-trivial stationary points of the Euclidean action

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{\rm Pl}^2}{16\pi} \mathcal{R} + \frac{f_a^2}{2} (\partial \right.$$

# Non-perturbative solutions

These non-perturbative euclidean solutions exist:
Wormholes

S. B. Giddings and A. Strominger, Nucl. Phys. B306, 890 (1988)

1) Euclidean metric
2) Scalar field configuration
3) Action

# 1) Euclidean metric

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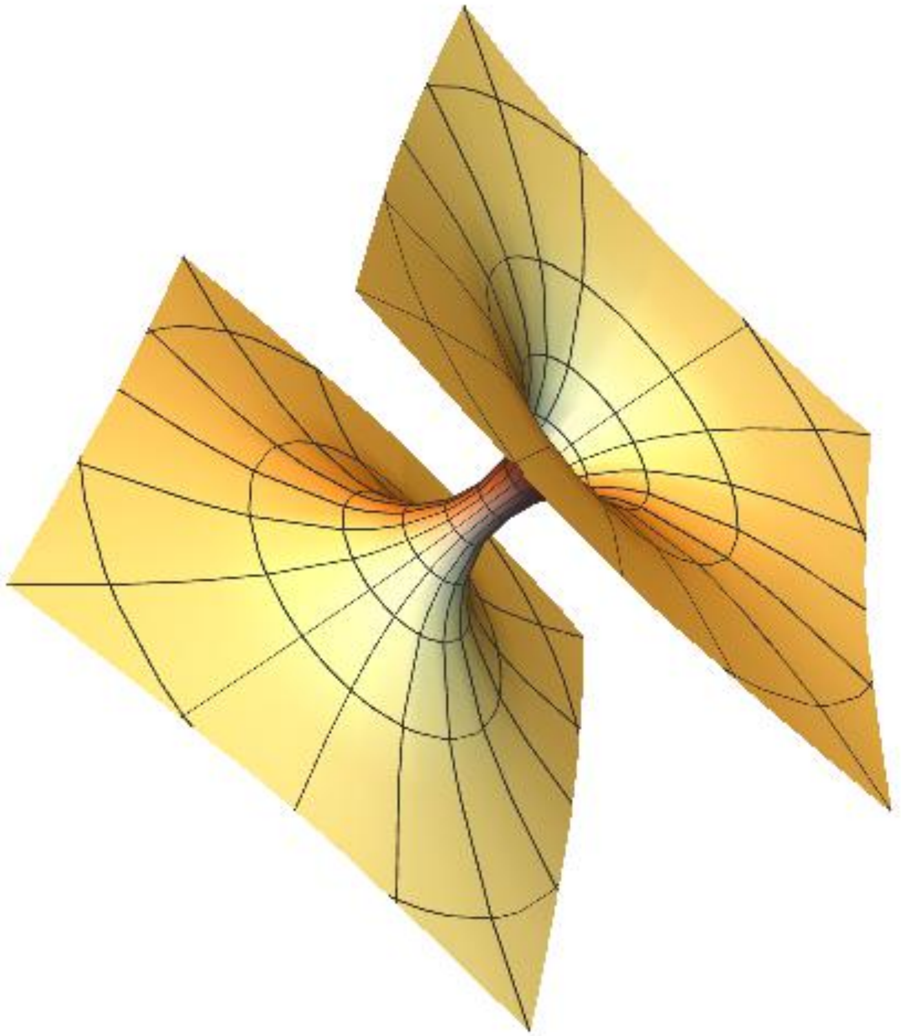
$$ds^2 = a^2(\tau)\left(d\tau^2 + \right.$$

$$a^2(\tau) = L^2 \cosh($$

$$L \equiv \left(\frac{n^2}{3\pi^3 M_{\rm Pl}^2 f_a^2}\right.$$



# 1) Euclidean metric

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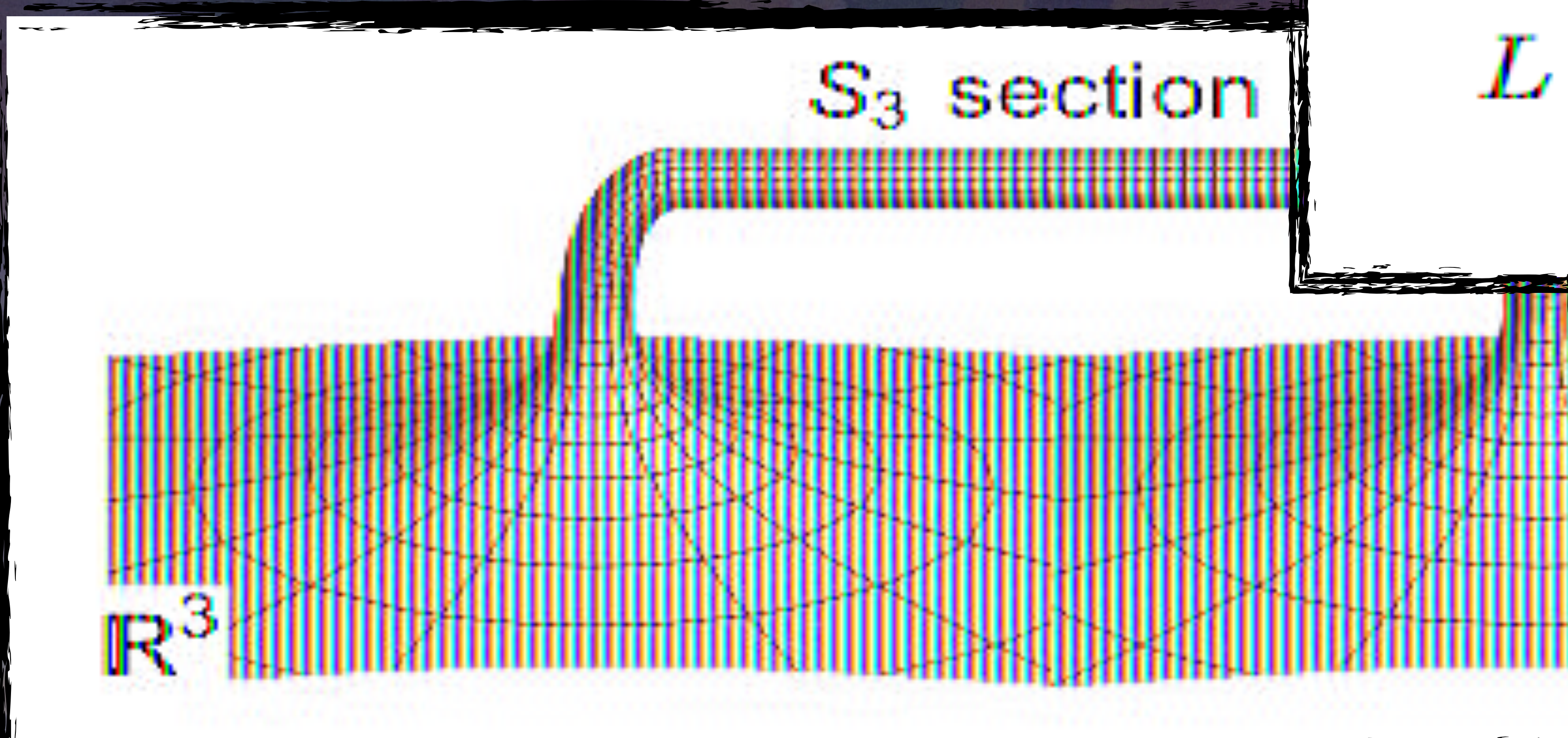

# 2) Axion field

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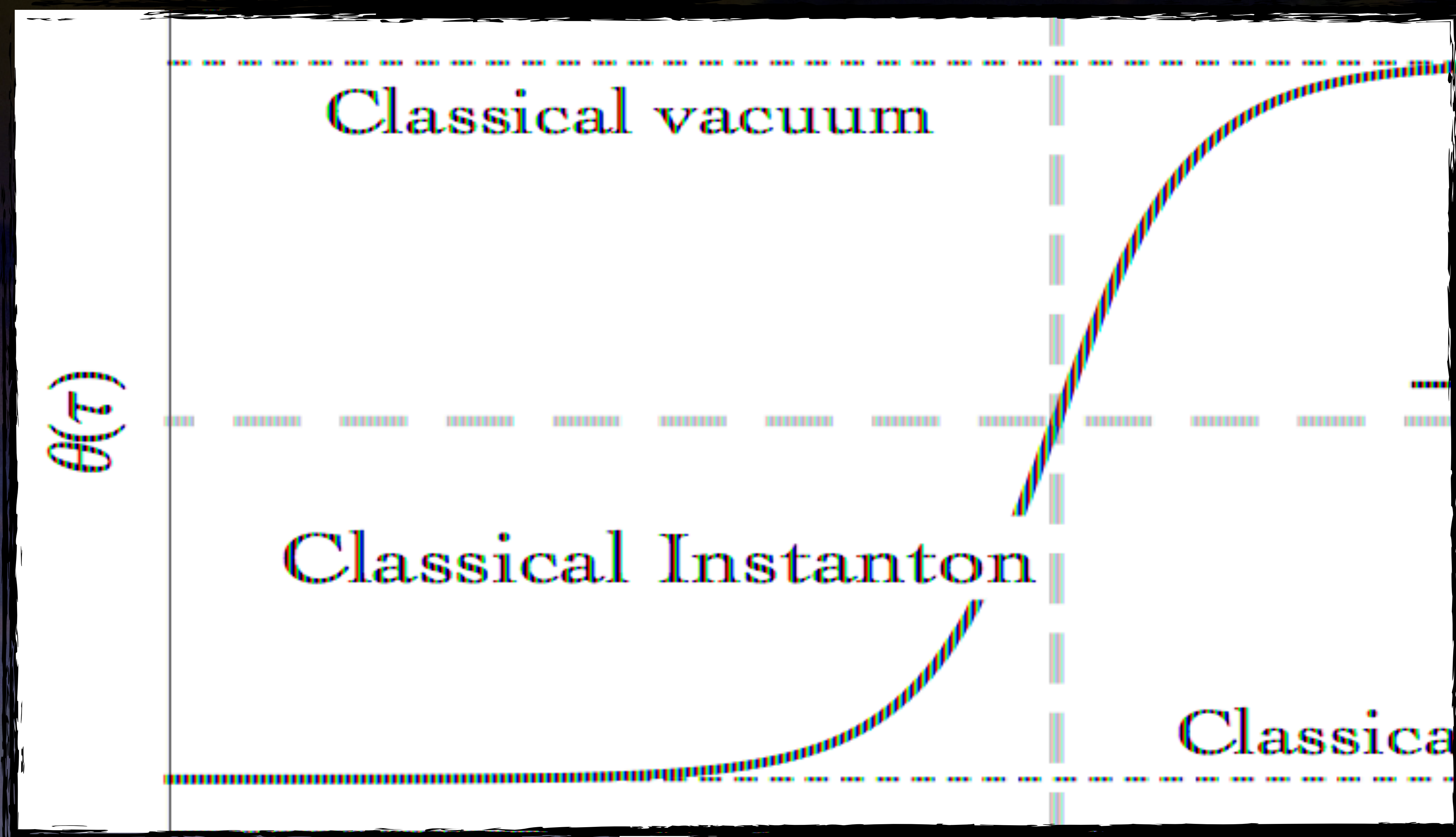


$\theta(\tau) \sim \mathrm{arctan}[\mathrm{tanh}$

# 3) Wormhole action

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$$S_{\text{wormhole}} = \frac{\sqrt{3\pi n}\,M}{8 f_a}$$

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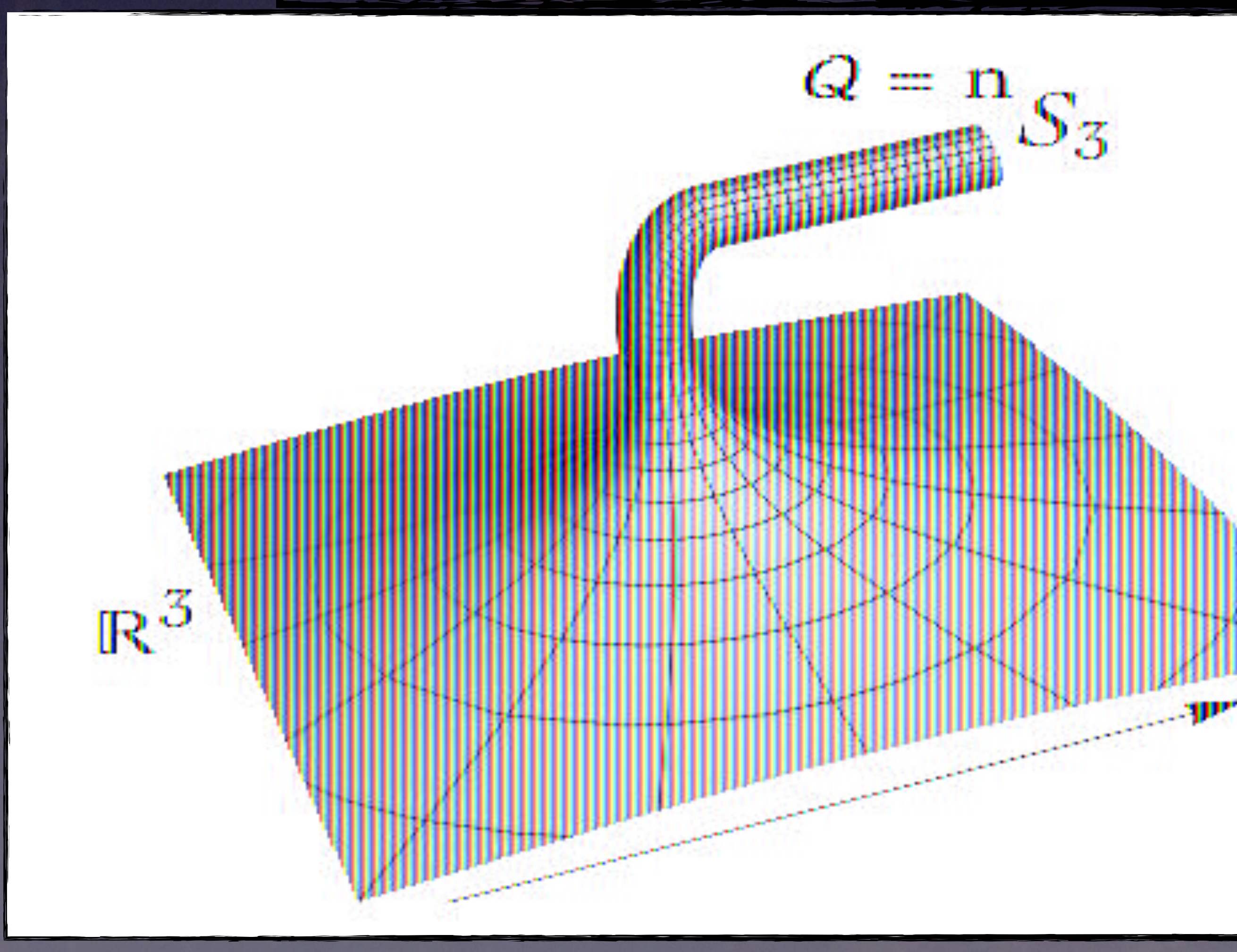


$$Q \equiv \int_{S_3} J^0 \, d\Omega_3$$

# Effective potential

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$$\langle M \rangle = \frac{1}{Z} \int \mathcal{D}g \mathcal{D}\phi \, M \, e$$

# Effective potential

Integration over both metric
and matter field degrees of freedom

$$\langle M \rangle = \frac{1}{Z} \int \mathcal{D}g\mathcal{D}\phi \, M \, e$$

# Effective potential

The gravitational part of the path integral involves an integration over all possible Euclidean four-geometries. In particular, it includes wormholes in all possible combinations.

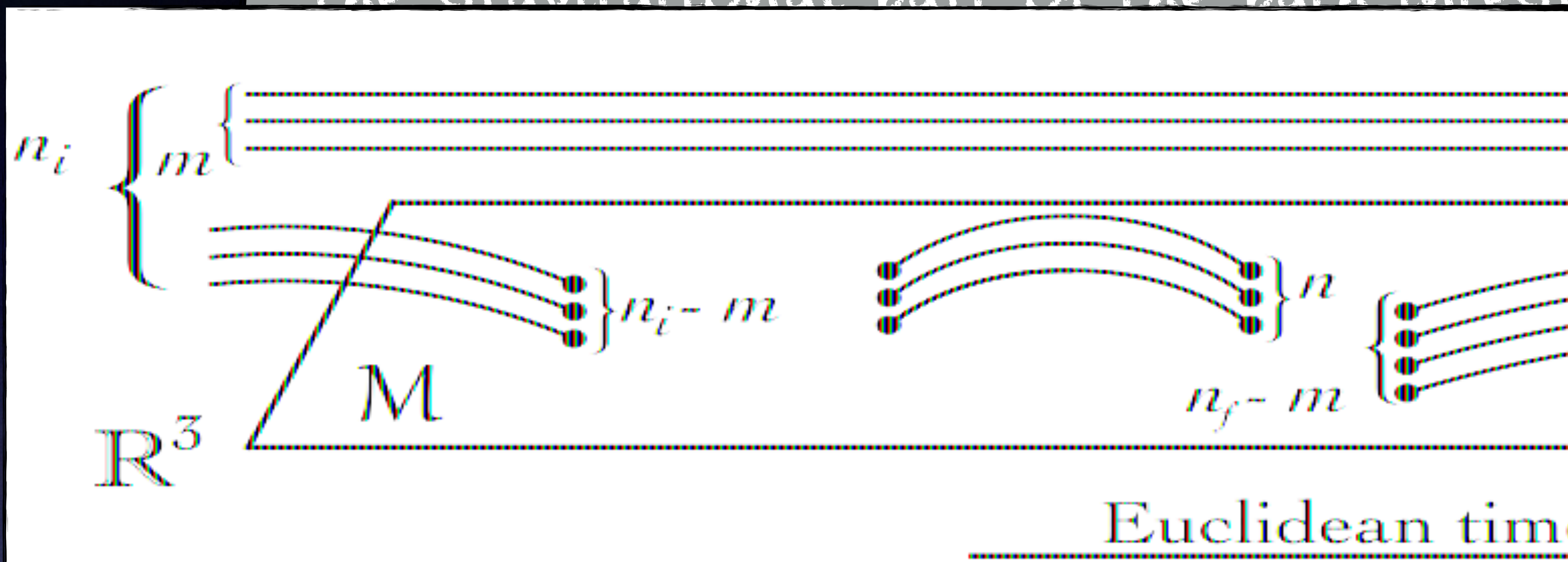
$$\langle M \rangle = \frac{1}{Z} \int \mathcal{D}g\mathcal{D}\phi \, M \, e$$

# Effective potential

The gravitational part of the path integral

r,

mbinations.

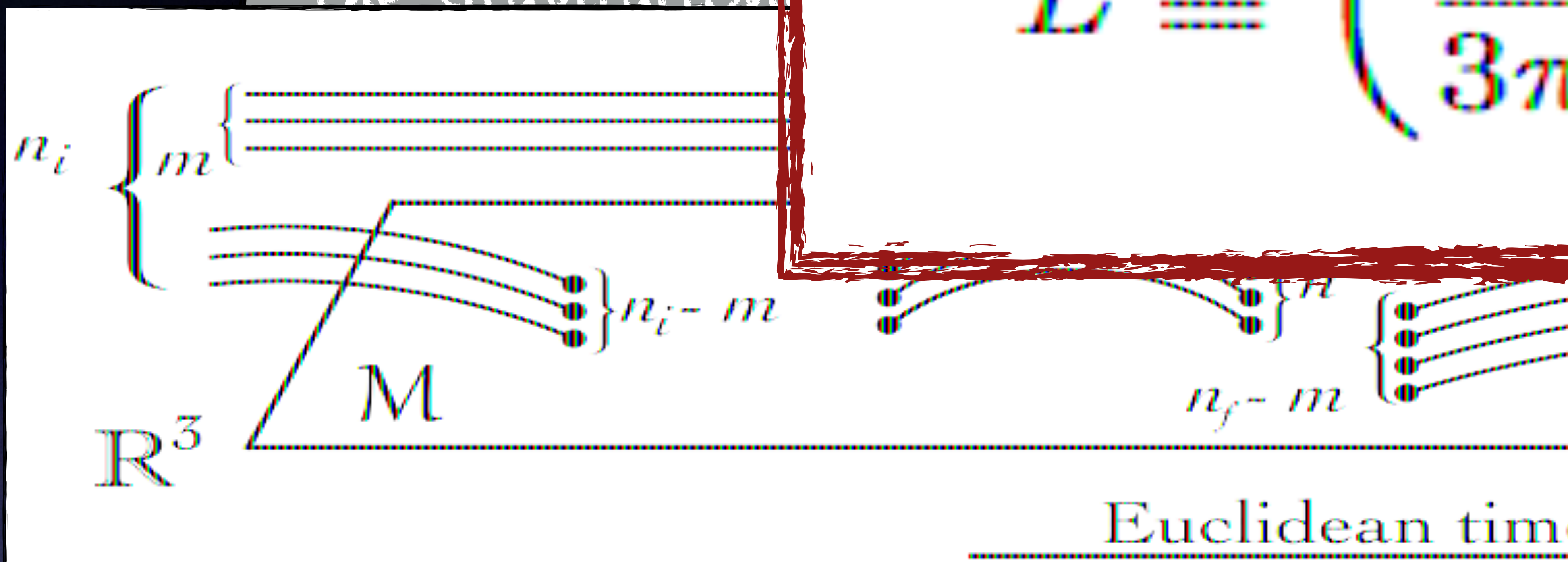



$$\langle M \rangle = \frac{1}{Z} \int \mathcal{D}g \mathcal{D}\phi \, M \, e$$

# Effec

The gravitation

mbinations.




$$L \equiv \left( \frac{n^2}{3\pi^3 M_{\rm Pl}^2 f_a^2} \right.$$

$$\langle M \rangle = \frac{1}{Z} \int \mathcal{D}g \mathcal{D}\phi \, M \, e$$

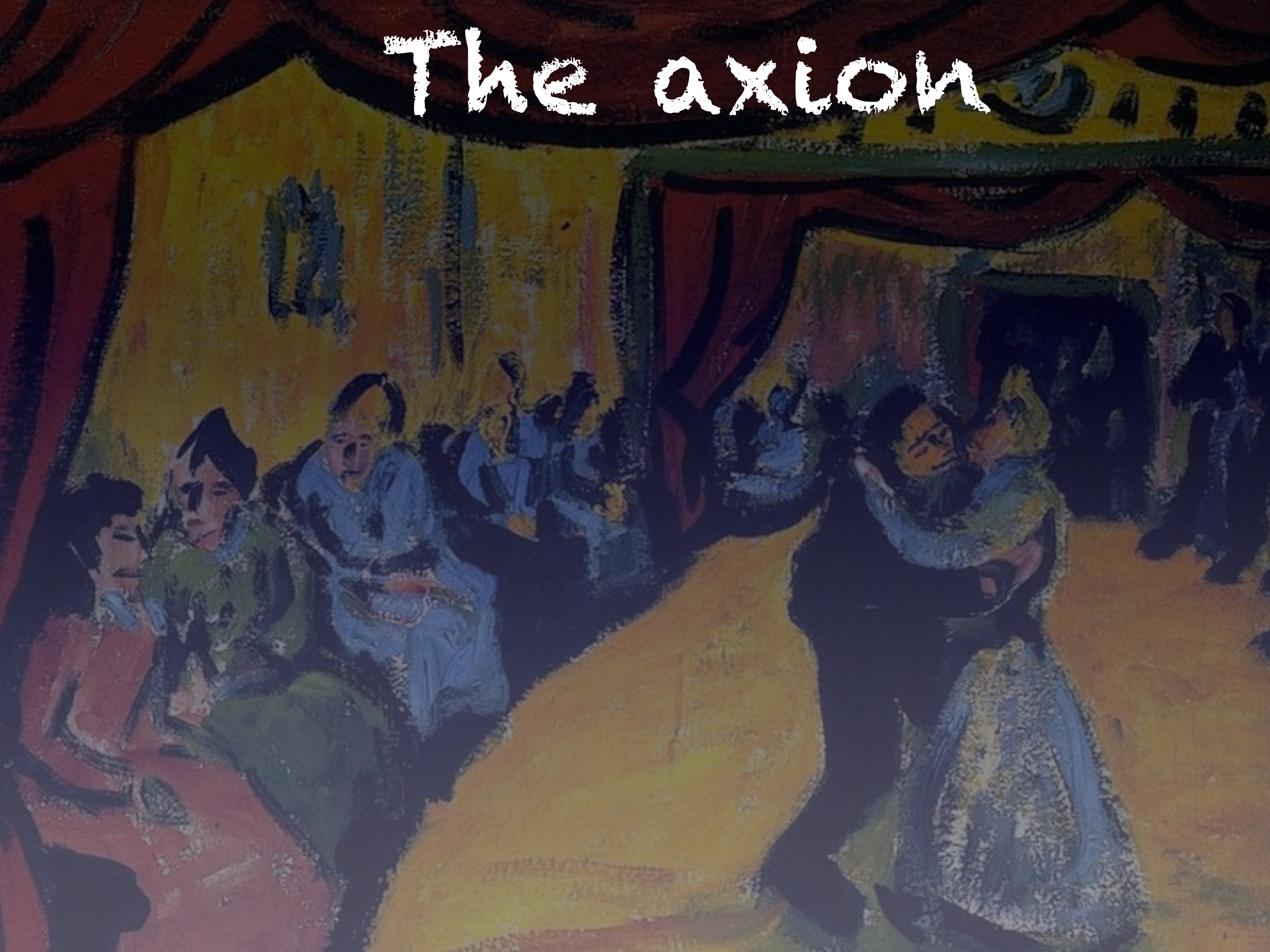
# Effective potential

Does not respect
the U(1) global symmetry

$$\langle M \rangle = \frac{1}{Z}\int \mathcal{D}\phi \, M \, e^{-S[\phi}$$

Wormhole fluctuations integrated out

# The axion



# The axion

$$V(\phi) = \Lambda_{\rm QCD}^4 \cos\left(\frac{\phi}{f_a}\right) + \frac{1}{L^4}\, e^{-S_{\rm wh}} \cos$$

$$S_{\rm wormhole} = \frac{\sqrt{3\pi n}}{8 f_a}$$

Gauge symmetry preserved

$$\phi \longrightarrow \phi + 2k\pi f_a, \quad k$$

# The axion

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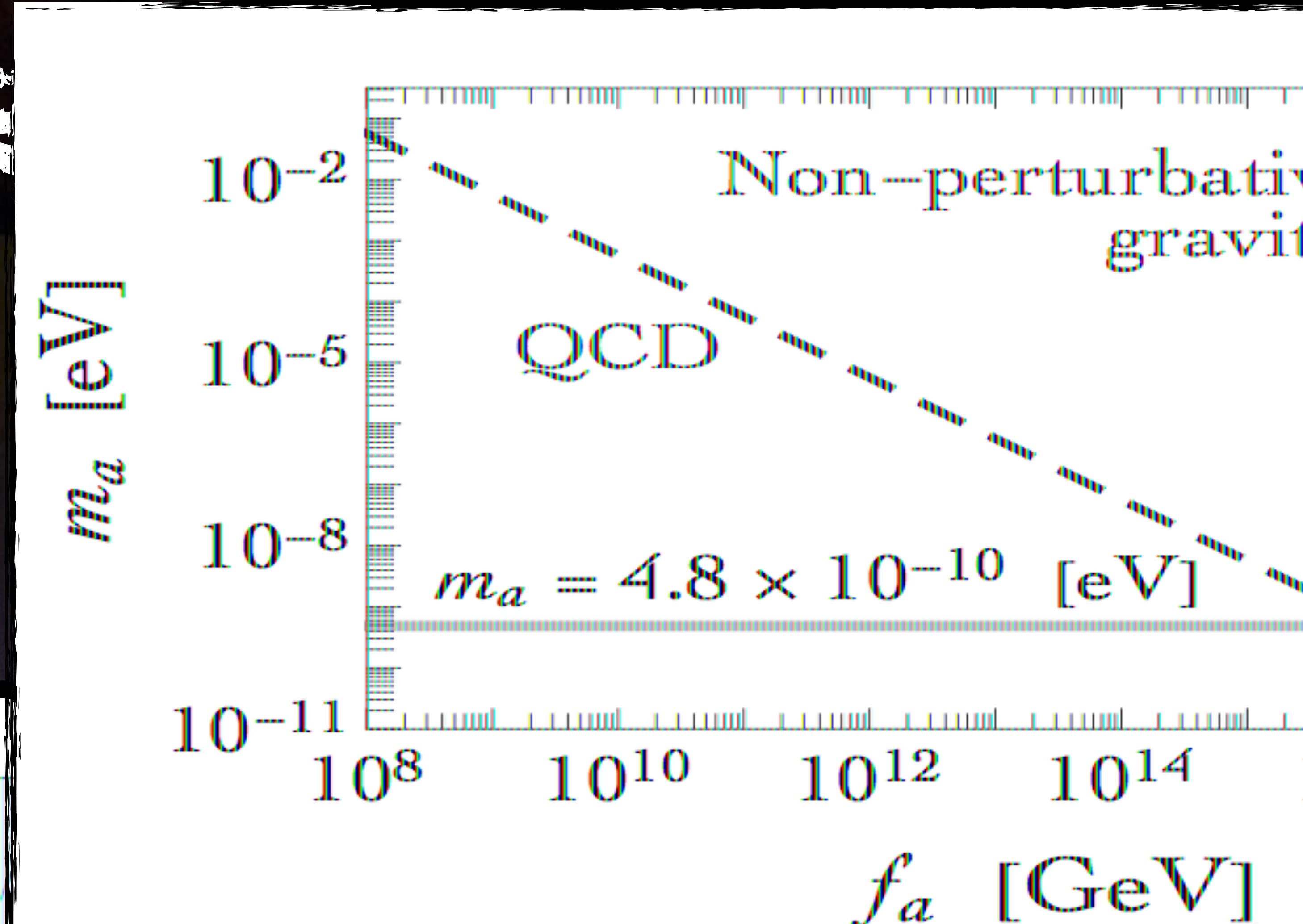
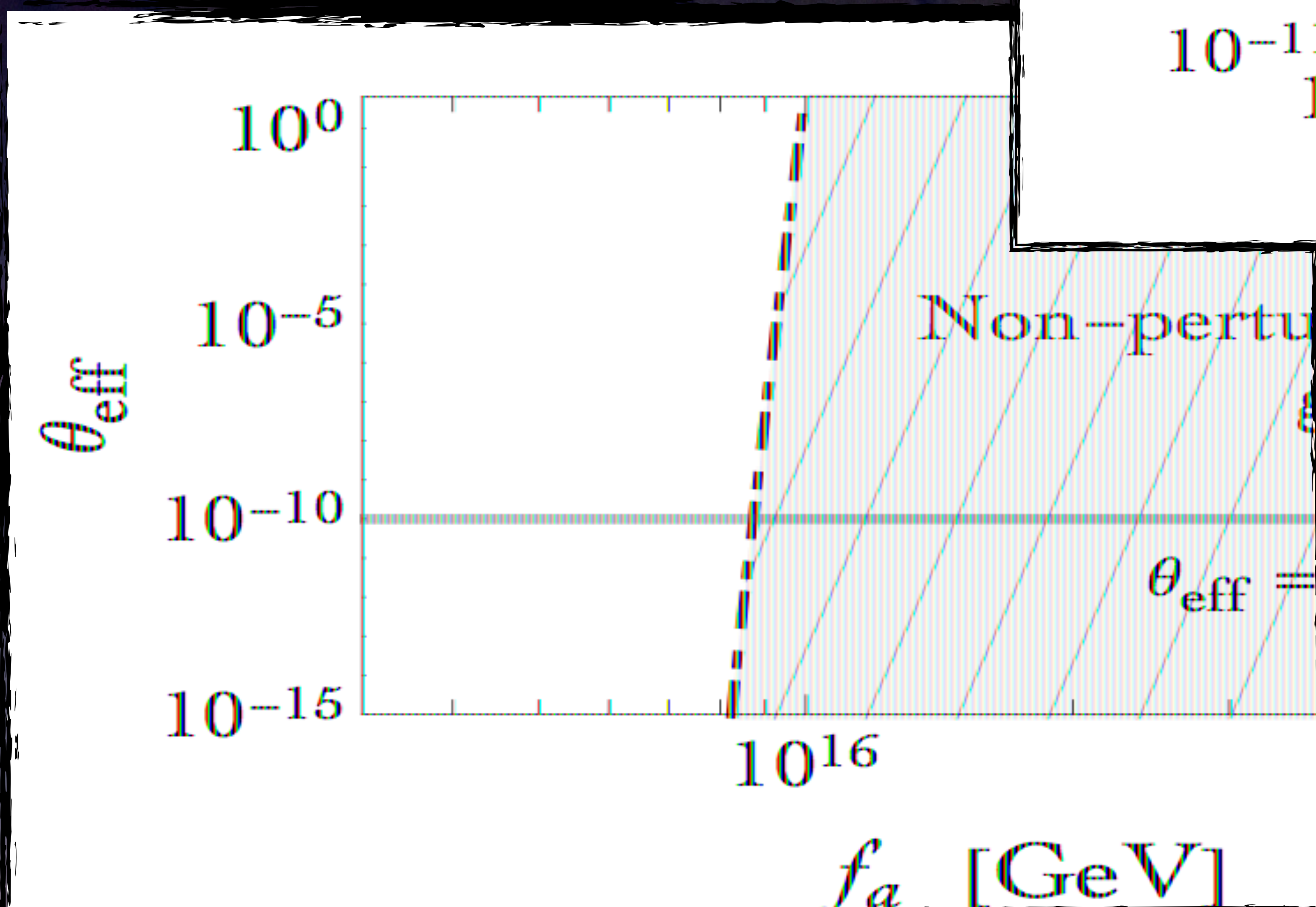
$$m_a^2 \simeq \frac{\Lambda^4_{\rm QCD}}{f_a^2} + \frac{(1/L)}{f_a^2}$$

$$\theta_{\rm eff} \simeq \frac{(1/L)^4}{\Lambda^4_{\rm QCD}} \sin\delta\, e^{-}$$

# The

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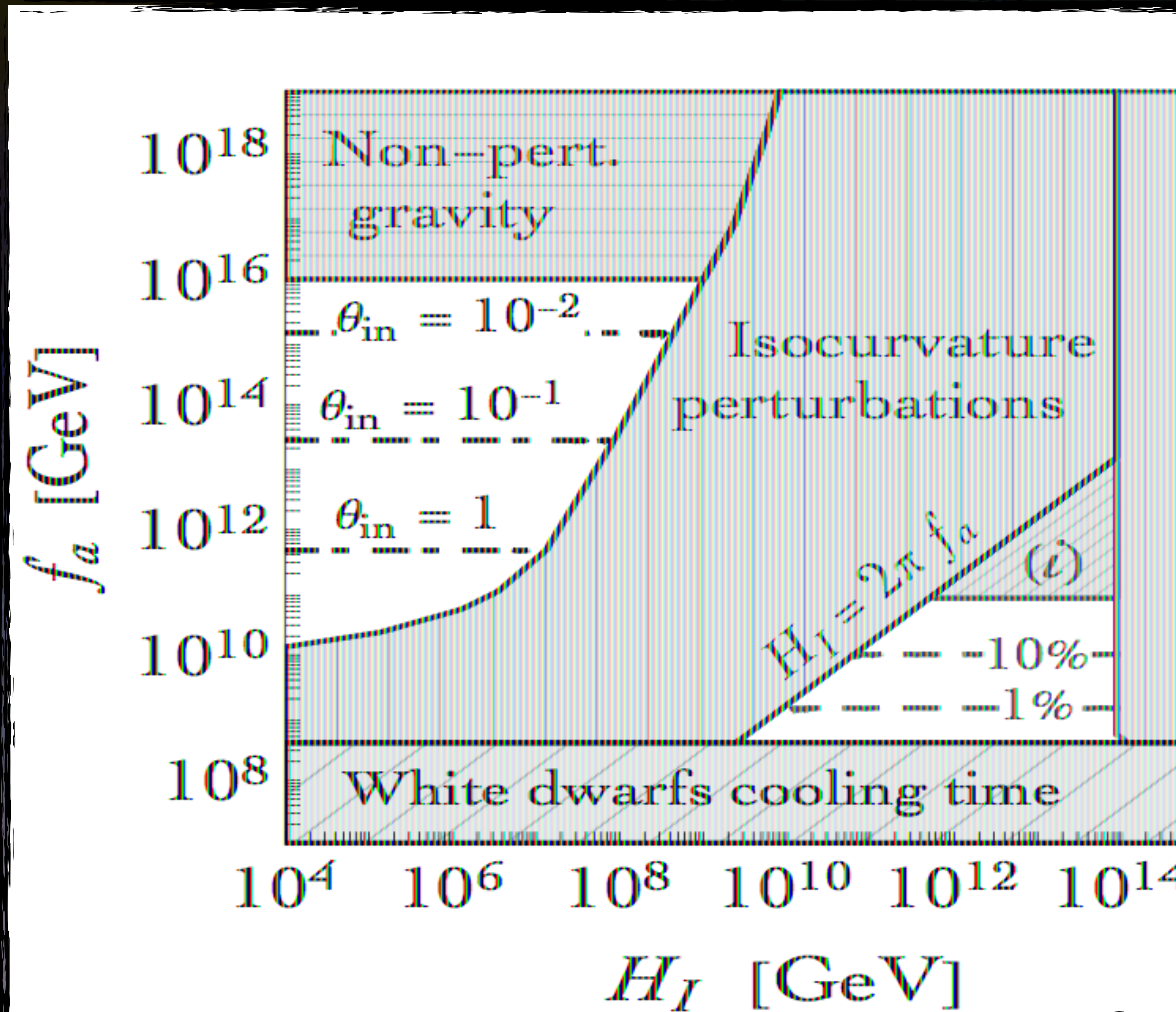
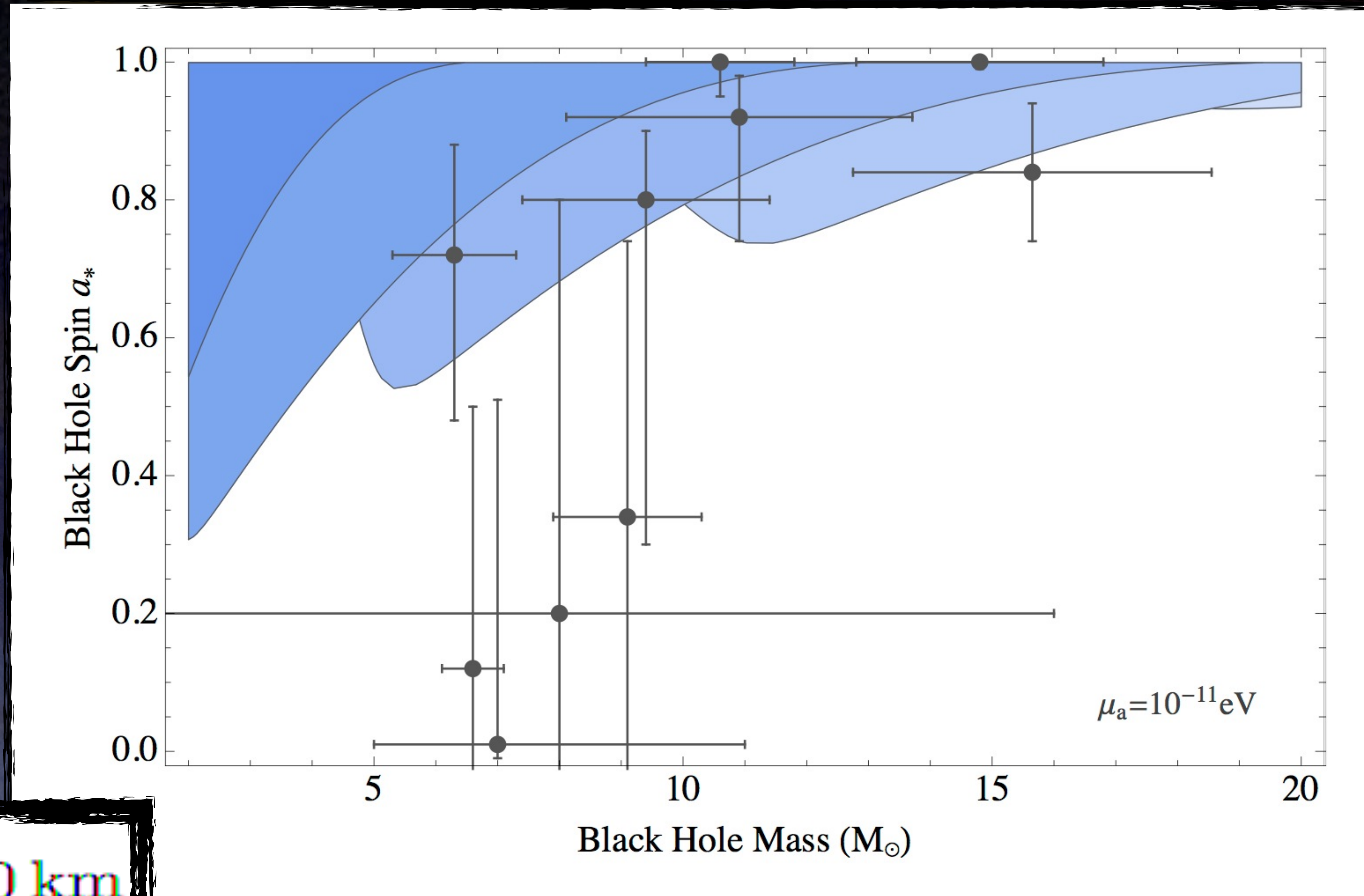


Non-pert. gravity
$\theta_{\rm in} = 10^{-2}$
$\theta_{\rm in} = 10^{-1}$
$\theta_{\rm in} = 1$
Isocurvature perturbations
$H_I = 2\pi f_a$
(i)
10%
1%
White dwarfs cooling time
$10^{18}$
$10^{16}$
$10^{14}$
$10^{12}$
$10^{10}$
$10^{8}$
$f_a$ [GeV]
$10^4$
$10^6$
$10^8$
$10^{10}$
$10^{12}$
$10^{14}$
$H_I$ [GeV]

# QCD superradiance

Arvanitaki, Baryakhtar, Huang, Phys.Rev. D91, 084011, [arxiv/1411.2263]




$$m_a \sim 6 \times 10^{-12} \left( \frac{30\text{ km}}{R} \right.$$

$$3 \times 10^{17} \lesssim f_a\,[\text{GeV}] \lesssim$$

$$6 \times 10^{-13} \lesssim m_a\,[\text{eV}] \lesssim 2 \times$$

# Ultra-light scalar as cosmological DM

Dark matter must behave sufficiently classically.

If we suppose dark matter to be a boson with mass $m_a$ and velocity v,

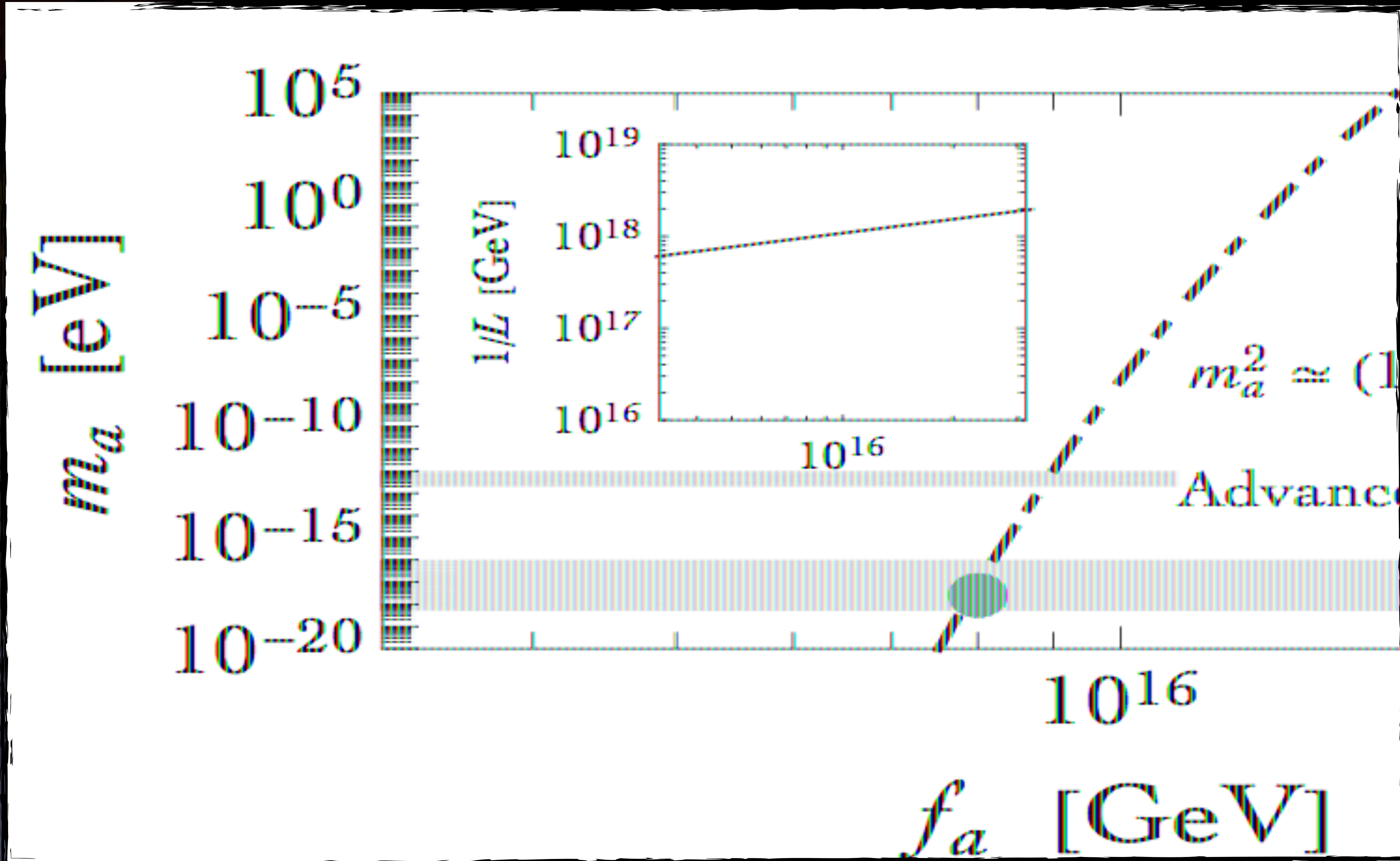
we can require to behave classically down to the typical size of Milky Way satellite galaxies, and obtain the condition:

$$\lambda_{\mathrm{De\ Broglie}} = \frac{1}{m_a v} \lesssim 1\,\mathrm{kpc} \quad \Longrightarrow \quad m_a \gtrsim 10^{-22}\ ($$

"Fuzzy" dark matter

$$\Omega_a h^2 \approx 0.1 \left(\frac{f_a}{10^{17}\,\mathrm{GeV}}\right)^2 \left(\frac{m}{10^{-22}}\right.$$

W. Hu, R. Barkana and A. Gruzinov, Phys. Rev. Lett. 85, 1158 [astro-ph/0003365]

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# UV completion of GR

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$$[v] = [G_F]^{-1/2} = \frac{[M}{[g}$$

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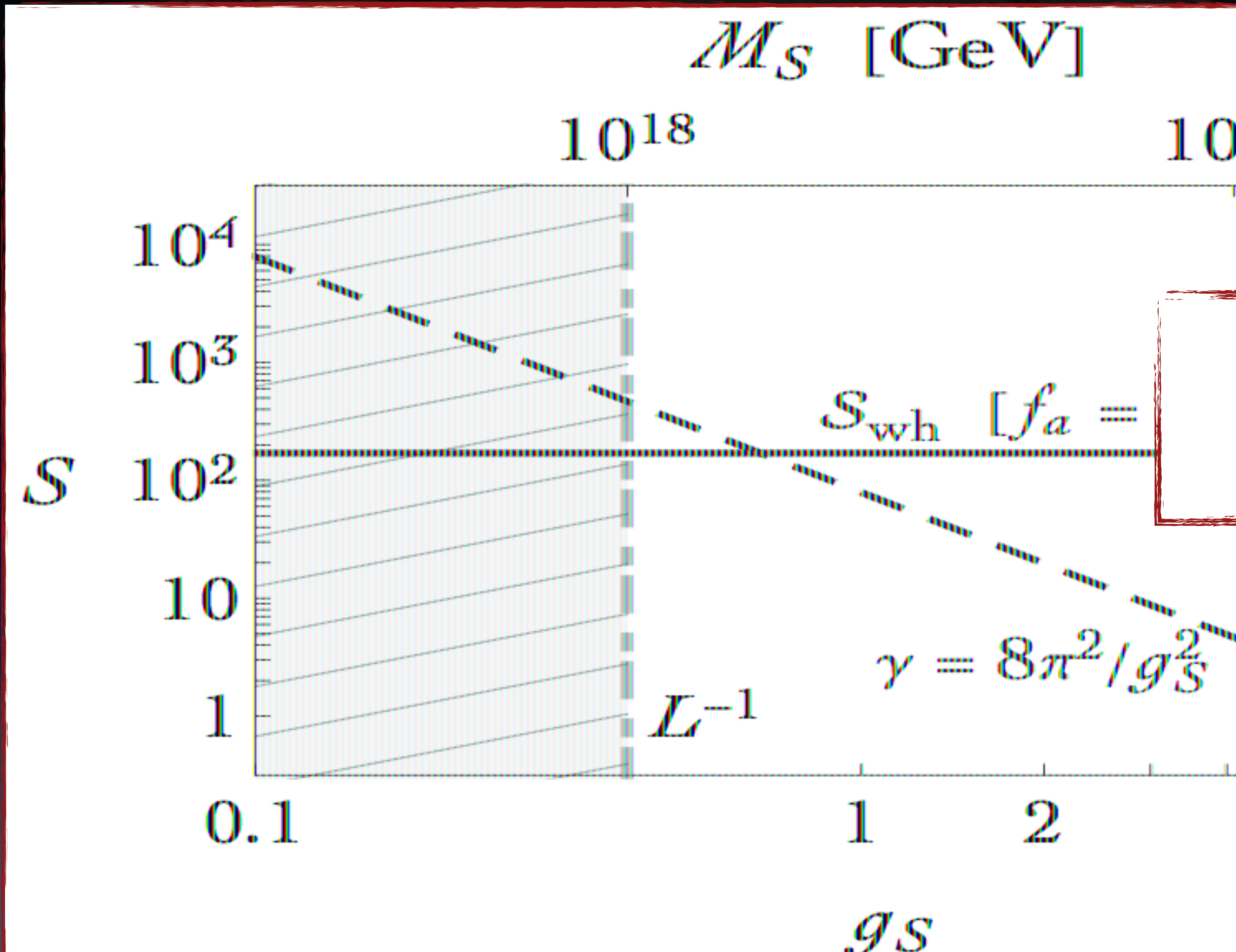
$$[M_{\mathrm{Pl}}] = [G_N]^{-1/2} =$$




$$S_{\rm wormhole} = \frac{\sqrt{3\pi n}}{8 f_a}$$

$$[M_{\rm Pl}] = [G_N]^{-1/2} =$$

# Outlook

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## Gravity breaks global symmetries non-perturbatively

$$V(\phi) = \Lambda_{\rm QCD}^4 \cos\left(\frac{\phi}{f_a}\right) + \frac{1}{L^4}\, e^{-S_{\rm wh}} \cos$$

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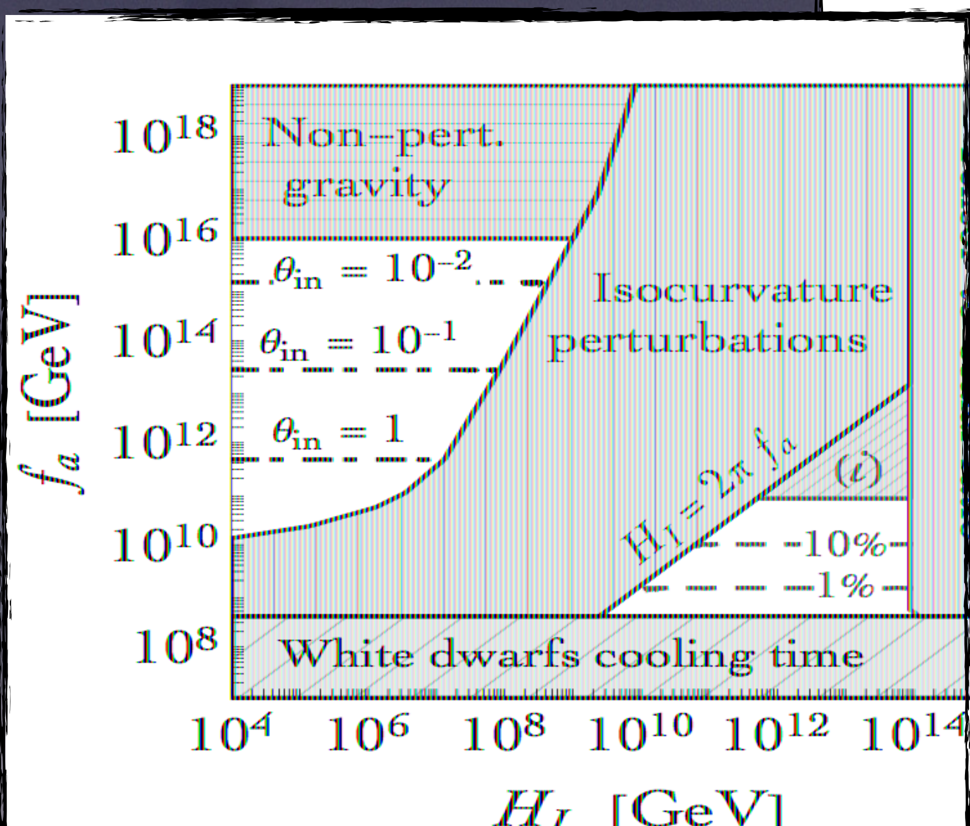


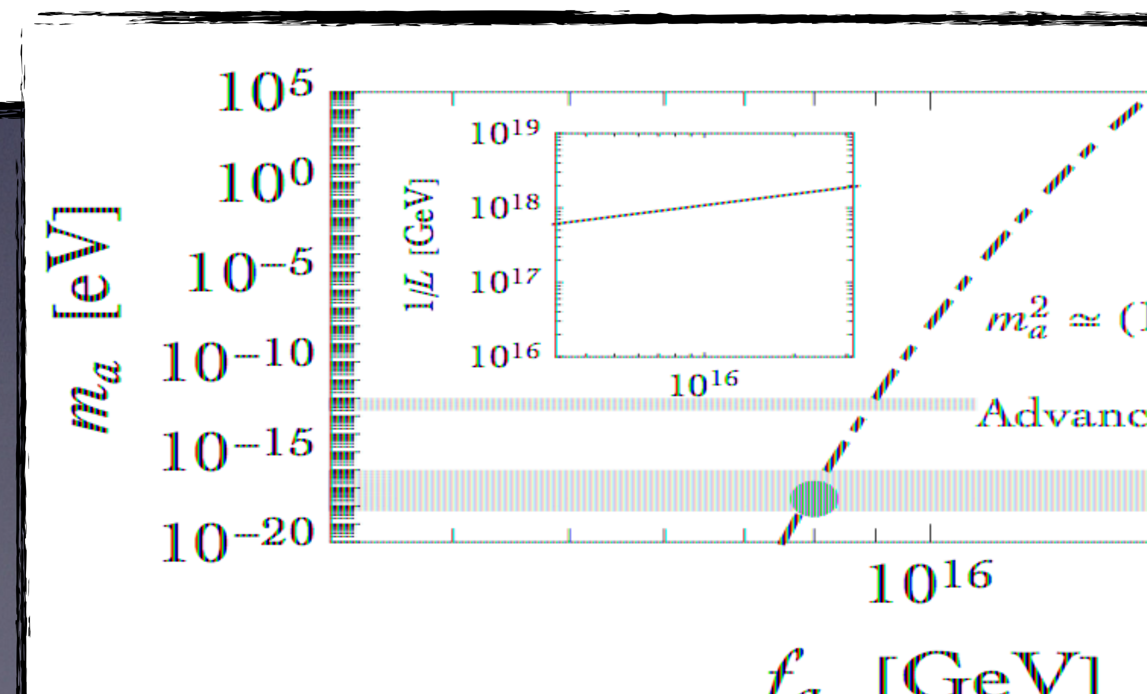

# Background material

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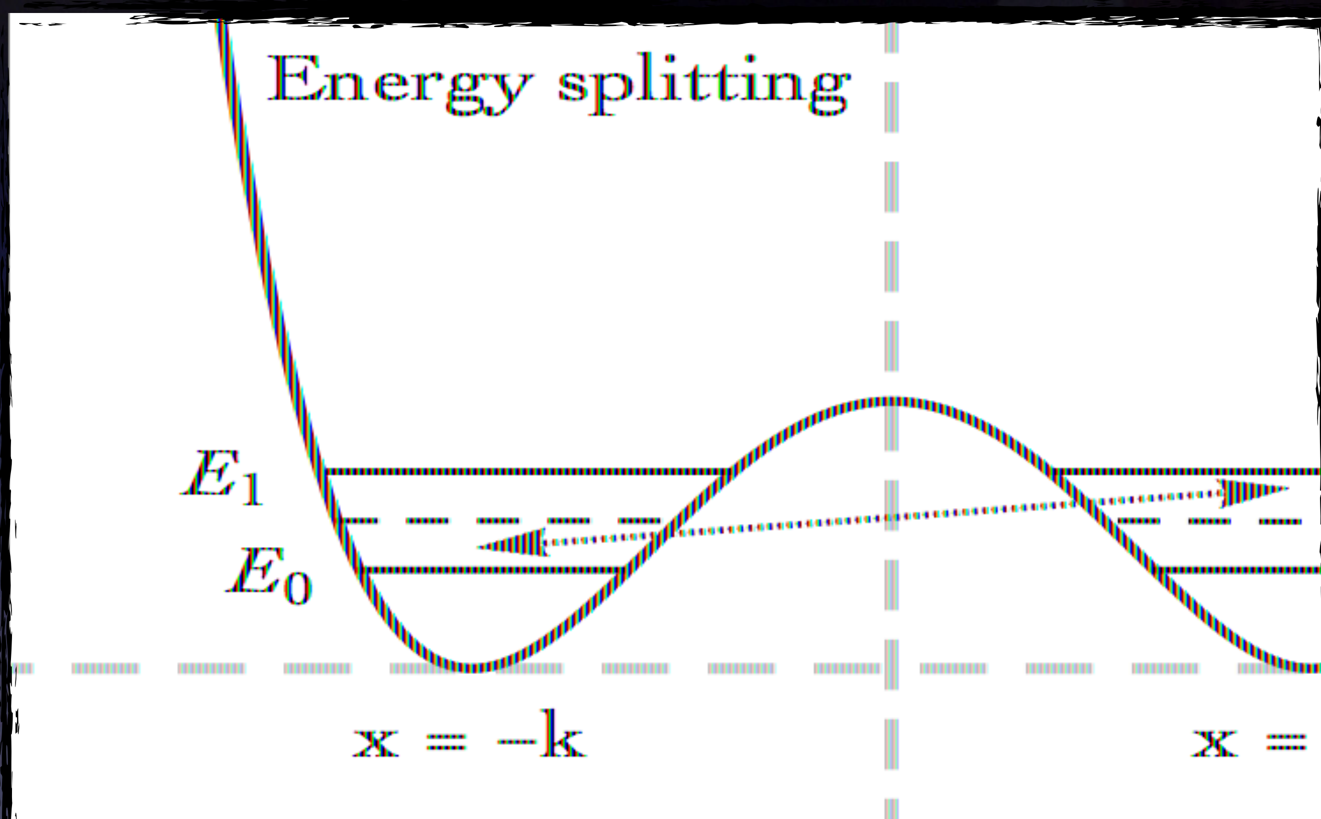


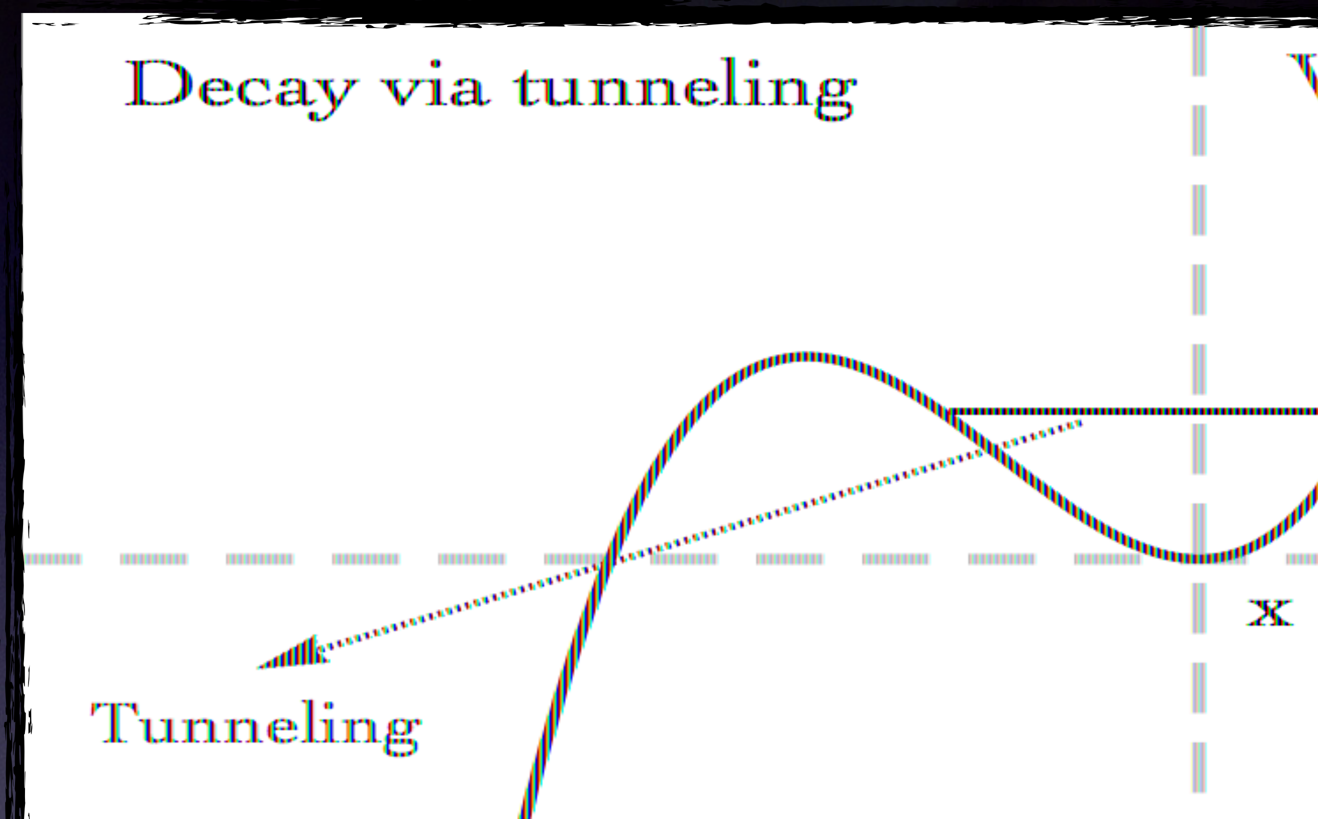


$$\langle f|e^{-TH}|i\rangle = \sum_n e^{-TE_n}\langle f|n\rangle\langle n|i\rangle \approx \frac{\mathcal{N}}{\sqrt{\det' (}}$$

# Background material