MMP II
Exercise Sheet 4

FS 19
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Exercise 1 [Regions (3 points)]

Let $f(z) : \mathbb{C} \to \mathbb{C}$ be the following function:

$$f(z) = \frac{iz^2 + 1}{1 - iz^2}.$$

Find the regions of $\mathbb{C}$ $U_<$, $U_=$ and $U_>$ for which

(a) $z \in U_< \iff |f(z)| < 1$,

(b) $z \in U_= \iff |f(z)| = 1$,

(c) $z \in U_> \iff |f(z)| > 1$.

Exercise 2 [Path integrals (6 points)]

Find the following integrals (for parts (b), (c) and (d) use Cauchy's integral formula):

(a) for a circle, C, centred around $z = 0$,

$$\frac{1}{2\pi i} \oint_C \frac{dz}{z},$$

(b) for a circle, C, with radius 2,

$$\oint_C \frac{\sin z}{2z - \pi} dz$$

(c) for a square, C, with vertices ± 1 and $\pm i$,

$$\oint_C \frac{\mathrm{e}^{3z}}{(z - \ln 2)^4} dz.$$

(d) for a circle, C, with radius 1 and $a, b \in \mathbb{R}, 0 < b < a$,

$$\oint_C \frac{d\theta}{a + b\cos\theta}$$

Exercise 3 [Laurent series (6 points)]

Expand the following functions into Laurent series:

(a) $f(z) = \frac{1}{1+z^2}$ about $z = 0$,

(b) $f(z) = \frac{z+2}{z+3}$ about $z = -1$,

(c) $f(z) = \frac{1}{z(z-1)(z-2)^2}$ about $z = 0$,

(d) $f(z) = \frac{\sin \pi z}{4z^2-1}$ about $z = \frac{1}{2}$ (you only need to give the first three terms for this one).