University of Zurich[UZH]

Solid State Physics
Exercise Sheet 7
Electron gas

HS19
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Exercise 1 *Chemical potential in two dimensions*

(a) Derive the density of states $D(\epsilon)$ of a two-dimensional gas of free electrons.

(b) Show that the chemical potential of a Fermi gas in two dimensions is given by:

$$\mu(T) = k_{\mathrm{B}} T \ln \left[\exp\left(\frac{\pi n \hbar^2}{m k_{\mathrm{B}} T} \right) - 1 \right], \tag{1}$$

where $n = N/L^2$ is the area density of particles. Hint: Use $N = \int_0^\infty D(\epsilon) f(\epsilon) d\epsilon$.

(c) In two dimensions, derive the Fermi energy E_{F} and express $\mu(0)$ in terms of the Fermi energy E_{F}.

(d) Make a plot of equation (1).

Exercise 2 *Fermi energy E_{F} and Fermi temperature T_{F}*
The atom ^{3}He has spin $\frac{1}{2}$ and is a fermion. The density of liquid ^{3}He is $0.081\,\mathrm{g/cm^3}$ near $T = 0\,\mathrm{K}$. Calculate the Fermi energy E_{F} and Fermi temperature T_{F}.

Exercise 3 *Ground-state properties of an electron gas*
Consider a three-dimensional gas of N free electrons at $0\,\mathrm{K}$.

(a) Show that the kinetic energy is $U = \frac{3}{5} N E_{\mathrm{F}}$.

(b) Calculate (i) the average velocity and (ii) the highest velocity of electrons when Fermi temperature is $T_{\mathrm{F}} = 10^4$ K.

(c) Express the bulk modulus $K = -V \frac{\partial P}{\partial V}$ of the electron gas in terms of $n = \frac{N}{V}$ and E_{F}. Note that the pressure P can be obtained through $P = -\frac{\partial U}{\partial V}$.

(d) Calculate the bulk modulus of Na using (c) and the density $0.917\,\mathrm{g/cm^3}$, and show that the result is consistent with the literature value $K = 6.3$ GPa.