Magnetism and Superconductivity

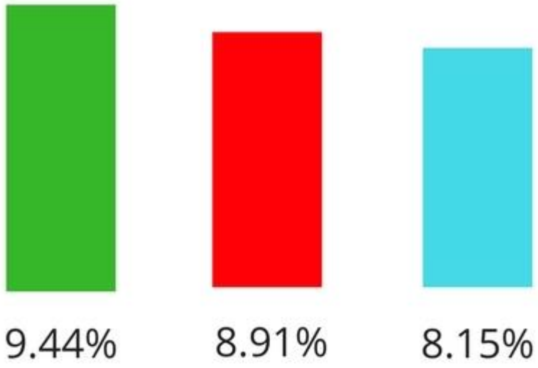
- fast overview -

Magnetism

Oxford - Simon Chapter 19-20

GLOBAL MAGNETIC MATERIALS MARKET BY APPLICATION

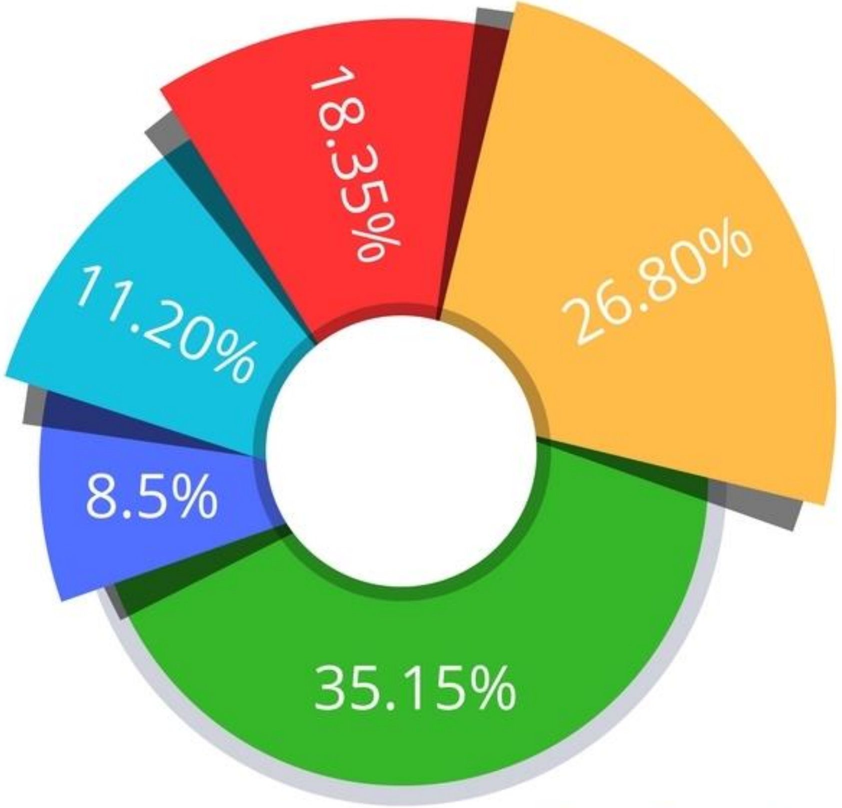
CAGR 2016-2021



LEGEND

- 🟩 **AUTOMOTIVE AND TRANSPORTATION**
- 🟥 **INDUSTRIAL**
- 🟦 **OTHERS**
- 🟨 **ELECTRONICS AND INSTRUMENTATION**
- 🟦 **ENERGY GENERATION**

2016 MARKET SHARE



technavio

Some definitions

- quantum-mechanics behaviour of electrons

$$\mu_B = \frac{e\hbar}{2m_e}$$

$\left(\frac{m_p}{m_e} \sim 1800 \rightarrow \text{contribution of nuclei is much smaller} \right)$

- Magnetization $\left(\frac{\text{magnetic moment}}{V} \right)$

$$\bar{M} = \chi \bar{H}$$

↑ susceptibility (dimensionless)

in a solid $\bar{B} = \mu_0 (\bar{H} + \bar{M})$

↑ permeability of free space

Paramagnet : material with $\chi > 0$

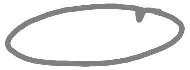
(magnetization in the direcction of applied field)

Ex. free spin

Diamagnet : $\chi < 0 \longrightarrow M$ is opposite to The field

In all materials but i
overcome by other stronger mag effects.

qualitatively-like Lenz-Law



Ferromagnet. $M \neq 0$ even when $B = 0$ (spontaneous magnetization)

Ex. iron (magnet fridges

A live frog levitates inside a 32 mm diameter vertical bore of a Bitter solenoid in a magnetic field of about 16 T at the Nijmegen High Field Magnet Laboratory.

"Everyone's Magnetism" by A. Geim, Physics Today, 36 (1999)

Atomic magnetism: Hund's rules

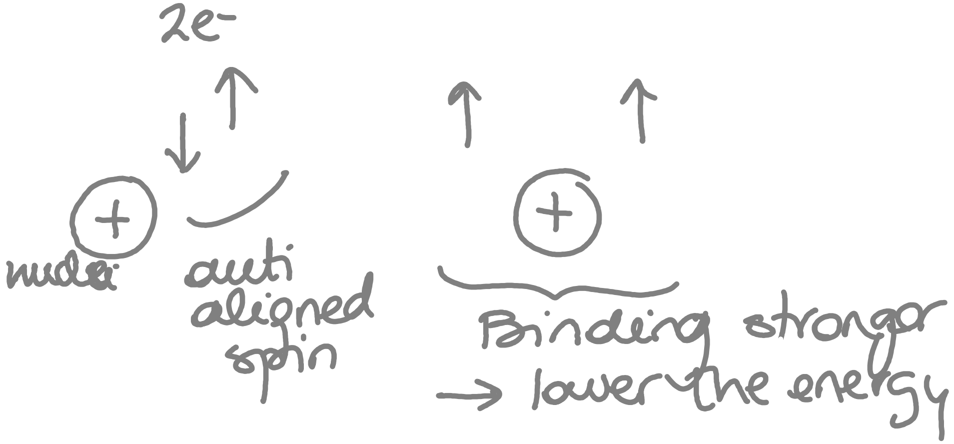
e- in isolated atoms → $|n, l, l_z, \sigma_z\rangle$

interested for magnetism in partially filled shells

We previously saw Aufau principle and Madelung's rule for n and l shells, now l_z & σ_z

∃ set of rules: Hund's rules

① Electrons try to align their spins →



② L is also maximized (subjected to ①)

③ $J = |L \pm S|$

⊕ more than half filled shell

⊖ less than " " "

reason: spin-orbit coupling

Ex: Pr ($Z = 59$) → 56 filled shells

→ $3e^-$ in f-shells

$f \to L = 3$

$2L+1$

$$\begin{array}{cccccccc} & - & - & - & - & \uparrow & \uparrow & \uparrow \\ l_z & -3 & -2 & -1 & 0 & 1 & 2 & 3 \end{array}$$

$J = L - S = 9/2$

$L_z = 6$

$S = 3/2$

vector potential $\nabla\times A = B$

Atomic magnetism

How e- couple to external field?

electron g-factor (2)

Bohr magnetron $\mu_B = \frac{e\hbar}{2m}$

$$\mathcal{H} = \underbrace{\frac{(\bar{p}+e\bar{A})^2}{2m}}_{\text{Kinetic term}} + \underbrace{V(r)}_{\text{attraction potential}} + \underbrace{g\mu_B \bar{B}\bar{\sigma}}_{\text{zeeman}}$$

e- spin

For a uniform mag field: $\bar{A} = \frac{1}{2}\bar{B}\times\bar{r}$ ($\Rightarrow$ $\bar{p}$ and $\bar{A}$ commute)

[...]

$$\mathcal{H} = \underbrace{\frac{p^2}{2m} + V(r)}_{\mathcal{H}(B=0)} + \underbrace{\mu_B \bar{B}(\bar{l} + g\bar{\sigma})}_{\text{PARAMAGNETIC term}} + \underbrace{\frac{e^2}{2m}\bar{A}^2}_{\text{DIAMAGNETIC term}}$$

angular moment $\hbar l = \bar{r}\times\bar{p}$

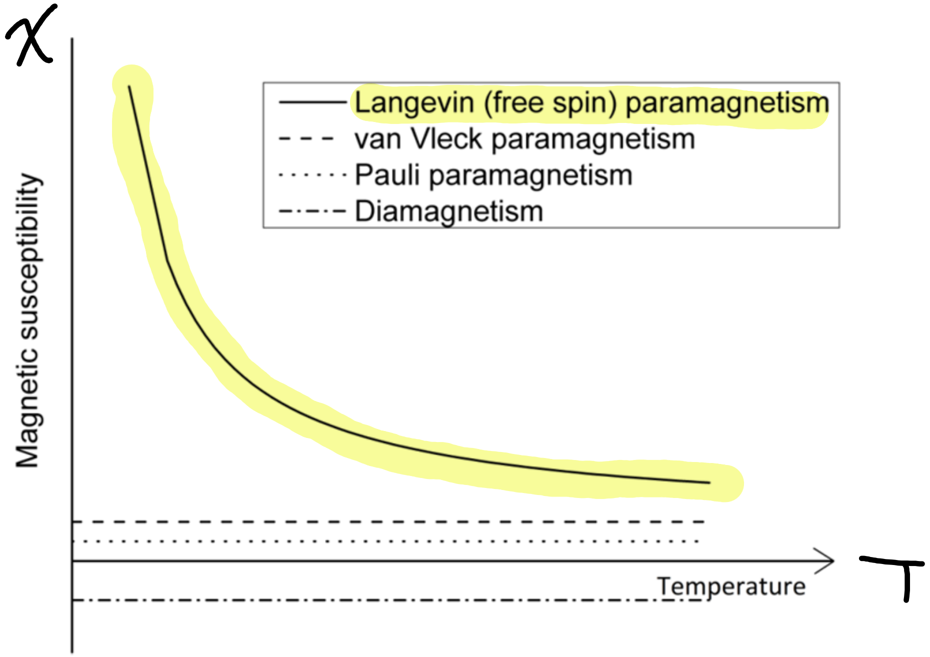
(more relevant in gL)

Free spin Langevin paramagnetism

$$\boxed{\chi = \frac{n \mu_0 \mu_B^2}{k_B T}}$$

n : # of spins per unit volume

"CURIE LAW" $\chi \sim \frac{C}{T}$



What is different in a solid compared to isolated atoms?

Pauli paramagnetism in metals

$$\epsilon(\bar{K},\uparrow) = \frac{\hbar^2|K|^2}{2m} + \mu_B B$$

$$\epsilon(\bar{K},\downarrow) = \frac{\hbar^2|K|^2}{2m} - \mu_B B$$

$$M = -\frac{1}{V}\frac{dE}{dB} = -\left([\#\text{up spins}] - [\#\text{down spins}]\right)\cdot \mu_B/V$$

In a mag. field, energy of e- with spin up (same direction field) and spin down

→ more spins ↓

→ M in the direction of the field

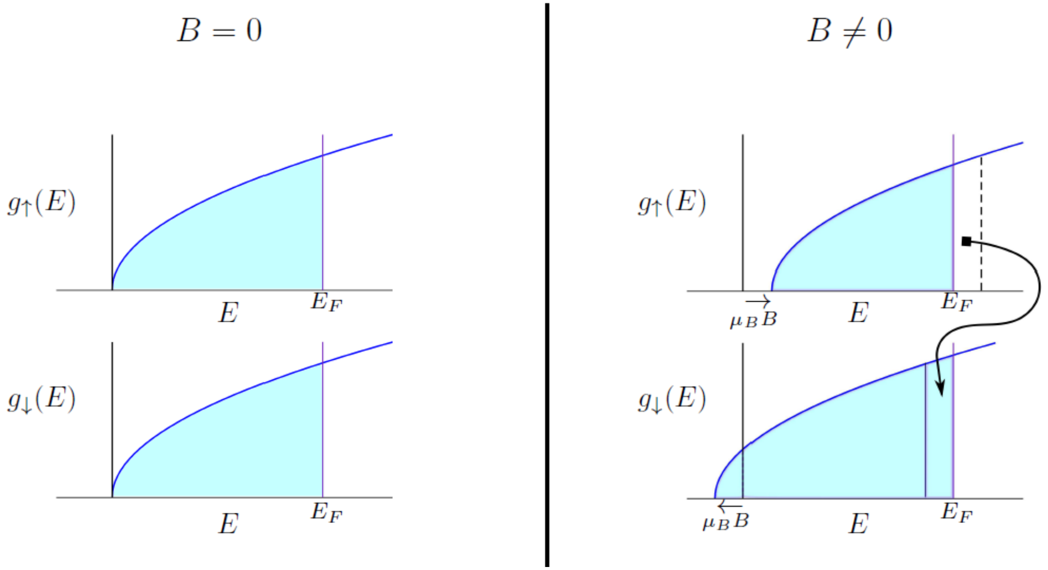
(remember: since e- charge is negative, e- dipole moment is actually opposite to the direction of spin)

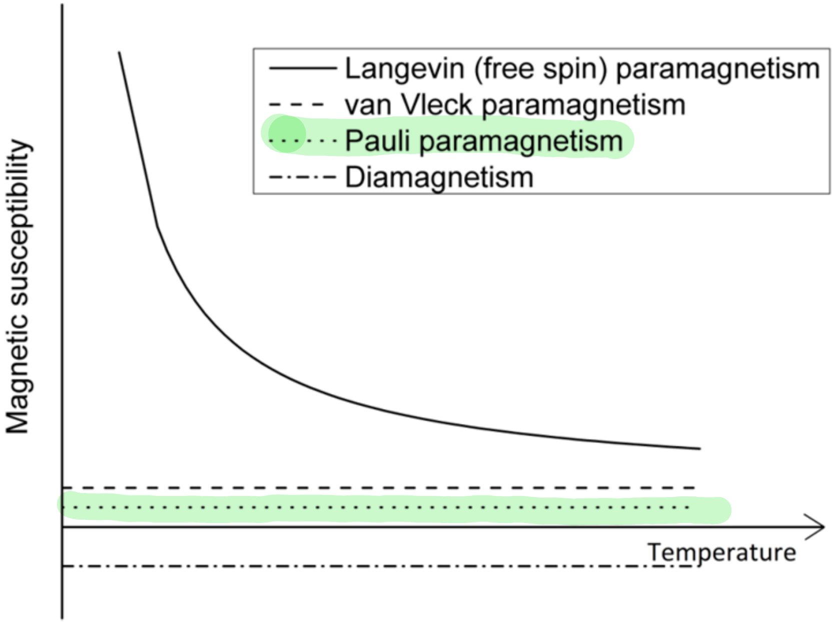
$$\boxed{\chi_{PAULI} = \mu_0 \mu_B^2 D(E_F)}$$

smaller than $\chi_{\text{free spin}}$ by $\sim \frac{K_B T}{E_F}$

PAULI PARAMAGNETISM

$D =$





Diamagnetism

↳ observed if $J=0$

→ Filled shell $L=S=J=0$ (noble gas)

→ $J=0$ because $L=S$

<u>Diamag. term</u> $\frac{e^2}{2m}\bar{A}^2 = \frac{e^2}{2m}\left(\frac{1}{2}(B\times r)\right)^2$

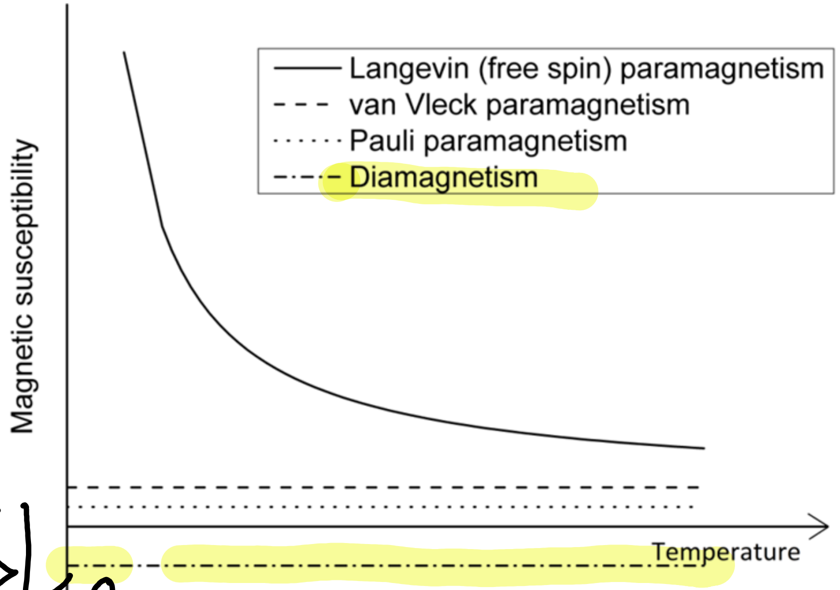
$$\delta E = \frac{e^2}{8m}\langle|B\times r|^2\rangle \underset{\text{if } \bar{B}=B\hat{z}}{=} \frac{e^2B^2}{8m}\langle x^2+y^2\rangle$$

mag mom per e- $\mu = -\frac{\partial E}{\partial B} = -\frac{Be^2}{4m}\underbrace{\langle x^2+y^2\rangle}_{\frac{2}{3}\langle x^2+y^2+z^2\rangle}$

$$\mu = -\frac{Be^2}{6m}\langle r^2\rangle$$

$$M = \underbrace{\rho}_{\text{density } e^-}\cdot\mu \longrightarrow \chi = -\frac{\rho e^2\mu_0\langle r^2\rangle}{6m}$$

a solid (i.e. noble gas atoms crystal) $\longrightarrow \rho = \underbrace{n}_{\text{density of } e^-}\underbrace{Z}_{\#e^- \text{ per atom}} \longrightarrow \boxed{\chi_{\text{Larmor}} = -\frac{Zne^2\mu_0\langle r^2\rangle}{6m}} < 0$



Collective magnetism

* no e- hopping assumption (insulator)

"spin" S_i on atom i

$$\mathcal{H} = \underbrace{\sum_i g\mu_B \bar{B} S_i}_{\text{coupling B and spin}} - \underbrace{\frac{1}{2}\sum_{ij} J_{ij} S_i S_j}_{\text{Interaction energy}}$$

to avoid double counting

J_{ij} = interaction between atoms i & j = J_{ji}

= **EXCHANGE**

(energy difference of having spins aligned versus antialigned)

simplification

nearest neighbour interaction

$$\begin{cases} J_{ij} = J & \text{if } i,j \text{ are neighbours} \\ J_{ij} = 0 & \text{otherwise} \end{cases}$$

let's assume $B = 0$:

$$\mathcal{H} = -\frac{1}{2}\sum_{\langle i,j \rangle} J\, \bar{S}_i \cdot \bar{S}_j$$

Heisenberg Hamiltonian

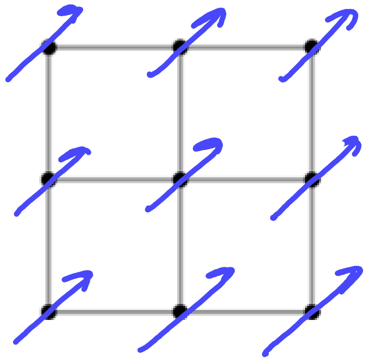
1rst neigh.

"rotationally symmetric" (it does not care in which direction spin points) → However, this is not normally the case and anisotropic term should be added: i.e. $\delta H = -K\sum_i |S_i^z|^2$

Magnetic order

→ spontaneous order, even the absence of magnetic field.

$J>0$

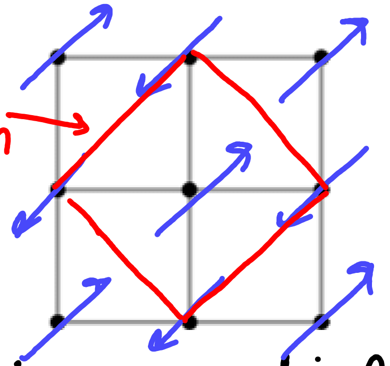


$\bar{M} \neq 0$

spins align (to minimize E)

⇒ **FERROMAGNETISM**

$J<0$



it can be seen by neutron diffraction

notice that magnetic unit cell is larger than "crystal" u.c.

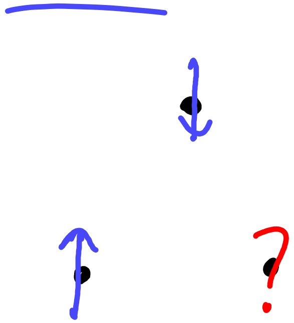
there is magnetic order but no net magnetization

neighbours antialign

⇒ **ANTIFERROMAGNETISM**

Note 1

triangular lattice

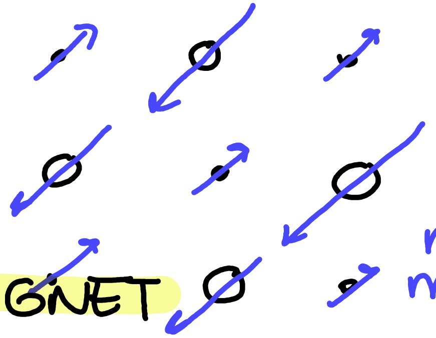


unhappy spin!!

FRUSTRATED AFM.

Note 2

2 (or more) atom basis



$S_{big} \neq S_{small}$ atom

⇓

net magnetization

FERRIMAGNET

Ferromagnet

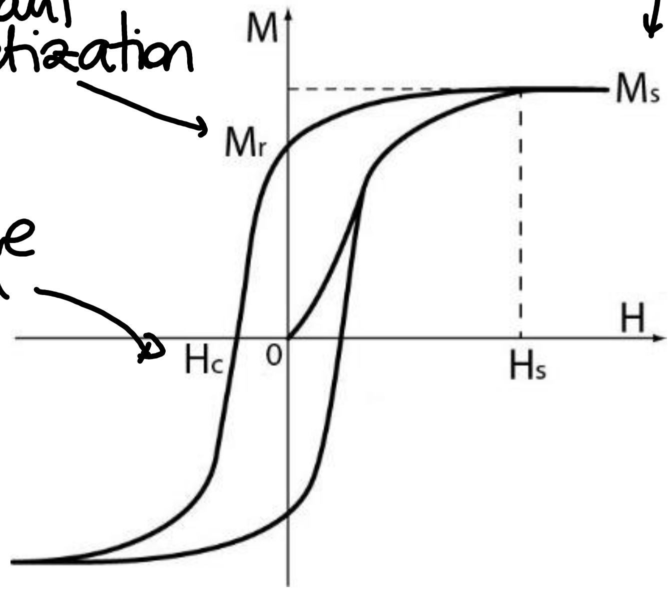
M(H)

saturation magnetization (intrinsic)

Extrinsic properties

Remnant magnetization

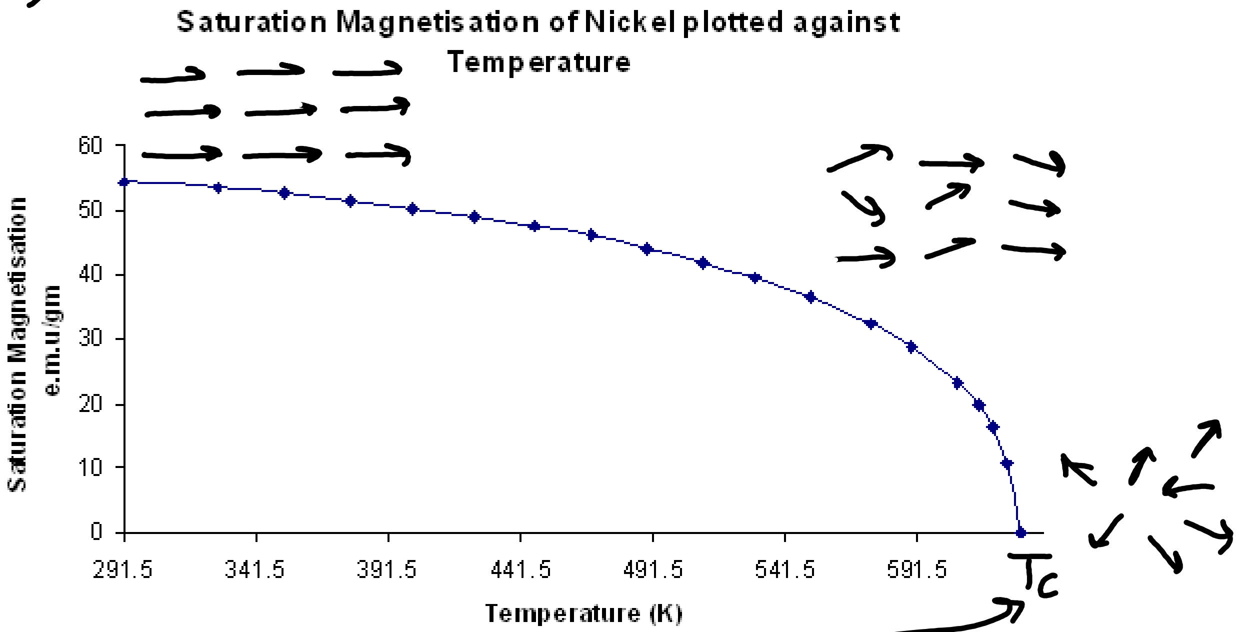
Coercive field



Hard FM: $M \neq 0$ @ $B = 0$
(broad square loops)

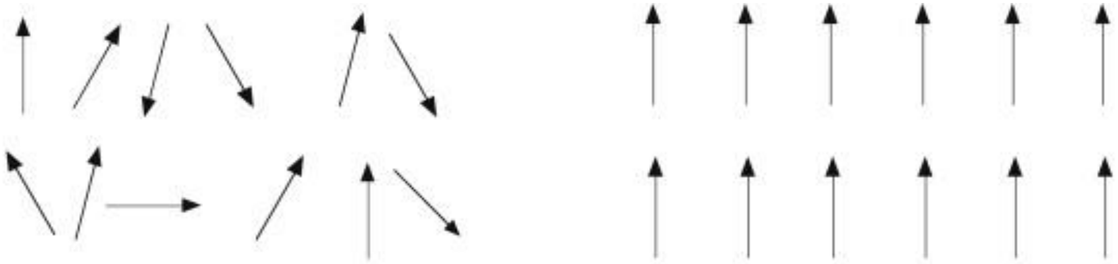
Soft FM: narrow loops
temporary magnets
($M \to 0$ as field is removed

M(T)



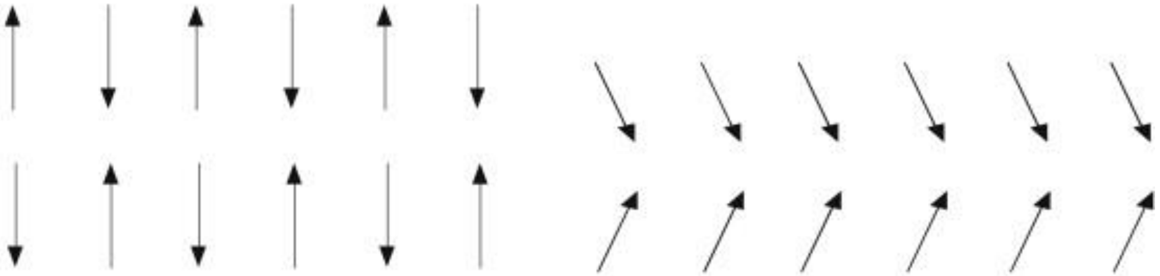
Curie Temperature
(@ T_c, $M \to 0$)

Substance	T_{Curie} [K]
Iron	1043
Cobalt	1394
Nickel	631
Gadolinium	317
Fe_2O_3	893



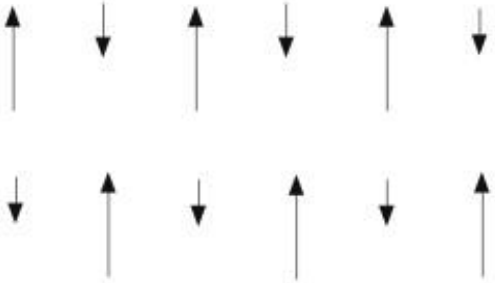
(a) Paramagnet with random spins

(b) Ferromagnetic ordering of spins

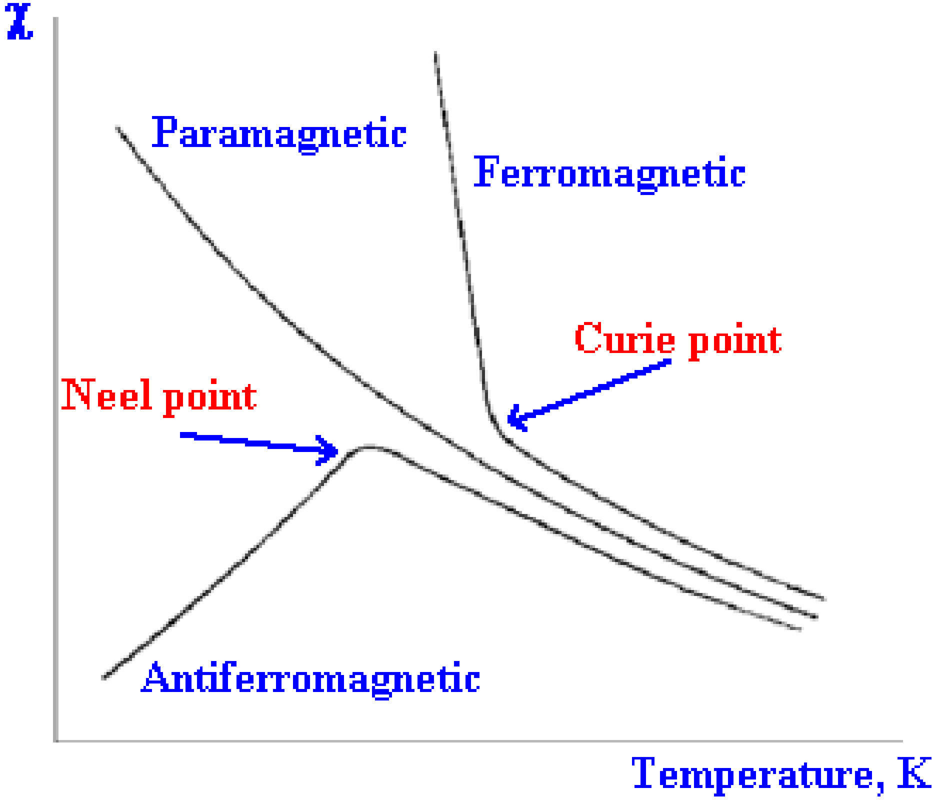


(c) Antiferromagnetic ordering

(d) Weak ferromagnet (or Canted antiferromagnet)



(e) Ferrimagnetic ordering

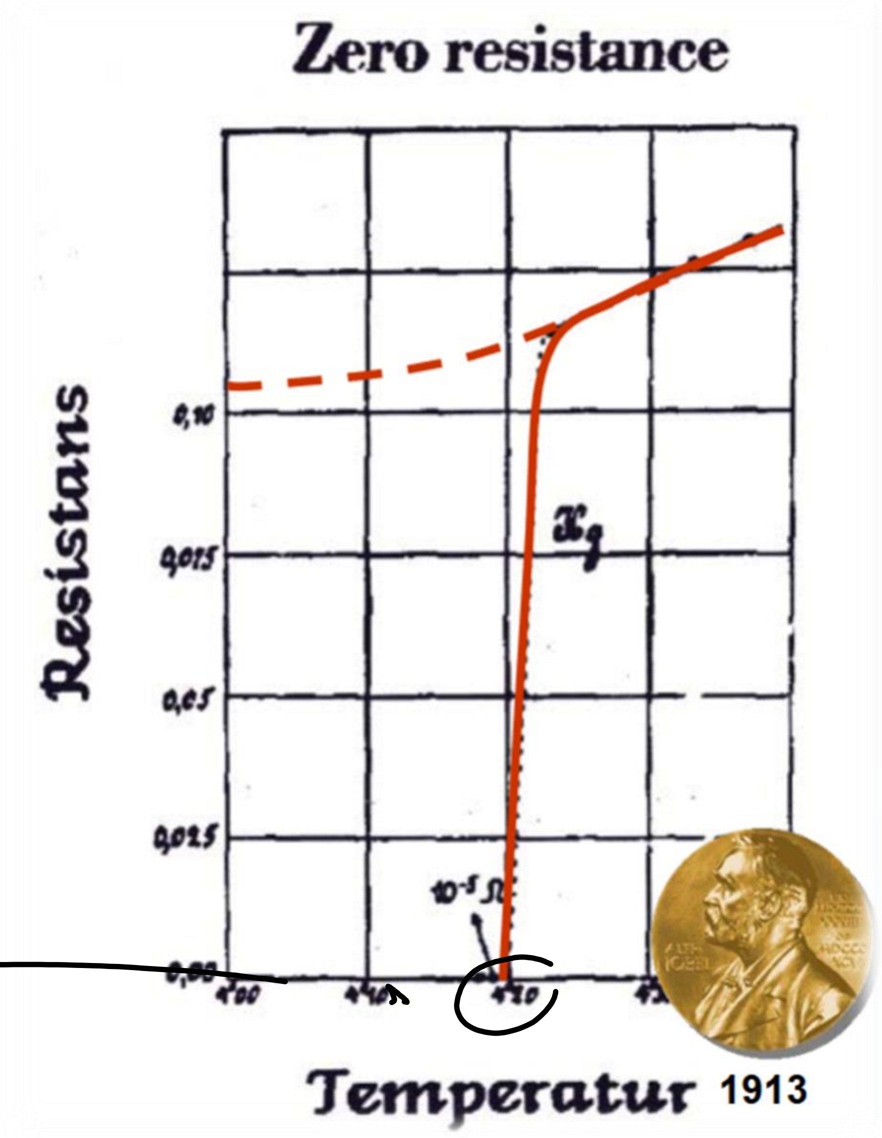

χ
Paramagnetic
Ferromagnetic
Curie point
Neel point
Antiferromagnetic
Temperature, K

Superconductivity

(very basic overview here!)

Zero electrical resistivity

Superconductivity was discovered by
Kamerlingh Onnes in Leiden in 1911

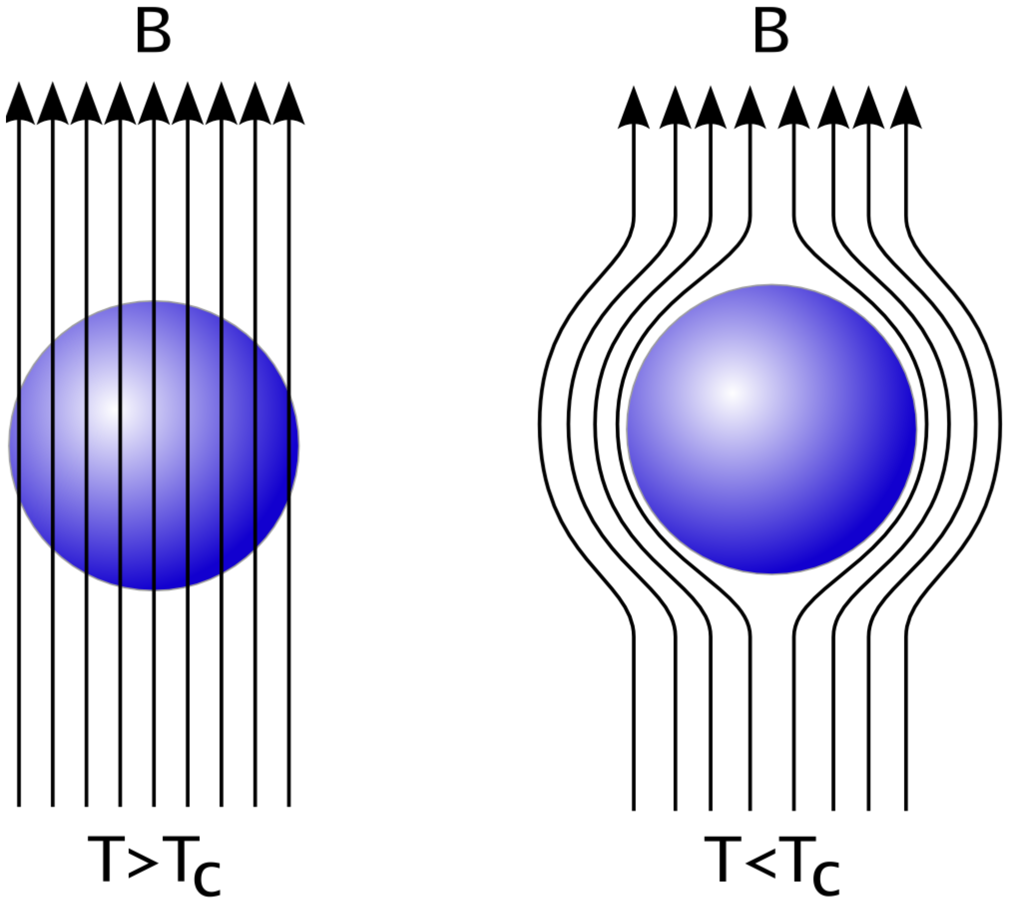


temperature at which
ρ drops to zero:

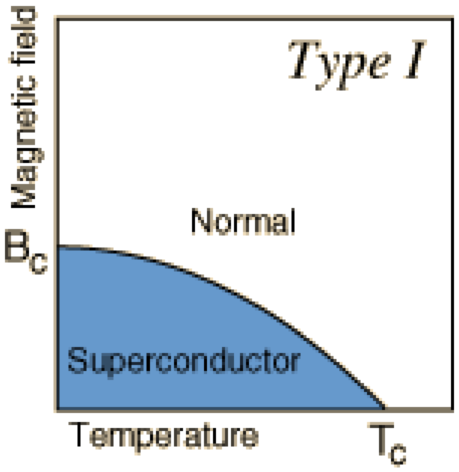
CRITICAL TEMPERATURE

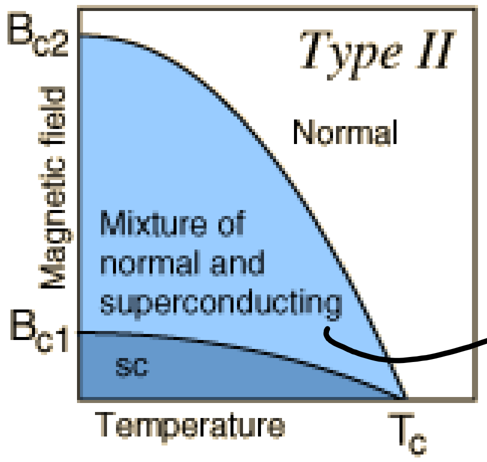
Perfect diamagnet

Meissner effect



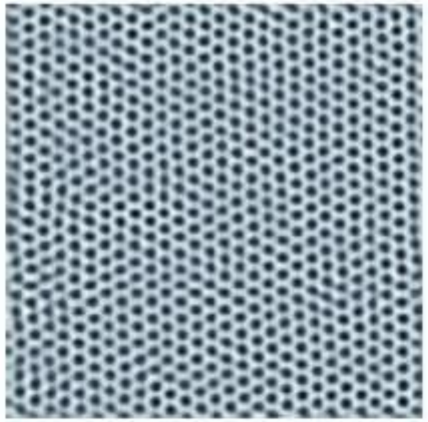
Type I and type II superconductors

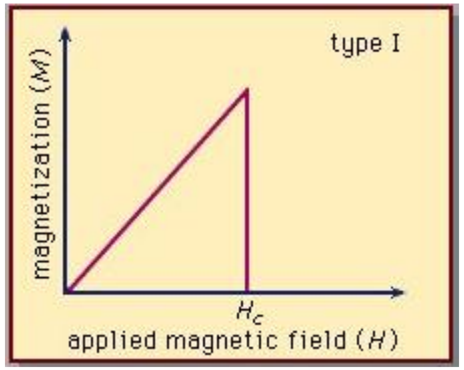


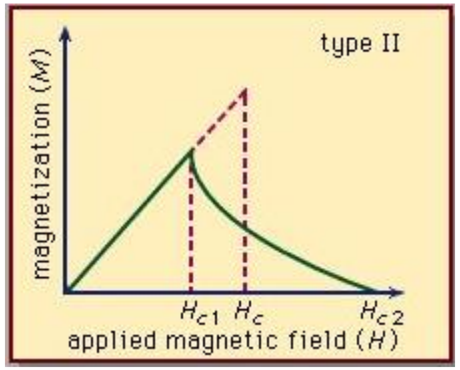


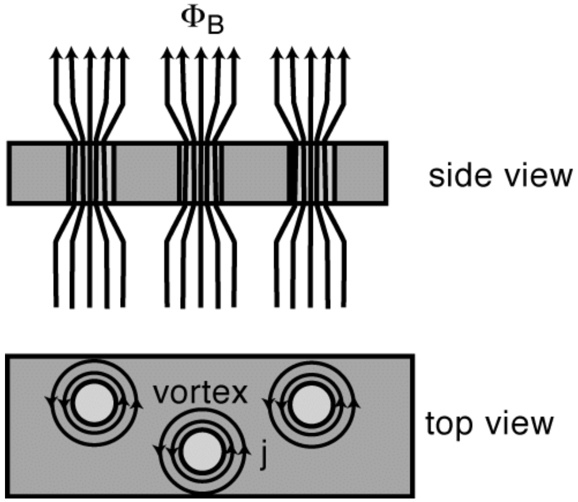
"bundles" of flux penetrate

vortices

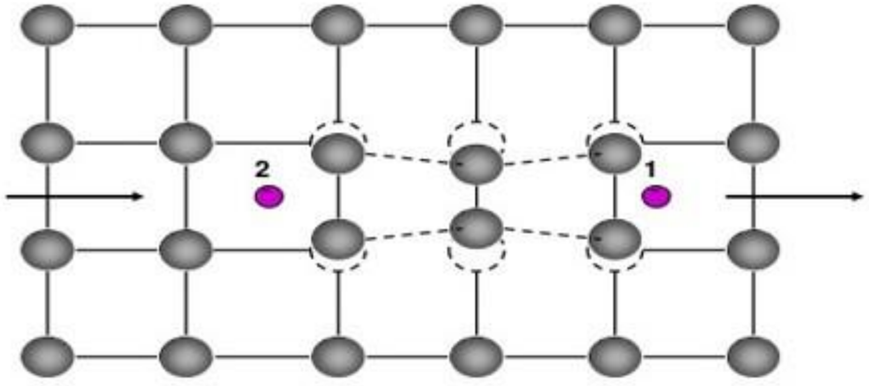


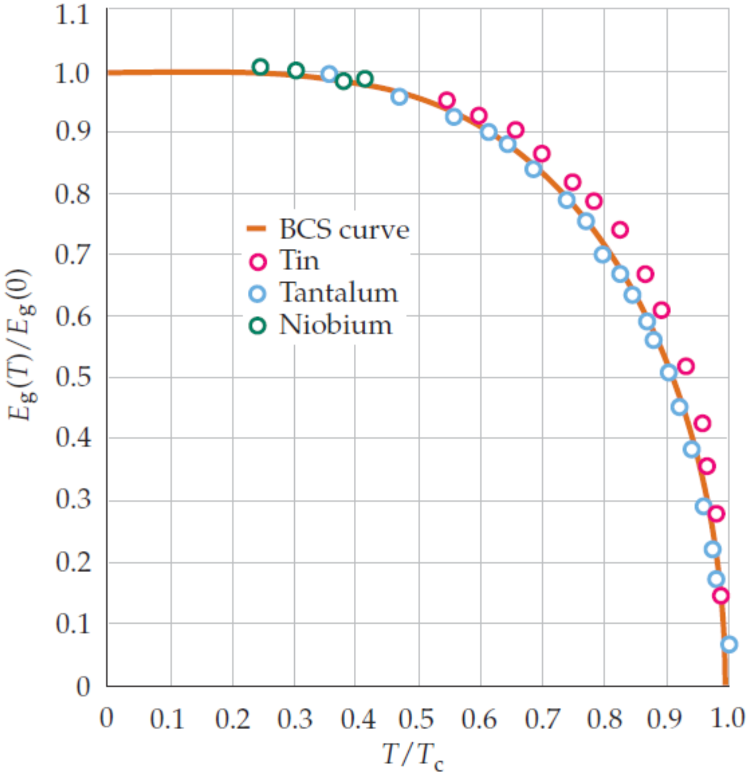






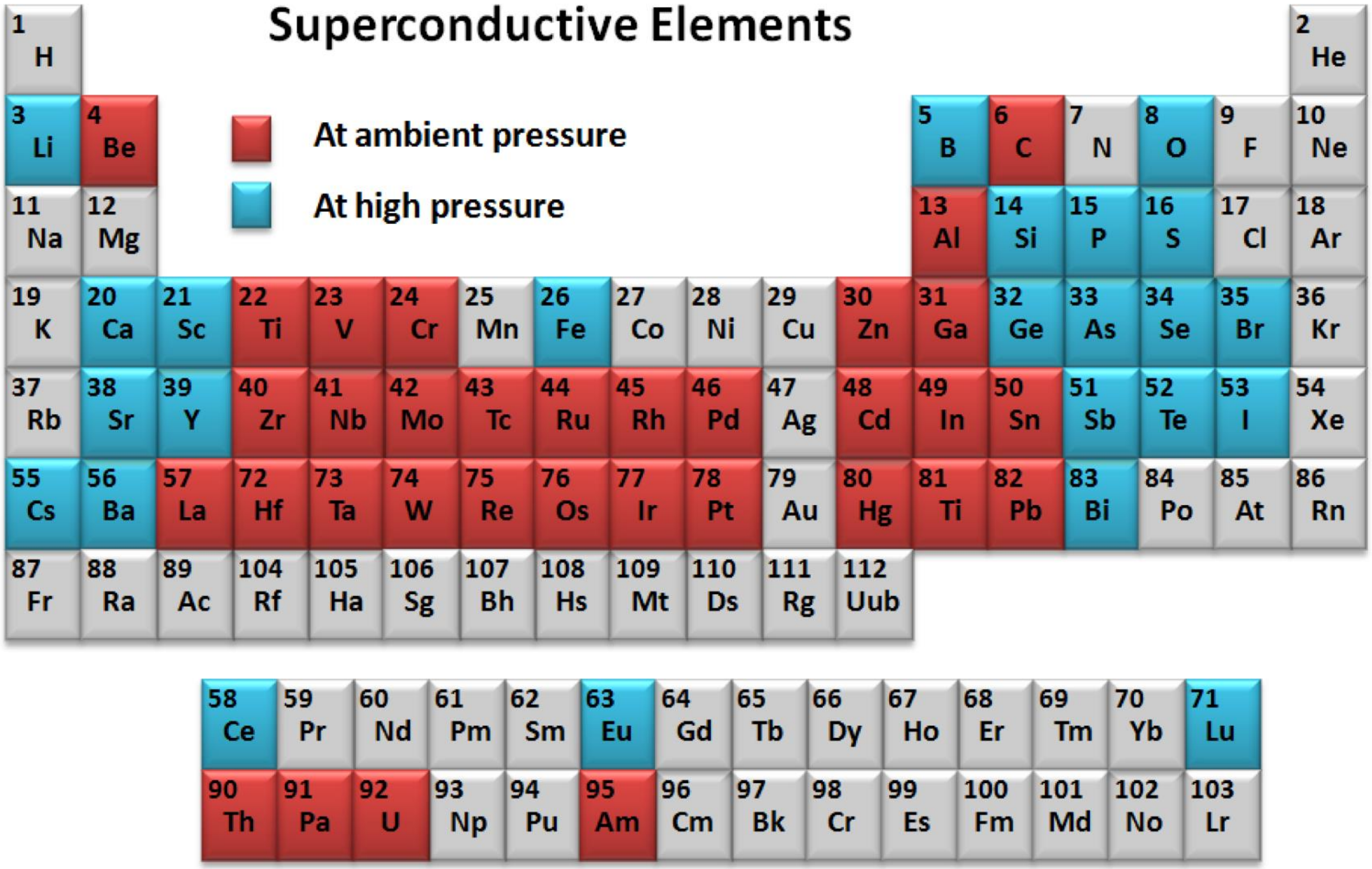
Bardeen, Cooper and Schrieffer (BCS) theory





- Cooper pairs

phonon interaction



Superconductive Elements

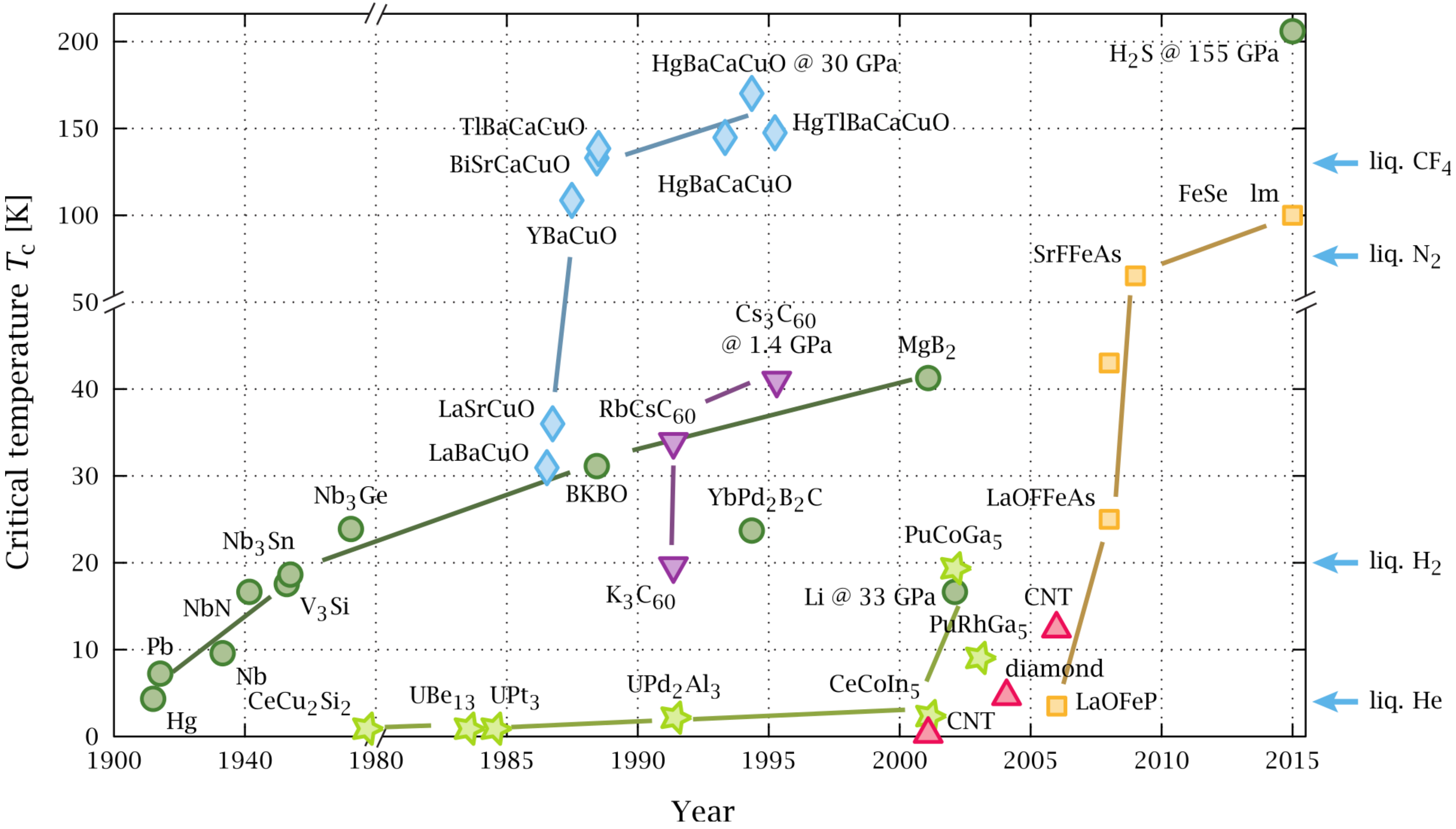
🟥 At ambient pressure

🟦 At high pressure

1 H																	2 He
🟦 3 Li	🟥 4 Be											🟦 5 B	🟥 6 C	7 N	🟦 8 O	9 F	10 Ne
11 Na	12 Mg											🟥 13 Al	🟦 14 Si	🟦 15 P	🟦 16 S	17 Cl	18 Ar
19 K	🟦 20 Ca	🟦 21 Sc	🟥 22 Ti	🟥 23 V	🟥 24 Cr	25 Mn	🟦 26 Fe	27 Co	28 Ni	29 Cu	🟥 30 Zn	🟥 31 Ga	🟦 32 Ge	🟦 33 As	🟦 34 Se	🟦 35 Br	36 Kr
37 Rb	🟦 38 Sr	🟦 39 Y	🟥 40 Zr	🟥 41 Nb	🟥 42 Mo	🟥 43 Tc	🟥 44 Ru	🟥 45 Rh	🟥 46 Pd	47 Ag	🟥 48 Cd	🟥 49 In	🟥 50 Sn	🟦 51 Sb	🟦 52 Te	🟦 53 I	54 Xe
🟦 55 Cs	🟦 56 Ba	🟥 57 La	🟥 72 Hf	🟥 73 Ta	🟥 74 W	🟥 75 Re	🟥 76 Os	🟥 77 Ir	🟥 78 Pt	79 Au	🟥 80 Hg	🟥 81 Ti	🟥 82 Pb	🟦 83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89 Ac	104 Rf	105 Ha	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Uub						

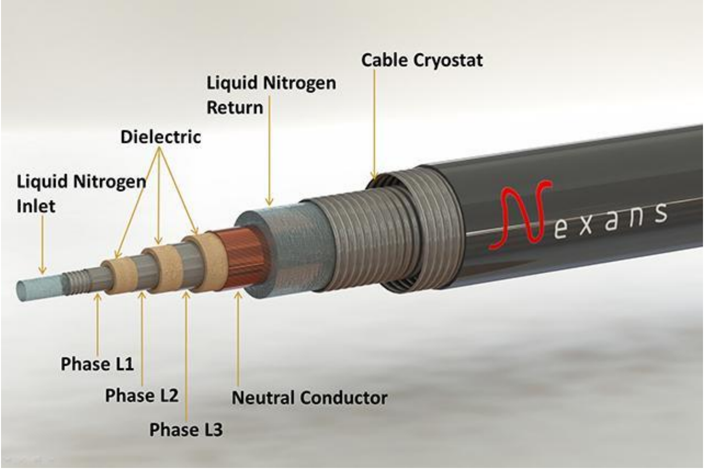
🟦 58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	🟦 63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	🟦 71 Lu
🟥 90 Th	🟥 91 Pa	🟥 92 U	93 Np	94 Pu	🟥 95 Am	96 Cm	97 Bk	98 Cr	99 Es	100 Fm	101 Md	102 No	103 Lr

Guo, J., Wu, Q. & Sun, L. (2018). *Chinese Physics B* **27**, 077402.


Critical temperature T_c [K]
Year
0
10
20
30
40
50
100
150
200
1900
1940
1980
1985
1990
1995
2000
2005
2010
2015
HgBaCaCuO @ 30 GPa
H_2S @ 155 GPa
TlBaCaCuO
HgTlBaCaCuO
BiSrCaCuO
HgBaCaCuO
liq. CF_4
FeSe film
YBaCuO
SrFFeAs
liq. N_2
Cs_3C_{60}
@ 1.4 GPa
MgB_2
LaSrCuO
$RbCsC_{60}$
LaBaCuO
BKBO
Nb_3Ge
YbPd$_2$B$_2$C
LaOFFeAs
Nb_3Sn
PuCoGa$_5$
liq. H_2
NbN
V_3Si
K_3C_{60}
Li @ 33 GPa
CNT
PuRhGa$_5$
Pb
Nb
diamond
CeCu$_2$Si$_2$
UBe$_{13}$
UPt$_3$
UPd$_2$Al$_3$
CeCoIn$_5$
LaOFeP
liq. He
Hg
CNT

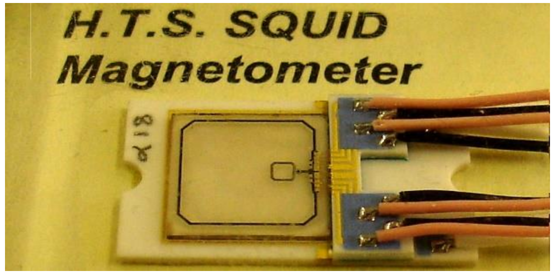
Some applications of superconductors...


Magnetic Resonance Imaging (MRI)



Power lines


Large Hadron Collider (LHC)



Superconducting Quantum Interference Device (SQUID)

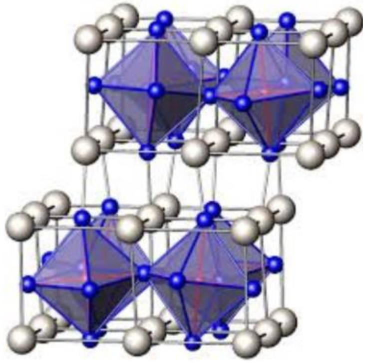

Maglev trains


Superconducting motors

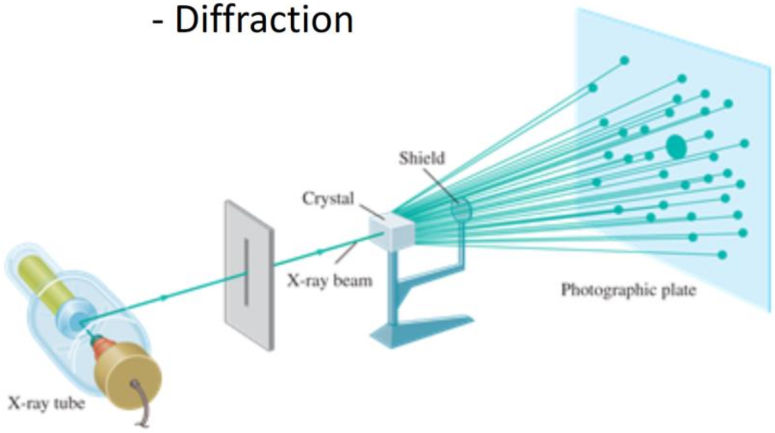
Summary

Summary of the course

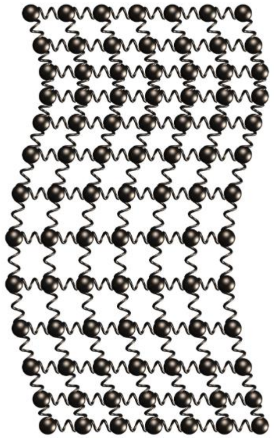
- Crystal Structures
- Crystal Bindings



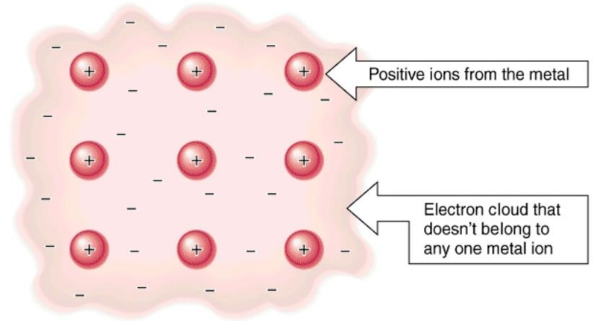
- Reciprocal Space
- Diffraction



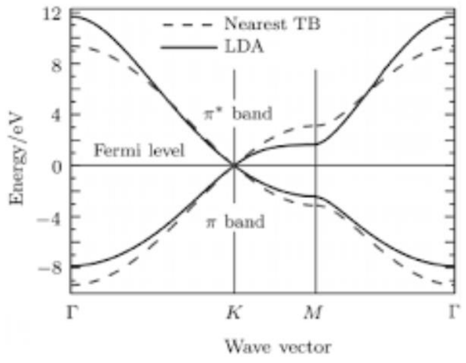
- Phonons
- Heat Capacity



- Free Electron Gas
- Electronic heat capacity
- Electronic properties

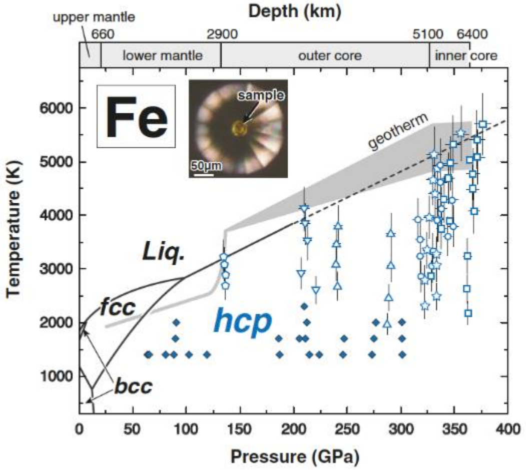


- Band structure
- Fermi Surfaces
- Electronic Material Classification

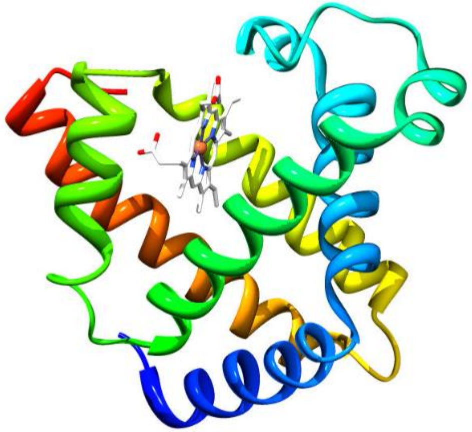


Why we care?

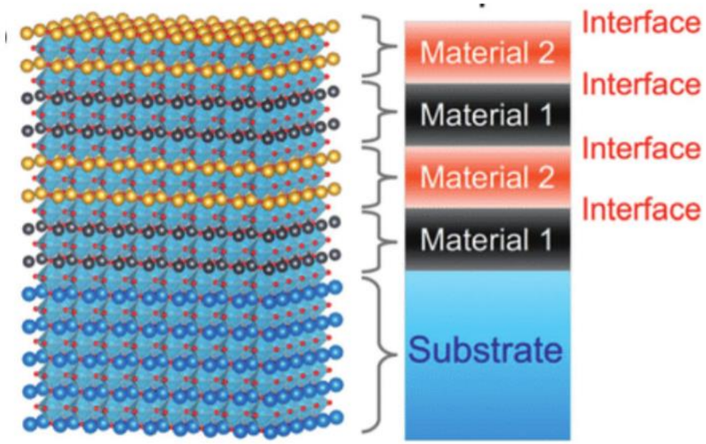
Planetary Science



Protein Structures



Material Design



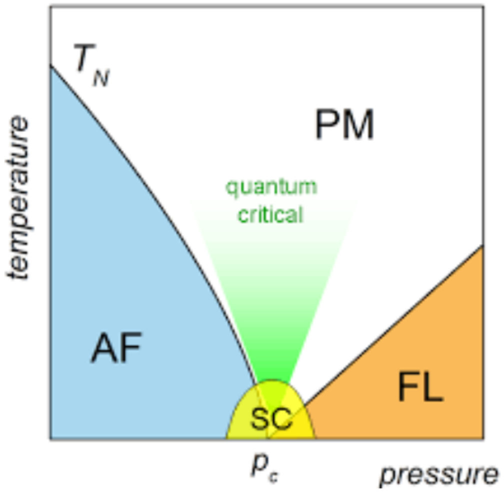
Data Storage

Semiconductor technology

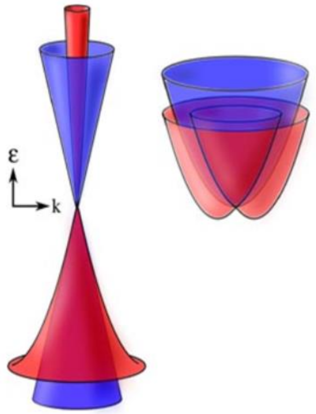
Power Transmission

Other condensed matter topics

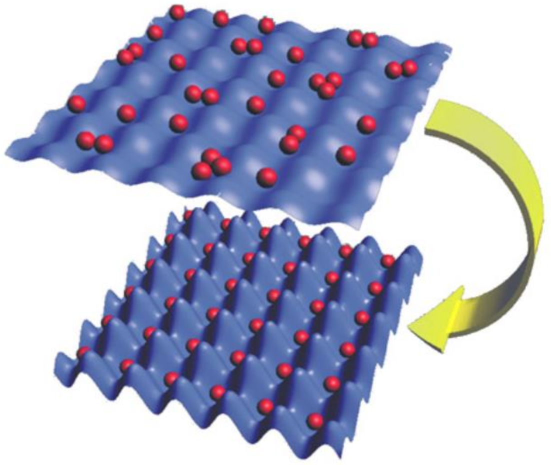
Phase transitions



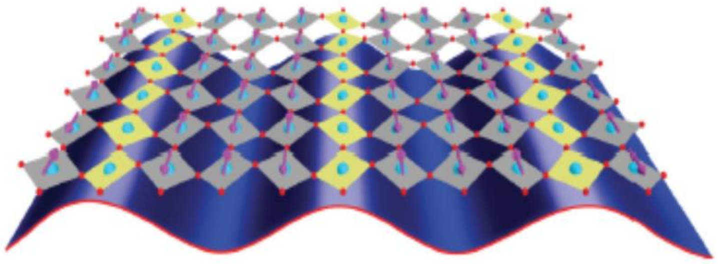
Topology



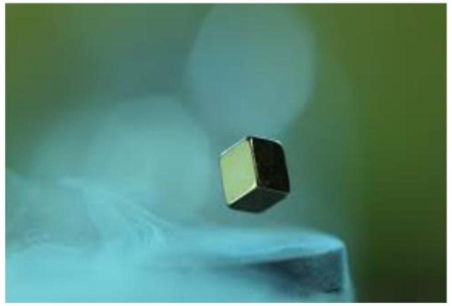
Electron driven insulators



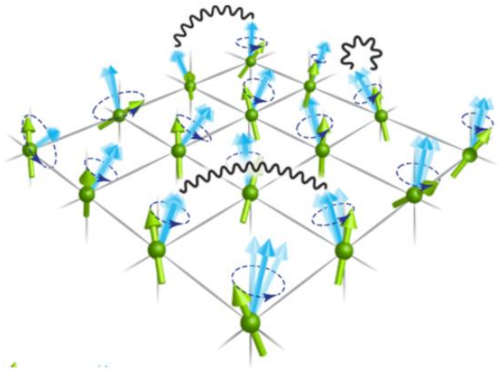
Charge order



Superconductivity



Magnetism



Physik-Nobelpreisträger der Universität Zürich