
Exercise 1 *Electron / hole density and Hall effect in GaAs*
In the lecture, we derived for the electron density $n = 2\left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{3/2} \exp[(\mu - E_c)/k_B T]$ and a similar formula for the hole density p.

(a) Use the effective mass and the band gap (see table values of the lecture slides) to estimate n and p (for light holes).

(b) Calculate the Hall coefficient if only holes or electrons contribute.

Exercise 2 *Hall effect: Multiband scenario*
In the lecture we derived single-band expressions for the resistivity $\rho = m/ne^2\tau$ and the Hall coefficient $R_H = -1/ne$. It is convenient to write the relation between the current density $\mathbf{j}$ and the electric field $\mathbf{E}$ as $\mathbf{E} = \boldsymbol{\rho}\mathbf{j}$ where:

$$\boldsymbol{\rho} = \begin{pmatrix} \rho & -R_H B \\ R_H B & \rho \end{pmatrix} \tag{1}$$

(a) Let us consider a metal where more than one band crosses the Fermi level. When applying an electric field $\mathbf{E}$, the current $\mathbf{j}_n$ on the n^{th} band is: $\mathbf{j}_n = \boldsymbol{\rho}_n^{-1}\mathbf{E}$ where

$$\boldsymbol{\rho}_n = \begin{pmatrix} \rho_n & -R_{H,n} B \\ R_{H,n} B & \rho_n \end{pmatrix}. \tag{2}$$

Show that the total induced current $\mathbf{j}$ is given by $\mathbf{E} = \boldsymbol{\rho}\mathbf{j}$ where $\boldsymbol{\rho} = \left(\sum \boldsymbol{\rho}_n^{-1}\right)^{-1}$.

(b) If only two bands are crossing the Fermi level, show that:

$$R_H = \frac{R_{H,1}\rho_2^2 + R_{H,2}\rho_1^2 + R_{H,1}R_{H,2}(R_{H,1} + R_{H,2})B^2}{(\rho_1 + \rho_2)^2 + (R_{H,1} + R_{H,2})^2 B^2} \tag{3}$$

$$\rho = \frac{\rho_1\rho_2(\rho_1 + \rho_2) + (\rho_1 R_{H,2}^2 + \rho_2 R_{H,1}^2)B^2}{(\rho_1 + \rho_2)^2 + (R_{H,1} + R_{H,2})^2 B^2} \tag{4}$$

Hint: It is allowed to use Mathematica. If you do so, print out the code and the output.

(c) Magnetic field dependence of resistivity is called magneto-resistance. If the two-band system has both electron-like and hole-like carries so that $|R_{H,1}| \approx |R_{H,2}|$, what is the field dependence of ρ.

(d) The mobility can be expressed as $\mu = e\tau/m$. Use the low-field limits of equations 3 and 4 to derive the following two expressions:

$$\sigma = ne\mu_{\mathrm{e}} + pe\mu_{\mathrm{h}} \tag{5}$$

$$R_{\mathrm{H}} = \frac{p\mu_{\mathrm{h}}^2 - n\mu_{\mathrm{e}}^2}{e(p\mu_{\mathrm{h}} + n\mu_{\mathrm{e}})^2}. \tag{6}$$