MORSE POTENTIAL:

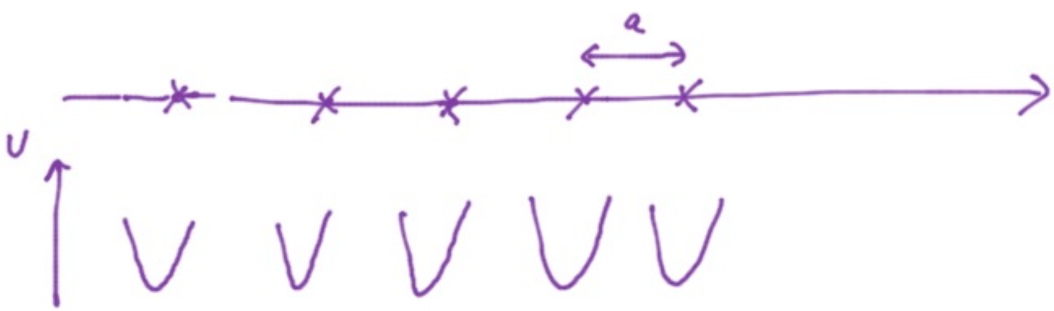
$$U = D \cdot \{1 - \exp(-a(R - R_0))\}^2$$

$\delta \equiv R - R_0$

$=$ distance to R_0

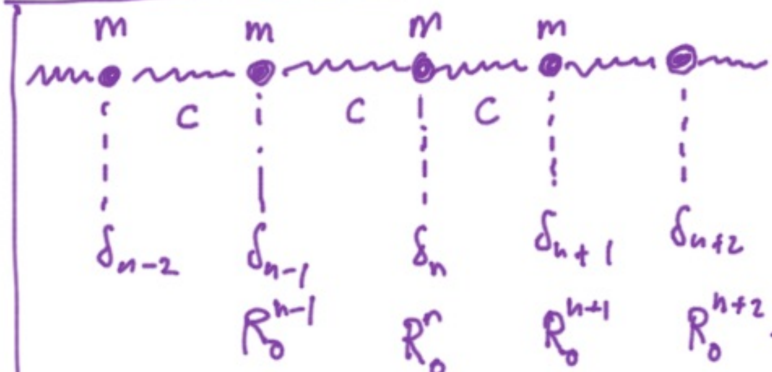
APPROXIMATE BY PARABOLIC FUNCTION

$$U \propto (R - R_0)^2 \propto \delta^2.$$



linear chain of atoms

Potential

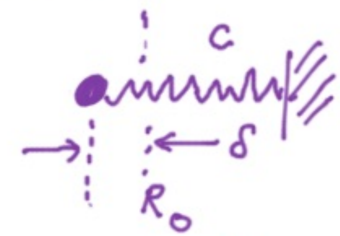


$\rightarrow R_0^j = ja$

$$F_n = c\{\delta_{n+1} - \delta_n\} + c\{\delta_{n-1} - \delta_n\}$$
$$= c \cdot \{\delta_{n+1} + \delta_{n-1} - 2\delta_n\} = m \cdot \frac{d^2\delta_n}{dt^2}$$

Model

c = spring constant
m = atom mass



R_0 = equilibrium distance

$F = c \cdot \delta$
Hookes Law

$$F_n = C\{\delta_{n+1} - \delta_n\} + C\{\delta_{n-1} - \delta_n\}$$

$$= C \cdot \{\delta_{n+1} + \delta_{n-1} - 2\delta_n\} = m \cdot \frac{d^2\delta_n}{dt^2}$$

ANSATZ: δ_n = Plane wave $= A \cdot \{e^{i\omega t - ikR_n^0}\} = A \cdot e^{i\omega t - ikna}$

Test: $-m \cdot \omega^2 \cdot \delta_n = C \cdot A \cdot e^{i\omega t}\left[e^{-ik(n+1)a} + e^{-ik(n-1)a} - 2e^{-ikna}\right]$

$$= C \cdot A\, e^{i\omega t - ikna}\left[e^{-ika} + e^{ika} - 2\right]$$

$$= -C\delta_n\left[2 - 2\cos(ka)\right]$$

$\Downarrow$

$$\omega^2 = \frac{2C}{m}\left[1 - \cos ka\right] = \frac{4C}{m} \cdot \sin^2 \tfrac{1}{2}ka$$

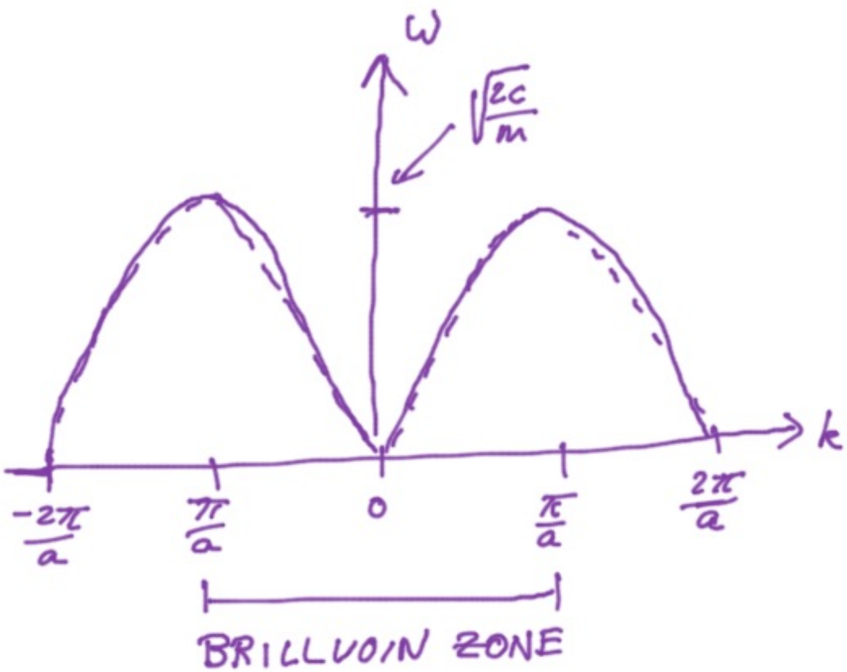
$\Downarrow$

$$\omega = \sqrt{\frac{4C}{m}}\, \sin \tfrac{1}{2}ka$$

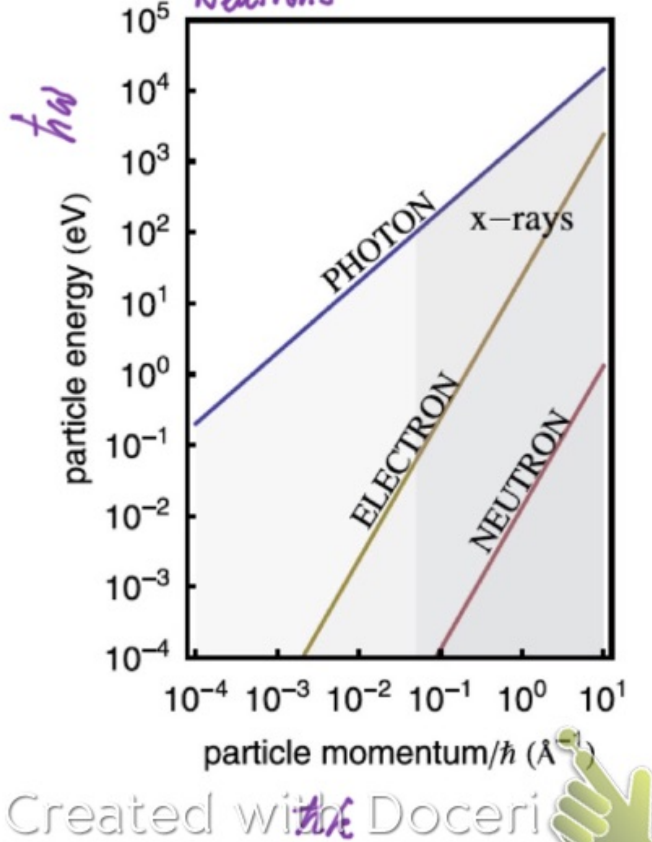
Dispersion Relation

Phonons - Quasi Particle

$$\omega = \sqrt{\frac{4C}{m}} \sin \tfrac{1}{2} ka$$



Electron's
Photon's
Neutron's
– FUNDAMENTAL PARTICLES



Wavelength Notation

$$\lambda = \frac{2\pi}{k}$$

Short wave length

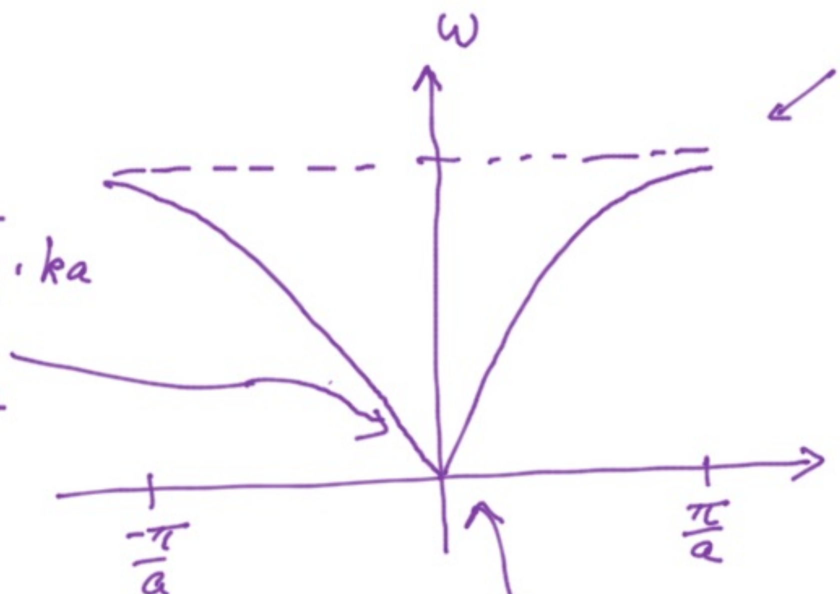
$\lambda = 2a$

$$\omega = \sqrt{\frac{c}{m}} \cdot ka$$

$$= v \cdot a$$

where

$$v = \sqrt{\frac{c}{m}} \cdot a$$



long wave length limit

$\lambda(k \to 0) = \infty$