



(Semi)leptonic B decays: Form factors and New Physics Searches

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Introduction and Motivation



QCD in “Low Energy” Processes

One focus of theoretical physics at UZH is QCD in high-energy processes, such as in higgs or top events at the LHC.

Conversely, my interests focus on QCD in low-energy processes, as needed in b hadron decays.

For this talk I will focus on the two simplest classes of b hadron decays with a $b \rightarrow u\ell\nu$ transition:

- purely leptonic decays, e.g.: $B^- \rightarrow \tau^- \bar{\nu}$
- semileptonic decays, e.g.: $\bar{B}^0 \rightarrow \pi^+ \mu^- \bar{\nu}$

Beyond these *exclusive* decay modes, it is also interesting to study the corresponding *inclusive* decay: $B \rightarrow X_u \mu \bar{\nu}$



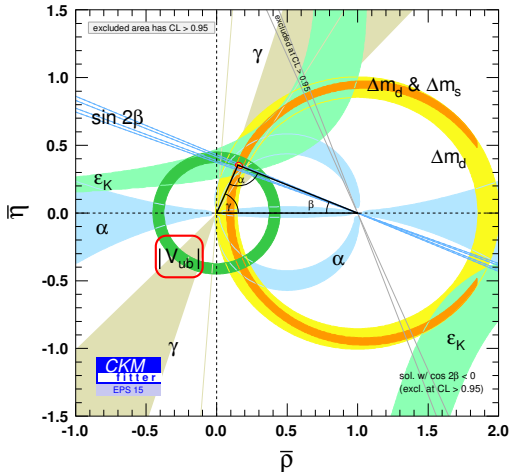
Motivation: The CKM element V_{ub}

Why are these decays interesting?

These decays can be used to determine the value of $|V_{ub}|$, which is an important input to CKM fits.

[CKMfitter Group: J. Charles *et al.*

hep-ph/0406184]





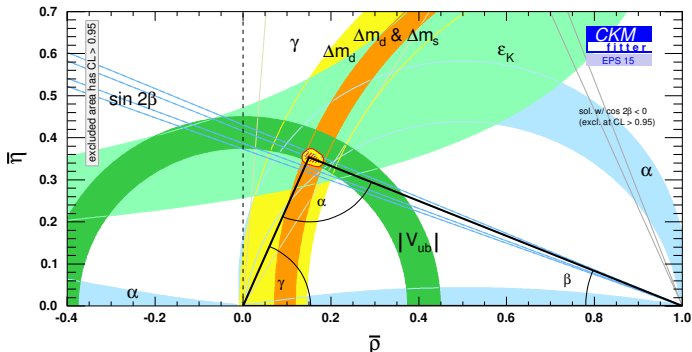
Motivation: A Tension between V_{ub} Determinations

However, the determinations of $|V_{ub}|$ from the individual decay channels do not agree well:

[HFAG 2014, 1412.7515]

$$V_{ub}^{B \rightarrow \pi \mu \nu} = (3.28 \pm 0.29) \times 10^{-3} \quad V_{ub}^{B \rightarrow X_u \mu \nu} = (4.41 \pm 0.21) \times 10^{-3}$$

This has a non-negligible impact on the CKM fit.





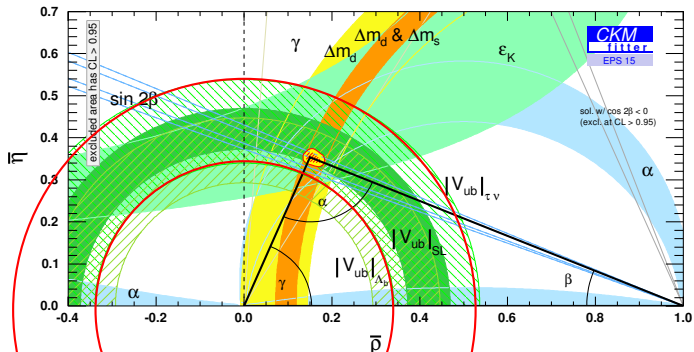
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Effective Operator

It is convenient to use an effective theory from which W has been removed. Schematically, this can be illustrated by replacing the W propagator (here in Feynman/'t Hoft gauge):

$$\frac{-ig^{\mu\nu}}{q^2 - M_W^2} \rightarrow \frac{ig^{\mu\nu}}{M_W^2} \left[1 + \mathcal{O}\left(\frac{q^2}{M_W^2}\right) \right].$$

In the SM, the expansion gives rise to exactly one universal local effective operator of mass dimension six:

$$\mathcal{O}_{LL} \equiv [\bar{u}(x)\gamma^\mu(1 - \gamma_5)b(x)] [\bar{\ell}(x)\gamma_\mu(1 - \gamma_5)\nu(x)]$$

which enters the effective Lagrangian via

$$\mathcal{L}^{\text{eff}} \supseteq \frac{G_F}{\sqrt{2}} V_{ub} \mathcal{O}_{LL} \quad \text{reminder: } G_F = \frac{g^2}{4\sqrt{2}M_W^2}$$



Hadronic Matrix Elements

In order to compute the partial decay widths one needs knowledge of the process dependent hadronic matrix element of the operator.

For the purely leptonic decay, one defines a *B-meson decay constant* f_B via

$$\langle 0 | \bar{u}(x) \gamma^\mu \gamma_5 b(x) | \bar{B}(p) \rangle \equiv i f_B p^\mu$$

For the semileptonic decay $B \rightarrow \pi \mu^- \bar{\nu}$, the hadronic matrix element is not a constant anymore, but rather depends on the momentum transfer $q \equiv p - k$. One parametrizes commonly

$$\langle \pi(k) | \bar{u}(x) \gamma^\mu b(x) | \bar{B}(p) \rangle \equiv f_+(q^2) (p+k)^\mu + \text{terms} \propto q^\mu$$

The function f_+ is called the vector form factor for this transitions. For massless leptons the unspecified terms are irrelevant, since $q^\mu [\bar{\ell} \gamma_\mu (1 - \gamma_5) \nu] = 0$.

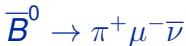


$$B^- \rightarrow \tau^- \bar{\nu}$$

In $B^- \rightarrow \tau^- \bar{\nu}$ only one observable, the partial decay width Γ , emerges:

$$\Gamma(B^- \rightarrow \tau^- \bar{\nu}) = \frac{G_F^2 |V_{ub}|^2}{8\pi} f_B^2 M_B M_\tau^2 \left(1 - \frac{M_\tau^2}{M_B^2}\right)^2.$$

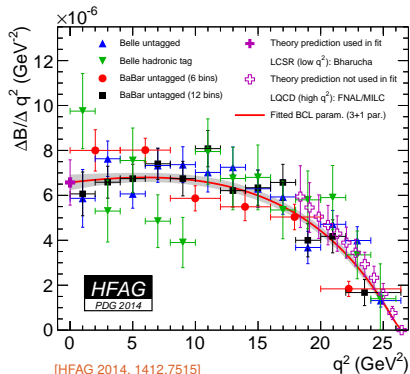
- decay most frequent for τ final state due to **helicity suppression**
- value f_B crucial for V_{ub} determination
 - available from lattice QCD or QCD sum rules (this talk)



In the semileptonic decay we have the partial differential decay width

$$\frac{d\Gamma(\bar{B}^0 \rightarrow \pi^+ \mu^- \bar{\nu})}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3} \lambda(M_B^2, M_\pi^2, q^2)^{\frac{3}{2}} [f_+(q^2)]^2.$$

- all experiments bin rate in q^2
- rate reduces at large q^2
- value and shape of $f_+(q^2)$ crucial
 - lattice QCD works only for large q^2
 - Light-Cone Sum Rules (this talk) only at small q^2





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Determining Hadronic Matrix Elements from QCD Sum Rules





Setup

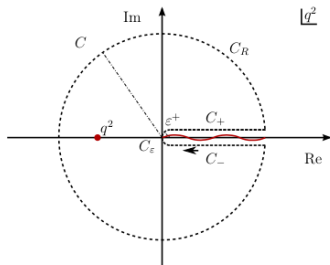
The current $j_5(x) \equiv m_b \bar{u}(x) i\gamma_5 b(x)$ matches the quantum numbers of the B meson. It is suitable to **interpolate** the B in a hadronic matrix element.

In QCD 2-point sum rules, one considers a specific 2-point function:

$$F(p^2) = \int \frac{d^4x}{(2\pi)^4} e^{-ip \cdot x} \langle 0 | \mathcal{T} \{ j_5(x), j_5(0) \} | 0 \rangle$$

$F(p^2)$ has poles and branchcuts at $\text{Re} p^2 > 0$, corresponding to on-shell states that are compatible with the interpolating current.

Suggests a dispersive representation of F .





Dispersive Representations

Use two independent representations for F :

- $F^{\text{OPE}}(p^2)$, expressed in terms of perturbative coefficients and vacuum condensates
 - valid if $p^2 \ll m_b^2$, $|p^2| \simeq m_b \Lambda_{\text{QCD}}$
- F^{had} , expressed in terms of hadronic matrix elements (e.g. f_B)

$$F(p^2) = \int \frac{d^4x}{(2\pi)^4} e^{-ip \cdot x} \langle 0 | \mathcal{T} \{ j_5(x), j_5(0) \} | 0 \rangle$$

$$F^{\text{OPE}}(p^2) = \int_{m_b^2}^{\infty} \frac{\rho^{\text{OPE}}(s) ds}{s - p^2}$$

$$F^{\text{had}}(p^2) = \frac{f_B^2 M_B^2}{M_B^2 - p^2} + \int_{(M_B + 2M_\pi)^2}^{\infty} \frac{\rho(s) ds}{s - p^2}$$



OPE Side

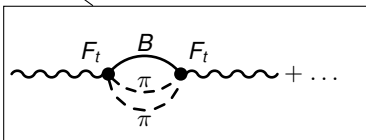
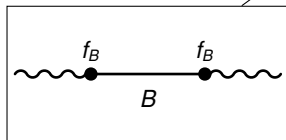
$$\rho^{\text{OPE}}(s) = \sum_k C_k(s) \langle 0 | O_k | 0 \rangle$$

- expansion in **universal vacuum condensates** $\langle 0 | O_k | 0 \rangle$ of increasing mass dimension, beginning with
 - unity $\langle 0 | 1 | 0 \rangle$
 - quark-quark condensate $\langle 0 | \bar{q}q | 0 \rangle$, with $q = u, d$
 - gluon condensate $\frac{\alpha_s}{\pi} \langle 0 | G_{\mu\nu} G^{\mu\nu} | 0 \rangle$
- take values of condensates from low(er)-energy QCD processes
- perturbative coefficients C_k known up to NNLO
- biggest sources of uncertainty: $\alpha_s(\mu)$, \bar{m}_b

Calculation has “saturated”: Results quite reliable, and improvements are work intensive and small.

Hadronic Side

$$F^{\text{had}}(p^2) = \frac{f_B^2 M_B^2}{M_B^2 - p^2} + \int_{(M_B + 2M_\pi)^2}^{\infty} \frac{\rho(s) ds}{s - p^2}$$



- parametrizes correlator in terms of **unknown** constants and functions
 - e.g. F_t : one of the $B \rightarrow \pi\pi$ form factors



Relating both Sides

Quark-Hadron Duality (QHD)

[M. Shifman hep-ph/0009131]

“[QHD] was first formulated at the dawn of the QCD era by Poggio, Quinn and Weinberg [1], who suggested that certain inclusive hadronic cross sections at high energies, **being appropriately averaged over an energy range**, had to (approximately) coincide with the cross sections one could calculate in the quark-gluon perturbation theory.” (emphasis mine)



Relating both Sides

Find some artificial threshold $s_0 > m_b^2$, from which on the integral over the OPE spectral density ρ^{OPE} coincides with the hadronic continuum:

$$\int_{(M_B+2M_\pi)^2}^{\infty} ds \frac{\rho(s)}{s-p^2} \stackrel{!}{=} \int_{s_0}^{\infty} ds \frac{\rho^{\text{OPE}}(s)}{s-p^2}$$

Consequently:

$$\frac{f_B^2 M_B^2}{M_B^2 - p^2} = \int_{m_b^2}^{s_0} \frac{\rho^{\text{OPE}}(s)}{s-p^2}$$

Also, apply Borel transform from p^2 to M^2 to increase numeric stability of the sum rule. Obtain $F(p^2) \mapsto F(1/M^2)$ by

$$\frac{1}{X-p^2} \mapsto e^{-\frac{X}{M^2}}, \quad [p^2]^n \mapsto 0 \text{ if } n \geq 0$$

Final 2-point sum rule (2ptSR):

$$f_B^2 = \frac{1}{M_B^2} \int_{m_b^2}^{s_0} ds e^{\frac{M_B^2-s}{M^2}} \rho^{\text{OPE}}(s)$$



How to determine s_0 ?

Sum rule:

$$f_B^2 = \frac{1}{M_B^2} \int_{m_b^2}^{s_0} ds e^{\frac{M_B^2 - s}{M^2}} \rho^{\text{OPE}}(s)$$

- result for f_B^2 depends artificially on Borel parameter M^2 !
- differentiate both sides w.r.t. $1/M^2$ and rewrite

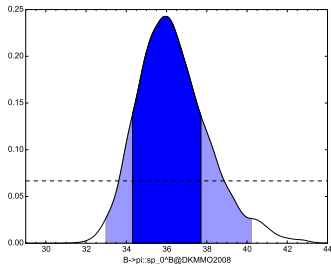
$$M_B^2 = \frac{\int_{m_b^2}^{s_0} ds s \rho^{\text{OPE}}(s) \exp^{-\frac{s}{M^2}}}{\int_{m_b^2}^{s_0} ds \rho^{\text{OPE}}(s) \exp^{-\frac{s}{M^2}}}$$



Bayesian Approach

[Imsong,Khodjamirian,Mannel,DvD 1409.7816]

- use informative priors for M^2 , as well as the QCD and OPE parameters
- construct a (purely theoretical) likelihood, which constrains the first moment of the sum rule to within 1% of M_B^2



- the posterior shows the priors mostly unchanged, and a peaking structure for s_0
- computer posterior-predictive distribution of f_B using the full posterior PDF



Light-Cone Sum Rules (LCSRs)

When switching from 2ptSRs to LCSRs, we change the correlation function:

$$F(q^2, (p+q)^2) = \int \frac{d^4x}{(2\pi)^4} e^{-iq \cdot x} \langle \pi(p) | \mathcal{T} \{ J_\mu(x), j_5(0) \} | 0 \rangle \Big|_{\rho_\mu \text{ coeff. only}}$$

$$F^{\text{had}}(q^2, (p+q)^2) = \frac{f_B f_+(q^2) M_B^2}{M_B^2 - (p+q)^2} + \int_0^\infty \frac{\rho(q^2, s) ds}{s - (p+q)^2}$$

$$F^{\text{OPE}}(q^2, (p+q)^2) = \int_{m_b^2}^\infty \frac{\rho^{\text{OPE}}(q^2, s) ds}{s - (p+q)^2}$$

Changes:

- replace one j_5 with weak current J_μ
- final state is **on-shell** π now
- dispersive integrals now in $(p+q)^2$
- OPE in $x^2 \simeq 0$ now
- vacuum condensates replaced by Light-Cone Distribution Amplitudes, similar to PDFs of π



Form Factor Results

[Imsong,Khodjamirian,Mannel,DvD 1409.7816]

LCSR

- $f_+(q^2)$, as well as 1st and 2nd derivatives w.r.t. q^2
- $q^2 = 0$ and $q^2 = 10 \text{ GeV}^2$
- includes 6-by-6 correlation matrix

e.g.

$$f_+(0) = 0.310 \pm 0.020,$$

$$f_+(10 \text{ GeV}^2) = 0.562 \pm 0.032,$$

with a correlation coefficient of 0.925

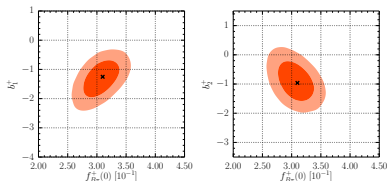
- correlation smaller between f_+ and derivatives

Fit to BCL Ansatz

$$f_+(q^2) = \frac{f_+(0)}{1 - \frac{q^2}{M_{B^*}^2}} \times \left[1 + b_1(z(q^2) - z(0)) + b_2 \mathcal{O}(z^2) \right]$$

3 parameters:

- normalization $f_+(0)$
- two shape params b_1, b_2

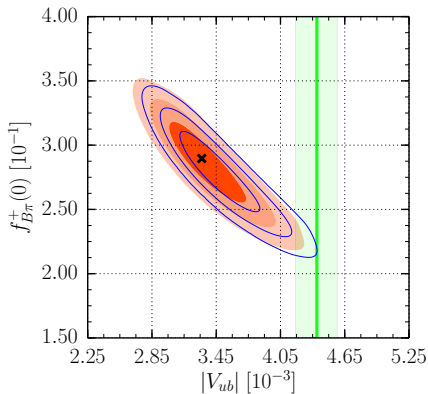


parametrization as in [Bourely,Caprini,Lellouch 0807.2722]



V_{ub} Results

[Imsong,Khodjamirian,Mannel,DvD 1409.7816]



- 2010 data: Belle+BaBar, 6 bins $q^2 \leq 12 \text{ GeV}^2$
- 2010 data vs inclusive: barely compatible @ 99% prob.

- 2013 data: Belle+BaBar, 6 bins $q^2 \leq 12 \text{ GeV}^2$
- 2013 data increases tension

- 1D marginals:

- $|V_{ub}|^{2010} = (3.43^{+0.27}_{-0.23}) \cdot 10^{-3}$
- $|V_{ub}|^{2013} = (3.32^{+0.26}_{-0.22}) \cdot 10^{-3}$

68%, 95%, 99% prob. contours for 2010 data

68%, 95%, 99% prob. contours for 2013 data

central value and 68% CL interval for GGOU [HFAG 2014, 1412.7215]



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Global Analysis of Data on $b \rightarrow u$ Transitions



Search for NP in Semileptonic $b \rightarrow u$ Transitions

Effective Field Theory

Modify the effective $b \rightarrow u \ell \nu$ vertex by adding a different chirality in the hadronic current:

$$\mathcal{L}_{b \rightarrow u}^{\text{eff}} = \frac{G_F V_{ub}^{\text{eff}}}{\sqrt{2}} \left[C_{V,LL} \mathcal{O}_{V,LL} + C_{V,RL} \mathcal{O}_{V,RL} \right]$$

where

$$\mathcal{O}_{V,RL} = [\bar{u}(x) \gamma^\mu (1 + \gamma_5) b(x)] [\bar{\ell}(x) \gamma_\mu (1 - \gamma_5) \nu(x)]$$

Consequently:

- $|V_{ub}^{B \rightarrow \tau \nu}|^2 \rightarrow |V_{ub}^{\text{eff}}|^2 |C_{V,L} - C_{V,R}|^2$
- $|V_{ub}^{B \rightarrow \pi \ell \nu}|^2 \rightarrow |V_{ub}^{\text{eff}}|^2 |C_{V,L} + C_{V,R}|^2$
- $|V_{ub}^{\text{incl.}}|^2 \rightarrow |V_{ub}^{\text{eff}}|^2 (|C_{V,L}|^2 + |C_{V,R}|^2)$

$B^- \rightarrow \tau^- \bar{\nu}_\tau$	
BaBar 2009	SL tag
BaBar 2012	had. tag
Belle 2012	had. tag
Belle 2015	SL tag
$\bar{B}^0 \rightarrow \pi^+ \mu^- \bar{\nu}_\mu$	
BaBar 2010	untagged
BaBar 2012	untagged
Belle 2010	untagged
Belle 2013	full recon.
$B \rightarrow X_u \ell \bar{\nu}$	
BaBar+Belle	HFAG avg. (GGOU)



Strategy

[Feldmann, Müller, DvD 1503.09063]

- fit $C_{V,L}$ and $C_{V,R}$ from data
 - choose global phase as phase of $V_{ub}^{\text{eff}} \cdot C_{V,L}$
 - $C_{V,L}$ is real-valued in the fit
- introduce nuisance parameters for exclusive decays, using informative priors
 - $B^- \rightarrow \tau^- \bar{\nu}$: B decay constant f_B from 2ptSRs
[Gelhausen,Khodjamirian,Pivovarov,Rosenthal 1404.0891]
 - $\bar{B}^0 \rightarrow \pi^+ \mu^- \bar{\nu}$: form factor $f_+(q^2)$ from LCSRs
[Imsong,Khodjamirian,Mannel,DvD 1409.7816]
- use several fit scenarios and perform statistical comparison
 - 1 no right-handed currents: $C_{V,R} = 0$, fit $C_{V,L}$
 - equivalent to determination of $|V_{ub}| = |V_{ub}^{\text{eff}} C_{V,L}|$
 - 2 real-valued right-handed currents
 - 3 complex-valued right-handed currents



Search for NP in Semileptonic $b \rightarrow u$ Transitions

Result: Scenario 1

[Feldmann, Müller, DvD 1503.09063]

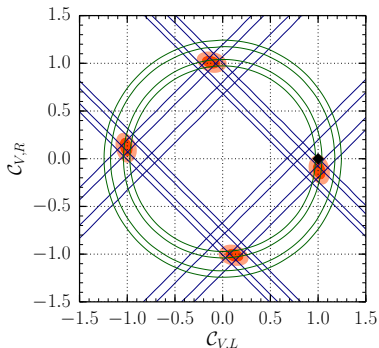
- find $|C_{V,L}| = 1.02 \pm 0.05$ at 68% probability
- corresponds to $|V_{ub}| = (4.07 \pm 0.20) \cdot 10^{-3}$ at 68% prob.
- $\chi^2 = 18.54$ for 28 degrees of freedom
 - excellent fit with p value of 91%
- however: form factor pull does not enter χ^2
 - 3 form factor parameters
 - pull of $\sim 3\sigma$



Search for NP in Semileptonic $b \rightarrow u$ Transitions

Result: Scenario 2 (w/ real-valued $C_{V,R}$)

[Feldmann, Müller, DvD 1503.09063]



contours at 68% and 95% prob.

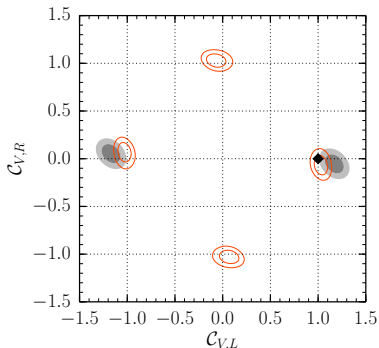
- blue stripes, negative slope: $B \rightarrow \pi \mu \nu$
- blue stripes, positive slope: $B \rightarrow \tau \nu$
- green rings: $B \rightarrow X_u \ell \nu$
- orange areas: combination
- gray area: add. hypo. measurement of A_{FB} in $\bar{B}_s \rightarrow K^{*+} (\rightarrow K \pi) \ell \nu$ (SM $\pm 10\%$)



Search for NP in Semileptonic $b \rightarrow u$ Transitions

Result: Scenario 2 (w/ real-valued $C_{V,R}$)

[Feldmann, Müller, DvD 1503.09063]



contours at 68% and 95% prob.

- blue stripes, negative slope:
 $B \rightarrow \pi \mu \nu$
- blue stripes, positive slope:
 $B \rightarrow \tau \nu$
- green rings: $B \rightarrow X_u \ell \nu$
- orange areas: combination
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Search for NP in Semileptonic $b \rightarrow u$ Transitions

Result: Scenario 2 (w/ real-valued $C_{V,R}$)

[Feldmann, Müller, DvD 1503.09063]

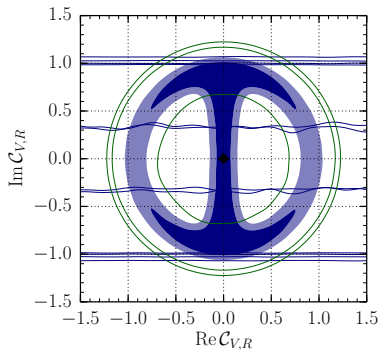
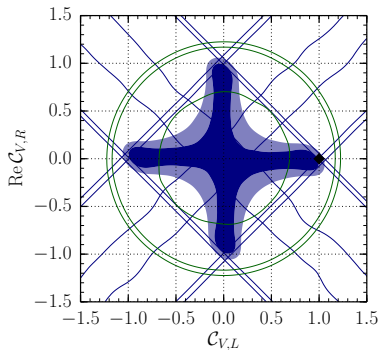
- best fit point: $C_{V,L} = 1.025$, $\text{Re}(C_{V,R}) = -0.079$
- χ^2 increases to 20.47 for 27 degrees of freedom
 - still **very good** fit with p value of 81%
- form factor pull decreases to $\sim 2\sigma$
 - $C_{V,R}$ compensates need to adjust form factor
- solutions Wilson coefficients:
 - $|C_{V,L}| = 1.02 \pm 0.05$ and $|\text{Re}(C_{V,R})| \leq 0.10$ at 68% probability
 - $\text{Re}(C_{V,R}) = 1.02 \pm 0.05$ and $|C_{V,L}| \leq 0.10$ at 68% probability
- loses against scenario 1 in posterior odds with 1 : 27.8



Search for NP in Semileptonic $b \rightarrow u$ Transitions

Result: Scenario 3 (w/ complex-valued $C_{V,R}$)

[Feldmann, Müller, DvD 1503.09063]

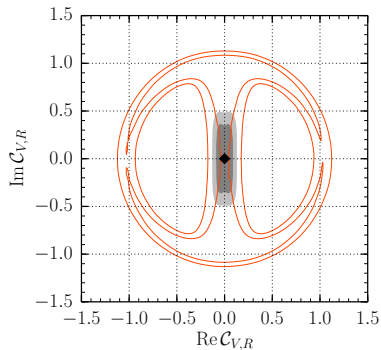
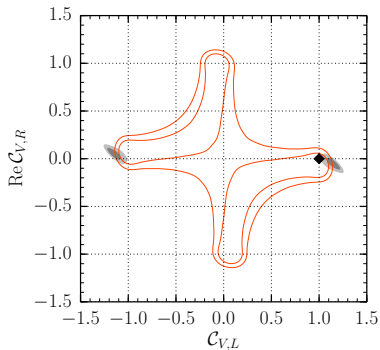




Search for NP in Semileptonic $b \rightarrow u$ Transitions

Result: Scenario 3 (w/ complex-valued $C_{V,R}$)

[Feldmann, Müller, DvD 1503.09063]





Search for NP in Semileptonic $b \rightarrow u$ Transitions

Result: Scenario 3 (w/ complex-valued $C_{V,R}$)

[Feldmann, Müller, DvD 1503.09063]

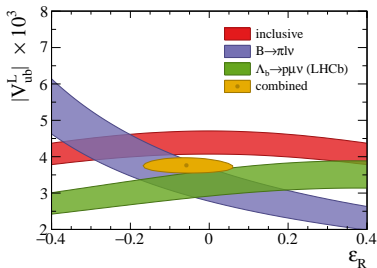
- best fit point: $C_{V,L} = 1.025$, $C_{V,R} = -0.079 + i0.000$
- χ^2 remains at 20.47 for now 26 degrees of freedom
 - still **very good** fit with p value of 77%
- form factor pull stays as $\sim 2\sigma$
 - $C_{V,R}$ compensates need to adjust form factor
- marginal solutions of the Wilson coefficients are not disjoint
- loses against scenario 2 in posterior odds with 1 : 3.62
(against scenario 1 with 1 : 100)



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Comments on V_{ub} from $\Lambda_b \rightarrow p\ell\bar{\nu}$



[LHCb 1504.01568]

The LHCb measurement of V_{ub} in $\Lambda_b \rightarrow p \ell \bar{\nu}$ is the first measurement of V_{ub} in a baryon decay.

The different spin structure between meson and baryon decays gives rise to complementary information on the Wilson coefficients $\mathcal{C}_{V,L(R)}$.

But: The V_{ub} extraction relies heavily on the normalization to the decay $\Lambda_b \rightarrow \Lambda_c \mu \bar{\nu}$! In fact, LHCb is only sensitive to the ratio V_{ub}/V_{cb} ! Accurate and precise predictions of the ratio of (binned) hadronic matrix elements are required, and currently only available from [lattice QCD](#).

However: We can test the lattice results by applying continuum QCD methods.



Testing the $\Lambda_b \rightarrow \Lambda_c$ Form Factors

Inclusive calculation of 2pt correlation function

[T. Mannel, DvD 1506.08780]

$$T_{\Gamma}(\varepsilon) \sim \int d^4x e^{i(v \cdot x)\varepsilon} \langle \Lambda_b | \mathcal{T} \left\{ \bar{b}_v(x) \Gamma c_v(x), \bar{c}_v(0) \Gamma b_v(0) \right\} | \Lambda_b \rangle$$

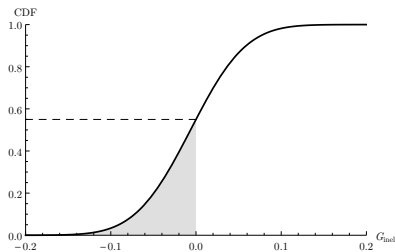
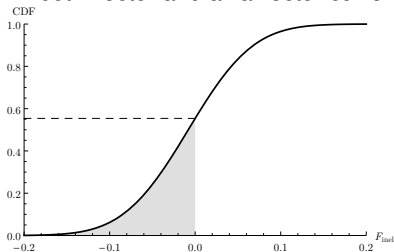
- v : velocity of the b and c quark in the Λ_b rest frame
- ε : excitation energy of the intermediate state

Contour integral of $T(\varepsilon)$ encompasses *all*, i.e. the elastic and all inelastic, contributions to the imaginary part of the forward matrix element \Rightarrow bounds the sum of exclusive form factors at the zero recoil point (when Λ_c rests in Λ_b rest frame).



Zero-Recoil Sum Rule

Difference between inclusive bounds and lattice values at zero recoil for both vector and axialvector currents:



For both currents we find that the lattice results for the $\Lambda_b \rightarrow \Lambda_c$ transitions **exceed the inclusive bounds at 55% probability**.



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Conclusion



Conclusion

- there is a tension between exclusive and inclusive determinations of $|V_{ub}|$
- V_{ub} from exclusive decays requires knowledge of hadronic decay constants and form factor
 - QCD sum rules can calculate these quantities
 - parametric correlations have been studied for the first time
- the tension could be due to New Physics
 - global analysis suggests that right-handed currents alone cannot explain the tension
- LHCb's measurement of $\Lambda_b \rightarrow p\mu^-\bar{\nu}$ important for V_{ub}/V_{cb} determination; complementary probe of NP
 - however: zero-recoil sum rules suggest that lattice form factors are too large