Exercise 1 *Topological Hall Effect (THE) in a Skyrmion Lattice Material*

Figure 1 shows the Hall resistivity ρ_{xy} as measured for the prototypical skyrmion lattice material MnSi.

(a) Discuss the data sets in the left panel of figure 1. Which contributions to the Hall effect can be distinguished? What is the meaning of the kink in data sets for temperature T that are smaller than the Curie temperature T_C?

(b) Determine the normal Hall constant R_0 and the corresponding charge carrier concentration for MnSi (Hint: Use the room temperature data set).

(c) The right panel of figure 1 clearly shows that within the skyrmion lattice phase an additional contribution to the Hall resistivity ρ_{xy} arise due to the THE. The THE in skyrmion materials emerges because the motion of conduction electrons follows the topologically nontrivial spin structure of the skyrmion lattice, and thus the charge carriers collect a Berry phase. This Berry phase may be viewed as an Aharonov-Bohm phase arising from a fictitious effective field $\vec{B}_{eff} = \Phi_0 \vec{\Phi}$. Here $\Phi_0 = \frac{h}{e}$ is flux quantum for a single electron. $\vec{\Phi}$ is given by the skyrmion density. Calculate the magnitude of the effective field $\vec{B}_{eff}$ for MnSi (Hint: In the hexagonal skyrmion lattice phase there is one skyrmion per unit cell. In reciprocal space the period of this lattice is given by $2\pi/\lambda$. For MnSi $\lambda = 165$ Å).

(d) Nominally, the exact contribution due to the THE depends on the electronic band structure, and thus is complicated to calculate. However, the same is true for the normal Hall effect, for which these factors enter the Hall constant R_0. Thus, in the adiabatic limit, where the spin polarization of charge carriers with infinite lifetime smoothly follows the spin texture M, the topological Hall signal may be expressed as $\Delta\rho_{xy}^{THE} = PR_0B_{eff}$. Here P is a polarization factor that arises because majority- and minority-spin carriers collect Berry phases of opposite sign. Use this to estimate $\Delta\rho_{xy}^{THE}$ for MnSi (Hint: P maybe estimate from the ratio of the ordered magnetic moment skyrmion lattice phase $\mu_{SKX} = 0.2 \pm 0.05\mu_B$ to the saturated magnetic moment $\mu_{sat} = 2.2 \pm 0.2\mu_B$).

(e) Compare your calculation with the measured contribution to the Hall resistivity. What could be the reason that experimental value is different from the estimate above?

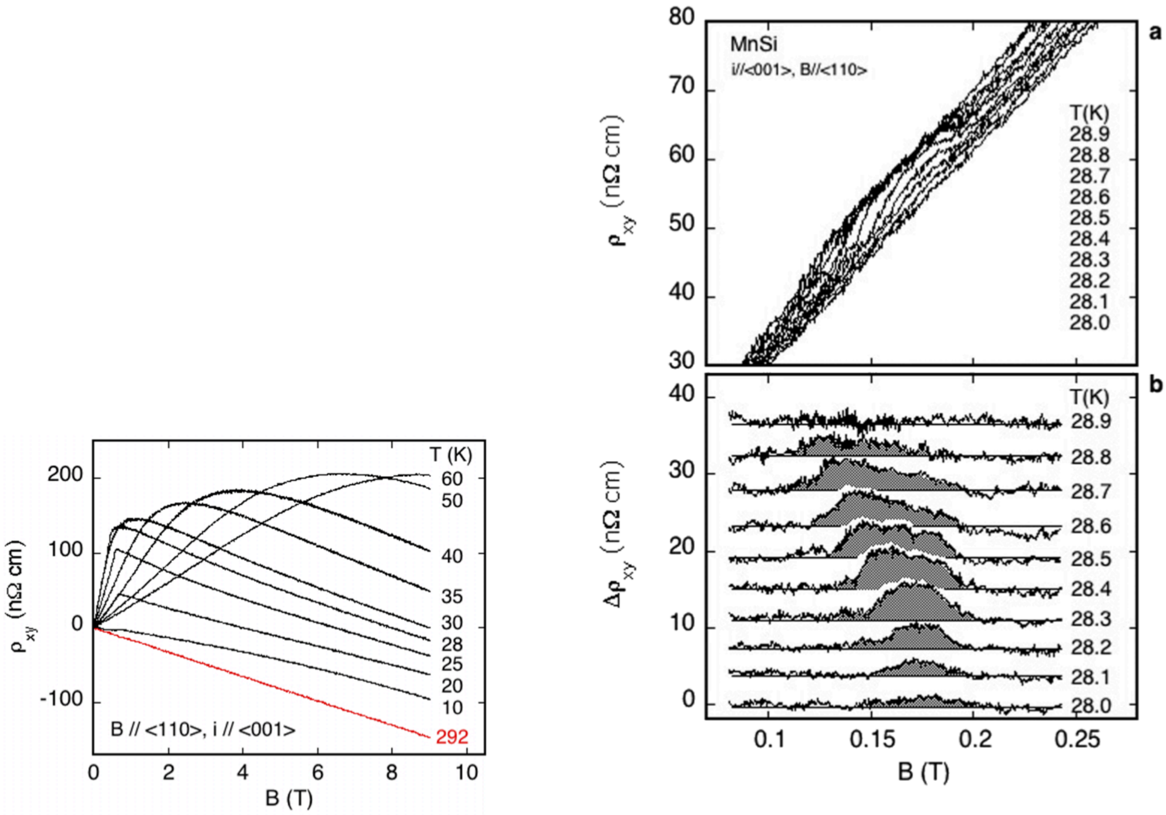

Figure 1: Hall resistivity ρ_{xy} for a single crystal of MnSi is shown where the magnetic field B was applied parallel to [110] and the current was applied along [001]. (left) Data for temperatures T ranging from 10 K to room temperature are shown. (right) A detailed view of the Hall resistivity for temperatures T and magnetic fields B within the skyrmion lattice phase are shown. For the lower panel only the contributions $\Delta\rho_{xy}$ to the Hall effect due to the THE are shown (other contributions were subtracted).

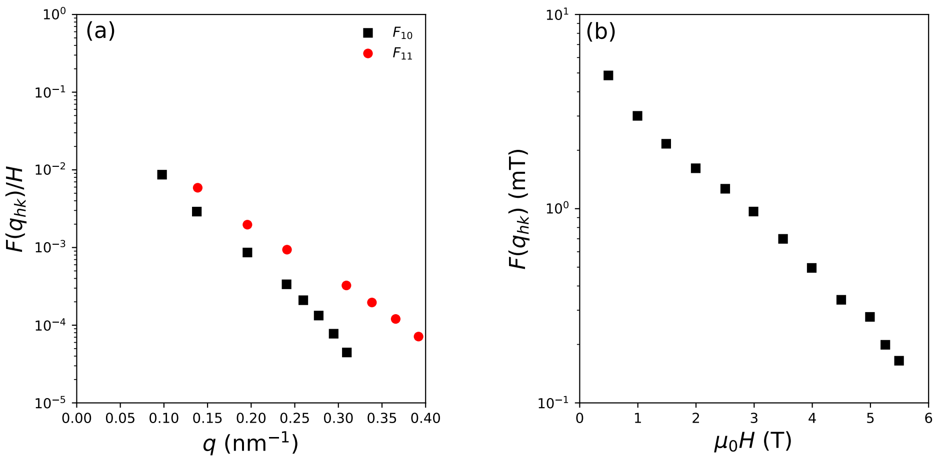

Figure 2: The vortex lattice (VL) form factor for the superconducting material $LuNi_2B_2C$ determined by small angle neutron scattering is shown. Here (a) shows the dependence with respect to the momentum transfer q for two VL Bragg reflections, and (b) shows the magnetic field dependence of the reflection (1,0).

Exercise 2 *Superconducting coherence length and penetration depth from SANS*
Figure 2 shows the vortex lattice (VL) form factor for the superconducting material $LuNi_2B_2C$ determined by small angle neutron scattering.

(a) Can you qualitatively explain why the form factor falls off with increasing momentum q?

(b) The simplest approach to describe the VL is based on the London model, extended by a Gaussian cut-off to take into account the finite extent of the vortex cores for which $F(q) = \frac{B}{1+(\lambda q)^2} \exp -c(\xi q^2)$. Here B is the magnetic field, λ is the superconducting penetration depth, and ξ is the superconducting coherence length. c is a constant that is typically between 1/4 and 2. Use this expression and the provided data sets (https://drive.switch.ch/index.php/s/2OFolV74HpsipZz) to determine λ and ξ for $LuNi_2B_2C$ (Hint: Use that the reciprocal lattice vector for the square VL of this material is given by $q_{hk} = (h^2 + k^2)^{1/2} q_0$; where $q0 = 2\pi B/\phi_0$ and $\phi_0 = h/2e = 2070$ T/nm^2 is the flux quantum. Further assume that $\lambda q \gg 1$ for all fields and use $c = 1/2$).

Exercise 3 *Critical Spin Fluctuation*
The magnetic neutron scattering cross-section is related to the imaginary part of the dynamical magnetic susceptibility $\chi''_{ij}(\mathbf{Q}, \omega)$ via

$$\frac{d^2\sigma}{d\Omega d\omega} \propto \frac{k_0}{k_f}(\delta_{ij} - \hat{q}_i \hat{q}_j)|F_{\mathbf{q}}|^2[n(\omega) + 1]\chi''_{ij}(\mathbf{q}, \omega), \tag{1}$$

where k_0 and k_f are the wave vector of the incident and scattered neutrons, respectively. $\hat{q}$ is a unit vector parallel to the scattering vector $\mathbf{q}$ and $n(\omega)$ is the Bose function so that $[n(\omega) + 1] = \frac{1}{1-e^{-\beta\hbar\omega}}$ with $\beta = k_B T$. $F_{\mathbf{q}}$ is the magnetic form factor. The dynamical magnetic susceptibility for critical spin fluctuations is given by:

$$\frac{\chi''(\mathbf{q},\omega)}{\omega} = \chi(\mathbf{q})\frac{\Gamma_q}{\Gamma_q^2+\omega^2} \tag{2}$$

$$\chi(\mathbf{q}) = \frac{\chi_0}{1+(\xi q)^2}, \tag{3}$$

where Γ_q and χ_0 are the momentum dependent relaxation frequency and the static magnetic susceptibility, respectively. ξ is the correlation length of the spin fluctuations.

(a) Show that for $\hbar\omega \ll k_B T$ the scattering function $S(\mathbf{q},\omega) = [n(\omega)+1]\chi''_{ij}(\mathbf{q},\omega)$ simplifies to $S(\mathbf{q},\omega) \approx \chi''_{\alpha\beta}(\mathbf{q},\omega)\frac{k_B T}{\hbar\omega}$.

(b) The so-called *quasi-static approximation* says that when the energy of incident neutrons E_i is larger than the relaxation frequency $\hbar\Gamma$ of the spin fluctuations that are being investigated the energy-integrated neutron scattering cross-section is given by

$$\frac{d\sigma}{d\Omega} \propto (\delta_{\alpha\beta} - \hat{Q}_\alpha\hat{Q}_\beta)|F_{\mathbf{Q}}|^2 S(\mathbf{q},0), \tag{4}$$

where

$$S(\mathbf{q},t) = \hbar\int S(\mathbf{q},\omega)\exp(i\omega t)d\omega, \tag{5}$$

Use the *quasi-static approximation* and the result from (a) to show that the energy-integrated neutron cross-section is given by

$$\frac{d\sigma}{d\Omega} \propto \pi\frac{k_B T\chi_0}{1+(\xi q)^2}. \tag{6}$$

(c) How can this be used to obtain ξ and χ_0. Why is this advantageous compared to dealing with the full expression from equation (1)?