University of Zurich UZH

HS19
Prof. Dr. Marta Gibert

Solid State Physics
Exercise Sheet 10
Electronic band structure

Assistant: Masafumi Horio

Received on 22nd November
Discussed on 3rd December

Exercise 1 *Nearly free electron bands in two dimensions*

An electron of mass m moves in a square lattice of lattice spacing a. Consider the limit of a very weak potential where there is no band gap at the Brillouine-zone boundary.

(a) Sketch $E(\boldsymbol{k})$ along the (0, 0)–(π/a, π/a) and (0, 0)–(π/a, 0) lines.
Hint: Consider 4 paraboloids originating at the following Brillouin-zone centers: (0, 0), ($2\pi/a$, 0), (0, $2\pi/a$), and ($2\pi/a$, $2\pi/a$).

(b) With one electron per site in the crystal, draw the Fermi surface in the 1st Brillouin zone. Is this a metal or an insulator ?

(c) With two electrons per site, draw the Fermi surface in the 1st Brillouin zone. Is this a metal or an insulator ?

Exercise 2 *Band gap in two dimensions*

We have a two-dimensional nearly free-electron system on a square lattice of lattice spacing a. The Fourier transform of the weak lattice potential is $V_{\boldsymbol{G}}$. We want to investigate the band structure around the (π/a, π/a) point in the reciprocal space using the central equation.

(a) Show that without perturbation the bands are four times degenerated at (π/a, π/a).

(b) From the central equation, construct a set of 4 equations using V_G with $\boldsymbol{G} = (0, 2\pi/a)$ and $\boldsymbol{G} = (2\pi/a, 2\pi/a)$ (Assume $V_{(0,0)} = 0$ for simplicity and neglect the other V_G terms as they are very small).

(c) Calculate the gap at (π/a, π/a) when $V_{(0,2\pi/a)} = 0$ and $V_{(2\pi/a,2\pi/a)} = V$.

Exercise 3 *Instability at the Fermi wavenumber*

Assume that a weak potential in the form of $V(x) = V_0 \cos(2k_\mathrm{F} x)$ is created for a one-dimensional nearly free electron system of lattice spacing a (k_F is the Fermi wavenumber and smaller than π/a).

(a) Derive the Fourier component V_G with $G = 2k_\mathrm{F}$, and calculate $E(k)$ around $k = k_\mathrm{F}$.

(b) Sketch $E(k)$ within $-\pi/a \leqq k \leqq \pi/a$. Compare qualitatively the total energy of electrons with and without the potential $V(x)$.