

Exercise 1 Electronic specific heat in two dimensions

Layered crystal structures often have electronic structures that can approximately be considered two-dimensional. The high-temperature superconductor $\text{Tl}_2\text{Ba}_2\text{CuO}_{6+\delta}$ is one such example.

(a) The electronic heat capacity is give by $C_{\rm el} = \gamma T$. Show that in two dimensions the Sommerfeld parameter γ can be written as $\gamma = \frac{A\pi k_{\rm B}^2}{3\hbar^2}m$ where m is the electronic mass and A is the total area. What is the unit of $C_{\rm el}$? Hint: Use $C_{\rm el} = \frac{1}{3}\pi^2 D(\epsilon_{\rm F})k_{\rm B}^2T$ and derive the density of state (DOS) in two dimensions.

(b) The crystal structure of $\text{Tl}_2\text{Ba}_2\text{CuO}_{6+\delta}$ consists of stacked layers of CuO_2 . Within a layer, the CuO_2 forms a square lattice with a Cu-O lattice distance of a = 3.8 Å. The sample area can thus be written as $A = a^2N$ where N is the number of Cu-O squares. The electronic **specific** heat capacity is measured in units $\text{J} \text{ mol}^{-1} \text{ K}^{-1}$. Show that $\gamma = \frac{N_A a^2 \pi k_B^2}{3\hbar^2} m$ where N_A is the Avogadro number.

(c) $\text{Tl}_2\text{Ba}_2\text{CuO}_{6+\delta}$ is observed to have the Sommerfeld parameter $\gamma = 6 \text{ mJ mol}^{-1} \text{ K}^{-2}$. Using the result of (b), what is the electronic mass m for $\text{Tl}_2\text{Ba}_2\text{CuO}_{6+\delta}$? How does it compare to the free electron mass?

Exercise 2 Extraction of electronic and phononic specific heat

In figure 1 the data from a specific heat experiment on Sr_2RuO_4 is shown (adapted from Mackenzie et al. JPSJ 67, 385 (1998)).

(a) Extract the electronic Sommerfeld parameter γ and the phonon coefficient α in $C_{\rm ph} = \alpha T^3$.

(b) The crystal structure of Sr_2RuO_4 is similar to the one of $Tl_2Ba_2CuO_{6+\delta}$. Which of the systems would have the larger electronic mass m.

(c) What is the Debye temperature for Sr_2RuO_4 ?



Figure 1: The total specific heat divided by temperature of Sr_2RuO_4 between T_c and 14 K in zero field (filled squares) and a magnetic field of 14 T (open circles) applied parallel to the c-axis.

Exercise 3 Specific heat of copper

Copper has a density of $\rho = 8.94 \,\mathrm{g \, cm^{-3}}$ and a molar mass of $m_{\rm mol} = 63.55 \,\mathrm{g \, mol^{-1}}$. Use the measured values for the specific heat of copper given below to:

(a) determine the electron mass. Remember that this is a three dimensional electronic system. Compare this value to the literature value for a free electron.

(b) determine the Debye temperature of copper.

 $T[\mathbf{K}] = 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00, 2.25, 2.50, 2.75, 3.00, 3.25, 3.50, 3.75, 4.00, 4.50$

$$\begin{split} C_{\rm V}[{\rm mJ\,mol^{-1}\,K^{-1}}] = \\ 0.17, 0.35, 0.54, 0.74, 0.96, 1.21, 1.47, 1.78, 2.11, 2.50, 2.91, 3.35, 3.91, 4.46, 5.15, 5.87, 7.49 \end{split}$$