Kern- und Teilchenphysik II
Spring Term 2015

Exercise Sheet 1

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1. Two body scattering

Consider a two-body scattering:

a) Derive the following equation in the centre-of-mass frame:

$$\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2)^2} = (E_1 + E_2)|p_1|$$

b) Derive the same formula in the lab frame (target particle at rest).

3 pt

2. Pauli Matrices

Verify the following commutation relationships of Pauli matrices:

a) $$[\sigma_x, \sigma_y] = 2i\sigma_z$$

b) $$[\sigma_y, \sigma_z] = 2i\sigma_x$$

c) $$[\sigma_z, \sigma_x] = 2i\sigma_y$$

d) $$[\sigma^2, \sigma_i] = 0$$

2 pt

3. Dirac Matrices

Show that the Dirac matrices have the following properties:

a) $$\{\gamma^5, \gamma^0\} = 0$$

b) $$\gamma^{5\dagger} = \gamma^5, \gamma^{0\dagger} = \gamma^0$$

c) $$\gamma^{\mu\dagger} = -\gamma^\mu$$

d) $$\gamma^\mu \gamma_\mu = 4\mathbf{I}$$

e) $$(\gamma^5)^2 = \mathbf{I}, (\gamma^0)^2 = \mathbf{I}, (\gamma^\mu)^2 = -\mathbf{I}$$

2 pt

4. Dirac Hamiltonian

Consider the free-particle Hamiltonian of the Dirac equation:

$$H = \vec{\alpha} \cdot \vec{p} + \beta m$$

a) Compute the commutator [H, γ^5]. What does it happen if the particle is massless?

1 pt

5. Dirac Equation

The Dirac equation for the particle spinor $u(\mathrm{p})$ is:

$$(\gamma^\mu p_\mu - m)\ u(\mathrm{p}) = 0$$

a) Defining $\not{p} \equiv \gamma^\mu p_\mu$, write the equivalent equation for the antiparticle spinor $v(\mathrm{p})$.

1 pt

b) Write the Dirac equation for the adjoint of the particle spinor, $\bar{u}(\mathrm{p})$ (which is defined as $\bar{u}(\mathrm{p}) = u^\dagger(\mathrm{p})\gamma^0$), and for the antiparticle spinor $\bar{v}(\mathrm{p})$.

[$Hint$: Use the third property of the Dirac matrices listed in the previous exercise]

2 pt

c) Verify the orthogonality of particle and antiparticle spinors:

$$u^{\dagger(1)}u^{(2)} = 0, \qquad v^{\dagger(1)}v^{(2)} = 0$$

2 pt

d) Show that:

$$\bar{u}u = 2mc, \qquad \bar{v}v = -2mc$$

2 pt

e) Show that they are complete:

$$\sum_{s=1,2} u^{(s)}\bar{u}^{(s)} = (\gamma^\mu p_\mu + mc), \qquad \sum_{s=1,2} v^{(s)}\bar{v}^{(s)} = (\gamma^\mu p_\mu - mc)$$

4 pt

Considering that $u^{(s)}$ and $v^{(s)}$ are the canonical solutions to the Dirac equation.

6. Coulomb Scattering

Consider the case of Coulomb scattering of a charged spin-0 particle from an external field A, which is a static field of a point charge Ze located in the origin:

$$A_{mu} = (V, \bar{A}) = (V, 0)$$

The potential is given by:

$$V(x) = \frac{Ze}{4\pi \mid \mathbf{x} \mid}$$

Compute:

a) The transition amplitude T_{fi} for this process:

$$T_{fi} = -i \int \mathrm{d}^4 x j_f^{\mu} i A_{\mu}$$

Hint : Make use of Fourier Transformation

$$\frac{1}{\mid (\bar{p_f} - \bar{p_i}) \mid^2} = \int \mathrm{d}^3 x \ \mathrm{e}^{i(\bar{p_f} - \bar{p_i})\bar{x}} \frac{1}{4\pi \mid \bar{x} \mid}$$

3 pt

b) Its transition probability:

$$\omega_{fi} = \lim_{T \to \infty} \frac{\mid T_{fi} \mid^2}{T}$$

Hint : Assume that the interaction occurs during a time period T from $t = -T/2$ up to $t = +T/2$

3 pt

c) The cross section:

$$\mathrm{d}\sigma = \frac{\omega_{fi}}{flux_i} \mathrm{d}LIPS$$

2 pt

d) Derive the Rutherford scattering cross section (i.e. consider the non-relativistic limit of the cross section computed in the previous point)

1 pt