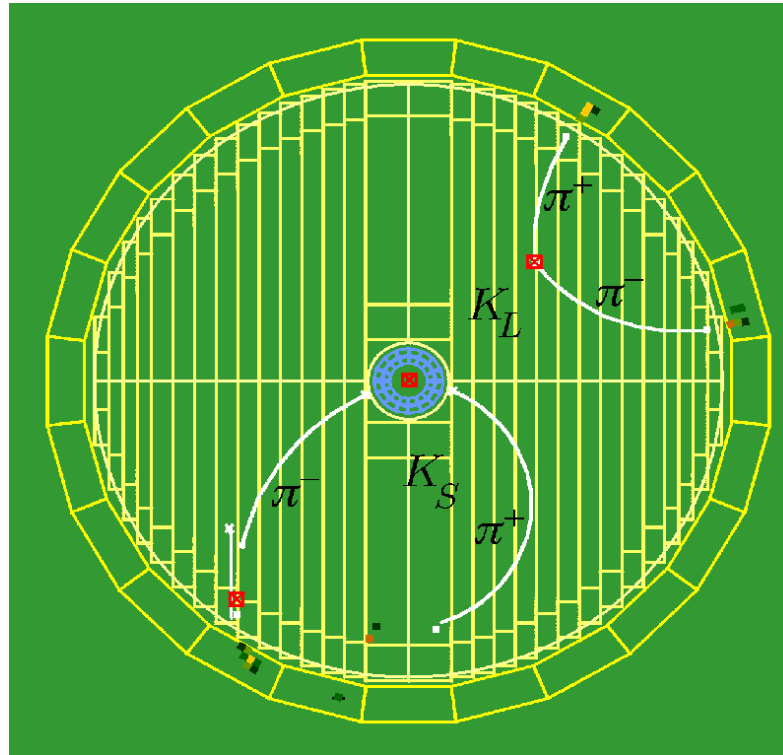# Particle Physics

Handout from Prof. Mark Thomson's lectures
Adapted to UZH by Prof. Canelli and Prof. Serra

Handout 12 : The CKM Matrix and CP Violation

CP Violation in the Early Universe

- **Very early in the universe might expect equal numbers of baryons and anti-baryons**
- **However, today the universe is matter dominated (no evidence for anti-galaxies, etc.)**
- **From "Big Bang Nucleosynthesis" obtain the matter/anti-matter asymmetry**

$$\xi = \frac{n_B - n_{\overline{B}}}{n_\gamma} \approx \frac{n_B}{n_\gamma} \approx 10^{-9}$$

i.e. for every baryon in the universe today there are 10^9 photons

- **How did this happen?**

★ **Early in the universe need to create a very small asymmetry between baryons and anti-baryons**

  e.g. for every 10^9 anti-baryons there were 10^9+1 baryons
 **baryons/anti-baryons annihilate** ⟹
  1 baryon + ~10^9 photons + no anti-baryons

★ <u>**To generate**</u> **this initial asymmetry three conditions must be met (Sakharov, 1967):**
  ❶ **"Baryon number violation", i.e.** $n_B - n_{\overline{B}}$ **is not constant**
  ❷ **"C and CP violation", if CP is conserved for a reaction which generates a net number of baryons over anti-baryons there would be a CP conjugate reaction generating a net number of anti-baryons**
  ❸ **"Departure from thermal equilibrium", in thermal equilibrium any baryon number violating process will be balanced by the inverse reaction**
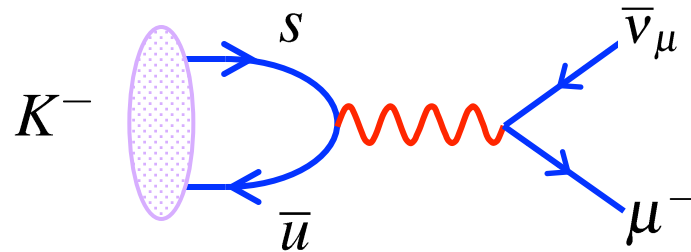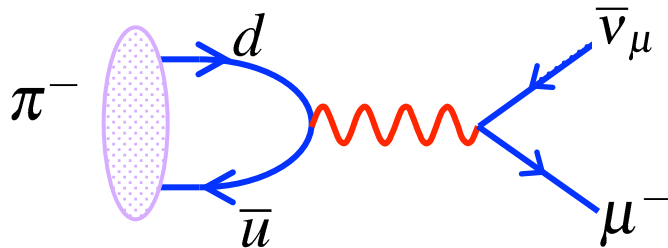
The Weak Interaction of Quarks

★ **Slightly different values of G$_F$ measured in μ decay and nuclear β decay:**
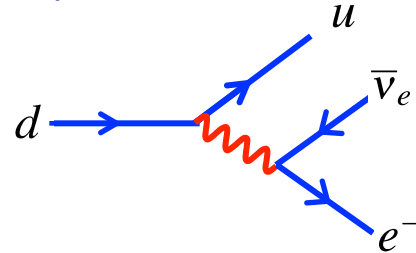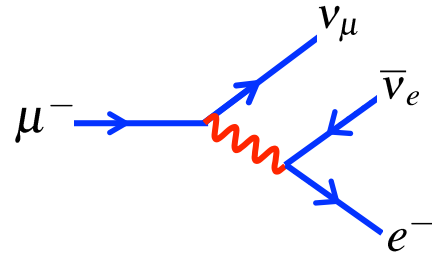
$$G_F^\mu = (1.16632 \pm 0.00002) \times 10^{-5}\,\text{GeV}^{-2} \qquad G_F^\beta = (1.136 \pm 0.003) \times 10^{-5}\,\text{GeV}^{-2}$$

★ **In addition, certain hadronic decay modes are observed to be suppressed, e.g. compare $K^- \to \mu^- \overline{\nu}_\mu$ and $\pi^- \to \mu^- \overline{\nu}_\mu$. Kaon decay rate suppressed factor 20 compared to the expectation assuming a universal weak interaction for quarks.**


• **Both observations explained by Cabibbo hypothesis (1963): weak eigenstates are different from mass eigenstates, i.e. weak interactions of quarks have same strength as for leptons but a u-quark couples to a linear combination of s and d**

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

GIM Mechanism

★ **In the weak interaction have couplings between both** ud **and** us **which implies that neutral mesons can decay via box diagrams, e.g.**
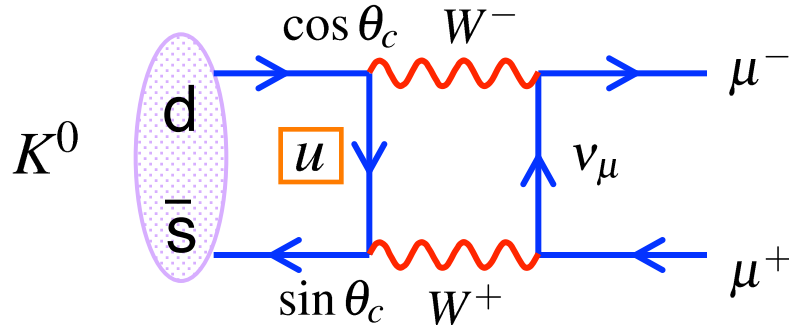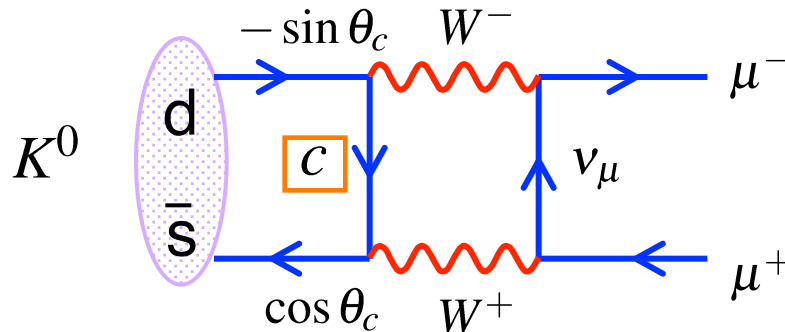

$$M_1 \propto g_W^4 \cos\theta_c \sin\theta_c$$

•**Historically, the observed branching was much smaller than predicted**

★ **Led Glashow, Illiopoulos and Maiani to postulate existence of an extra quark - before discovery of charm quark in 1974. Weak interaction couplings become**

$$\cos\theta_c \frac{g_W}{\sqrt{2}} \quad \sin\theta_c \frac{g_W}{\sqrt{2}} \quad -\sin\theta_c \frac{g_W}{\sqrt{2}} \quad \cos\theta_c \frac{g_W}{\sqrt{2}}$$
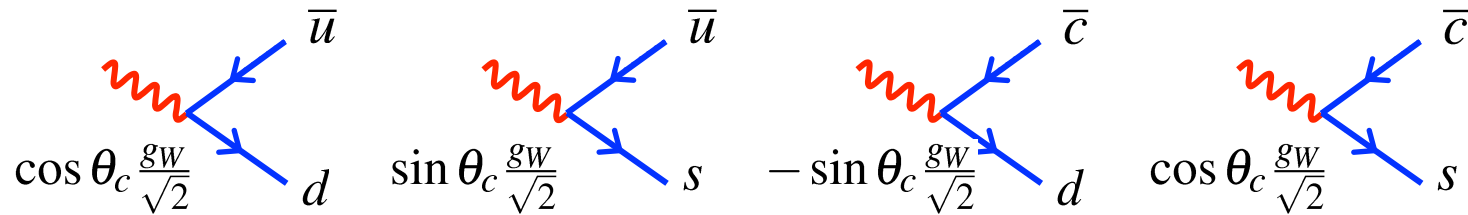
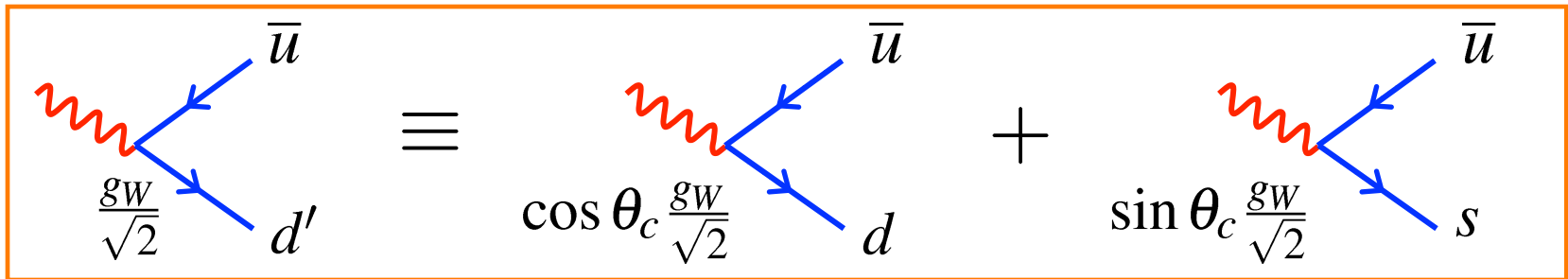★ **Gives another box diagram for** $K^0 \to \mu^+\mu^-$

$$M_2 \propto -g_W^4 \cos\theta_c \sin\theta_c$$

•**Same final state so sum amplitudes**

$$|M|^2 = |M_1 + M_2|^2 \approx 0$$

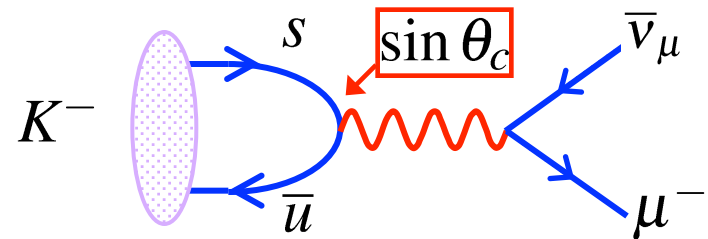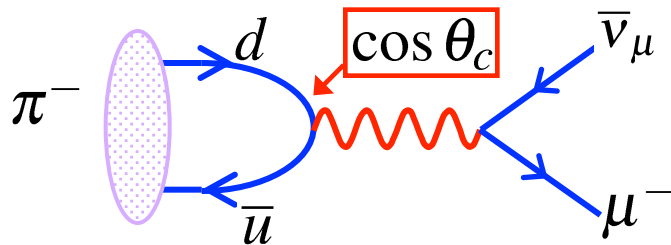•**Cancellation not exact because** $m_u \neq m_c$

i.e. weak interaction couples different generations of quarks

$$\frac{g_W}{\sqrt{2}} \quad \equiv \quad \cos\theta_c \frac{g_W}{\sqrt{2}} \quad + \quad \sin\theta_c \frac{g_W}{\sqrt{2}}$$

(The same is true for leptons e.g. $e^- \nu_1$, $e^- \nu_2$, $e^- \nu_3$ couplings – connect different generations)

★ **Can explain the observations on the previous pages with $\theta_c = 13.1°$**

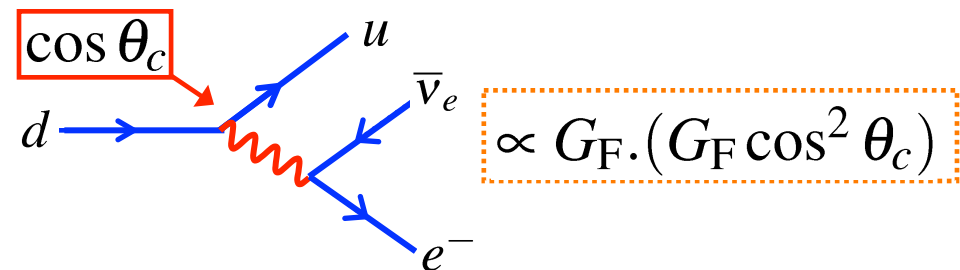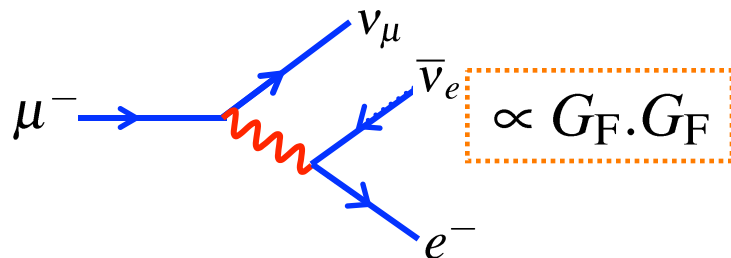 • **Kaon decay suppressed by a factor of** $\tan^2\theta_c \approx 0.05$ **relative to pion decay**

$$\Gamma(\pi^- \to \mu^- \overline{\nu}_\mu) \propto |M|^2 \propto \cos^2\theta_c \qquad \Gamma(K^- \to \mu^- \overline{\nu}_\mu) \propto |M|^2 \propto \sin^2\theta_c$$

• **Hence expect** $G_F^\beta = G_F^\mu \cos\theta_c$

$$\propto G_F \cdot G_F \qquad \propto G_F \cdot (G_F \cos^2\theta_c)$$

CKM Matrix

★ **Extend ideas to three quark flavours (analogue of three flavour neutrino treatment)**

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

By convention CKM matrix defined as acting on quarks with charge $-\frac{1}{3}e$
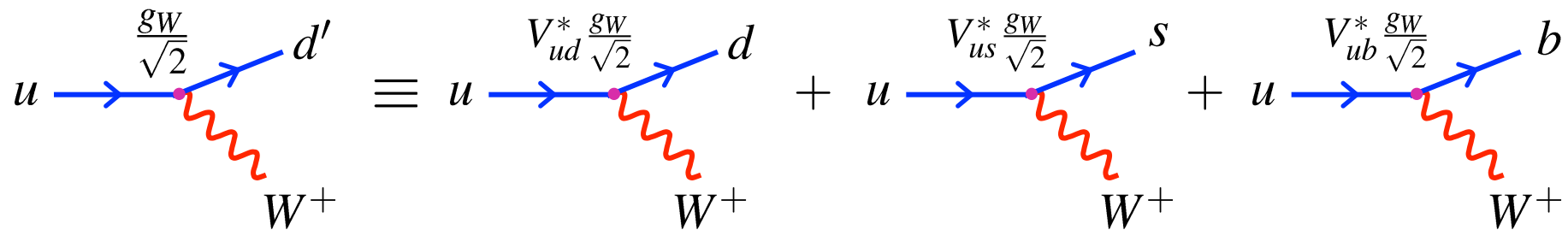
Weak eigenstates   **CKM Matrix**   **Mass Eigenstates**

**(Cabibbo, Kobayashi, Maskawa)**

★ **e.g. Weak eigenstate** d' **is produced in weak decay of an up quark:**

- **The CKM matrix elements** V_{ij} **are <u>complex constants</u>**
- **The CKM matrix is <u>unitary</u>**
- **The** V_{ij} **are not predicted by the SM – have to determined from experiment**

Feynman Rules

- **Depending on the order of the interaction, $u \to d$ or $d \to u$, the CKM matrix enters as either V_{ud} or V_{ud}^***

- **Writing the interaction in terms of the WEAK eigenstates**
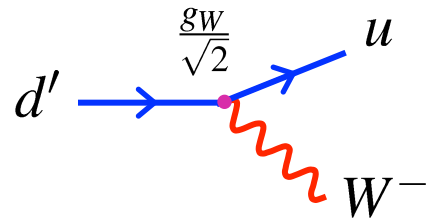
NOTE: ū is the adjoint spinor not the anti-up quark

$$j_{d'u} = \overline{u} \left[-i\frac{g_W}{\sqrt{2}} \gamma^\mu \tfrac{1}{2}(1-\gamma^5) \right] d'$$

- **Giving the $\boxed{d \to u}$ weak current:**

$$j_{du} = \overline{u} \left[-i\frac{g_W}{\sqrt{2}} \gamma^\mu \tfrac{1}{2}(1-\gamma^5) \right] V_{ud} d$$

- **For $u \to d'$ the weak current is:**
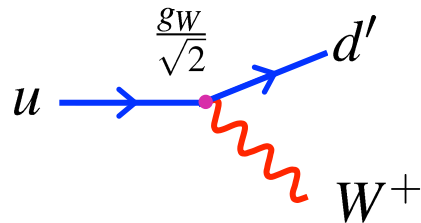
$$j_{ud'} = \overline{d}' \left[-i\frac{g_W}{\sqrt{2}} \gamma^\mu \tfrac{1}{2}(1-\gamma^5) \right] u$$

- **In terms of the mass eigenstates** $\quad \overline{d}' = d'^\dagger \gamma^0 \to (V_{ud}d)^\dagger \gamma^0 = V_{ud}^* d^\dagger \gamma^0 = V_{ud}^* \overline{d}$

- **Giving the $\boxed{u \to d}$ weak current:**

$$j_{ud} = \overline{d} V_{ud}^* \left[-i\frac{g_W}{\sqrt{2}} \gamma^\mu \tfrac{1}{2}(1-\gamma^5) \right] u$$

- **Hence, when the charge $-\frac{1}{3}$ quark enters as the adjoint spinor, the complex conjugate of the CKM matrix is used**

★ **The vertex factor the following diagrams:**
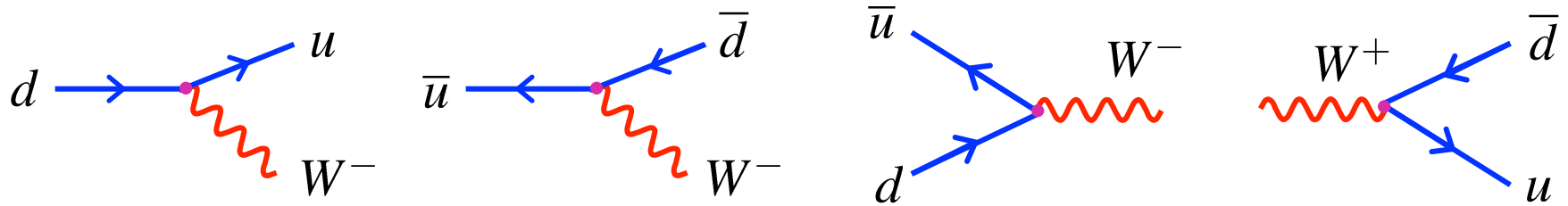
is
$$-i\frac{g_W}{\sqrt{2}}V_{ud}\,\gamma^\mu\tfrac{1}{2}(1-\gamma^5)$$

★ **Whereas, the vertex factor for:**

is
$$-i\frac{g_W}{\sqrt{2}}V_{ud}^*\,\gamma^\mu\tfrac{1}{2}(1-\gamma^5)$$
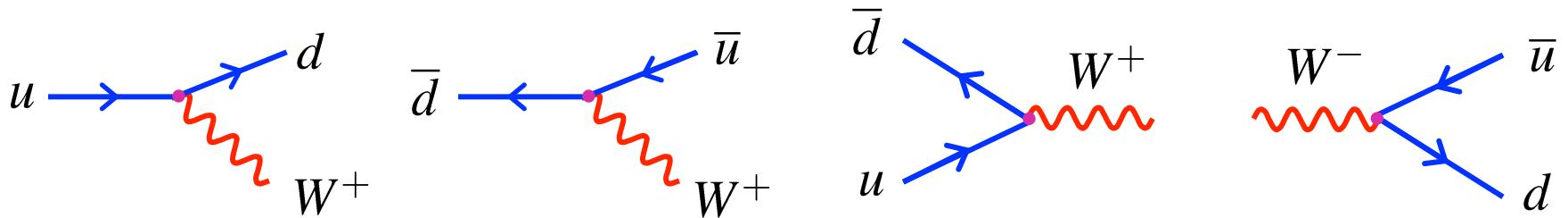
Cabibbo matrix

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.23 & 0.96 & 0.04 \\ 0.01 & 0.04 & 0.999 \end{pmatrix}$$

★ **NOTE: within the SM,** the charged current, $W^{\pm}$, weak interaction:
 ① **Provides the only way to change flavour !**
 ② **only way to change from one generation of quarks or leptons to another !**

★ **However,** the off-diagonal elements of the CKM matrix are relatively small.
 • **Weak interaction largest between quarks of the same generation.**
 • **Coupling between first and third generation quarks is very small !**

★ **The number of free parameters in the CKM matrix are three real parameters and one imaginary phase**

★**The presence of an imaginary phase is source of CP violation!**

The Neutral Kaon System

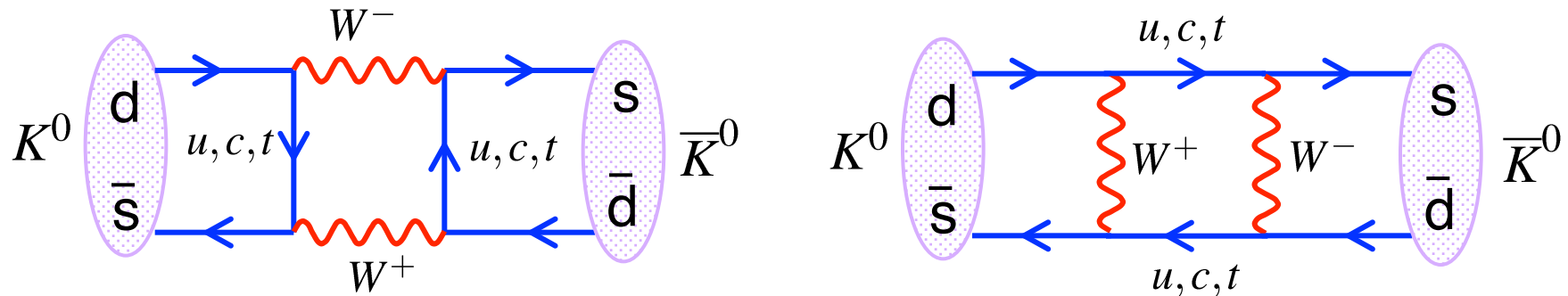- **Neutral Kaons** decay **via the** **weak interaction**
- **The Weak Interaction also allows mixing of neutral kaons via "box diagrams"**

- **The fact that for the quarks eigenstates of flavour are NOT eigenstates of mass implies that the u couples not only with the d, but with the s as well (this would not happen otherwise)**

- **This allows transitions between the strong eigenstates states** $K^0, \overline{K}^0$
- **Consequently, the neutral kaons propagate as eigenstates of the overall strong + weak interaction; i.e. as linear combinations of** $K^0, \overline{K}^0$

- **These neutral kaon states are called the "K-short"** K_S **and the "K-long"** K_L

- **These states have approximately the same mass** $m(K_S) \approx m(K_L) \approx 498\,\text{MeV}$

- **But very different lifetimes:** $\boxed{\tau(K_S) = 0.9 \times 10^{-10}\,\text{s}}$ $\boxed{\tau(K_L) = 0.5 \times 10^{-7}\,\text{s}}$

CP Violation and the CKM Matrix

★ **How can we explain** $\Gamma(\overline{K}^0_{t=0} \to K^0) \neq \Gamma(K^0_{t=0} \to \overline{K}^0)$ **in terms of the CKM matrix ?**

★ **Consider the box diagrams responsible for mixing, i.e.**
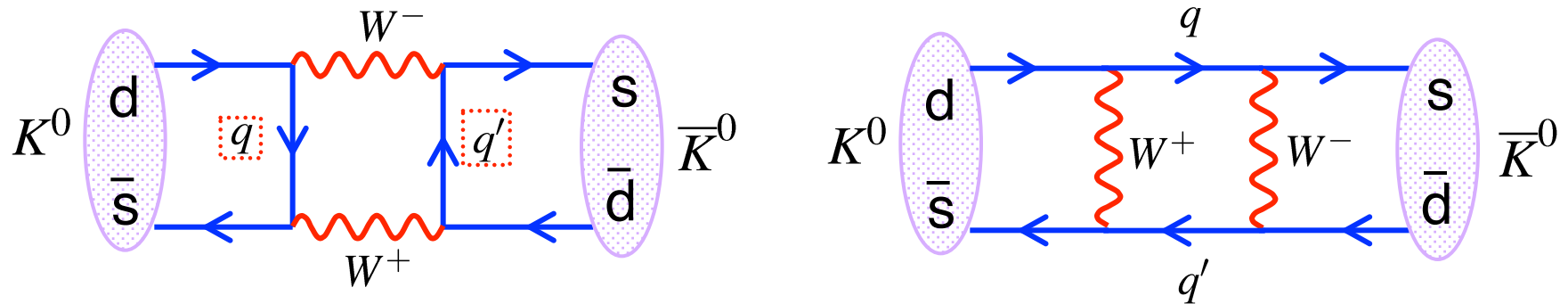
where $q = \{u, c, t\}, \qquad q' = \{u, c, t\}$

★ **Have to sum over all possible quark exchanges in the box. For simplicity consider just one diagram**
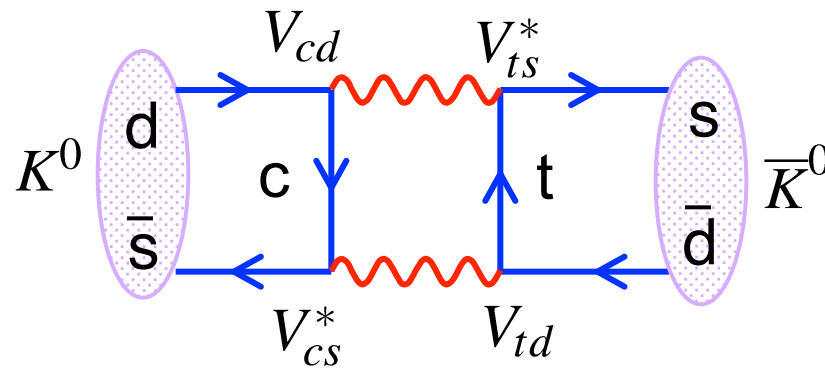
$$M_{fi} \propto A_{ct} V_{cd} V_{cs}^* V_{td} V_{ts}^*$$

A constant related to integrating over virtual momenta

★ **Compare the equivalent box diagrams for $K^0 \to \overline{K}^0$ and $\overline{K}^0 \to K^0$**
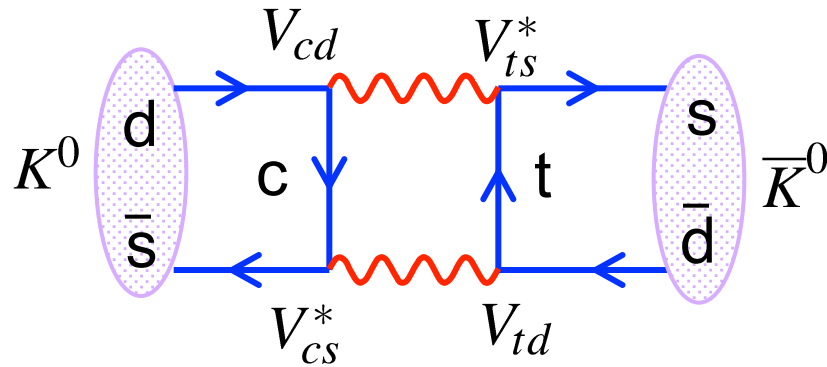
$$M_{fi} \propto A_{ct} V_{cd} V_{cs}^* V_{td} V_{ts}^*$$
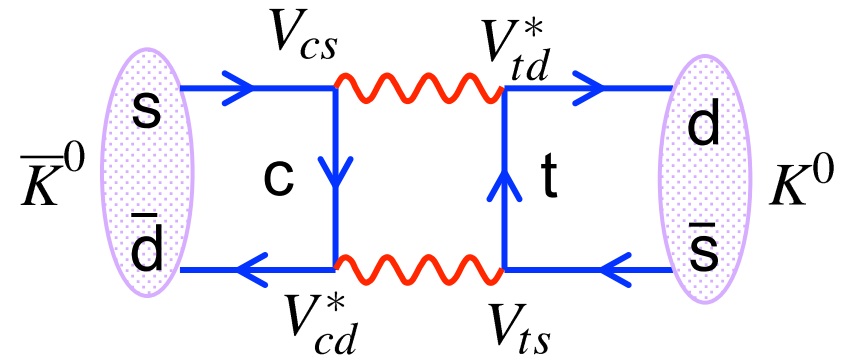
$$M'_{fi} \propto A_{ct} V_{cd}^* V_{cs} V_{td}^* V_{ts} = M_{fi}^*$$

★ **Therefore difference in rates**

$$\Gamma(K^0 \to \overline{K}^0) - \Gamma(\overline{K}^0 \to K^0) \propto M_{fi} - M_{fi}^* = 2\Im\{M_{fi}\}$$

★ **Hence the rates can only be different if the CKM matrix has imaginary component**

$$|\varepsilon| \propto \Im\{M_{fi}\}$$

★ **In the kaon system we can show**

$$|\varepsilon| \propto A_{ut}.\Im\{V_{ud}V_{us}^*V_{td}V_{ts}^*\} + A_{ct}.\Im\{V_{cd}V_{cs}^*V_{td}V_{ts}^*\} + A_{tt}.\Im\{V_{td}V_{ts}^*V_{td}V_{ts}^*\}$$

Shows that CP violation is related to the imaginary parts of the CKM matrix

CP Eigenstates

★ The K_S and K_L are closely related to eigenstates of the combined charge conjugation and parity operators: CP

• The strong eigenstates $K^0(d\overline{s})$ and $\overline{K}^0(s\overline{d})$ have $J^P = 0^-$

with $$\hat{P}|K^0\rangle = -|K^0\rangle, \qquad \hat{P}|\overline{K}^0\rangle = -|\overline{K}^0\rangle$$

• The charge conjugation operator changes particle into anti-particle and *vice versa*

$$\hat{C}|K^0\rangle = \hat{C}|d\overline{s}\rangle = +|s\overline{d}\rangle = |\overline{K}^0\rangle$$

similarly $$\hat{C}|\overline{K}^0\rangle = |K^0\rangle$$

> The + sign is purely conventional, could have used a - with no physical consequences

• Consequently

$$\hat{C}\hat{P}|K^0\rangle = -|\overline{K}^0\rangle \qquad \hat{C}\hat{P}|\overline{K}^0\rangle = -|K^0\rangle$$

i.e. neither K^0 or $\overline{K}^0$ are eigenstates of CP

• Form CP eigenstates from linear combinations:

$$|K_1\rangle = \tfrac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) \qquad \hat{C}\hat{P}|K_1\rangle = +|K_1\rangle$$

$$|K_2\rangle = \tfrac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle) \qquad \hat{C}\hat{P}|K_2\rangle = -|K_2\rangle$$

CP Eigenstates

- **Neutral kaons often decay to pions (the lightest hadrons)**
- **The kaon masses are approximately 498 MeV and the pion masses are approximately 140 MeV. Hence neutral kaons can decay to either 2 or 3 pions**
- **We already showed that particle and antiparticles have opposite P, therefore ground state scalar mesons have negative parity**

Decays to Two Pions:

- **The** $\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$ **is an eigenstate of** $\hat{C}$
- **It is easy to show that** $\hat{C}\hat{P}(\pi^+\pi^-) = +1$ **and** $CP(\pi^0\pi^0) = +1$

Decays to Three Pions (Assuming L=0):

- **Excited L are suppressed for the angular momentum barrier**

$$P(\pi^0\pi^0\pi^0) = -1. -1. -1.$$
$$C(\pi^0\pi^0\pi^0) = +1. +1. +1$$
$$\implies CP(\pi^0\pi^0\pi^0) = -1$$

$$P(\pi^+\pi^-\pi^0) = -1. -1. -1.$$
$$C(\pi^+\pi^-\pi^0) = +1.C(\pi^+\pi^-)$$
$$\implies CP(\pi^+\pi^-\pi^0) = -1.$$

- **So the two pion state is CP even, the three pion state is CP odd!**

★ **If CP were conserved in the Weak decays of neutral kaons, would expect decays to pions to occur from states of definite CP (i.e. the CP eigenstates K_1, K_2)**

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) \quad\quad \hat{C}\hat{P}|K_1\rangle = +|K_1\rangle \quad\quad K_1 \to \pi\pi \quad\quad \text{CP EVEN}$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle) \quad\quad \hat{C}\hat{P}|K_2\rangle = -|K_2\rangle \quad\quad K_2 \to \pi\pi\pi \quad\quad \text{CP ODD}$$

★ **Expect lifetimes of CP eigenstates to be very different**
  - **For two pion decay energy available:** $m_K - 2m_\pi \approx 220\,\text{MeV}$
  - **For three pion decay energy available:** $m_K - 3m_\pi \approx 80\,\text{MeV}$

★ **Expect decays to two pions to be more rapid than decays to three pions due to increased phase space**

★ **This is exactly what is observed: a short-lived state "K-short" which decays to (mainly) to two pions and a long-lived state "K-long" which decays to three pions**

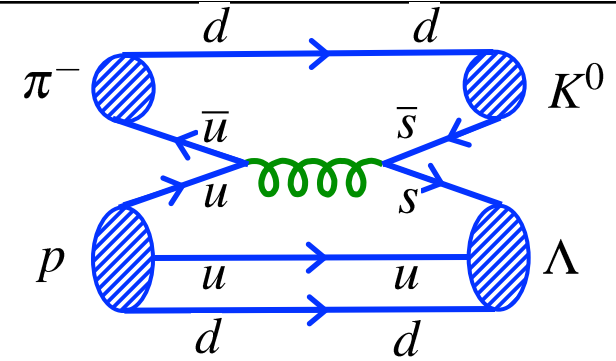★ **In the absence of CP violation we can identify**

$$|K_S\rangle = |K_1\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) \quad\quad \text{with decays:} \quad K_S \to \pi\pi$$

$$|K_L\rangle = |K_2\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle) \quad\quad \text{with decays:} \quad K_L \to \pi\pi\pi$$

Neutral Kaon Decays to pions

- **Consider the decays of a beam of** K^0
- **The decays to pions occur in states of definite CP**
- **If CP is conserved in the decay, need to express** K^0 **in terms of** K_S **and** K_L

$$|K_0\rangle = \frac{1}{\sqrt{2}}(|K_S\rangle + |K_L\rangle)$$

- **Hence from the point of view of decays to pions, a** K^0 **beam is a linear combination of CP eigenstates:**

  a rapidly decaying CP-even component and a long-lived CP-odd component
- **Therefore, expect to see predominantly two-pion decays near start of beam and predominantly three pion decays further downstream**

$K_S \rightarrow \pi\pi$

$K_L \rightarrow \pi\pi\pi$

At large distance left with pure K$_L$ beam

Log Intensity

Distance from K^0 production
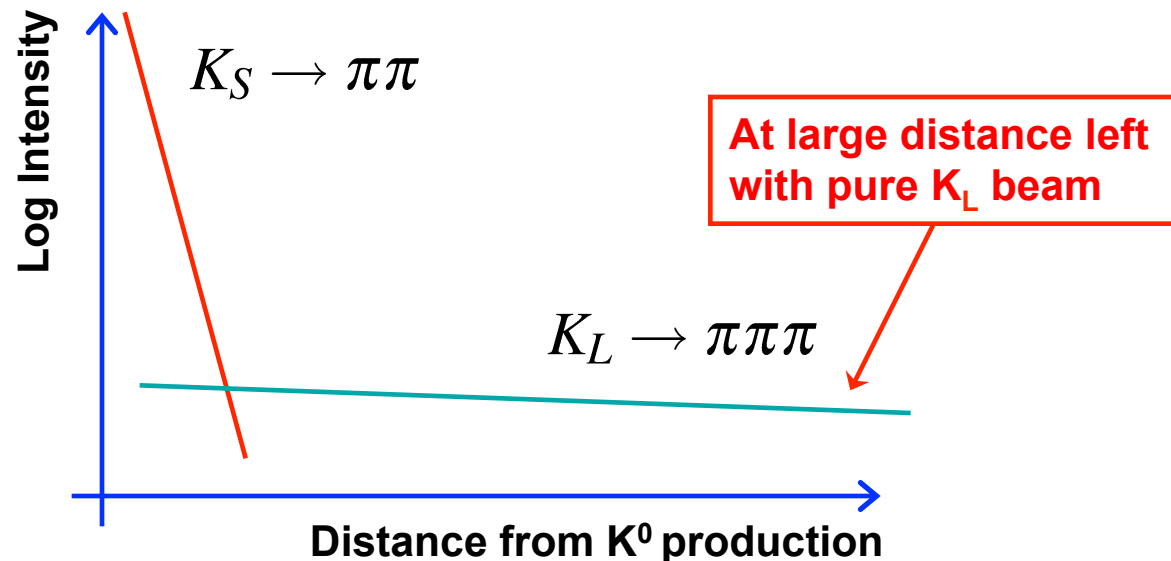
- **Experimentally we find:** $\boxed{\tau(K_S) = 0.9 \times 10^{-10}\,\text{s}}$ $\boxed{\tau(K_L) = 0.5 \times 10^{-7}\,\text{s}}$

  and $\boxed{\Delta m = (3.506 \pm 0.006) \times 10^{-15}\,\text{GeV}}$

  i.e. the K-long mass is greater than the K-short by 1 part in 10^{16}

- **The mass difference corresponds to an oscillation period of**

$$T_{osc} = \frac{2\pi\hbar}{\Delta m} \approx 1.2 \times 10^{-9}\,\text{s}$$

- **The oscillation period is relatively long compared to the K_S lifetime and consequently, do not observe very pronounced oscillations**

$K_{t=0}^0 \rightarrow K^0$

$K_{t=0}^0 \rightarrow \overline{K}^0$

$|K_L\rangle = \alpha |K^0\rangle + \beta |\overline{K}^0\rangle$

$\Gamma(K^0 \rightarrow \overline{K}^0) = |\beta|^2$ if $|\alpha| \neq |\beta|$ CPV

$\Gamma(\overline{K}^0 \rightarrow K^0) = |\alpha|^2$

After a few K_S lifetimes, left with a pure K_L beam which is half K^0 and half $\overline{K}^0$

CP Violation in the Kaon System

★ **So far we have ignored CP violation in the neutral kaon system**

★ **Identified the K-short as the CP-even state and the K-long as the CP-odd state**

$$|K_S\rangle = |K_1\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) \qquad \text{with decays:} \qquad K_S \to \pi\pi \qquad \boxed{\text{CP = +1}}$$
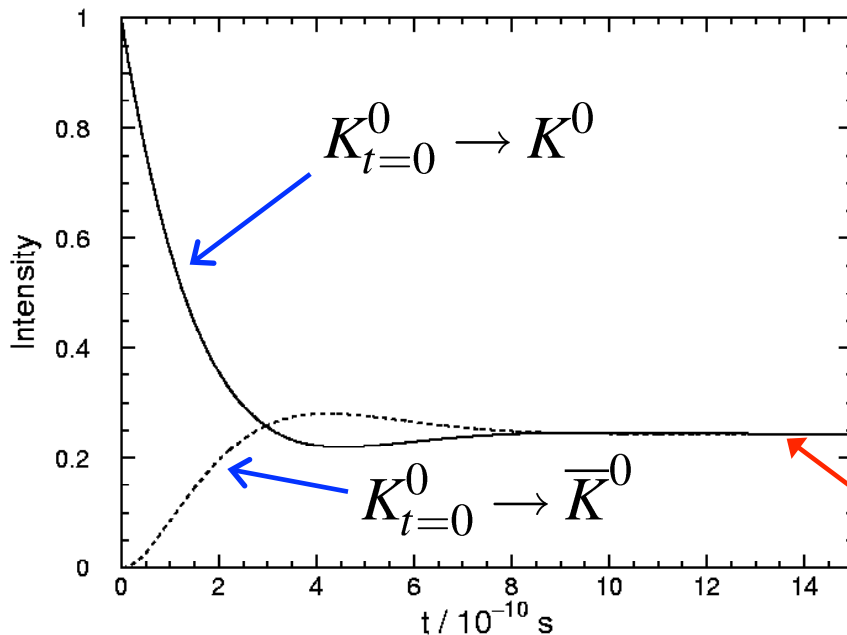
$$|K_L\rangle = |K_2\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle) \qquad \text{with decays:} \qquad K_L \to \pi\pi\pi \qquad \boxed{\text{CP = -1}}$$

★ **At a long distance from the production point a beam of neutral kaons will be 100% K-long (the K-short component will have decayed away). Hence, if CP is conserved, would expect to see only three-pion decays.**

★ **In 1964 Fitch & Cronin (joint Nobel prize) observed 45 $K_L \to \pi^+\pi^-$ decays in a sample of 22700 kaon decays a long distance from the production point**

⟹ **Weak interactions violate CP**

• **CP is violated in hadronic weak interactions, but only at the level of 2 parts in 1000**

K$_L$ to pion BRs:

$$
\begin{aligned}
K_L \quad &\to \quad \pi^+\pi^-\pi^0 \qquad BR = 12.6\% \qquad CP = -1 \\
&\to \quad \pi^0\pi^0\pi^0 \qquad BR = 19.6\% \qquad CP = -1 \\
&\to \quad \pi^+\pi^- \qquad BR = 0.20\% \qquad \boxed{CP = +1} \\
&\to \quad \pi^0\pi^0 \qquad BR = 0.08\% \qquad \boxed{CP = +1}
\end{aligned}
$$

★Two possible explanations of CP violation in the kaon system:

i) The K_S and K_L do not correspond exactly to the CP eigenstates K_1 and K_2

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}[|K_1\rangle + \varepsilon|K_2\rangle]$$ $$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}[|K_2\rangle + \varepsilon|K_1\rangle]$$

with $|\varepsilon| \sim 2 \times 10^{-3}$

•**In this case the observation of** $K_L \to \pi\pi$ **is accounted for by:**

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}[|K_2\rangle + \varepsilon|K_1\rangle]$$

$\longrightarrow \pi\pi$ CP = +1

$\longrightarrow \pi\pi\pi$ CP = -1

ii) and/or CP is violated in the decay

$$|K_L\rangle = |K_2\rangle \quad \boxed{\text{CP = -1}}$$

$\longrightarrow \pi\pi\pi$ CP = -1   **Parameterised by** ε'

$\longrightarrow \pi\pi$ CP = +1

★ Experimentally both known to contribute to the mechanism for CP violation in the kaon system but i) dominates: $\varepsilon'/\varepsilon = (1.7 \pm 0.3) \times 10^{-3}$ { NA48 (CERN) KTeV (FermiLab)

Summary

★ **The weak interactions of quarks are described by the CKM matrix**

★ **Similar structure to the lepton sector, we will introduce the PMNS matrix next time when we discuss neutrino oscillations**

★ **CP violation enters through via a complex phase in the CKM matrix**

★ **A great deal of experimental evidence for CP violation in the weak interactions of quarks**

★ **CP violation is needed to explain matter – anti-matter asymmetry in the Universe**

★ **HOWEVER, CP violation in the SM is not sufficient to explain the matter – anti-matter asymmetry. There is probably another mechanism.**