Elementary Particle Theory – PHY452

Fall Semester 2020

Exercise Sheet 3

<u>Exercise 1</u>: Feynman rules

Consider the following Lagrangian

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial_\mu \phi) + \frac{1}{2}\partial_\mu \rho\, \partial^\mu \rho - \frac{1}{2} m^2 \rho^2 + \lambda_1 \, \phi^\dagger \phi \rho + \frac{\lambda_2}{3!}\rho^3 \,, \tag{1}$$

where ρ is a massive scalar field interacting with a massless complex scalar field ϕ. Write down the Feynman rules for all vertices and propagatos in momentum space.

<u>Exercise 2</u>: Clifford algebra and bilinear covariants

The Dirac matrices satisfy the relation

$$\{\gamma^\mu, \gamma^\nu\} = 2 g^{\mu\nu} \,. \tag{2}$$

The anti-symmetric γ^μ product is defined by as

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] \,, \tag{3}$$

and the matrix γ^5 by

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = -\frac{i}{4}\varepsilon_{\mu\nu\sigma\rho}\gamma^\mu\gamma^\nu\gamma^\sigma\gamma^\rho \,, \tag{4}$$

where $\varepsilon_{\mu\nu\sigma\rho}$ denotes the fully anti-symmetric Levi-Civita tensor, with $\varepsilon^{0123} = 1$. These matrices satisfy the following rules for Hermitian conjugation,

$$\left(\gamma^5\right)^\dagger = \gamma^5 \,, \qquad \left(\gamma^\mu\right)^\dagger = \gamma^0\gamma^\mu\gamma^0 \,. \tag{5}$$

a) Compute the following traces

$$\mathrm{tr}\big(\gamma^5\big) = 0 \,, \tag{6}$$

$$\mathrm{tr}\big(\gamma^\mu\gamma^\nu\big) = 4g^{\mu\nu} \,, \tag{7}$$

$$\mathrm{tr}\big(\gamma^\mu\gamma^\nu\gamma^5\big) = 0 \,, \tag{8}$$

$$\mathrm{tr}\big(\gamma^\mu\gamma^\nu\gamma^\sigma\gamma^\rho\gamma^5\big) = -4i\varepsilon^{\mu\nu\sigma\rho} \,. \tag{9}$$

<u>Hints</u>: *Exploit the cyclicity of the trace. Try to obtain equations of the form $a = -a$.*

b) Show that the Hermitian conjugate of a general bilinear-covariant is given by

$$\big(\overline{\psi}_1 \, \Gamma \psi_2\big)^\dagger = \overline{\psi}_2 \, \overline{\Gamma} \, \psi_1 \,, \qquad \text{with} \qquad \overline{\Gamma} = \gamma^0 \, \Gamma^\dagger \, \gamma^0 \,, \tag{10}$$

where $\psi_{1,2}$ are fermionic spinors.

c) Derive an explicit expression for $\overline{\Gamma}$ for each element of $\Gamma \in \{1, \gamma^5, \gamma^\mu, \gamma^\mu\gamma^5, \sigma^{\mu\nu}\}$.

Exercise 3: Higgs-boson decay into fermions

The Lagrangian describing the interactions between the Higgs boson H and SM fermions f reads

$$\mathcal{L}_{\text{Yuk}} = -\sum_f \frac{m_f}{v} \overline{\psi}_f \psi_f H \,, \tag{11}$$

where $v = (\sqrt{2} G_F)^{-1/2} \approx 246$ GeV, and m_f is mass of the fermion f, which can be down-type quarks (d, s, b), up-type quarks (u, c, t), and charged leptons (e, μ, τ).

a) Derive the Feynman rules for $H \to f\bar{f}$.

b) Use the Feynman rules derived above to compute the decay amplitude $\mathcal{M}(H \to f\bar{f})$. Compute the partial decay widths $\Gamma(H \to \mu^+\mu^-)$ and $\Gamma(H \to b\bar{b})$ as a function of m_μ and m_b, respectively.

Hint: *The summation over fermion polarizations in the squared amplitude satisfy*

$$\sum_{\sigma_1,\sigma_2} |\bar{u}_{f_1}(p_1,\sigma_1)\, \Gamma\, v_{f_2}(p_2,\sigma_2)|^2 = \text{Tr}\left[(\not{p}_1 + m_{f_1})\, \Gamma\, (\not{p}_2 - m_{f_2})\, \overline{\Gamma} \right] \,, \tag{12}$$

where $f_{1,2}$ are fermions with momenta $p_{1,2}$, polarization $\sigma_{1,2}$ and masses $m_{f_{1(2)}}$, Γ is any combination of Dirac matrices, and $\overline{\Gamma} = \gamma^0\, \Gamma^\dagger\, \gamma^0$, cf. Eq. (10).

c) By considering only the Higgs decays into fermions, estimate the total Higgs width by using the particles masses available in the [Particle Data Group]. Use this expression to estimate the branching fractions $\mathcal{B}(H \to \mu^+\mu^-)$ and $\mathcal{B}(H \to b\bar{b})$ in the Standard Model. Which is the easiest decay mode to look for the Higgs boson experimentally?