

# Crystal Structure

# CRYSTALS

NaCl (Salt)



C (Diamond)



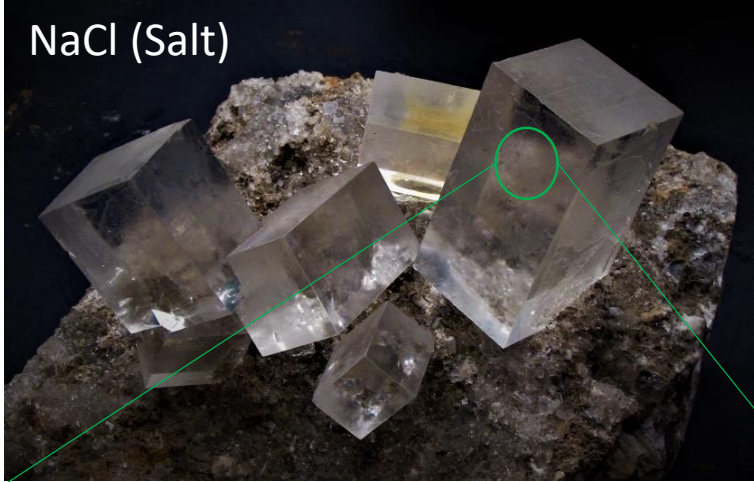
SiO<sub>2</sub> (Quartz)



YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7-x</sub> (superconductor)

A **crystal** is a periodic array of atoms or group of atoms

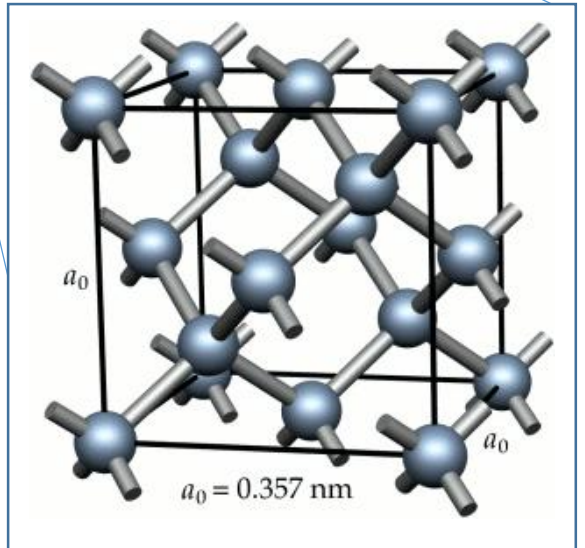
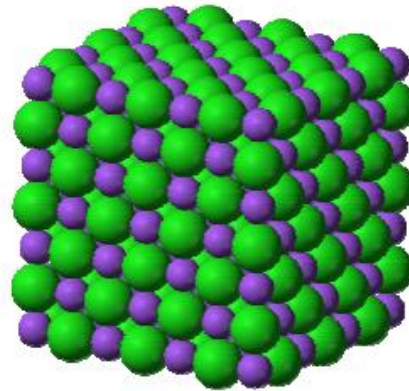
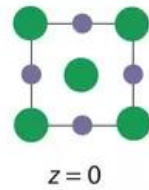
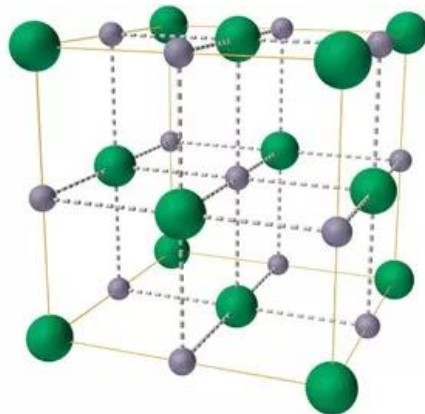
NaCl (Salt)



C (Diamond)

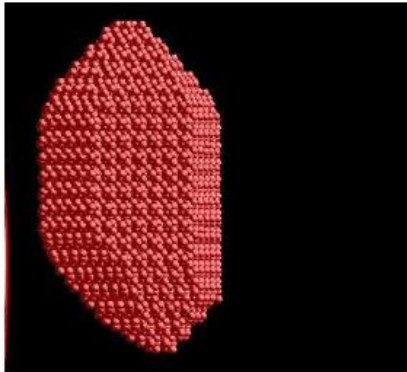
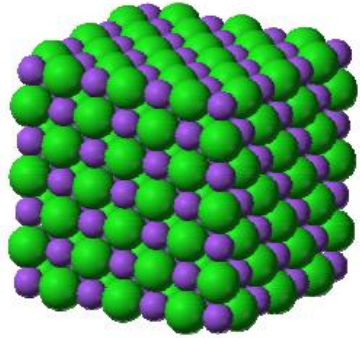


Key  
● Na<sup>+</sup>  
● Cl<sup>-</sup>

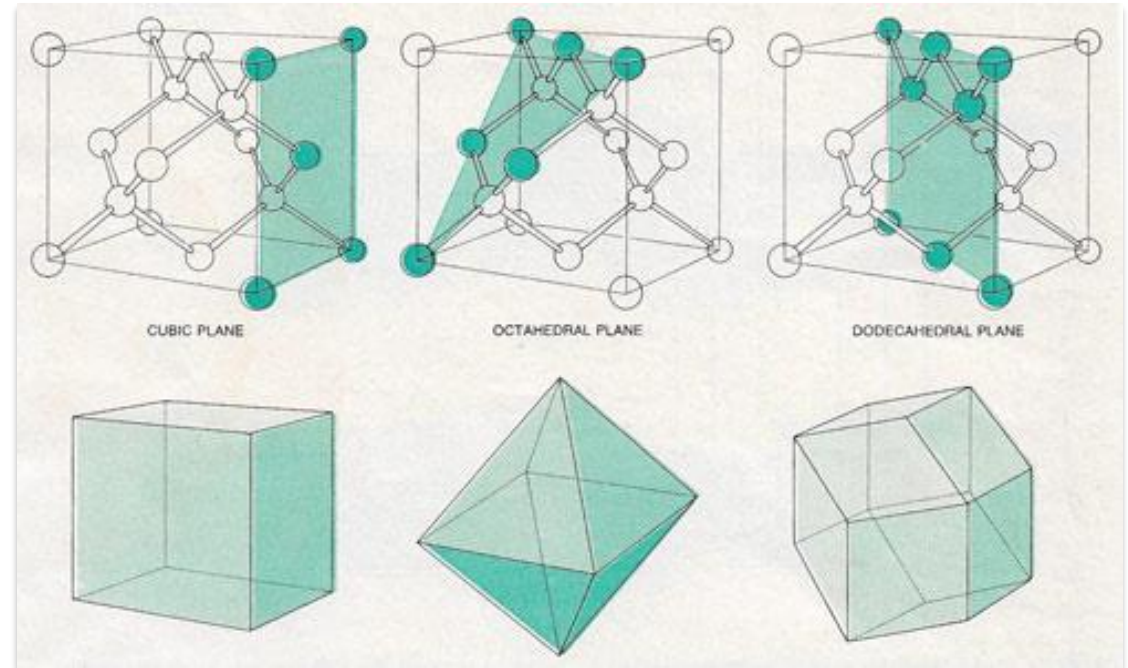


A **crystal** is a periodic array of atoms or group of atoms

NaCl (Salt)



C (Diamond)

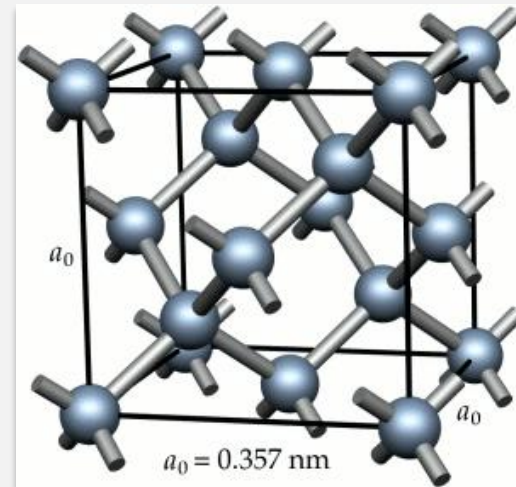


The macroscopic morphology of a crystal reflects its underlying structure

## Learning outcomes of the lecture

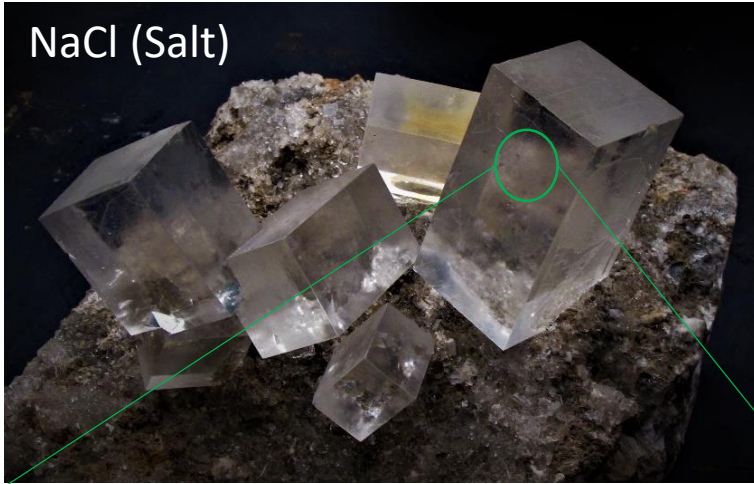
➤ Describe a crystal structure

1. the Lattice



A **crystal** is a periodic array of atoms or group of atoms

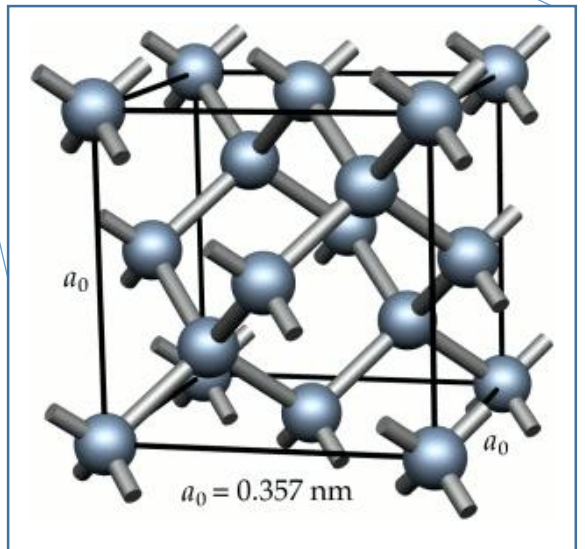
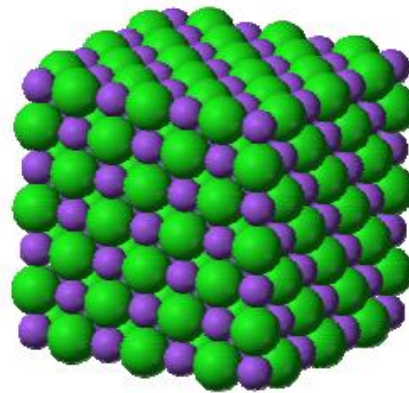
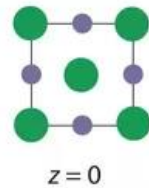
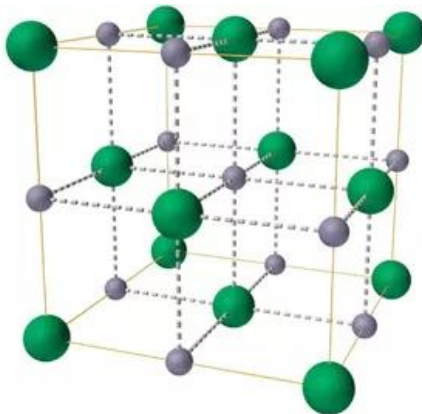
NaCl (Salt)



Diamond (C)



Key  
● Na<sup>+</sup>  
● Cl<sup>-</sup>



another definition

A **crystal** consists of a repeating pattern of objects (i.e. atoms or molecules) in an effectively infinite 3D array

A convenient way of describing a crystal :

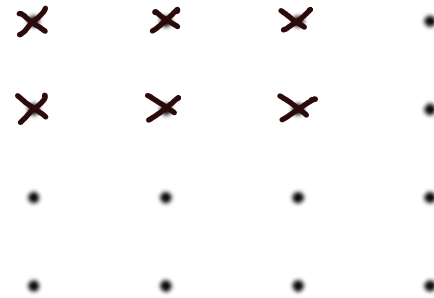
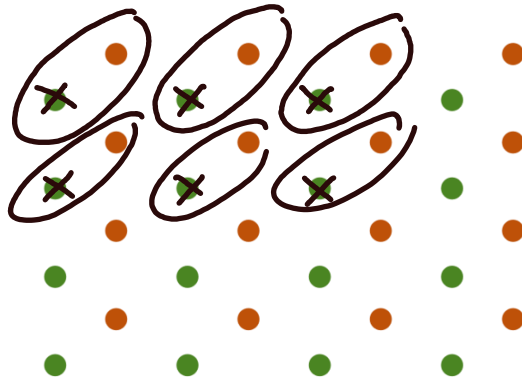
**Crystal**

=

**Lattice**

+

**Basis**



Lattice of points  
(Bravais Lattice)

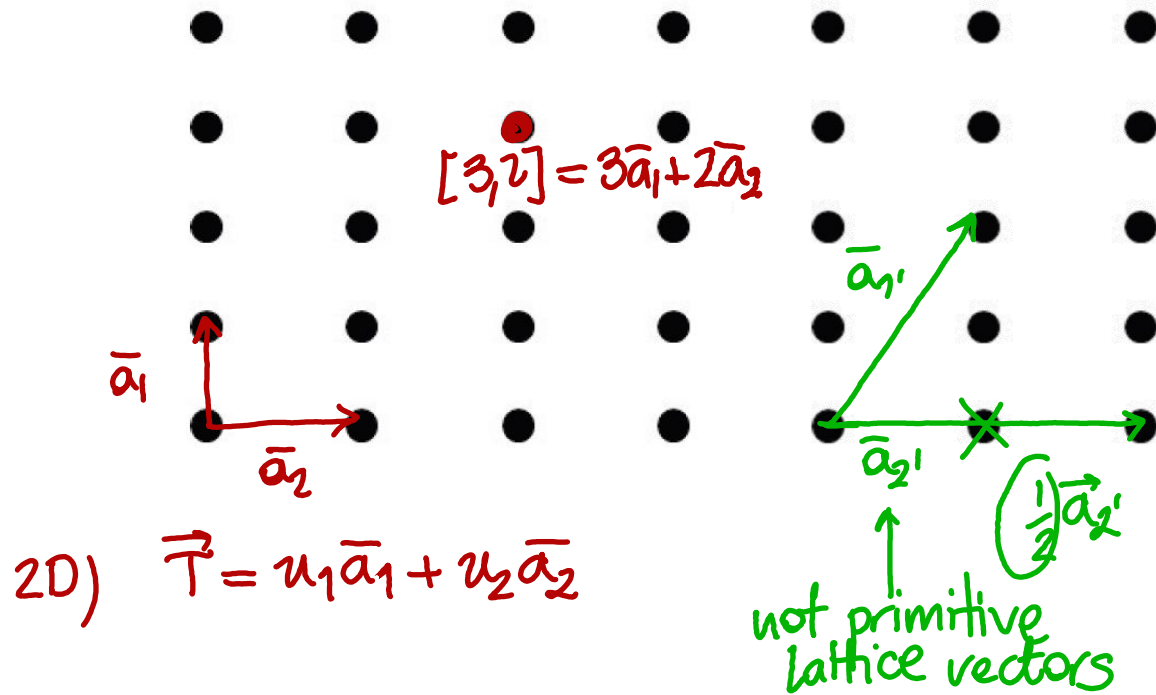
( set of mathematical  
points )

Basis of atoms

( physical object "attached"  
to the lattice point )

# Lattice

- Infinite array of points in the space, in which each point has identical surroundings to all other points



Lattice points are generated by translation vectors:

$$\vec{r}' = \vec{r} + \vec{T}$$

where  $\vec{T} = u_1 \bar{a}_1 + u_2 \bar{a}_2 + u_3 \bar{a}_3$

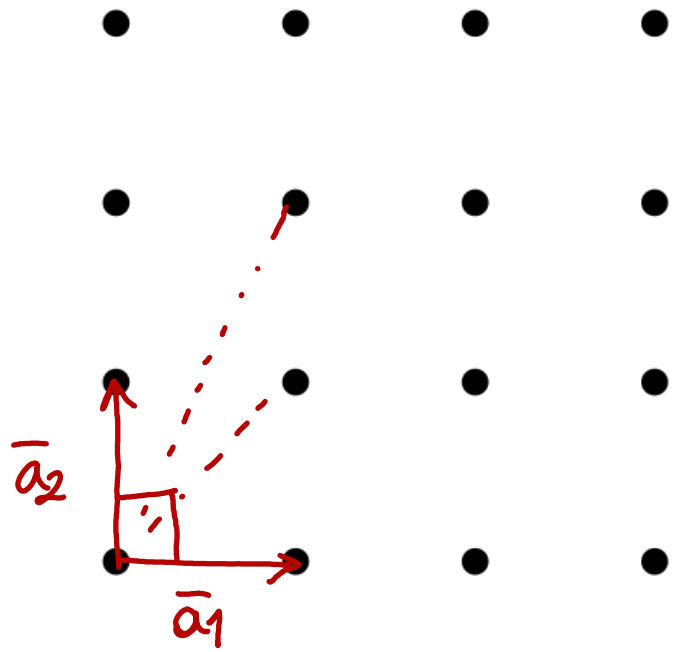
with  $\bar{a}_i$  : primitive lattice vectors  
 $u_i$  : arbitrary integers

property, i.e. electronic density  
 $\rho(\vec{r}) = \rho(\vec{r} + \vec{T})$

Alternative definition: A **lattice** is an infinite set of points defined by the integer sum of a set of linearly independent primitive lattice vectors

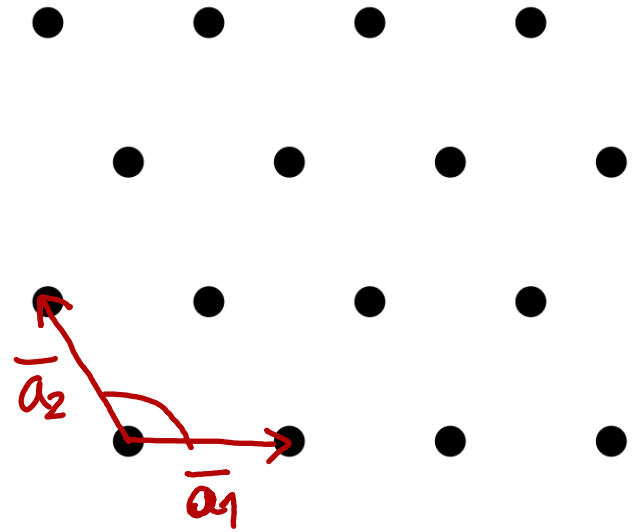


If all the points of the lattice, and only the lattice points, can be reached with  $n_i$  integers, the  $\vec{a}_i$  are called primitive lattice vectors



$$a_1 = a_2$$
$$\psi = 90^\circ$$

Square  
lattice



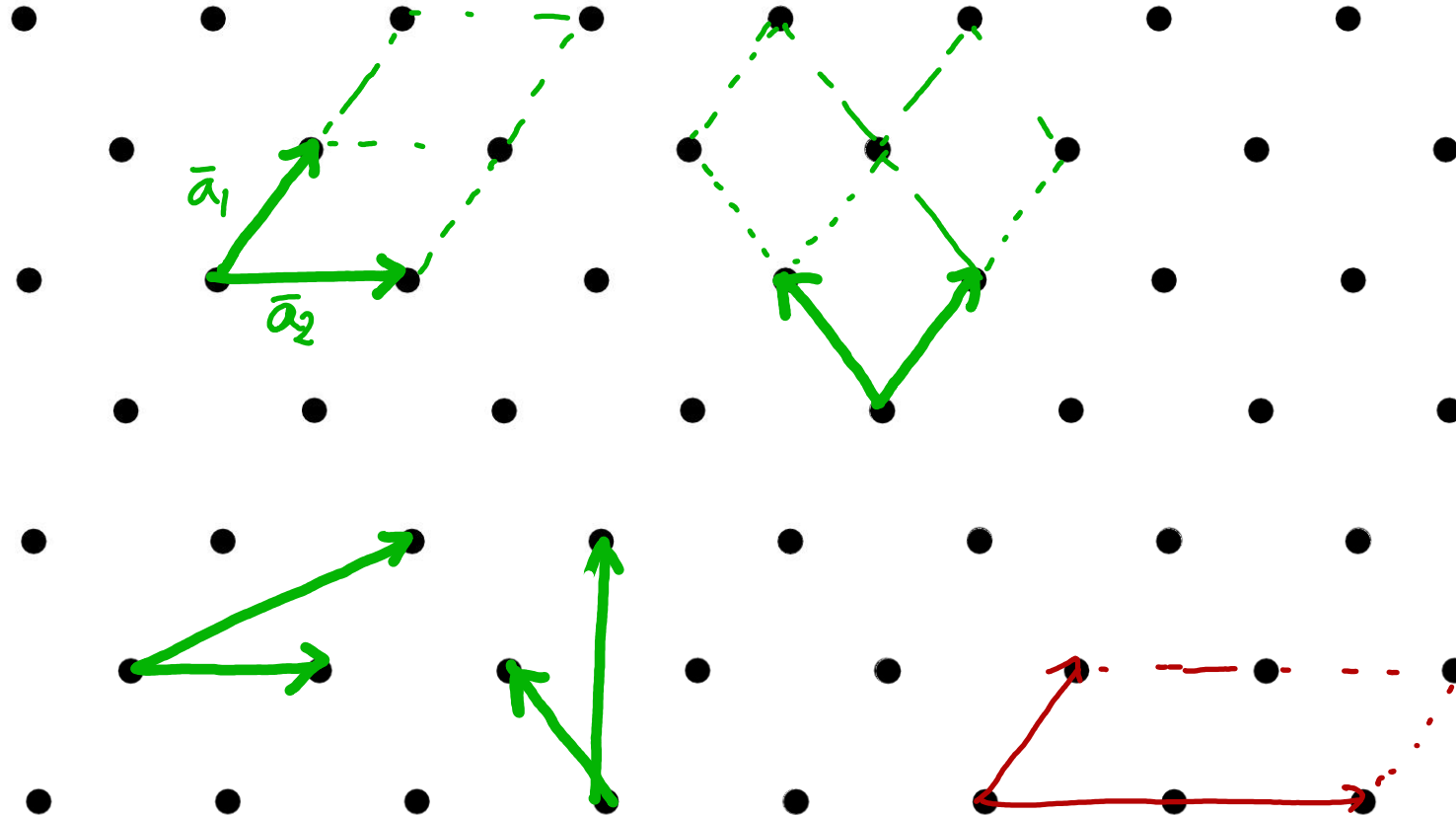
$$a_1 = a_2$$
$$\psi = 120^\circ$$

Hexagonal

Is the choice of primitive lattice vectors unique?

NO!

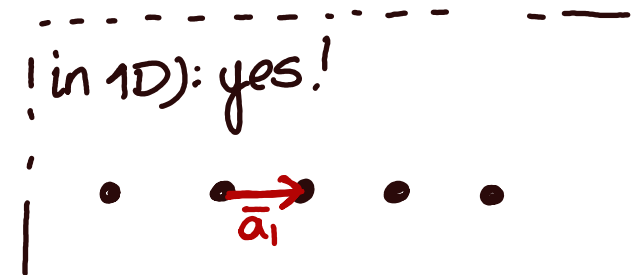
$$\vec{T} = u_1 \vec{a}_1 + u_2 \vec{a}_2$$



All valid sets of possible primitive lattice vectors

Not primitive translation vectors

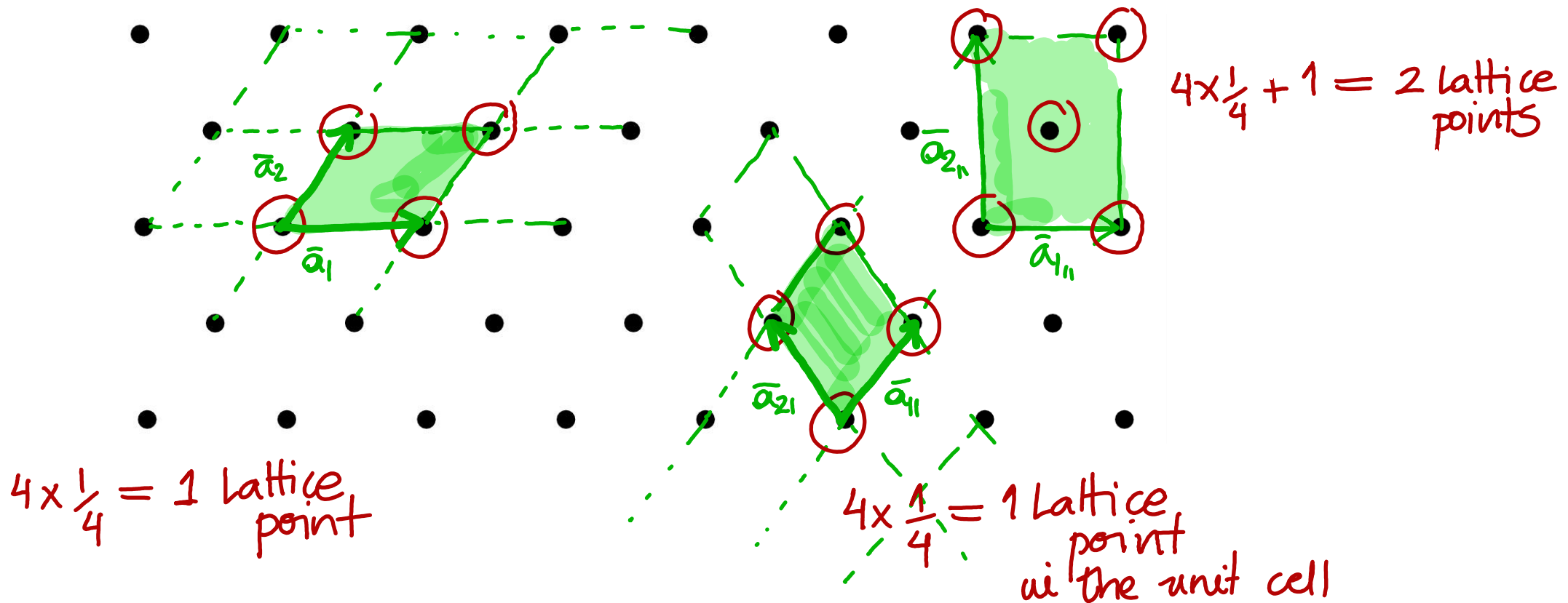
! in 1D): yes!



# Unit cell

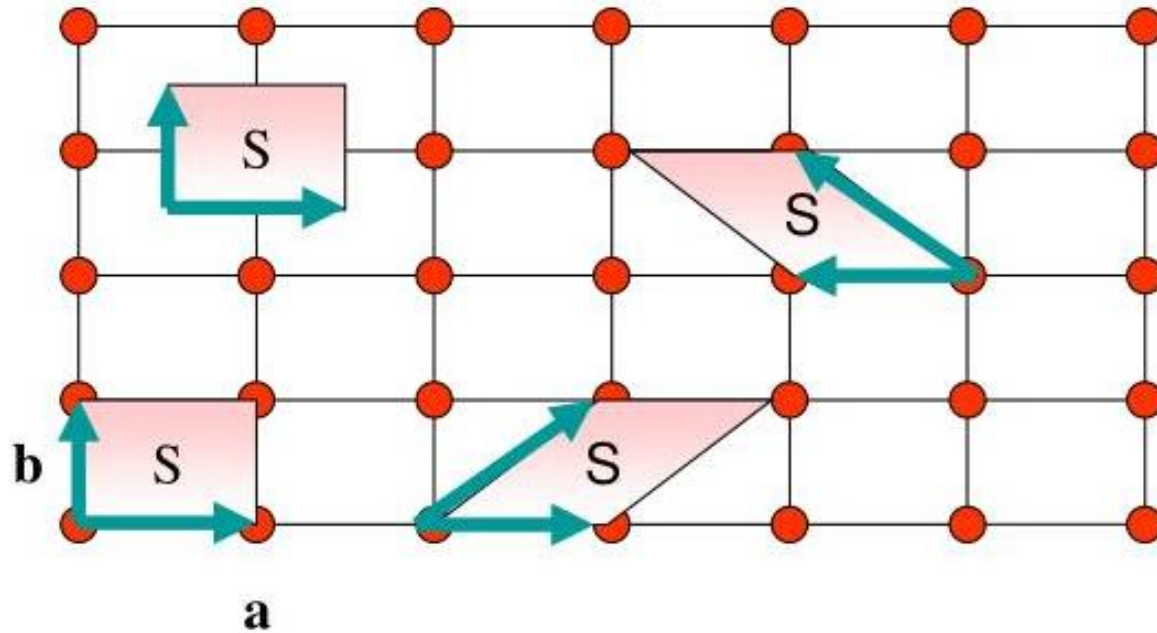
- A unit cell is a region of space that when repeated completely fills all of space and reconstructs the full structure

Equivalently: “the repeated motif which is the element building block of the periodic structure”



# Primitive Unit Cell

- A primitive unit cell for a periodic crystal is a unit cell containing exactly one lattice point

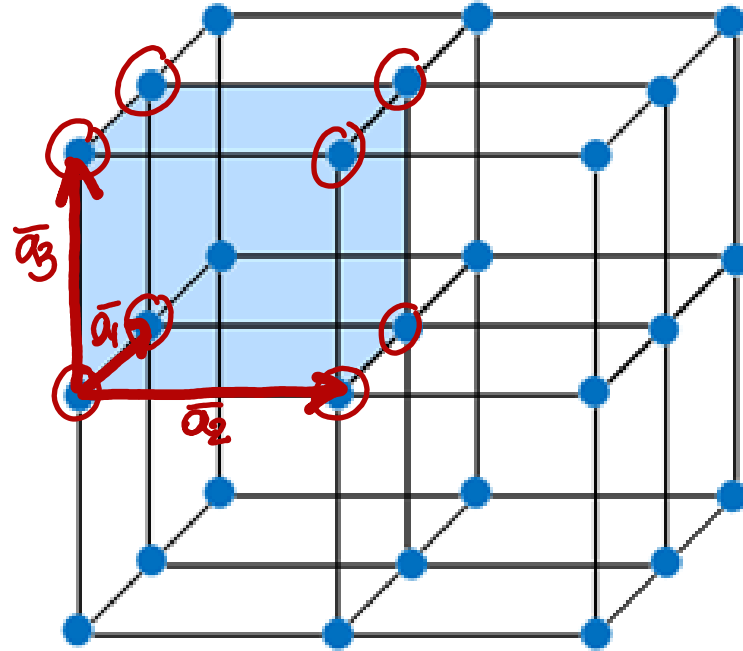


- there is no cell of smallest volume than  $|\bar{\mathbf{a}}_1 \cdot \bar{\mathbf{a}}_2 \times \bar{\mathbf{a}}_3|$

Primitive unit cell is not unique!

# Primitive Unit Cell in 3D

$$8 \times \frac{1}{8} = 1 \text{ Lattice point per primitive cell}$$



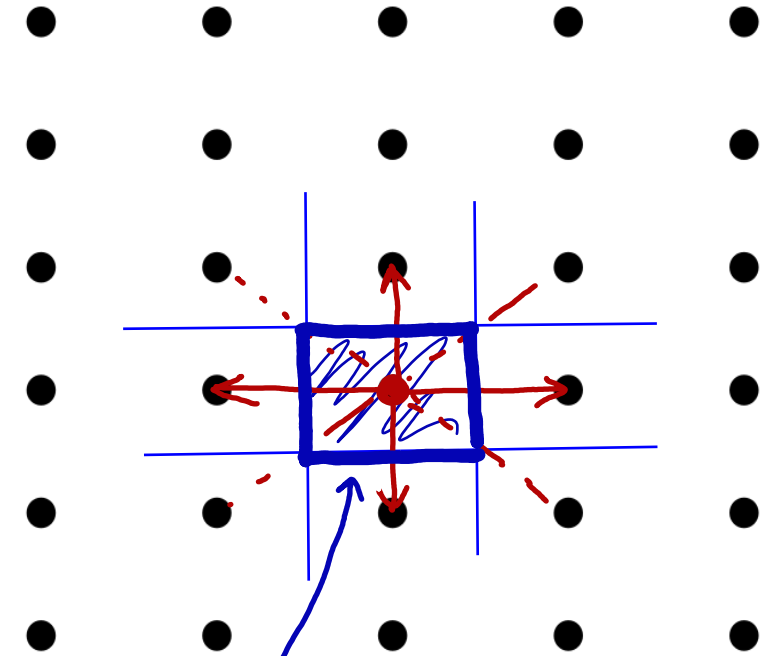
Primitive unit cell in 3D

# Wigner-Seitz method for defining a primitive unit cell

1. Choose a lattice point.
2. Construct vector to connect this point to all neighboring lattice points.
3. Planes are then constructed perpendicular to and passing through midpoints of these vectors

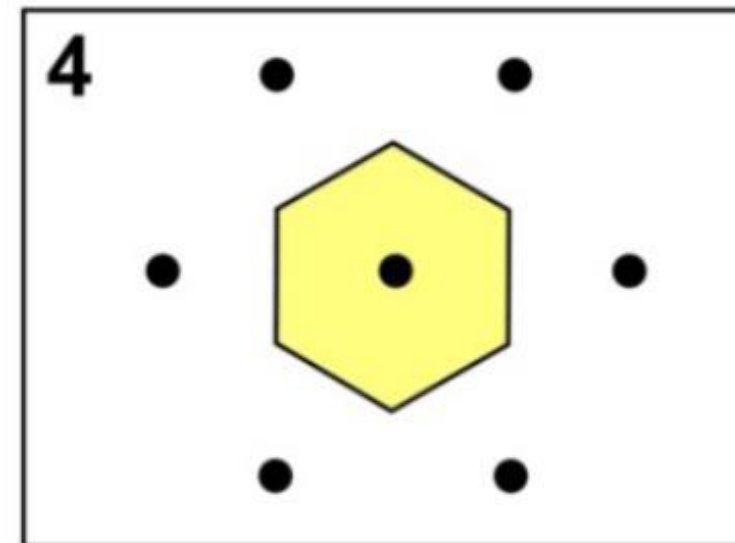
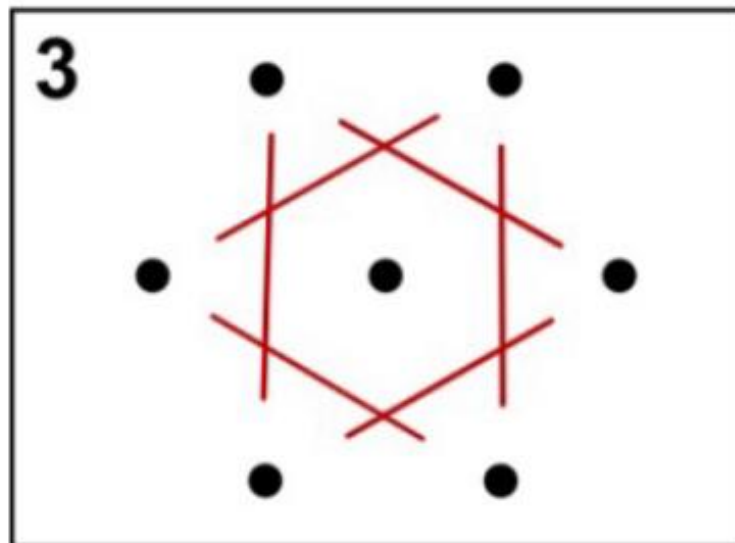
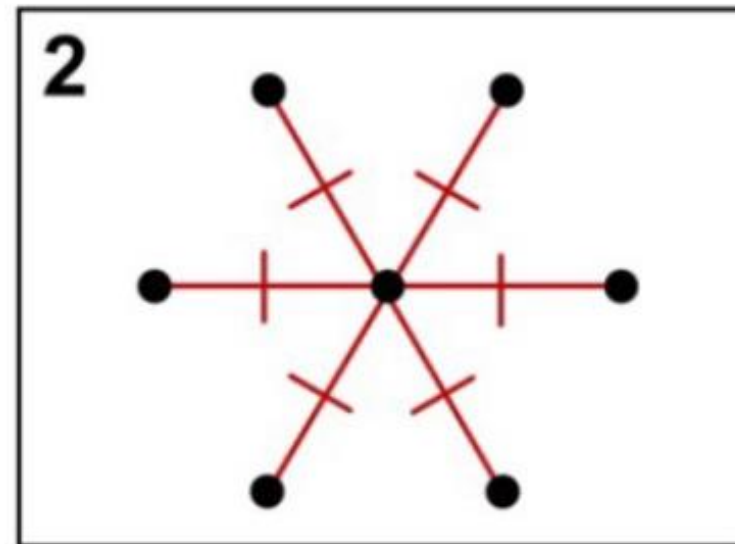
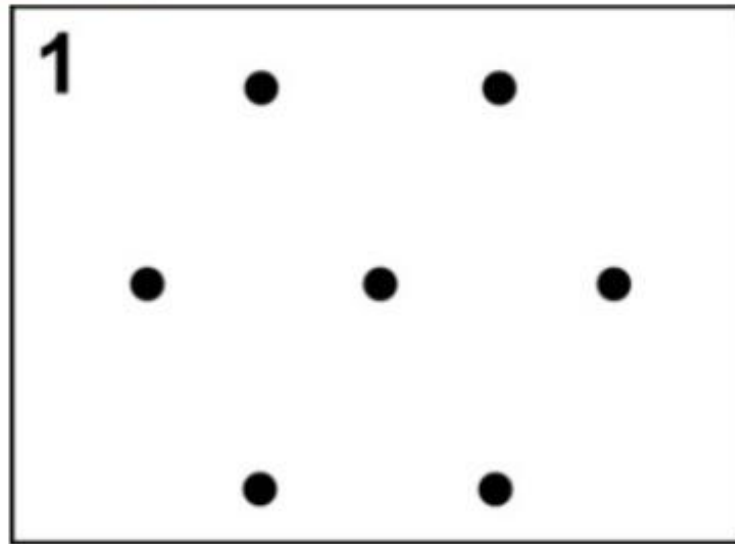


The smallest volume enclosed is called as a Wigner-Seitz cell.



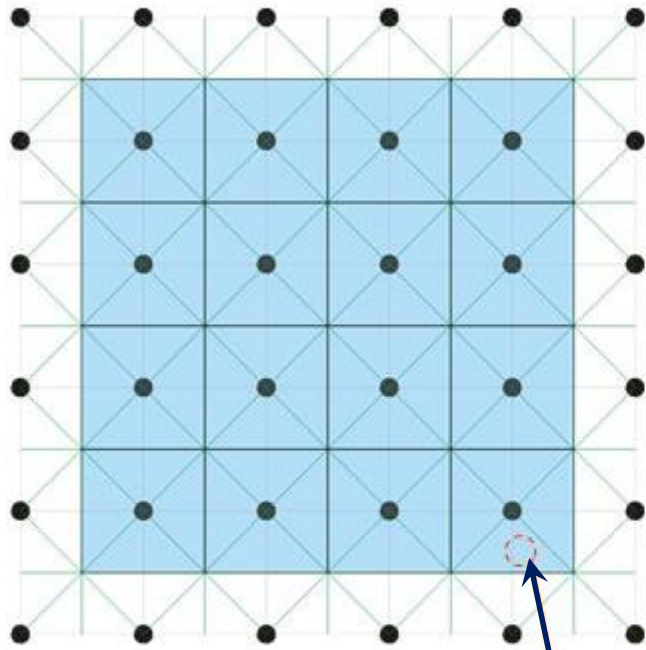
Wigner-Seitz cell  
(1 lattice point)

# Wigner-Seitz construction

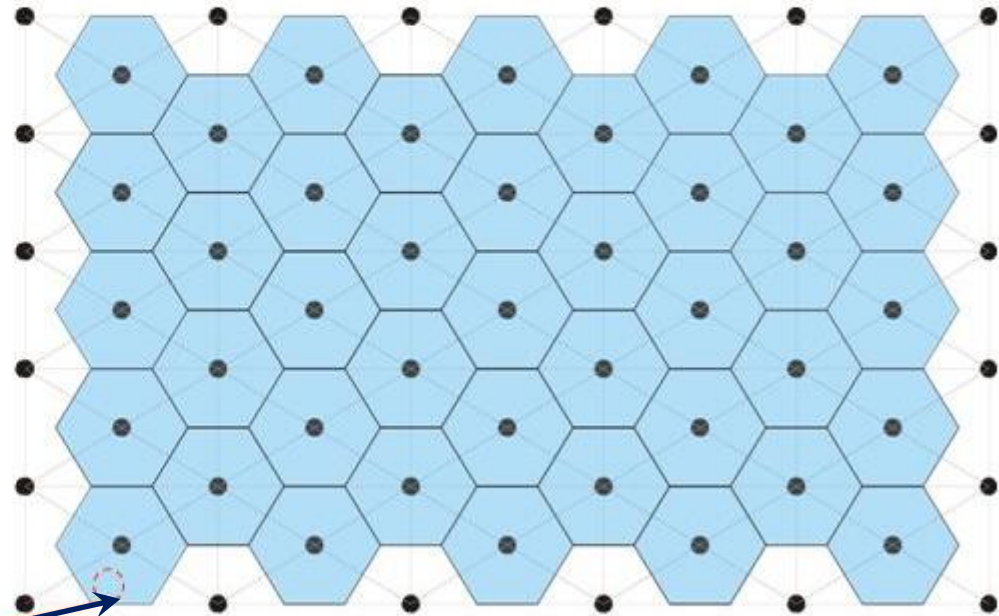


# Wigner-Seitz cells

Square lattice



Centred Rectangular lattice



Wigner-Seitz cells



Quiz: Which are primitive unit cells of the lattice?

