RESULTS FROM LAST WEEK

DENSITY-OF-STATES: $$D(\omega) = \frac{V}{2\pi^2} \frac{k^2}{\frac{d\omega}{dk}} \qquad (3D)$$

$$\Downarrow$$

$$D(\omega)\,d\omega = \frac{V}{2\pi^2} k^2\,dk = \frac{V}{(2\pi)^3} 4\pi k^2\,dk$$

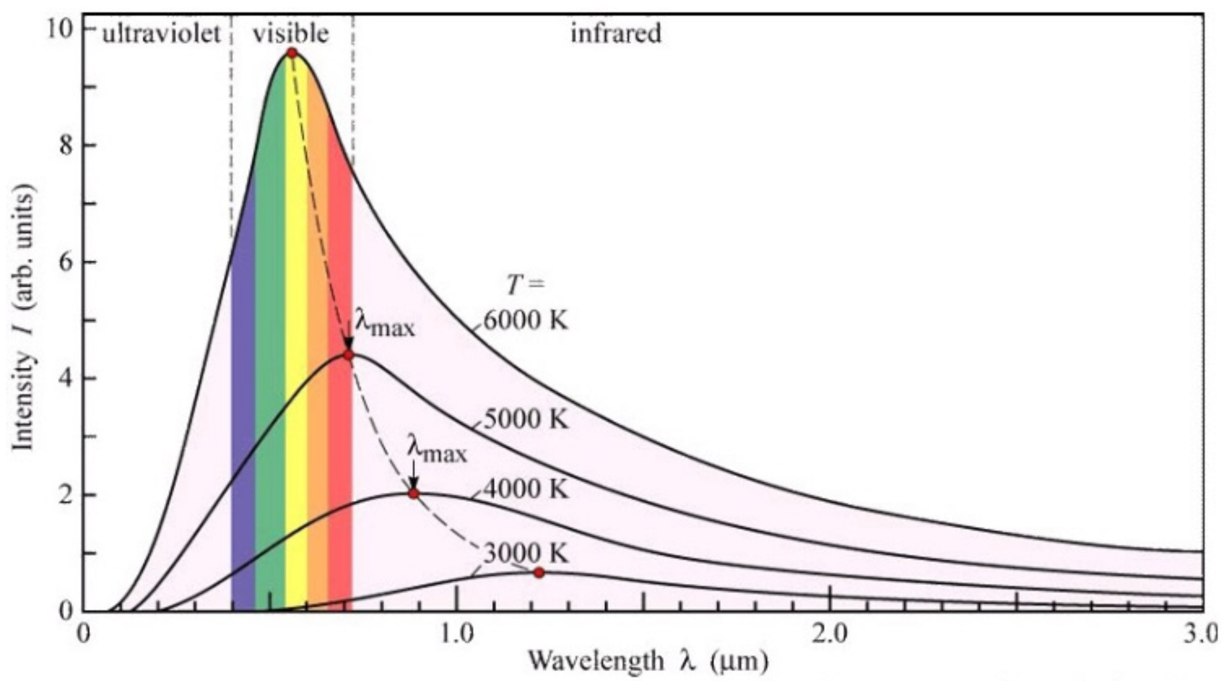
SUM → INTEGRAL RULE: $$\sum_{k} = \frac{V}{(2\pi)^3} \int d\bar{k} \qquad (3D)$$

$$= \frac{V}{(2\pi)^3} \int 4\pi k^2\,dk$$

$$= \int D(\omega)\,d\omega$$

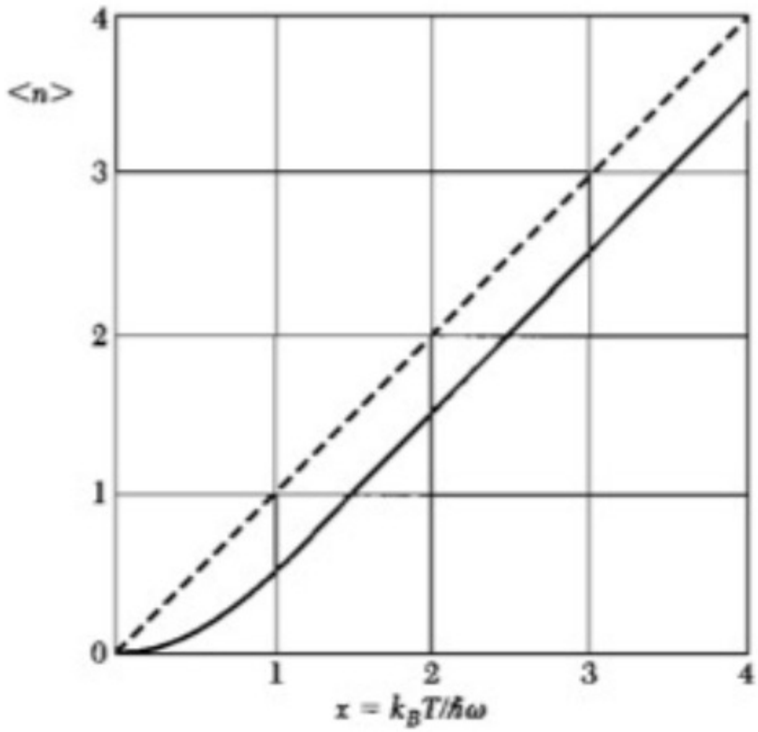
Planks Distribution:

$$P(k_BT, \hbar\omega) = \frac{1}{\exp\left(\frac{\hbar\omega}{k_BT}\right) - 1}$$



Planks Distribution:

$$P(k_BT, \hbar\omega) = \frac{1}{\exp\left(\frac{\hbar\omega}{k_BT}\right) - 1} = \frac{1}{\exp\left(\frac{1}{x}\right) - 1} = P(x)$$



SPECIFIC HEAT (HEAT CAPACITY)

$$C_V = \left(\frac{\Delta U}{\Delta T}\right)_V \rightarrow \Delta T = \frac{\Delta U}{C_V}$$

specific heat coefficient

T = temperature

U = Energy of system

$$C_V = C_{LATTICE} + C_{ELECTRONIC} + \ldots\ldots$$

Today

Coming lectures

LATTICE ENERGY

$$U_{LATTICE} = \sum_{\#\,modes} P(x)\cdot\hbar\omega_k = \sum_{\#BRANCHES}\sum_{k} P(x)\cdot\hbar\omega_k$$

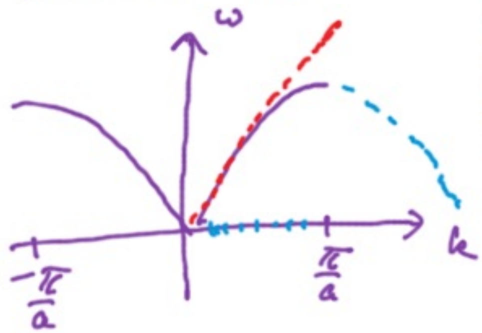
$$= \sum_{\#BRANCHES} \int D(\omega)\, P(\omega,T)\, \hbar\omega_k \, d\omega$$

$$\approx 3 \int D(\omega)\, P(\omega,T)\, \hbar\omega_k \, d\omega$$

Assume isotropic system

WE KNOW

Phonon Dispersion



$D(\omega) = \frac{V}{2\pi^2} \frac{k^2}{\frac{d\omega}{dk}}$ (3D)

DEBYE THEORY

$\omega_k = v \cdot k$

sound velocity

$D(\omega) = \frac{V}{2\pi^2}\left(\frac{\omega}{v}\right)^2 \frac{1}{v}$

$= \frac{V}{2\pi^2} \frac{\omega^2}{v^3}$

PROBLEM: $\int_0^\infty D(\omega)\, d\omega \neq N$

modes

"SOLUTION": DEBYE FREQUENCY (cut-off)

$$N = \int_0^{\omega_D} D(\omega)\, d\omega = \frac{V}{2\pi^2} \frac{1}{v^3}\left[\tfrac{1}{3}\omega^3\right]_0^{\omega_D} = \frac{V}{6\pi^2} \frac{\omega_D^3}{v^3}$$

$\Downarrow$

$\omega_D^3 = 6\pi^2 v^3 \frac{N}{V}$

DEBYE FREQ.

$k_D = \frac{\omega_D}{v} = \left(6\pi^2 v^2 \frac{N}{V}\right)^{1/3}$

DEBYE MOMENTUM

$T_D = \frac{\hbar\omega_D}{k_B} = \frac{\hbar v}{k_B}\left(\frac{6\pi^2 N}{V}\right)^{1/3}$

DEBYE TEMPERATURE

DEBYE THEORY

$$\beta \equiv \frac{\hbar\omega}{k_B T} \quad \& \quad \beta_D \equiv \frac{\hbar\omega_D}{k_B T} = \frac{T_D}{T}$$

$$U_{LATTICE}(DEBYE) = 3 \cdot \int_0^{\omega_D} D(\omega) \frac{1}{\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1} \hbar\omega \, d\omega$$

$$= 9Nk_BT \left(\frac{T}{T_D}\right)^3 \int_0^{\beta_D} \frac{\beta^3}{e^{\beta} - 1} d\beta$$

Low-Temperature limit:

$T \to 0$; $\beta_D \to \infty$

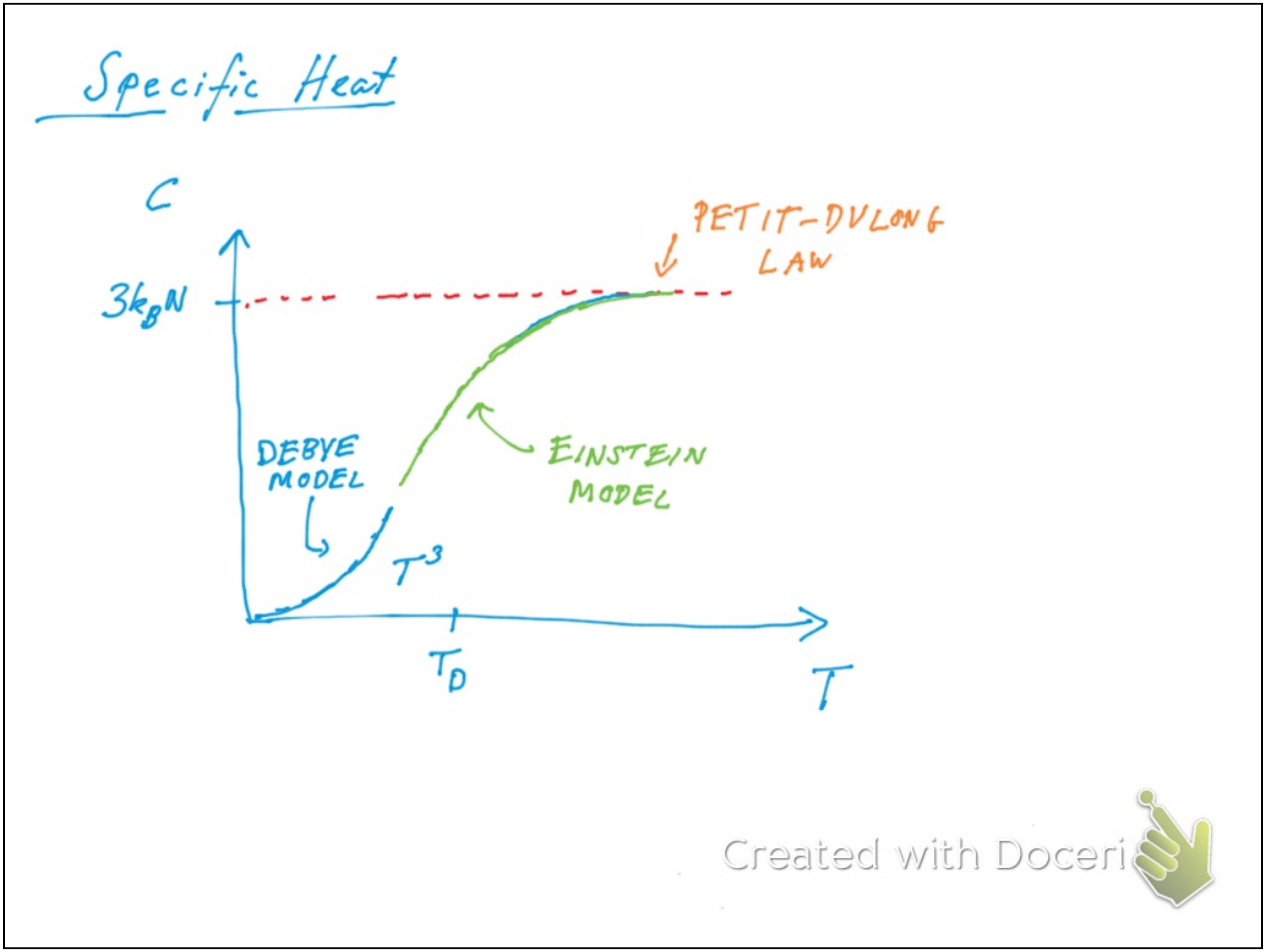
$$C_V = \frac{\partial U}{\partial T} = \frac{12\pi^4}{5} N k_B \left(\frac{T}{T_D}\right)^3$$

High-Temperature limit:

$T \gg T_D$; $\beta_D \to 0$

$$C_V = \frac{\partial U}{\partial T} = 3Nk_B$$

(Petit-Dulong law)


Specific Heat
C
$3k_BN$
PETIT-DULONG LAW
DEBYE MODEL
EINSTEIN MODEL
T^3
T_D
T
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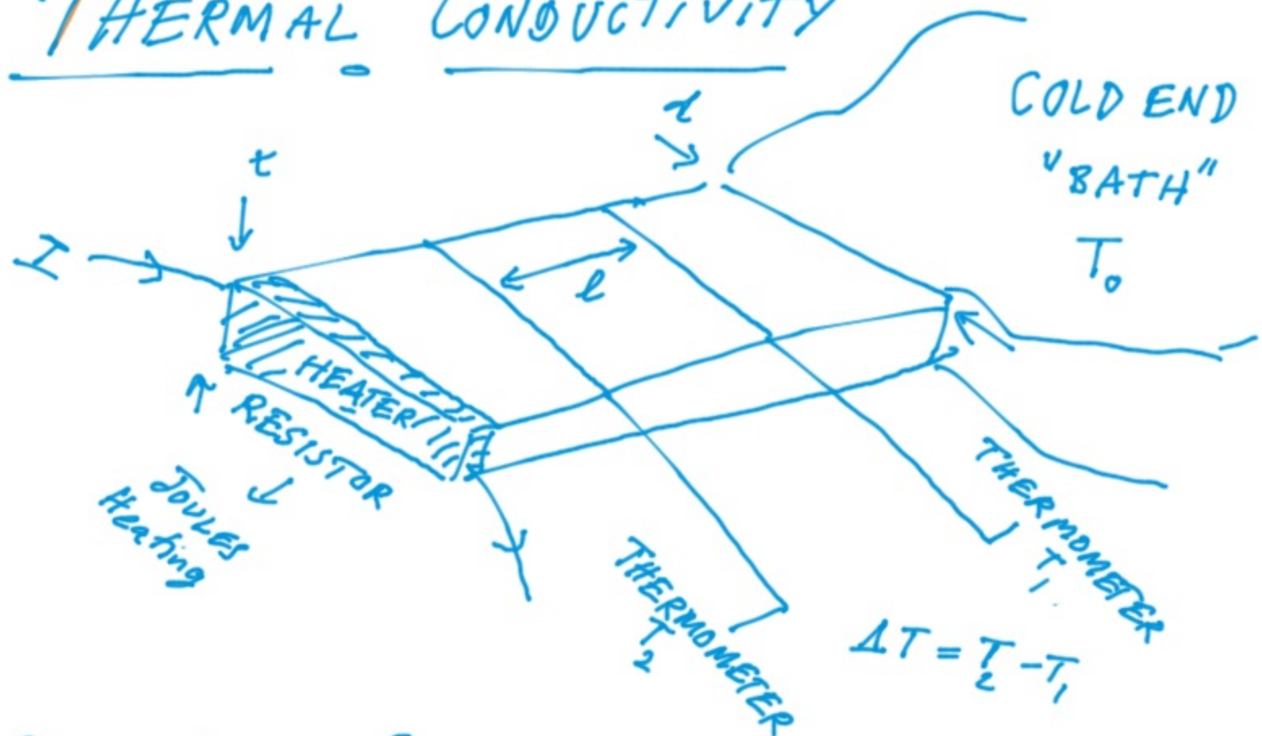
THERMAL CONDUCTIVITY



$P \equiv$ Power input $[\text{WATT} = \frac{J}{s}]$

$A =$ CROSS-SECTION $= t \cdot d$

$$\kappa = \text{thermal conductivity} = \frac{P/A}{\Delta T/\ell} = \frac{\ell}{A} \cdot \frac{P}{\Delta T}$$

$$\Downarrow$$

$$\Delta T = \frac{\ell}{A} \frac{P}{\kappa}$$

SPECIFIC HEAT & THERMAL CONDUCTIVITY

$$C_V = \left(\frac{\Delta U}{\Delta T}\right)_V \rightarrow \Delta T = \frac{\Delta U}{C_V}$$

specific heat coefficient

T = temperature

U = Energy of system

$$C_V = C_{LATTICE} + C_{ELECTRONIC} + \ldots\ldots$$

$$k = k_{LATTICE} + k_{ELECTRONIC}$$