University of Zurich UZH

Solid State Physics
Exercise Sheet 8
Specific heat of metals

HS19
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Exercise 1 *Specific heat of metals*
In metals, specific heat has contribution both from electrons and phonons.

(a) At temperature T much lower than the Debye temperature, express the ratio between phononic and electronic specific heat (C_{ph}/C_{el}) of a three-dimensional metal with Fermi temperature T_F and Debye temperature Θ_D.

(b) Calculate C_{ph}/C_{el} at $T = 1$ K and 10 K using typical values for metals: $T_\mathrm{F} = 50000$ K and $\Theta_\mathrm{D} = 300$ K. Discuss qualitatively the temperature dependence of C_{ph}/C_{el}.

Exercise 2 *Electronic specific heat in two dimensions*
Layered crystal structures often have electronic structures that can approximately be considered two-dimensional. The high-temperature superconductor $Tl_2Ba_2CuO_{6+\delta}$ is one such example.

(a) The electronic heat capacity is give by $C_\mathrm{el} = \gamma T$. Show that in two dimensions the Sommerfeld parameter γ can be written as $\gamma = \frac{A\pi k_\mathrm{B}^2}{3\hbar^2}m$ where m is the electronic mass and A is the total area. What is the unit of C_el?

(b) The crystal structure of $Tl_2Ba_2CuO_{6+\delta}$ consists of stacked layers of CuO_2. Within a layer, the CuO_2 forms a square lattice with a Cu-O lattice distance of $a = 3.8\,\text{Å}$. The electronic **specific** heat capacity is measured in units $\mathrm{J\,mol^{-1}\,K^{-1}}$. Show that $\gamma = \frac{N_\mathrm{A} a^2 \pi k_\mathrm{B}^2}{3\hbar^2}m$ where N_A is the Avogadro number.

(c) $Tl_2Ba_2CuO_{6+\delta}$ is observed to have the Sommerfeld parameter $\gamma = 6\,\mathrm{mJ\,mol^{-1}\,K^{-2}}$. Using the result of (b), what is the electronic mass m for $Tl_2Ba_2CuO_{6+\delta}$? How does it compare to the free electron mass?

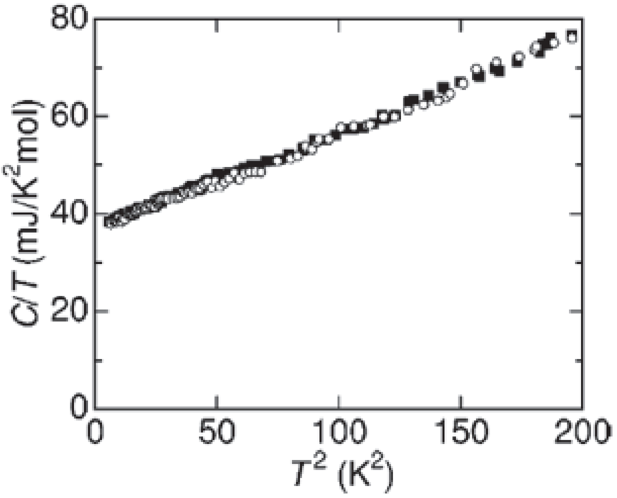

Figure 1: The total specific heat divided by temperature of Sr_2RuO_4 between T_c and 14 K in zero field (filled squares) and a magnetic field of 14 T (open circles) applied parallel to the c-axis.

Exercise 3 *Extraction of electronic and phononic specific heat*
In figure 1 the data from a specific heat experiment on Sr_2RuO_4 is shown (adapted from Mackenzie et al. JPSJ **67**, 385 (1998)).

(a) Extract the electronic Sommerfeld parameter γ and the phonon coefficient A in $C_{\mathrm{ph}} = AT^3$.

(b) The crystal structure of Sr_2RuO_4 is similar to the one of $Tl_2Ba_2CuO_{6+\delta}$. Which of the systems would have the larger electronic mass m?

(c) What is the Debye temperature for Sr_2RuO_4?

Exercise 4 *Specific heat of copper*
Copper has a density of $\rho = 8.94\,\mathrm{g\,cm^{-3}}$ and a molar mass of $m_{\mathrm{mol}} = 63.55\,\mathrm{g\,mol^{-1}}$. Use the measured values for the specific heat of copper given below to:

(a) determine the electron mass. Compare this value to the literature value for a free electron.

(b) determine the Debye temperature of copper.

$T[\mathrm{K}] =$
$0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00, 2.25, 2.50, 2.75, 3.00, 3.25, 3.50, 3.75, 4.00, 4.50$
$C_{\mathrm{V}}[\mathrm{mJ\,mol^{-1}\,K^{-1}}] =$
$0.17, 0.35, 0.54, 0.74, 0.96, 1.21, 1.47, 1.78, 2.11, 2.50, 2.91, 3.35, 3.91, 4.46, 5.15, 5.87, 7.49$