Silicon

- Most abundant element in Earth's Crust ⟶ SiO_2
- Primary Component in:
  - Semiconductor circuits (integrated circuits, computer chips, solar cells)
  - Stone, glass, concrete

Band Structure

- Crystalline solid described by 3 basis vectors $\vec{a}, \vec{b}, \vec{c}$

$$\vec{R} = m\vec{a} + n\vec{b} + p\vec{c}$$

where m, n, p are arbitrary int.

SHOW FIG 1

- Bloch's Theorem
  - For any $V(\vec{r})$ periodic in $\vec{r}$, the solutions to Schrödinger's Eq. are

$$\boxed{\psi_k(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} u_k(\vec{r})}$$

$\vec{k}$ is the wavevector
$\vec{r}$ is the position
$u_k(\vec{r})$ is periodic in $\vec{r}$

- Important remarks:
  1. Energy levels are semi-continuous
  2. Some Energy levels are forbidden
     → no sol.
     → Form BAND GAPS
  3. @ T = 300 K, $E_g = 1.12$ eV

SHOW FIG 2

- lower band is Valence band, E_v
  → valence electrons bound to parent atoms

- Upper band is Conduction band, E_c
  → Quasi-free states where electrons can move throughout the crystal

- Bands very with wavevector

SHOW FIG 3

→ Direct vs. Indirect.

Extrinsic vs. Intrinsic

- Main advantage to semicon come from ability to change electrical cond. by introducing Impurities called DOPANTS
  - → one more or less valence electrons
- Each Si atom shares 4 bonds w/ neighbors forming COVALENT BONDS

SHOW FIG 4

- P atoms have +1 ele. w/o covalent bond
  - → loosly bound, easily excited to E_c
  - → Reffered to as DONORS (N-TYPE)

- B atoms have -1 electron
  - → leaves a positively charged HOLE
    - → travels in E_v
  - → Called Acceptors (P-type)
- Si with dopants called EXTRINSIC

SPACE CHARGE

- Doponts are purposely added DEFECTS are more general
- Donors: neutral with e^-
  positive with h
- Acceptors: negative with e^-
  neutral with h

- Probability for a defect to be occupied by electron @ therm eq. given by Fermi-Dirac stats

$$f(E) = \frac{1}{1+e^{(E-E_F)/KT}}$$

$E \equiv$ energy of state
$E_F \equiv$ Fermi energy
$K \equiv$ Boltzmann's const.
$T \equiv$ Temperature in K

- For defect w/ N_t, E_t concentration of trapped electrons:

$$n_t = N_t f(E_t) = \frac{N_t}{1+e^{(E_t-E_F)/KT}}$$

trapped holes:

$$p_t = N_t(1-f(E_t)) = \frac{N_t}{1+e^{-(E_t-E_F)/KT}}$$

FIG 5

- charge state depends on 4 competing processes

1. emission of e^- to E_c

   $r_1 = e_n n_+$

2. capture of $\bar{e}$ to unoccupied energ state

   $r_2 = c_n n p_+$

3. Capture of holes into states occupied be e^- (same as $e^- \rightarrow E_v$)

   $r_3 = c_p p n_+$

4. emission of hole into E_v (Same as capture of e^- from E_v)

   $r_4 = e_p p_+$

$n, p \equiv$ concentration of free ele, holes in E_c, E_v

$e_{n,p} \equiv$ emmission rates for $\bar{e}$, h

$c_{n,p} \equiv$ Capture coeff

- $e_{n,p}$ related to $c_{n,p}$ by

$$e_{n,p} = c_{n,p}\, e^{\pm(E_{c,v}-E_t)/KT}\, N_{c,v}$$

where $N_{c,v}$ concentration of free states in $E_{c,v}$

- Define CAPTURE CROSS-SEC

$$\sigma_{n,p} = c_{n,p}/\bar{V}_{n,p}$$

where $\bar{V}_{n,p} \equiv$ thermal velocity

⇒ Lifetimes

$$\tau_{n,p} = \frac{1}{\sigma_{n,p}\bar{V}_{n,p}N_t}$$

- Electric field in device determind by Poisson's Equation

$$\boxed{\nabla^2 V = -\frac{\rho}{\varepsilon} = -\frac{q(N_{eff}+p-n)}{\varepsilon}}$$

$V \equiv$ electric potential
$q \equiv$ electron charge
$\varepsilon \equiv$ dielectric constant

$$N_{eff} = \sum_{\text{Donors}} p_t + \sum_{\text{acceptors}} n_t$$

Carrier Transport

- Under $\vec{E}$, electrons get accelerated

→ Unlike vacuum, no const acceleration but const. velocity, DRIFT VElOCITY

$$\vec{V} = \vec{E}\mu$$

$$\mu \equiv \text{mobility} = \frac{\mu_0}{\underbrace{1+bE}_{\text{leads to voltage saturation.}}}$$

→ hole movement more complicated

$$\mu_p \approx \frac{1}{3}\mu_e$$

→ @ $T = 300\,K$

$$\mu_p = 480\ cm^2/Vs$$

$$\mu_n = 1350\ cm^2/Vs$$

- ~~B~~ Besides drift, there is also diff. ~~defined~~ determined by Diff. coeff related to μ be Einstein Relation

$$D = \frac{KT}{q}\mu$$

- Current is sum of drift + Diffusion

$$\boxed{\begin{aligned}\vec{J}_n &= q\mu_n\vec{E} + qD_n\nabla n\\ \vec{J}_p &= q\mu_p\vec{E} - qD_p\nabla p\end{aligned}}$$

total current

$$\vec{J} = \vec{J}_n + \vec{J}_p$$

(neglects Hall effect)

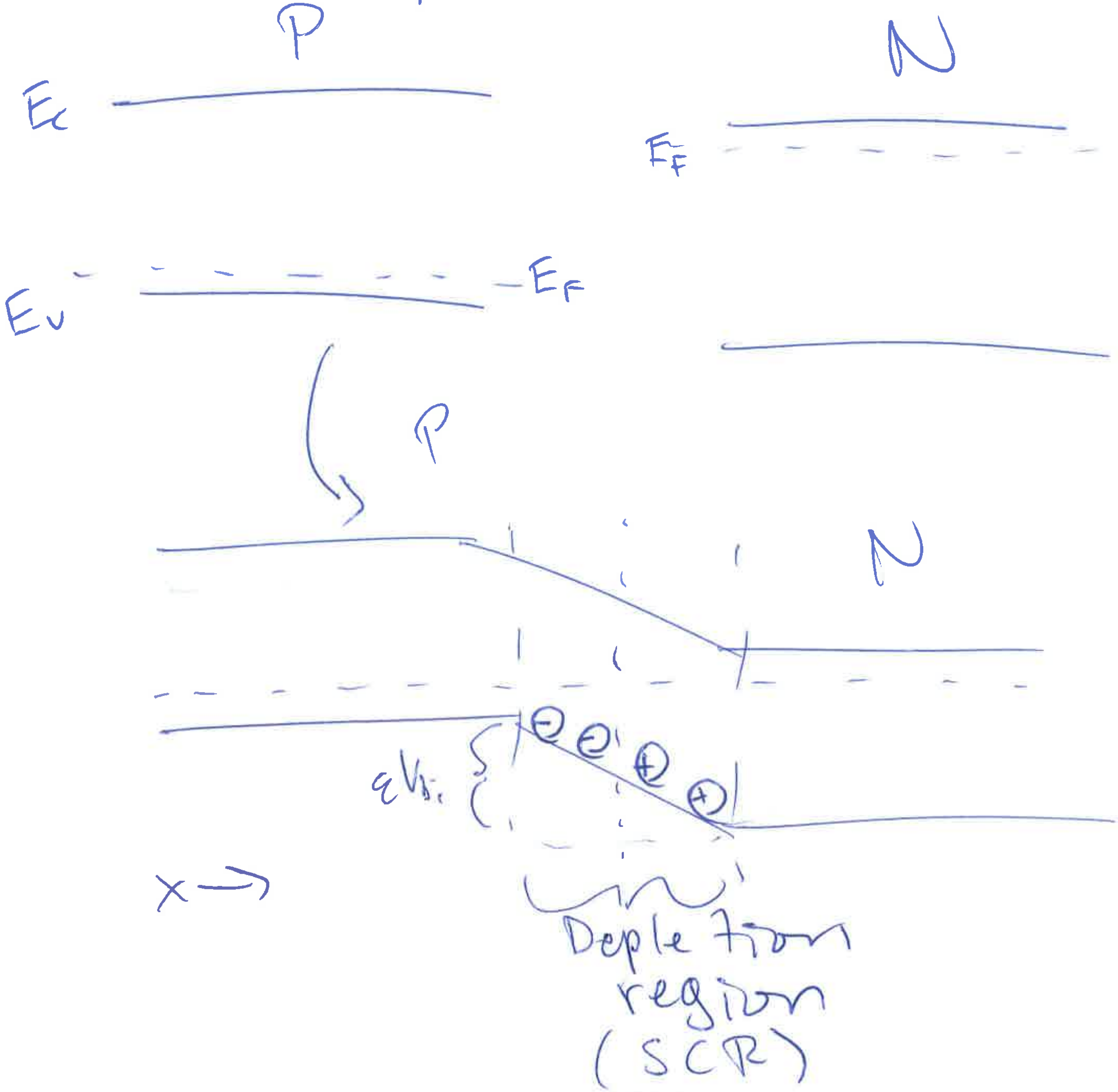
- Current must satisfy continuety equations

$$\boxed{\begin{aligned}\frac{\partial n}{\partial t} &= G_n - U_n + \frac{1}{q}\nabla\cdot\vec{J}_n\\ \frac{\partial p}{\partial t} &= G_p - U_p - \frac{1}{q}\nabla\cdot J_p\end{aligned}}$$

where G, U are generation, recombination rates

The P-N junction

- Two terminal device made by contactin p-type to n-type material



SHOW FIG 6

Current - Voltage

- Ideal case based on four assumptions

1. Abrupt depletion - layer approx; SCR is abrupt, outside this is assumed neutral
2. the Boltzmann approximation
3. Low-injection assumption: injected minority-carrier densities are small compared w/ majority-carrier densities.
4. No generation/recombination in depletion layer
   → electron + hole currents const. in dep. layer.

ⓦ Therm. equilibrium Boltzmann relation is

$$n = n_i \exp\left(\frac{E_F - E_i}{kT}\right)$$

$$p = n_i \exp\left(\frac{E_i - E_F}{kT}\right)$$

$n_i \equiv$ intrinsic concentr.

in therm eq. $n_i^2 = pn$

- When V is applied, minority-carrier densities change on both sides of the juction $\Rightarrow pn \neq n_i^2$

Now define quasi-Fermi (imref) levels as

$$n \equiv n_i \exp\left(\frac{E_{Fn} - E_i}{KT}\right)$$

$$p \equiv n_i \exp\left(\frac{E_i - E_{Fp}}{KT}\right)$$

or

$$E_{FN} = E_i + KT \ln\left(\frac{n}{n_i}\right)$$

$$E_{Fp} = E_i - KT \ln\left(\frac{p}{n_i}\right)$$

$$\Rightarrow pn = n_i^2 \exp\left(\frac{E_{Fn} - E_{Fp}}{KT}\right)$$

using the fact that

$\vec{E} = \nabla E_i / q$ [illegible] , $E_i = E_g/2$

$\vec{J}_n = q\mu_n \vec{E} n + q D_n \nabla n$

$= q\mu_n \left(n\vec{E} + \frac{KT}{q} \nabla n \right)$

$= \mu_n n \nabla E_i + \mu_n KT \left[\frac{n}{KT} (\nabla E_{Fn} - \nabla E_i) \right]$

$= \mu_n n \nabla E_{Fn}$

Similarly,

$\vec{J}_p = \mu_p p \nabla E_{Fp}$

⇒ current densities prop. to. (imref)

@ therm equi[illegible]

$E_{Fn} = E_{Fp} \simeq$ const.

$\Rightarrow J_n = J_p = J = 0$

SHOW FIG 7

- E_{Fn}, E_{Fp} remain ≈ const. in depletion layer because while free carrier consentrations are relatively large, current is fairly const ⇒ gradients of imref have to be small
- Depletion width much shorter than diff. length ⇒ drop in imref not significant in dep. reg.

$\Rightarrow qV = E_{Fn} - E_{Fp}$

electron density @ dep-layer edge on p-side ($x = -W_{Dp}$)

$$n_p(-W_{Dp}) = \frac{n_i^2}{p_p}\exp\left(\frac{qV}{KT}\right) \approx n_{po}\exp\left(\frac{qV}{KT}\right)$$

where $p_p \approx p_{po}$ for low-level injection and n_{po} is equilibrium ele. density on p-side.

Similarly

$$p_n(W_{Dn}) \approx p_{no} \exp\left(\frac{qV}{KT}\right)$$

at $x = W_{Dn}$ for n-type boundry

- From cont. eq. in n-side

$$-U + \mu_n \vec{E} \frac{dn_n}{dx} + \mu_n n \frac{d\vec{E}}{dx} + D_n \frac{d^2 n_n}{dx^2} = 0$$

$$-U - \mu_p \vec{E} \frac{dp_n}{dx} - \mu_p p_n \frac{d\vec{E}}{dx} + D_p \frac{d^2 p}{dx^2} = 0$$

where U is net recomb. rate

Multiplying first by $\mu_p p_n$, Second by $\mu_n n_n$ and w/ $D = (KT/q)\mu$

$\Rightarrow$

$$-\frac{p_n - p_{no}}{\tau_p} - \frac{n_n - p_n}{n_n/\mu_p + p_n/\mu_n}\,\frac{\vec{E}\,dp_n}{dx} + D_a\frac{d^2 p_n}{dx^2} = 0$$

where

$$D_a = \frac{n_n + p_n}{n_n/D_p + p_n/D_n}$$

$\equiv$ ambipolar diff. coeff.

and

$$\tau_p \equiv \frac{p_n - p_{no}}{U}$$

From low-injection assump.
[e.g. $p_n \ll (n_n \approx n_{no})$ in n-type]
we get

$$-\frac{p_n - p_{no}}{\tau_p} - \mu_p E\frac{dp_n}{dx} + D_p\frac{d^2 p_n}{dx^2} = 0$$

in neutral region where $\vec{E} = 0$
this reduces to

$$\frac{d^2 p_n}{dx^2} - \frac{p_n - p_{no}}{D_p \tau_p} = 0$$

with solution

$$p_n(x) - p_{no} = p_{no}\left[\exp\left(\frac{qV}{KT}\right)-1\right]\exp\left(-\frac{x-W_{Dn}}{L_p}\right)$$

where $L_p \equiv \sqrt{D_p \tau_p}$

@ $x = W_{Dn}$, hole current is

$$J_p = -qD_p\frac{dp_n}{dx}\Big|_{W_{Dn}} = \frac{qD_p p_{no}}{L_p}\left[\exp\left(\frac{aV}{KT}\right)-1\right]$$

Similarly electron current in p side

$$J_n = qD_n\frac{dn_p}{dx}\Big|_{-W_{Dp}} = q\frac{D_n n_{po}}{L_p}\left[\exp\left(\frac{aV}{KT}\right)-1\right]$$

$$\boxed{J = J_p + J_n = J_0\left[\exp\left(\frac{qV}{KT}\right)-1\right]}$$

Shockley Eq.

$$J_0 = \frac{qD_p p_{no}}{L_p} + \frac{qD_n n_{po}}{L_n} \equiv \frac{qD_p n_i^2}{L_p N_D} + \frac{qD_n n_i^2}{L_n N_A}$$

SHOW FIG 8

- In rev. bias, depletion region increases with V
- Size of this region, W, determined by solving Poisson's equation.

- For simplicity assume abrupt 1D junction @ $x=0$, and consider only one side.
- Also with low injection $p \approx n \approx 0$

$V_{bias} \equiv$ applied voltage

$V = V(x)$

$$\frac{d^2V}{dx^2} = \frac{qN_{eff}}{\varepsilon}$$

integrating and w/

$$\left.\frac{dV}{dx}\right|_{x=W} = -E(W) = 0, \quad V(W) = V_{bias}$$

$$W = \left(\frac{2\,\varepsilon}{q\,N_{eff}} V_{bias} \right)^{1/2}$$

Silicon Detectors

(21)

- essential to HEP exp.
- Provides primary means of momentum determination of charged particles
- Can resolve primary and secondary vertices to μm level

Basic Design

SHOW FIG 9

- Complicated diode struct.
  - → segmented p-n junc. in parallel
  - → For SSD, several n^+ (or p^+) strips on a p-type (or n-type) substrate
- Reverse bias leads to dep region and $\vec{E}$ field
  - → e/h generated by traversing particles
  - → Defines active region of device.

- layer of SiO_2 couples signal to Al readout strips and to readout electronics
- Detectors biased from backplane
  → strips held to grd bias ring via polysilicon resistor
- Guard rings surround bias ring to slowly drop voltage between back and surface
- Passivated w/ SiO_2 to protect from environ. effects
  → Small opening for elec. contacts.

Signal Formation

- Turn energy deposited by charged particles into electrical signals
  - → For Si, takes form of creation of e/h pairs from ionizing rad, which move toward readout under influence of $\vec{E}$ field in active volume
- # of e/h pairs created depends on Energy of incoming particle
- Avg energy loss determined from Bethe-Bloch formula

$$-\frac{dE}{dx} = 4\pi N_A r_e^2 m_e c^2 z^2 \frac{Z}{A}\frac{1}{\beta}\left[\frac{1}{2}\ln\left(\frac{2m_e c^2\beta^2\gamma^2 T_{max}}{I^2}\right) - \beta^2 - \frac{\delta(\gamma)}{2}\right]$$

SHOW FIG 10

$z \equiv$ charge of inc. particle
$T_{max} \equiv$ max KE that can be imparted to electron in collision
$I \equiv$ Avg excitation energy
$Z \equiv$ atomic number
$A \equiv$ Atomic mass
$N_A \equiv$ Avogradro's number
$m_e \equiv$ electron mass
$c \equiv$ speed of light
$r_e \equiv$ classical electron radius
$\beta \equiv v/c$
$\gamma = 1/\sqrt{1-\beta^2}$
$\delta \equiv$ density effect correction

- Particles with energy at minimum are termed Minimum Ionizing Particles (MIP)
- Avg energy loss 390 eV/um in Si with e/h pair needing 3.6 eV
  ⇒ 108 e/h pairs for each um
  ~~⇒ 76 e/h most mpv~~

- MPV of deposited charge is less than average due to statistical fluct.
  → described by ~~Landau~~distribution
  Laundau distribution (+gaus for noises)
- For Si, MPV ≈ 76 e/h per um

SHOW FIG 11

Leakage Current

- One of main design consideratons
- Represents a source of noise in final signal
  → Keep as low as possible
- Bulk current
  → generated in bulk (shockley eq)
- Surface current
  → From imperfection (scratches
  processing
  poor oxides
- Thermal Runaway → Cool!

Capacitance

- Many sources
  → Some beneficial
  Some detrimental
- Coupling Cap: larger means better coupling of signal
- InterStrip Cap: · Noise between strips
  · Charge Sharing
- Bulk Cap: · Between strips and backplane
  · Also add to noise

For 1D case, $C = \varepsilon A / w$

→ can measure dep voltage ~~and~~

$$\rightarrow \frac{1}{C^2} = \frac{2 V_{bias}}{q \varepsilon A N_{eff}}$$

→ Freq dep.

Radiation Damage

- At LHC large particle fluxes lead to large amount of radiation damage
  - → Bulk → main limiting factor
  - → Surface

- Bulk
  - Damage from displacement of atoms from lattice sites through non-ionizing energy loss (NIEL)
  - Leave behind energy states categorized as
    - interstitials (I)
    - vacancies (V)

SHOW FIG 12

- Higher energy coll.
  → recoil → further damage

- Areas w/ large # of defects called CLUSTERS
- Individual displacement POINT DEFECTS

SHOW FIG 13

- Energy states from rad. induced defects change electrical properties of device
  - larger leakage current by E_t near midgap
  - change in depletion or operating voltage due to changes in N_{eff}
  - Loss of signal/efficiency due to trapping by defect states

- NIEL hypothesis allows normalization of the displacement damage due to diff. radiation particle species w/ diff energy

$$D(E) = \sum_i \sigma_i(E_{kin}) \int_0^{E_{R,max}} f_i(E_{kin}, E_R)\, P(E_R)\, dE_R$$

where the Σ is over all possible interactions

$\sigma_i \equiv$ cross-section for process i

$f_i(E, E_R) \equiv$ prob. of a particle w/ energy E_{kin} transfering recoil energy E_R

$P(E_R) \equiv$ Lindhard partition function describing fraction of energy displacing a silicon atom

SHOW FIG 14

- Standard 1 MeV neutron equivalent fluence, or n_{eq}/cm^2 (n_{eq})

→ Scaling K to diff part spc. w/ diff. energy

$$K = \frac{\int D(E)\Phi(E)\,dE}{95\ \text{MeV mb} \cdot \Phi} = \frac{\Phi_{eq}}{\Phi}$$