Wilson Loop approach to Topological Photonic Crystals

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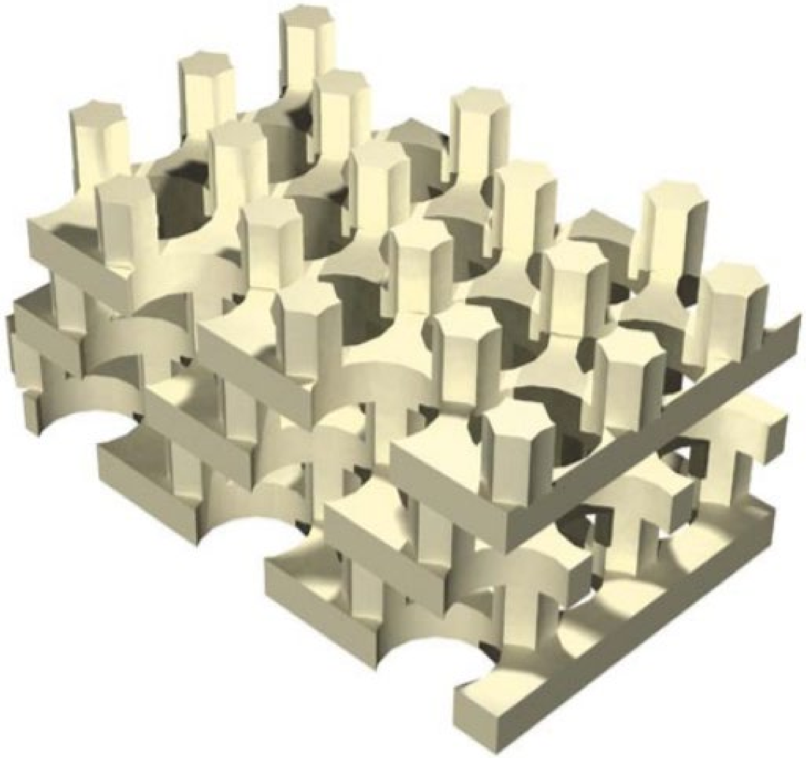
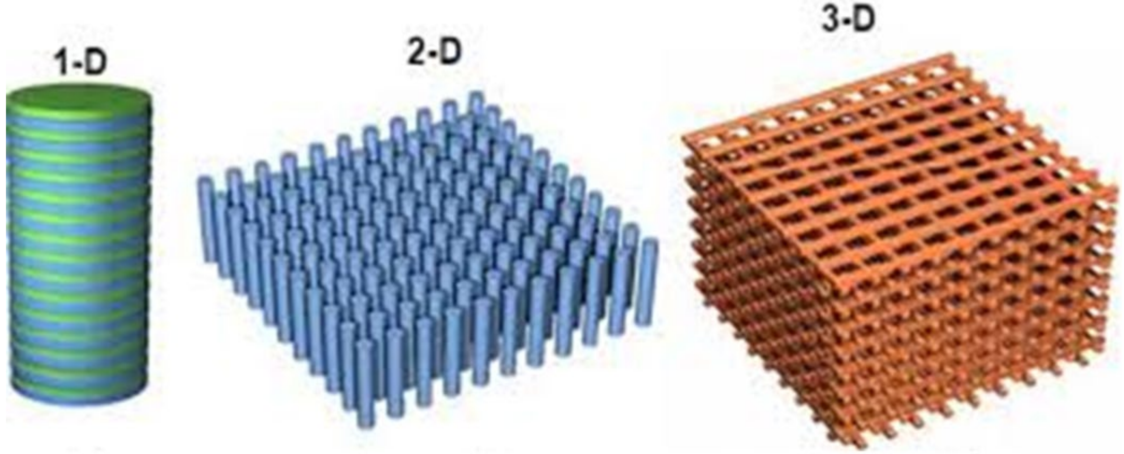
What is a Photonic Crystal?





Assumption 1 (Material weights)

$$W(\boldsymbol{r}) = \begin{pmatrix} \varepsilon(\boldsymbol{r}) & \chi(\boldsymbol{r}) \\ \chi^{\dagger}(\boldsymbol{r}) & \mu(\boldsymbol{r}) \end{pmatrix}$$

- Eigenvalues $(\widehat{W}) > 0$ (dielectric)
- $W = W^{\dagger}$ (lossless medium)
- $\widehat{W}(\omega) \sim \widehat{W}(\omega_0)$ (istantaneous response)
- $\widehat{W}$ periodic on a lattice $\Gamma \sim \mathbb{Z}^3$

A photonic crystal is to light what a crystalline solid is to an electron

Maxwell equations in the Schrodinger formalism

Complexification of the classical system

$$\overline{\boldsymbol{E}}, \overline{\boldsymbol{H}} = 2Re(\boldsymbol{E}, \boldsymbol{H})$$

- Systematic link to a complex Hilbert space is essential if one wants to apply methods from quantum mechanics

$$\begin{pmatrix} \boldsymbol{E}(\boldsymbol{r}, t) \\ \boldsymbol{H}\ (\boldsymbol{r}, t) \end{pmatrix} \in L^2_W(\mathbb{R}^3, \mathbb{C}^6)$$

- Real fields can be uniquely represented by complex waves composed solely of non-negative frequencies (taking the real part as the physical field):

$$\begin{pmatrix} \boldsymbol{E}(\boldsymbol{r}, t) \\ \boldsymbol{H}\ (\boldsymbol{r}, t) \end{pmatrix} = e^{-i\omega t} \begin{pmatrix} \boldsymbol{E}(\boldsymbol{r}, t) \\ \boldsymbol{H}\ (\boldsymbol{r}, t) \end{pmatrix}$$

$$\omega > 0$$

- $\omega > 0$ and $\omega < 0$ contributions are not independent degrees of freedom and one can be reconstructed from the other

QM approach

- Dynamical field Maxwell equations:

$$i\partial_t \begin{pmatrix} \boldsymbol{E}(\boldsymbol{r}, t) \\ \boldsymbol{H}\ (r, t) \end{pmatrix} = \widehat{M}(\boldsymbol{r}) \begin{pmatrix} \boldsymbol{E}(\boldsymbol{r}, t) \\ \boldsymbol{H}\ (\boldsymbol{r}, t) \end{pmatrix}$$

- Maxwell operator M playes the role of the Hamiltonian of the classical system:

$$\widehat{M} = W(\boldsymbol{r})^{-1} \begin{pmatrix} 0 & +i\boldsymbol{\nabla}^{\times} \\ -i\boldsymbol{\nabla}^{\times} & 0 \end{pmatrix}$$

- Absence of sources:

$$\begin{pmatrix} \nabla \cdot & 0 \\ 0 & \nabla \cdot \end{pmatrix} \left(W(\boldsymbol{r}) \begin{pmatrix} \boldsymbol{E}(\boldsymbol{r}, t) \\ \boldsymbol{H}\ (\boldsymbol{r}, t) \end{pmatrix} \right) = 0$$

Some observations

- Conformal invariance of the Hamiltonian:

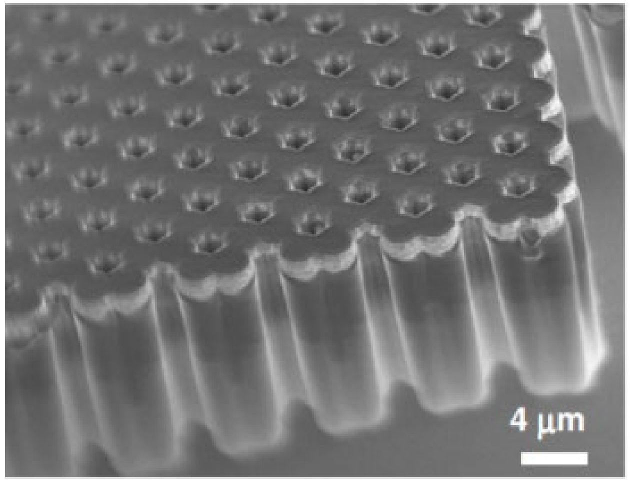
$$W(\boldsymbol{r}) \rightarrow W(\boldsymbol{r}/s)$$

$$\begin{pmatrix} \boldsymbol{E}(\boldsymbol{r}) \\ \boldsymbol{H}\,(\boldsymbol{r}) \end{pmatrix} \rightarrow \begin{pmatrix} \boldsymbol{E}(\boldsymbol{r}/s) \\ \boldsymbol{H}\,(\boldsymbol{r}/s) \end{pmatrix} \text{ and } \omega \rightarrow \omega/s$$

frequencies scalable with the lattice parameters

$a \sim 4\mu m$
$\upsilon \sim 100\, THz$
(MIR)

Challenge at optical frequencies and nanoscale



Klembt, Nature volume 562, (2018)

- Weighted energy scalar product:

$$\left\| \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} \right\|_W^2 = \left\langle \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} \middle| \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix} \right\rangle_W \equiv \int d^3\boldsymbol{r} \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}^\dagger W \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}$$

$$\langle U|V\rangle_W \equiv \int d^3\boldsymbol{r}\, U^\dagger \begin{pmatrix} \varepsilon & \chi \\ \chi^\dagger & \mu \end{pmatrix} V$$

vector field in the presence of a metric

Photonic band picture and Bloch light waves

- Master equation:

$$\hat{M}(\boldsymbol{k})U_{n,\boldsymbol{k}} = \omega(\boldsymbol{k})_n U_{n,\boldsymbol{k}} \qquad \omega(k)_n > 0$$

$$\hat{M}(\boldsymbol{k}) = W(\boldsymbol{r})^{-1}\begin{pmatrix} 0 & (-i\boldsymbol{\nabla}+\boldsymbol{k})^{\times} \\ (+i\boldsymbol{\nabla}-\boldsymbol{k})^{\times} & 0 \end{pmatrix}$$

Bloch wavefunction

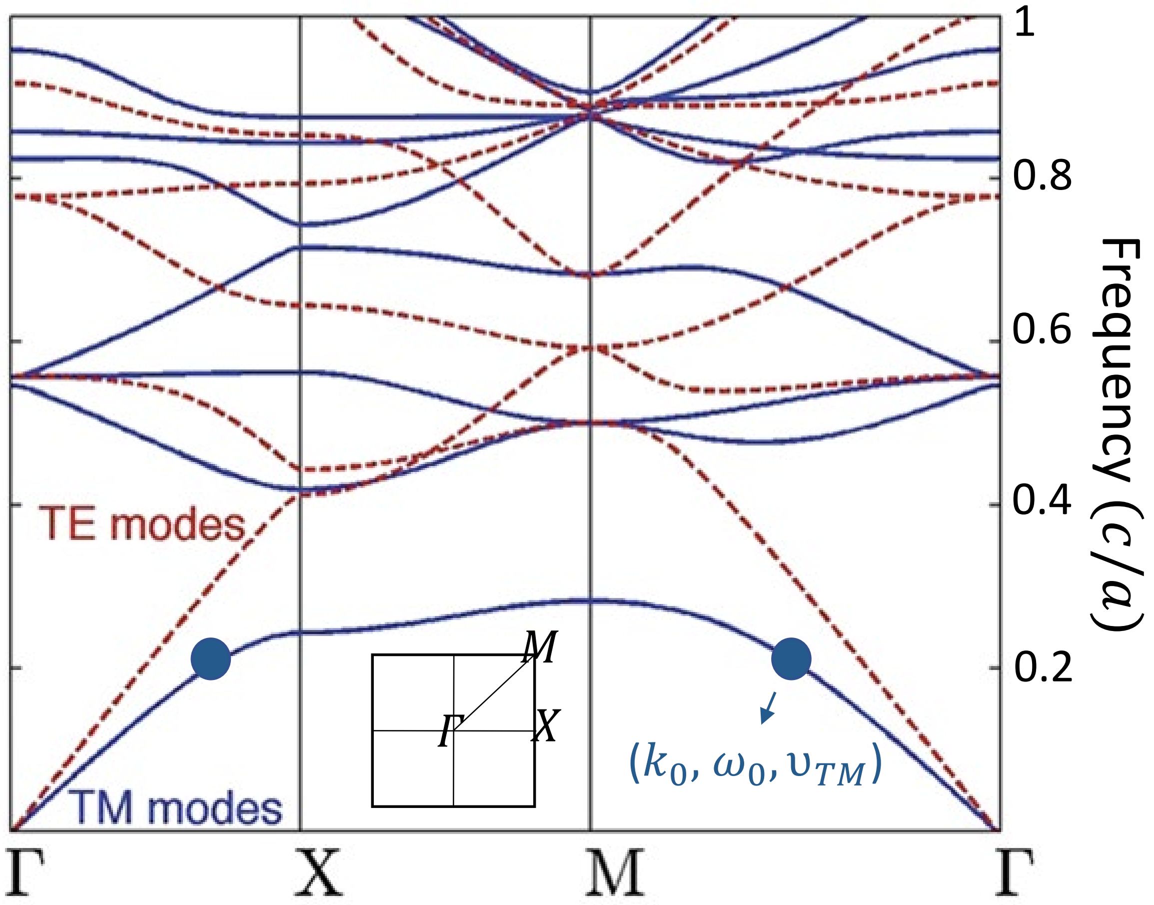
$$\Psi_{n,\boldsymbol{k}} = U_{n,\boldsymbol{k}} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}$$

Periodic part

$$U_{n,\boldsymbol{k}} = \begin{pmatrix} \boldsymbol{E}_{n,\boldsymbol{k}}(\boldsymbol{r}) \\ \boldsymbol{H}_{n,\boldsymbol{k}}(\boldsymbol{r}) \end{pmatrix}$$

- Transversality condition

$$\begin{pmatrix} (i\boldsymbol{k}+\boldsymbol{\nabla})\cdot & 0 \\ 0 & (i\boldsymbol{k}+\boldsymbol{\nabla})\cdot \end{pmatrix}\left(W(\boldsymbol{r})U_{n,\boldsymbol{k}}(\boldsymbol{r})\right) = 0$$



Electrons are conserved charged interacting fermions

Photons are neutral non-interacting non-conserved bosons →

(i) 2 Golstone linear dispersing modes at $\omega(k=0)=0$ for each υ_{pol}

(ii) Absence of Fermi level: multidirectional excitation by antennas or single mode $(k_0, \omega_0, \upsilon_{pol})$ excited by laser

Classification of Topological Photonic Crystals

Internal (local) symmetries:

$U_n = (\sigma_2 \otimes \mathbb{1}_{3\times3})\ n = 1,2,3$ unitary

$T_n = (\sigma_n \otimes \mathbb{1}_{3\times3})K\ n = 0,1,2,3$ anti-unitary

> "Suppose material weigths W satisfies Assumption 1, then all **admissible symmetries** of the Maxwell operator $\widehat{M}$, must **commute** with W"
> **De Nittis**, Ann. Phys. 396, 579-617 (2018)

- Absence of charge conjugation C operator (particle-hole)
  - Even $(T^2 = 0,1)$ time reversal symmetry

    $T\,(\sigma_1 \otimes \mathbb{1}_{3\times3})K\ , T_3 = (\sigma_3 \otimes \mathbb{1}_{3\times3})\ K$

    T_1, T_3 in T class (i.e. commuting and anti-unitary)
  - EM duality (out of the AZ scheme)

    $U_2 = (\sigma_2 \otimes \mathbb{1}_{3\times3})$

    $(\boldsymbol{E}, \boldsymbol{H}) \rightarrow i(-\boldsymbol{H}, \boldsymbol{E})$

    Topological protection?

"Ten-fold classification (AZ)"

Altland and **Zirnbauer**, Phys. Rev. B 55, 1142 (1997)

Symmetry				Dimension							
AZ	T	C	S	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	-1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

$\mathbb{Z}$

- Two-fold *vs* Ten-fold way (A,AI)
  - Chern phase allowed in 3D
- Absence of $\mathbb{Z}_2$ phases (pseudo-spin to be built out of e.g. Nonlocal crystalline symmetries, EM duality...)

Photonic bulk-edge correspondence

- Observables in the bulk and at the boundary approximately given in terms of topological quantities:

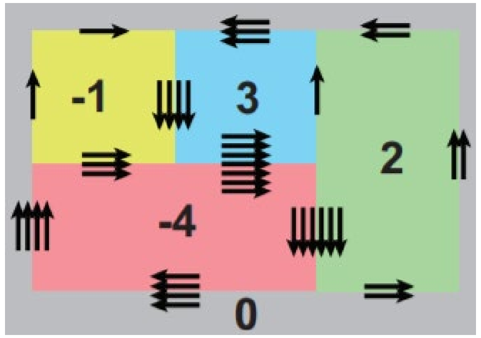
$$O_{bulk/edge} \approx T_{bulk/edge}$$

- Topological quantities necessarily agree with one another:

$$T_{bulk} = T_{edge}$$

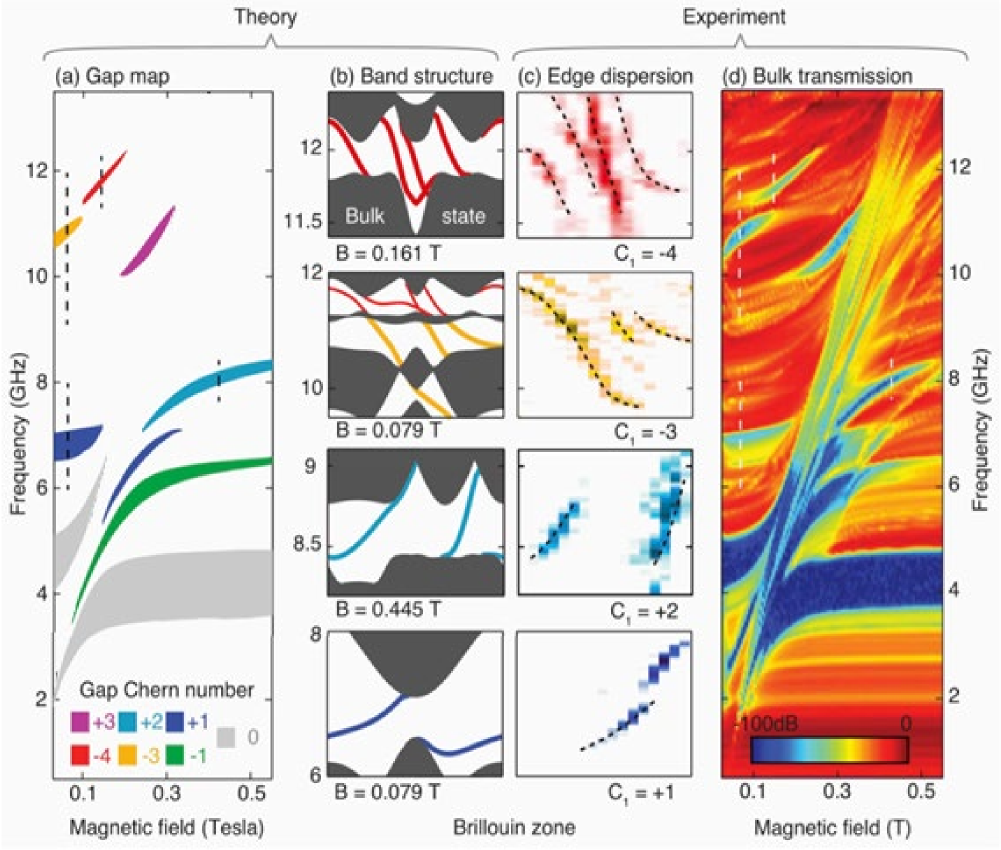
- Observables: $Re(f(\boldsymbol{E},\boldsymbol{H}))$, *e.g.* Pointing $S = Re(\boldsymbol{E} \times \boldsymbol{H})$
- Topologically protected one-way propagation at the boundary between crystals (l, r) with different gap Chern numbers:

"signed ♯ edge modes"=$\Delta C_{gap_{l,r}}$



- Gap Chern number, sum of Chern numbers of all the bands below the bandgap ($C_{gap} = \sum_{i\ below} C_i$), fixes the number of chiral edge modes

Skirlo, Phys. Rev. Lett. **115**, 253901, (2015)
Hafezi, Phys. Rev. Lett. **112**, 210405 (2014)



- Edge dispersion: Fourier transform of the edge-mode profiles
- Bandgap sign: directional dependent transmission between exciting and receving antenna

Detecting topology: Wilson loop approach

- Non-Abelian Berry curvature:

$$m, n = 1, \dots, N$$

relevant occupied subset of bands

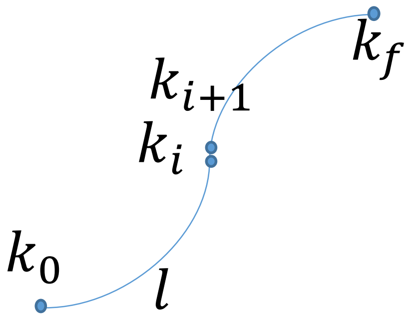
$$\boldsymbol{A}_{m,n,\boldsymbol{k}} = \langle U_{m,\boldsymbol{k}} | \boldsymbol{\nabla}_{\boldsymbol{k}} | U_{n,\boldsymbol{k}} \rangle^{*}_{W}$$

where $U_{n,\boldsymbol{k}} = \begin{pmatrix} \boldsymbol{E}_{n,\boldsymbol{k}}(\boldsymbol{r}) \\ \boldsymbol{H}_{n,\boldsymbol{k}}(\boldsymbol{r}) \end{pmatrix}$

$$\langle U_{n,\boldsymbol{k}} | V_{n,\boldsymbol{k}} \rangle_W \equiv \int_{unit\ cell} d^3\boldsymbol{r}\, U^{\dagger}_{n,k} \begin{pmatrix} \varepsilon & \chi \\ \chi^{\dagger} & \mu \end{pmatrix} V_{n,\boldsymbol{k}}$$

- Path ordered Wilson line:

$$M_{m,n} = \overline{exp}\left(-\int_{l} \boldsymbol{A}_{m,n,\boldsymbol{k}} \cdot d\boldsymbol{k}\right)$$



- Numerical implementation by overlap matrices:

$$M^{\boldsymbol{k}_i,\boldsymbol{k}_{i+1}}_{m,n} = \langle U_{m,\boldsymbol{k}_i} | U_{n,\boldsymbol{k}_{i+1}} \rangle_W$$

$$M_{m,n} = \overline{\prod_{k_i \epsilon l}} M^{\boldsymbol{k}_i,\boldsymbol{k}_{i+1}}_{m,n}$$

*missing i convention

Equivalence exact in the limit $d\boldsymbol{k} \to 0$

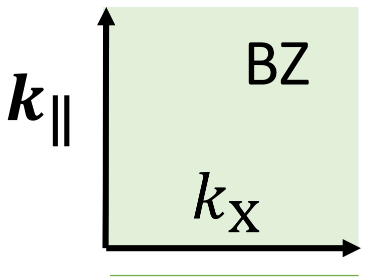
$$\langle U_{m,\boldsymbol{k}} | U_{n,\boldsymbol{k}+d\boldsymbol{k}} \rangle_W = \langle U_{m,\boldsymbol{k}} | \left(| U_{n,\boldsymbol{k}} \rangle + d\boldsymbol{k} \cdot \nabla_{\boldsymbol{k}} | U_{n,\boldsymbol{k}} \rangle \right) + O((d\boldsymbol{k})^2)$$

$$= \delta_{m,n} - \langle U_{m,\boldsymbol{k}} | \boldsymbol{\nabla}_{\boldsymbol{k}} | U_{n,\boldsymbol{k}} \rangle_W + O((d\boldsymbol{k})^2) \cong \exp((-\boldsymbol{A}_{m,n,\boldsymbol{k}} \cdot d\boldsymbol{k})$$

Hybrid energy centers

- Choice of a closed path l in the BZ (Berry phase) or across BZ (Zak phase) to ensure gauge invariance of the Wilson loop M spectrum:

$$\boldsymbol{k} = (k_{\mathrm{X}}, \boldsymbol{k}_{\parallel}) \in T^d$$



- Spectral equivalence with the projected position operator:

$$P(\boldsymbol{k}_{\parallel}) = \sum_n \int_{-\pi}^{\pi} \frac{dk_x}{2\pi} \, |U_{n,\boldsymbol{k}}\rangle W \langle U_{n,\boldsymbol{k}}|$$

$$-Im \log\big(M(\boldsymbol{k}_{\parallel})\big)/2\pi \quad \leftrightarrow P(\boldsymbol{k}_{\parallel}) x P(\boldsymbol{k}_{\parallel})$$

WL matrix eigenvalues

Transverse position operator projected over the occupied subset

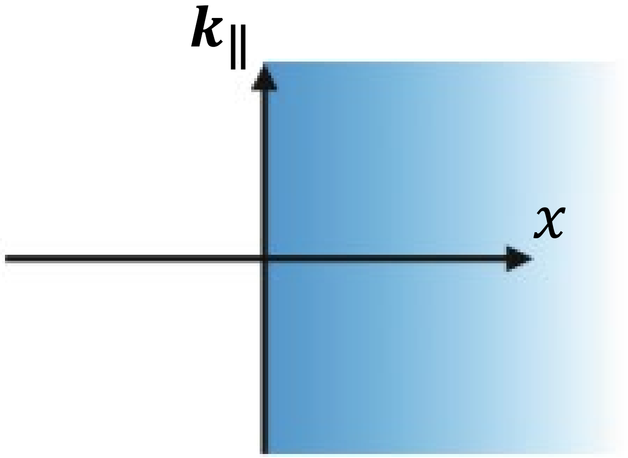
Eigenvalues:

$$\theta_n(\boldsymbol{k}_{\parallel})/2\pi$$
$$n = 1, .., N$$

Eigenvalues:

$$\frac{\theta_n(\boldsymbol{k}_{\parallel})}{2\pi} + z$$
$$n = 1, .., N, z \in \mathbb{Z}$$

e.g. Zak θ_n phase in 1D

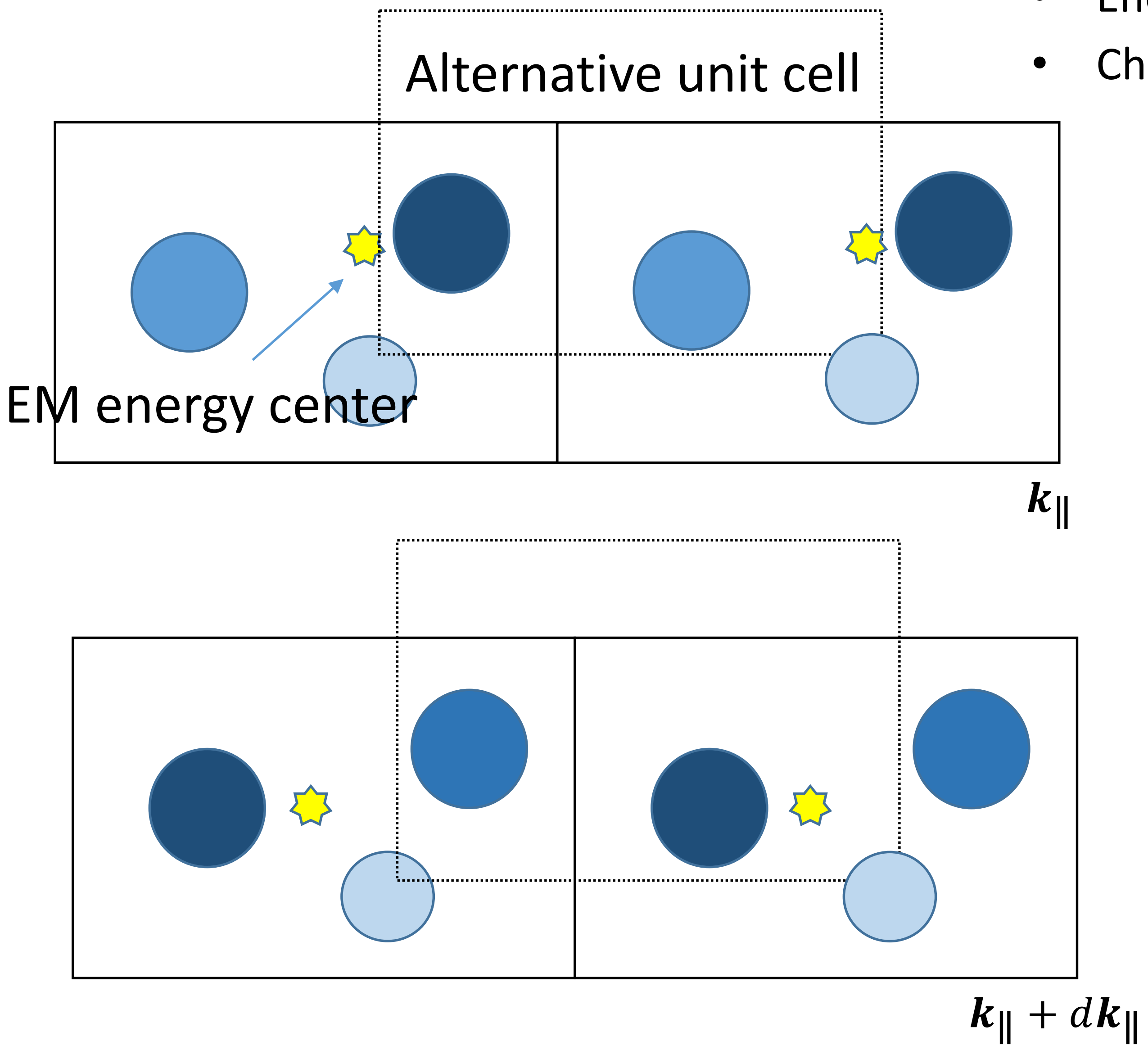


interpretation in terms of “energy centers” localized in the x-direction and plane waves perpendicular to it

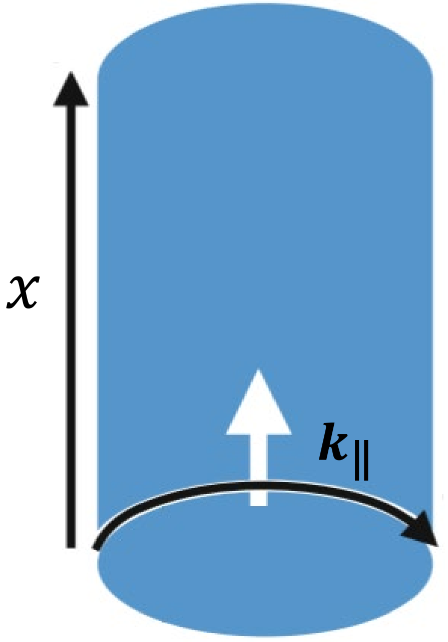
- Proof based on the covariant representation of $x^a \equiv \frac{\partial}{\partial k_a}$

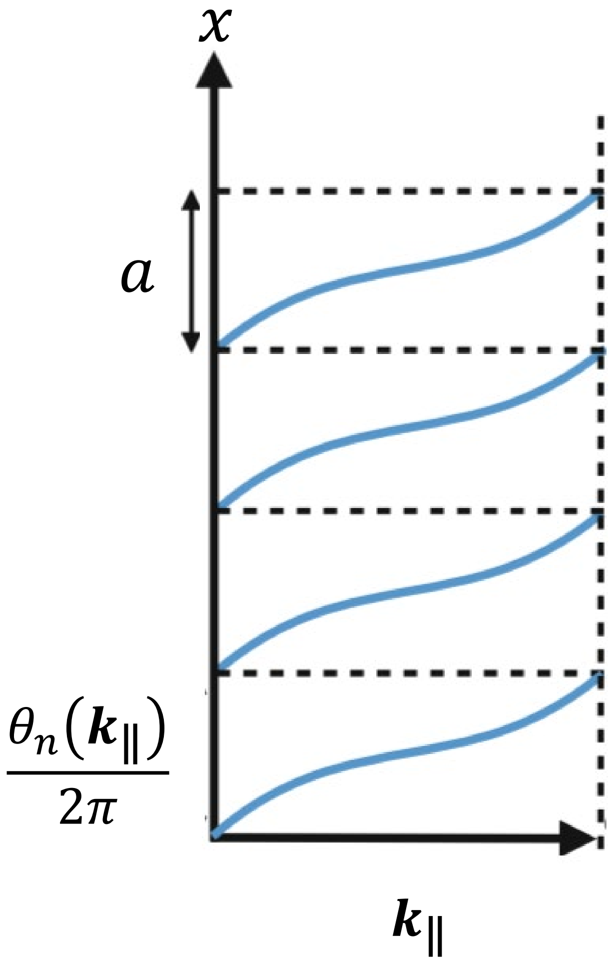
Spectral flow of energy centers

- Energy centers $\theta_n(\boldsymbol{k}_\parallel)$ depend on the choice of the unit cell
- Change in the energy centers is unambiguos



Transverse flow interpretation in $d \geq 2$





Example of WL applications for topology detection

1. Zak phase in a **1D dielectric stack**

2. Chern number in chiral **2D gyro-electric** photonic crystal

3. Chern number in chiral **2D gyro-magnetic** photonic crystal

4. Pseudospin Z_2 in **3D all-dielectric** photonic crystal

5. Weyl (I) semimetal in gyro-electric **3D liquid crystal blue phase** of rods

(1) Zak phase in 1D dielectric stack

$$\theta_n = -Im\log(m_n)$$

Zak phase

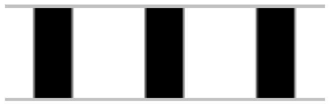
Inversion symmetry:

Quantized $\theta_{\bar{n}} = 0, \pi$

As pairs $\theta_n, \theta_{n+1} = \chi, -\chi$

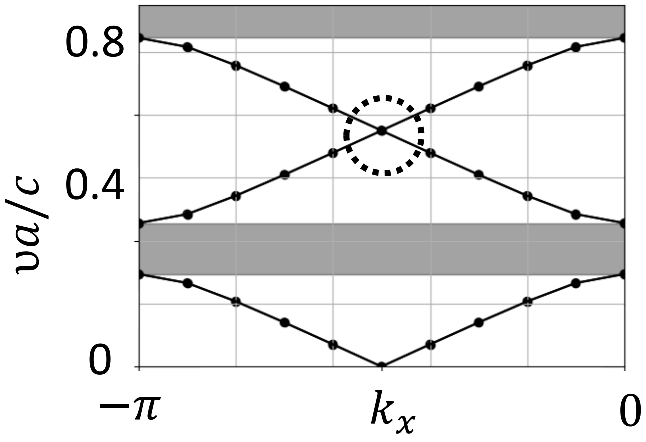
Quarter-wave stack

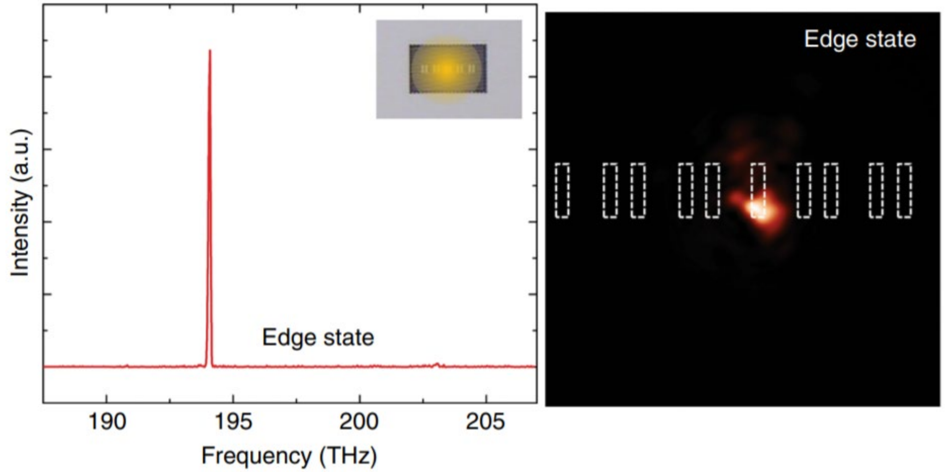
$$n_{high}d_{high} = n_{low}d_{low}$$



$n_{high} = 2$

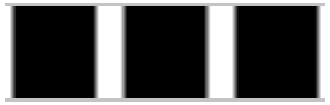
$n_{low} = 1$

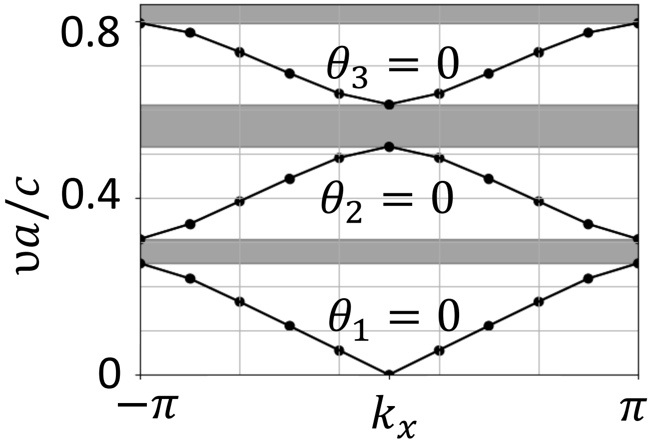




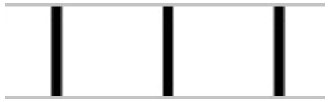
Han, Light: Science & Applications (2019) 8:40

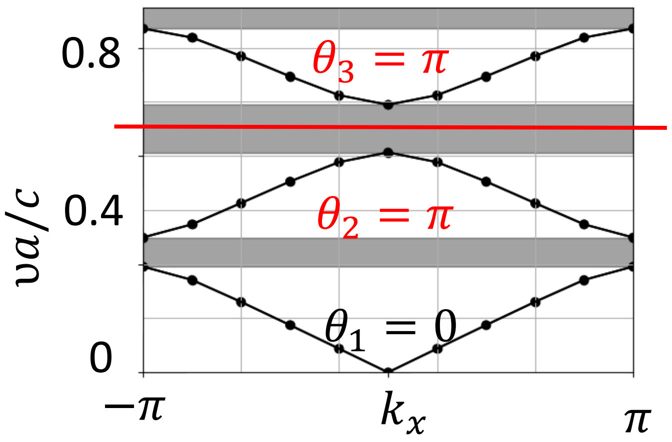
Phase0





Phase1



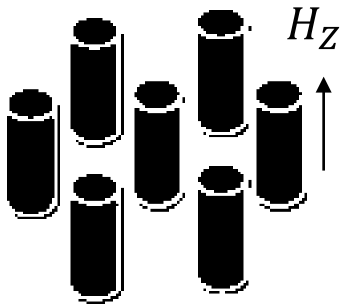


Wang, *et al* (2019)

New J. Phys. 21 093029

(2) Chiral 2D Gyroelectric ph.c.

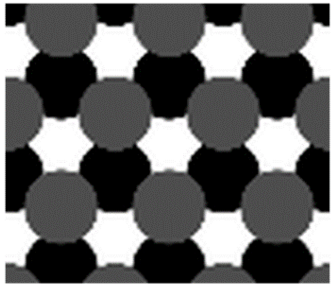
TE bands



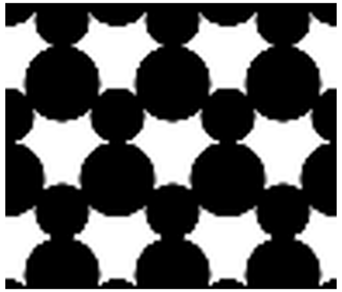
(E_X, E_Y, H_Z)

ε_{rods}=14

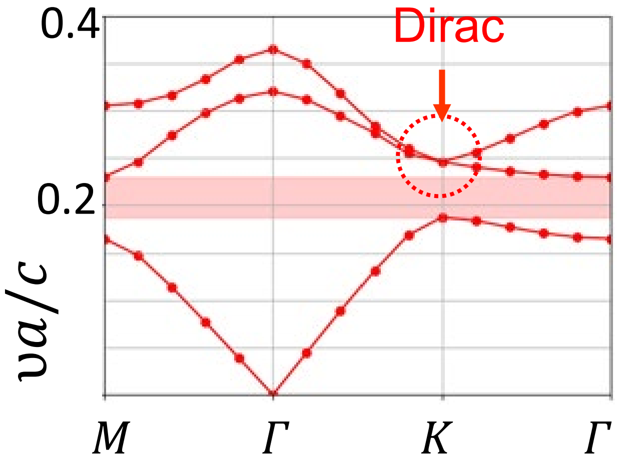
Broken P



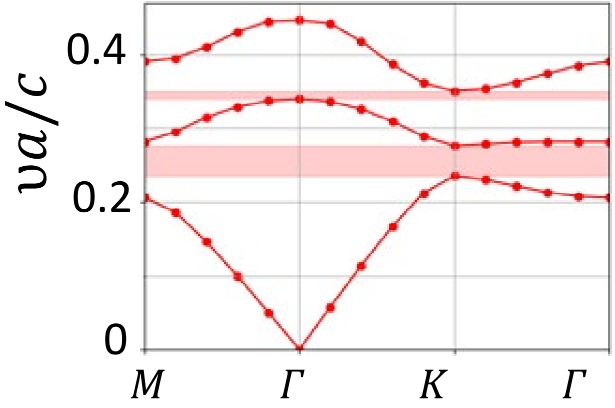
ε_{rods_A}=14
ε_{rods_B}=10



r_{rods_A}=0.34
r_{rods_B}=0.25



Trivial Gap Opening



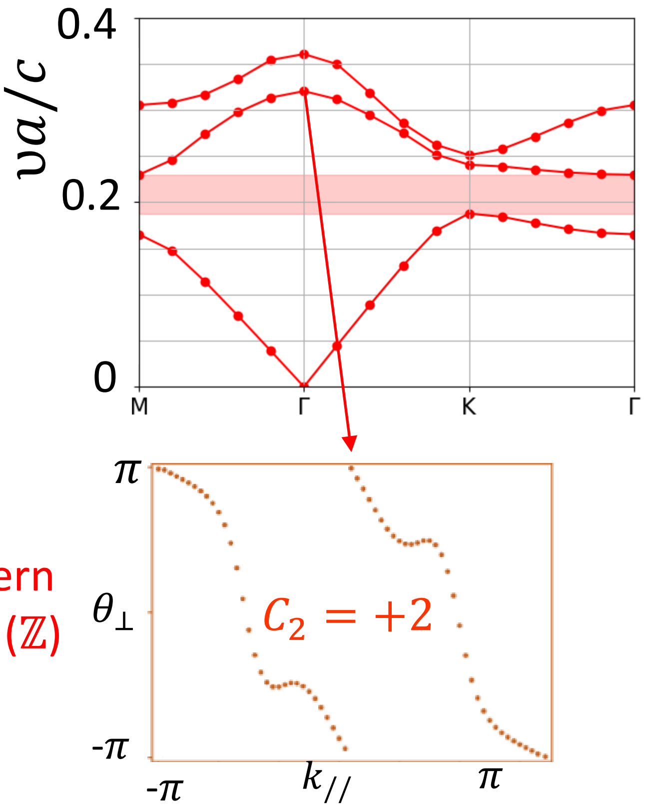
Broken T

Faraday-rotation effect in external B field

$$\hat{\varepsilon}_{BKG} = \begin{pmatrix} 1 & i\Lambda_\varepsilon & 0 \\ -i\Lambda_\varepsilon & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \Lambda_\varepsilon=0.6$$

Unlike electrons, the external magnetic field does not interact directly with photons: this makes the system an analogue of the QAHE

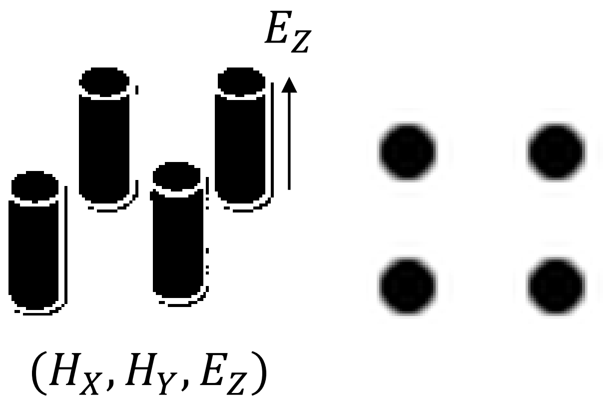
Raghu and **Haldane**, Phys. Rev. A 78, 033834 (2008)



First Chern number ($\mathbb{Z}$)

(3) Chiral 2D Gyromagnetic ph.c.

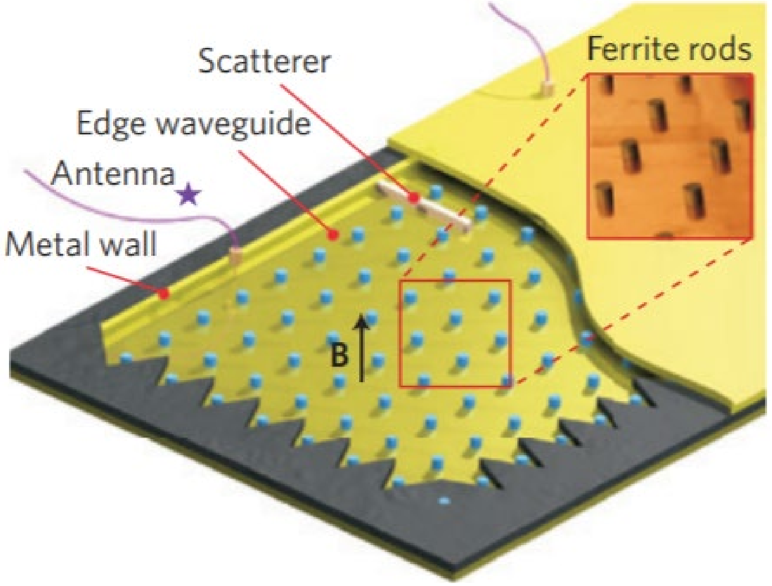
TM Bands

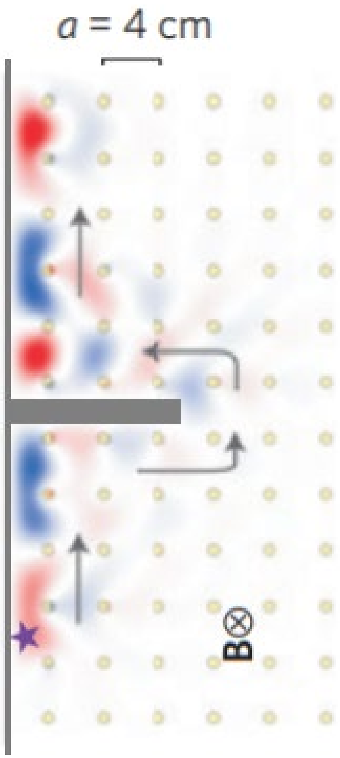


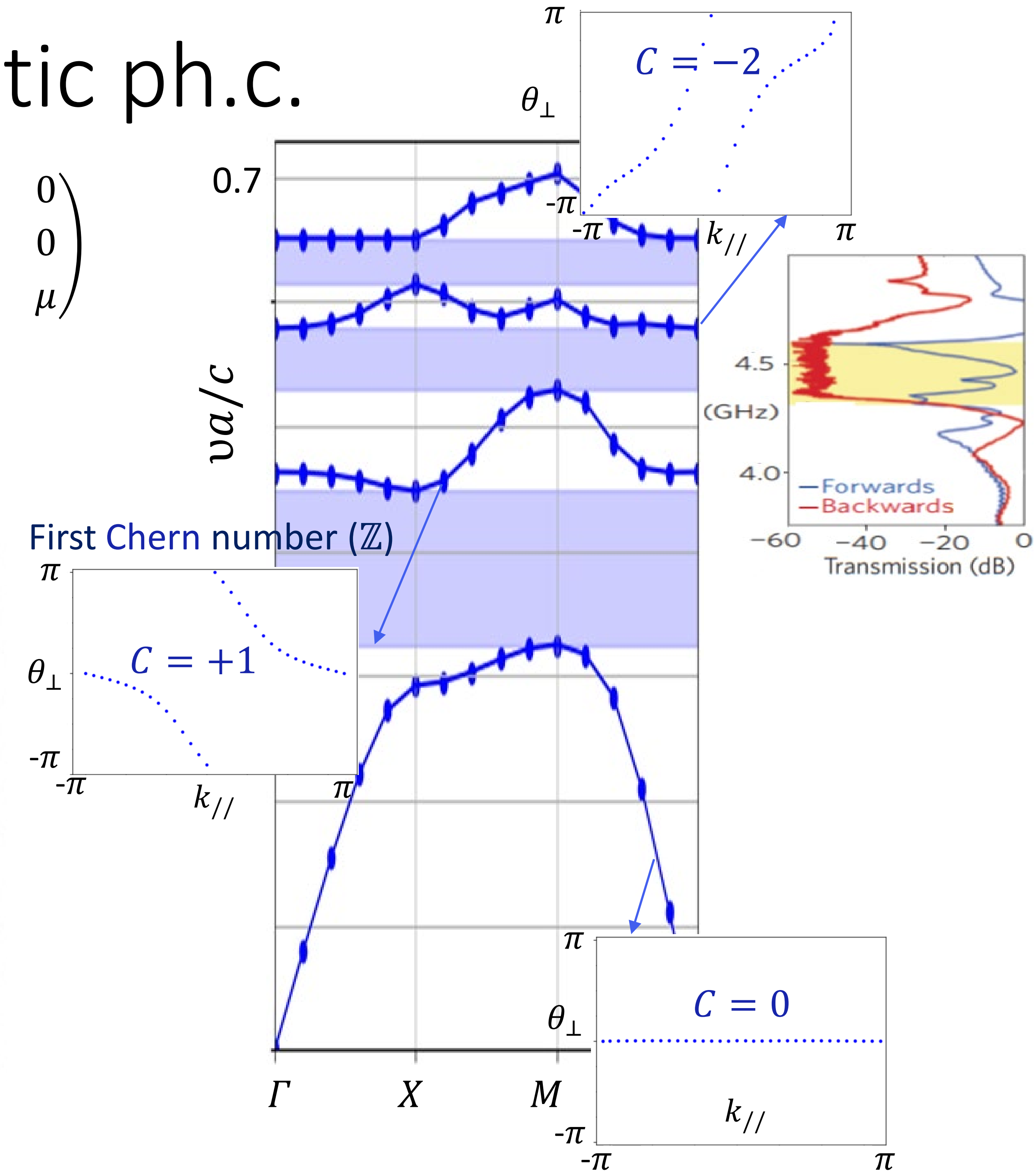
YIG Square Lattice ($\mu_{YIG} = 15$)

$$\hat{\mu}_{rods} = \begin{pmatrix} \mu & i\Lambda_\mu & 0 \\ -i\Lambda_\mu & \mu & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

$$\Lambda_{\mu_{YIG}} = 12.4$$



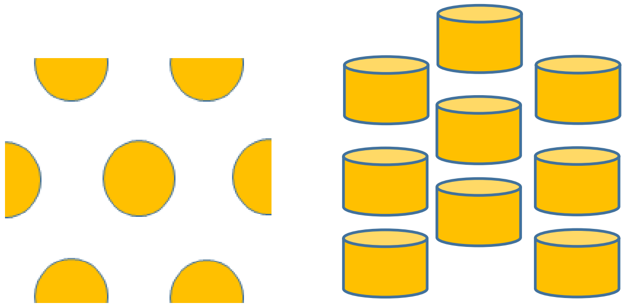




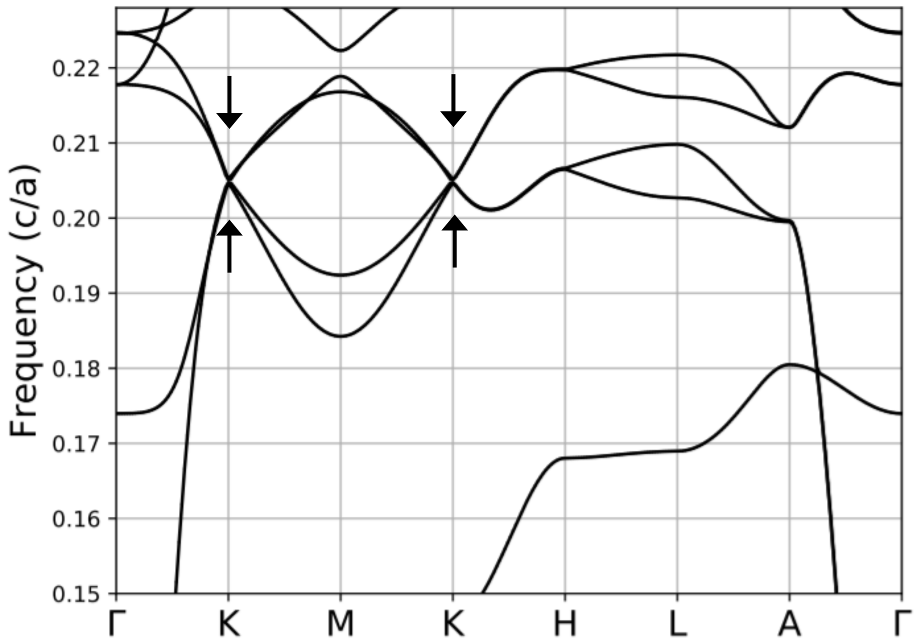
Soljacic, Phys. Rev. Lett. 100, 013905 (2008)

(4) Pseudospin Z_2 in all-dielectric 3d ph.c.

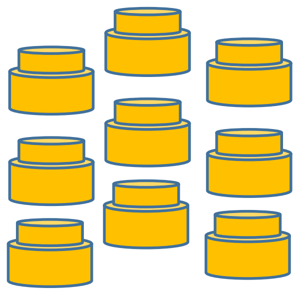
Enforced in-plane duality
$(E,H)_{x,y}$ by meta-atoms design
$\sigma_z \; (z \to -z)$

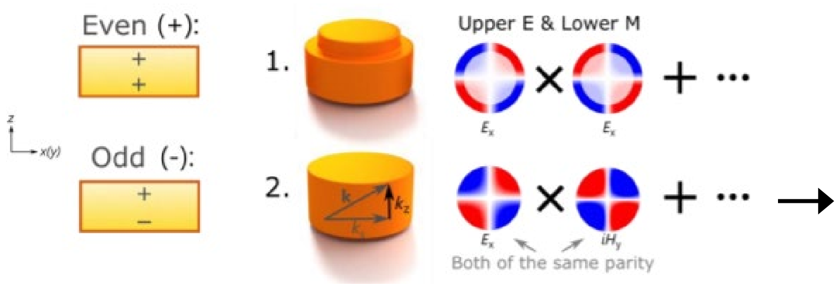


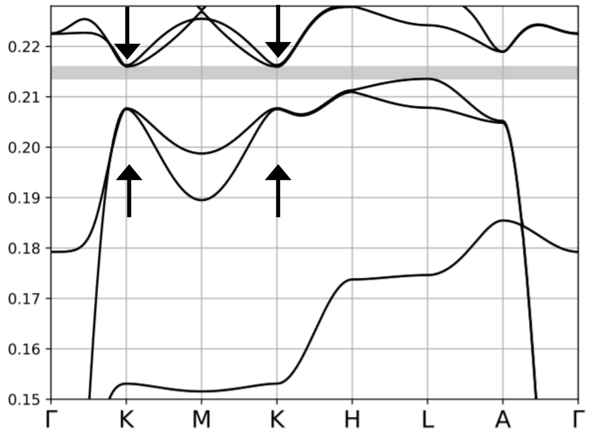
Slobozhanyuk, Nature Photon. vol11, 130–136 (2017)



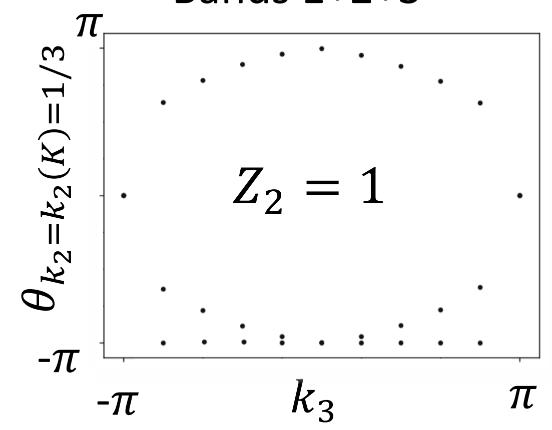
Dual symmetry reduction:
pseudo-SOC interaction

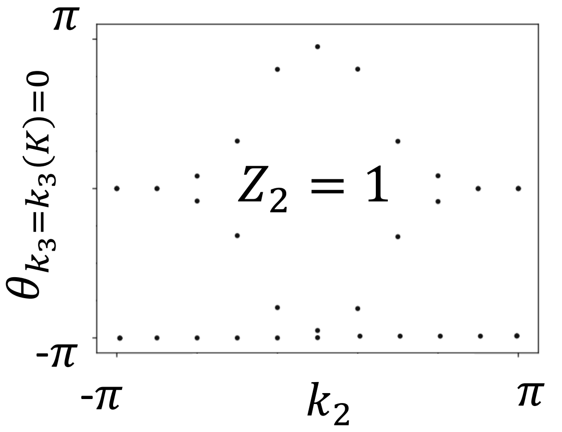






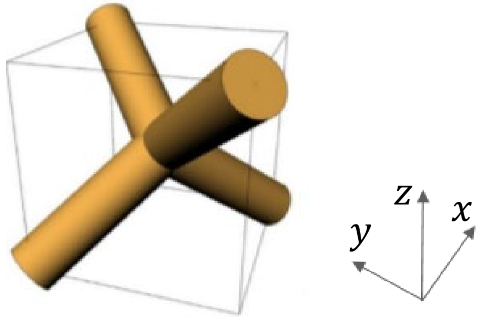
Bands 1+2+3





(5) Weyl (I) semimetal in gyro-electric 3d ph.c

Intrinsic P-Breaking + T-Breaking



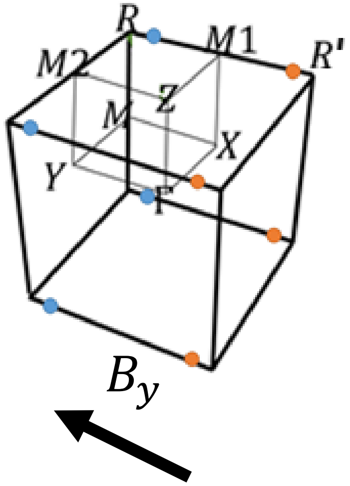
Chiral SG 208

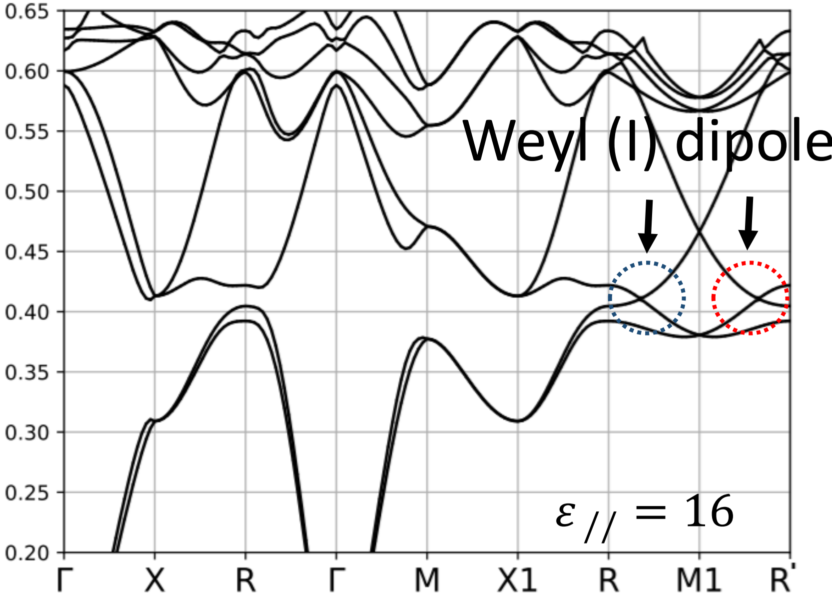
BPII (Liquid Crystal Blue Phase II)

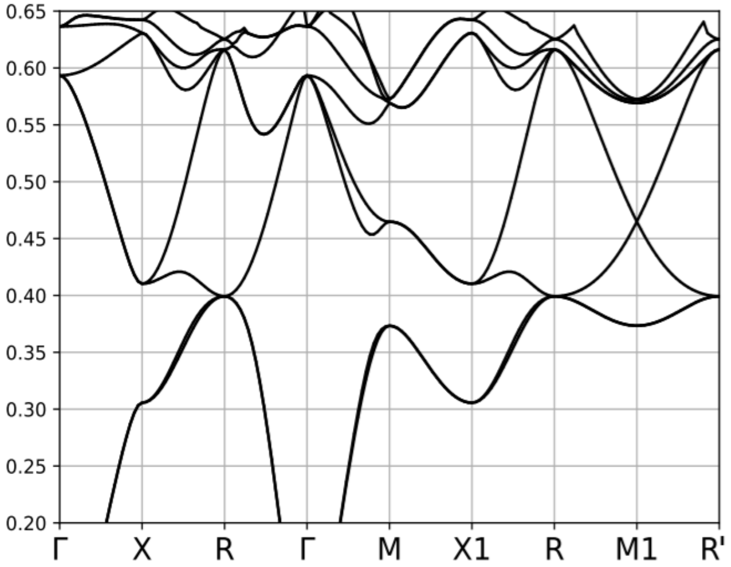
$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_{//} & 0 & i\Lambda_{\varepsilon_i} \\ 0 & \varepsilon_{//} & 0 \\ -i\Lambda_{\varepsilon_i} & 0 & \varepsilon_{//} \end{pmatrix}$$

$$\Lambda_{\varepsilon_{1,2,3,4}} = 10$$

Yang, Optics Express 15772, Vol. 25, No. 14 , 2017

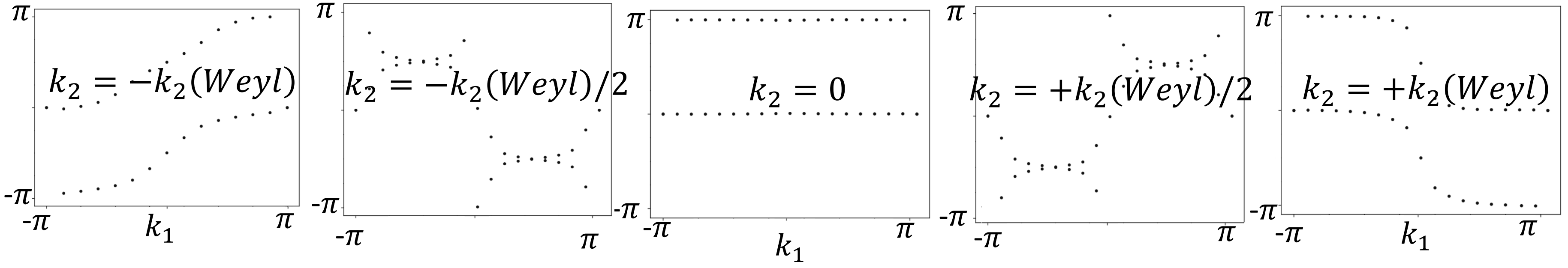






Change in Section Chern number: discontinuity at Weyl

interpret as **Oono,** Phys. Rev. B 94, 125125 (2016)



On-Going Research

i. How to enforce a $\mathbb{Z}_2$ phase by constructing a polarization **TE-TM pseudo-spin**?

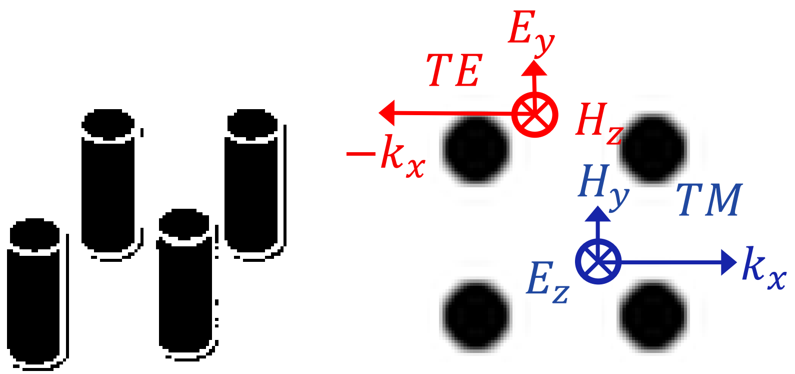
ii. Can **EM duality** U_2 and **mode parity** be used as protecting internal symmetries?

iii. How to realize **3D Chern** $\mathbb{Z}$ photonic phases with no analogue in electronics in chiral woodpiles or chiral liquid crystal blue phases?

iv. Which is the possible WL characterization of topological properties of multi-fold degeneracies in''new-fermions'' **beyond Weyl and Dirac**?

(i) TE/TM pseudo-spin in 2D

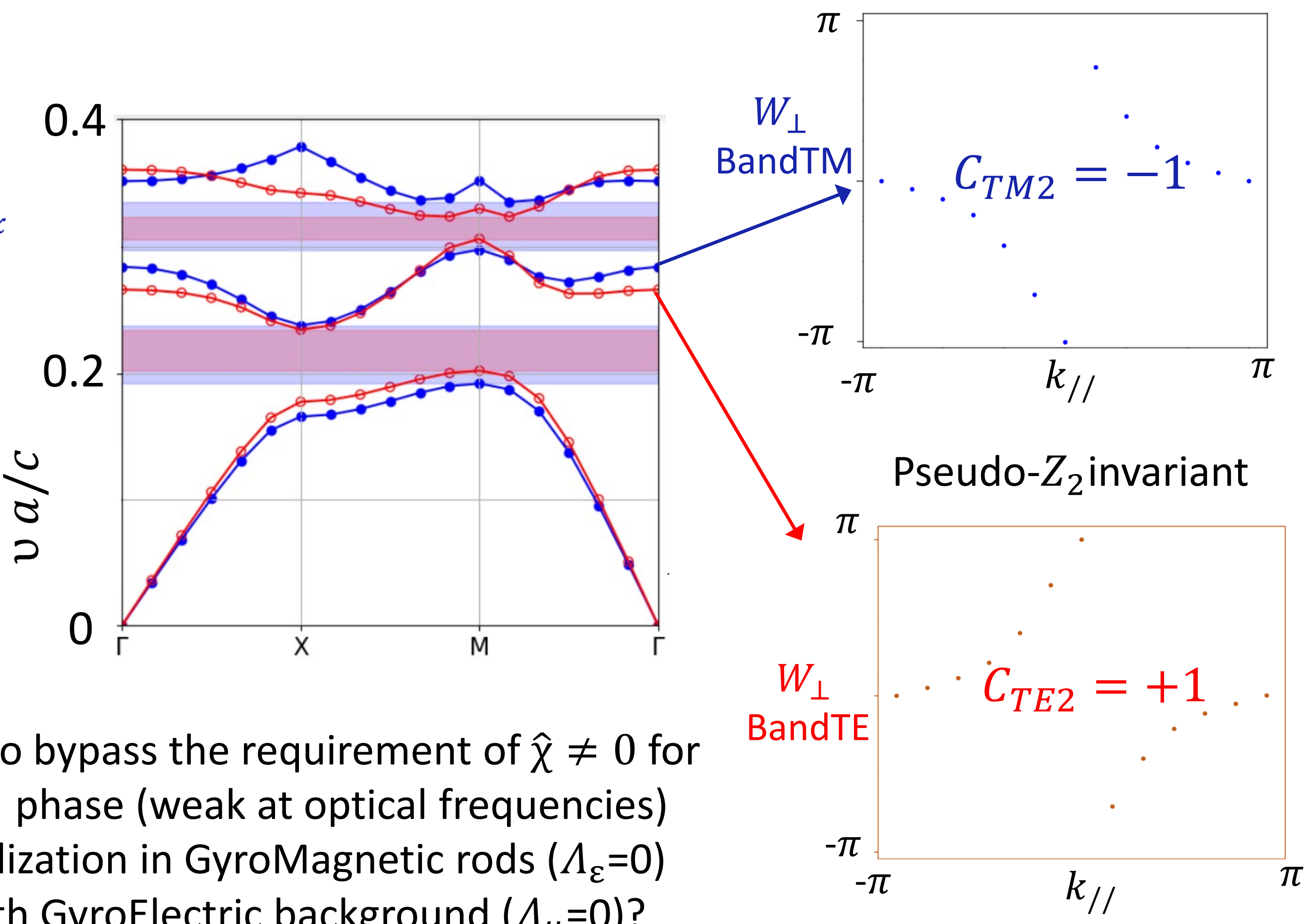
(i) How to enforce a $\mathbb{Z}_2$ phase by constructing a pseudo-spin for TE-TM polarization?



Polarization Transport with $\hat{\chi} = 0$

$$\hat{W} = \begin{pmatrix} \hat{\varepsilon} & \hat{0} \\ \hat{0} & \hat{\mu} \end{pmatrix} \text{ with } \Lambda_\mu \Lambda_\varepsilon < 0$$

$$\mu_{diag} = 9$$
$$\mu_{off} = i\Lambda_\mu = 8.5i$$
$$\varepsilon_{diag} = 15$$
$$\varepsilon_{off} = i\Lambda_\varepsilon = -12i$$

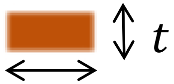


- Way to bypass the requirement of $\hat{\chi} \neq 0$ for a Z_2 phase (weak at optical frequencies)
- Realization in GyroMagnetic rods (Λ_ε=0) with GyroElectric background (Λ_μ=0)?

(ii) Even/Odd Complementary Slabs

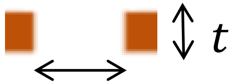
(ii) Can EM duality and mode parity be used as protecting internal symmetries?

u =upper



TM modes
(first 2 bands)
Disconnected Geometry

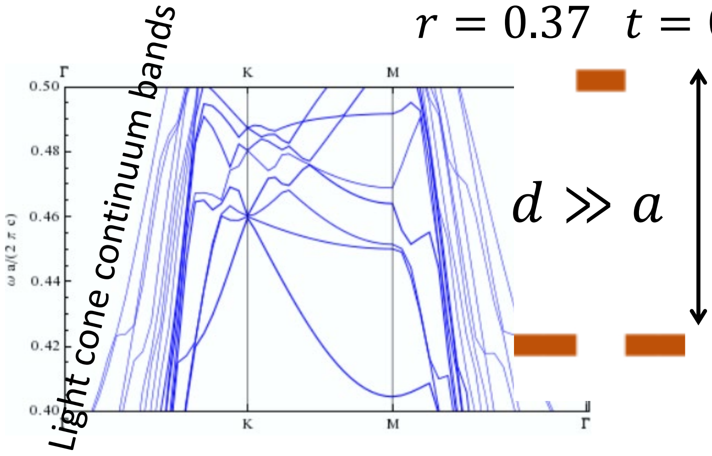
l =lower



TE modes
(first 2 bands)
Connected Geometry

Distant layers limit

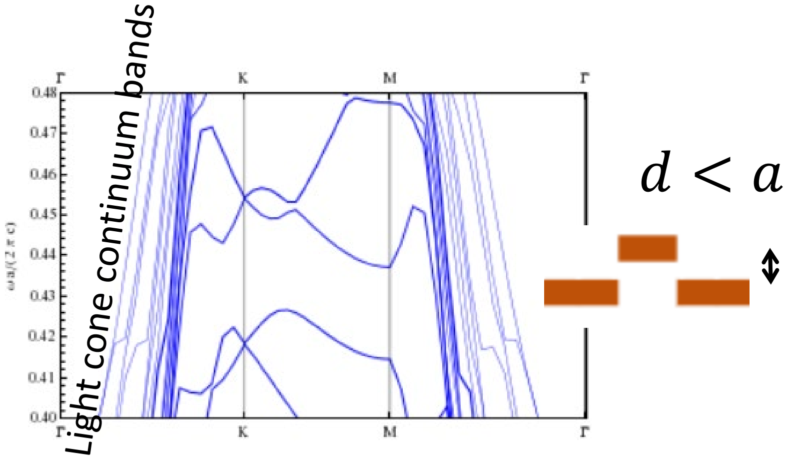
$r = 0.37 \quad t = 0.32$



$$\omega_{TM(k=K)} = \omega_{TE(k=K)^c}$$

Babinet principle approach:
Mutually complementary layers
$u = l^c$
$(u \neq u, l \neq l^c)$

Facing layers

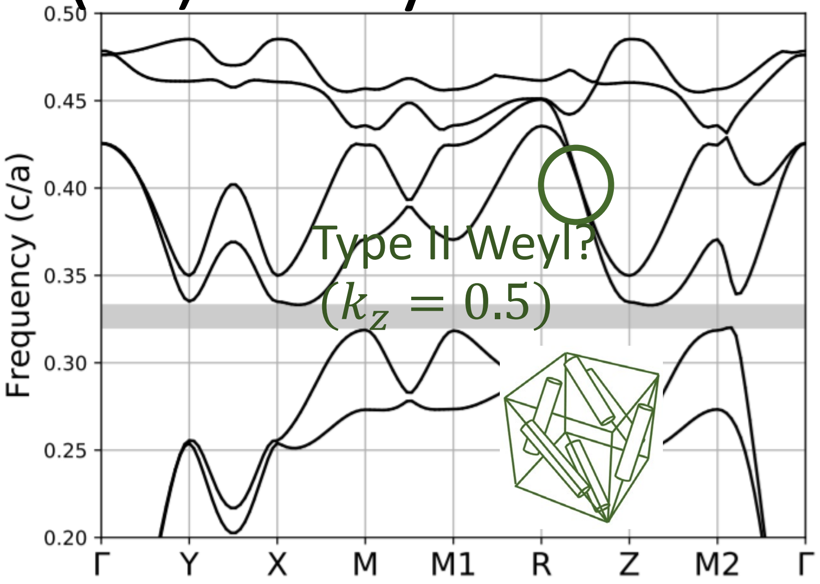


Broken σ_z → even and odd modes coupling
$(u + l) = (u + l)^c$ →Dual pseudospin Z_2?

Detection:

- Character sharing and inversion?
- Light cone continuum bands: Wilson loop in finite systems?

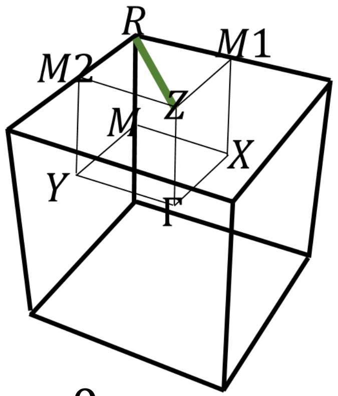
(iii) Weyls in Chiral Gyro-magnetic phc. of Rods



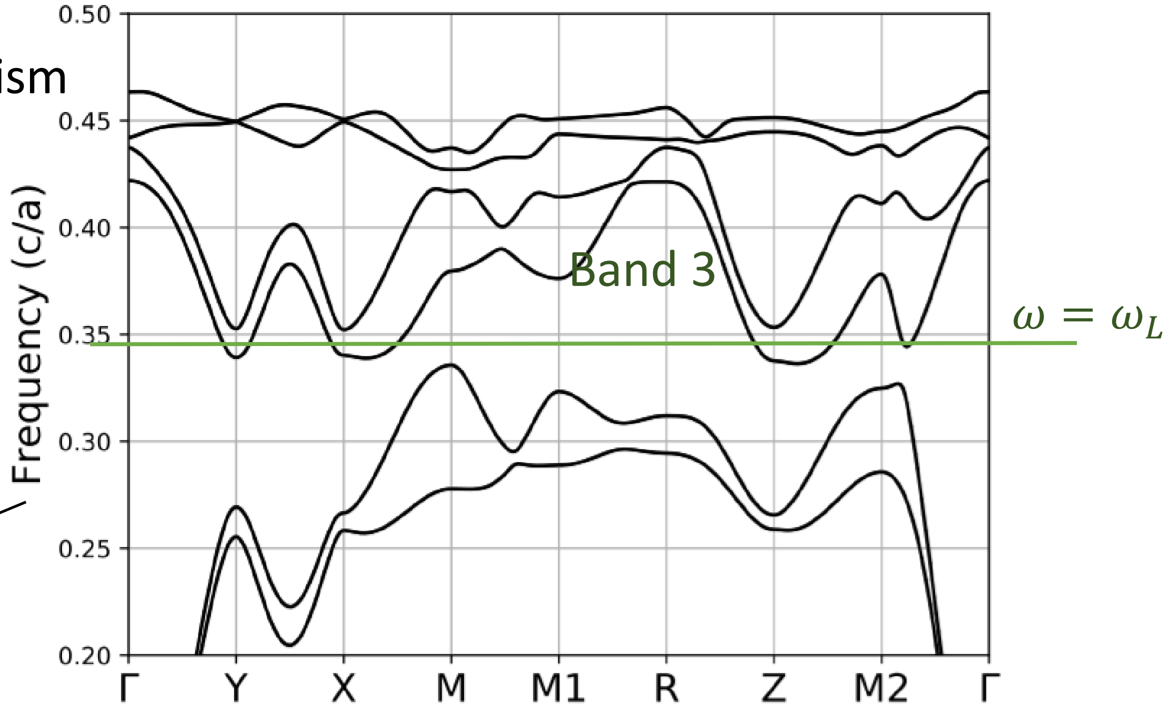
Liquid Crystal of Rods
Chiral SG 209

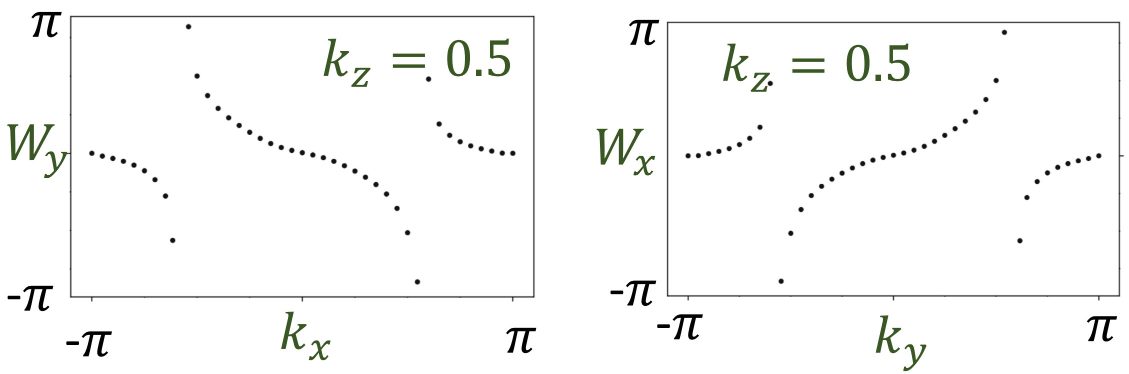
(iii) How to realize 3D Chern $\mathbb{Z}$ photonic phases with no analogue in electronics in chiral woodpiles or chiral liquid crystal blue phases?

Faraday gyromagnetism

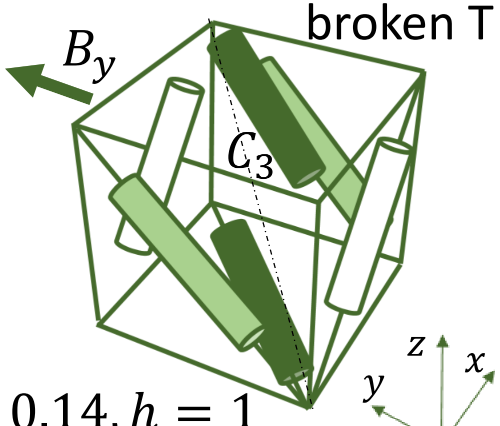


$\Lambda_{\varepsilon_1} = \Lambda_{\varepsilon_2} = \Lambda_{\varepsilon_3} = 0$





$$\hat{\varepsilon_i} = \begin{pmatrix} \sqrt{\varepsilon_{//}^2 + \Lambda_{\varepsilon_i}^2} & 0 & i\Lambda_{\varepsilon_i} \\ 0 & \varepsilon_{//} & 0 \\ -i\Lambda_{\varepsilon_i} & 0 & \sqrt{\varepsilon_{//}^2 + \Lambda_{\varepsilon_i}^2} \end{pmatrix}$$

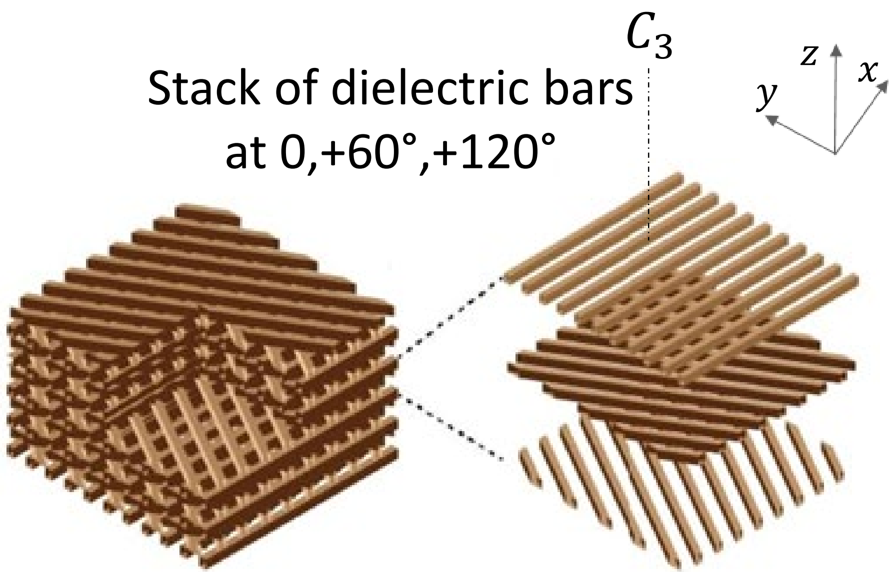


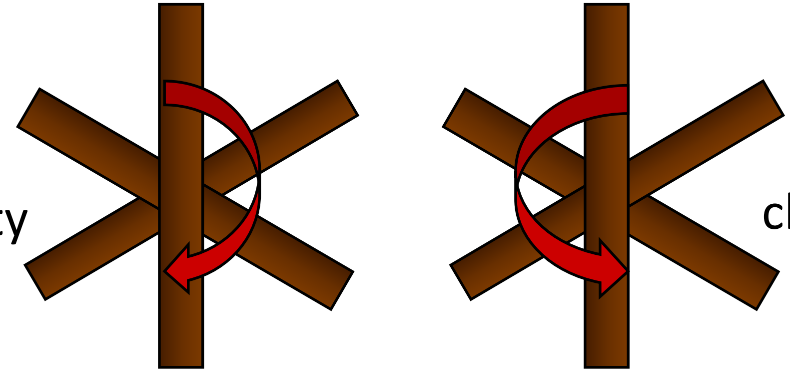
- Control on TRS breaking: amount of Λ_{ε_i} limited by $\varepsilon_{//}$
- How to get the gap in the full BZ?

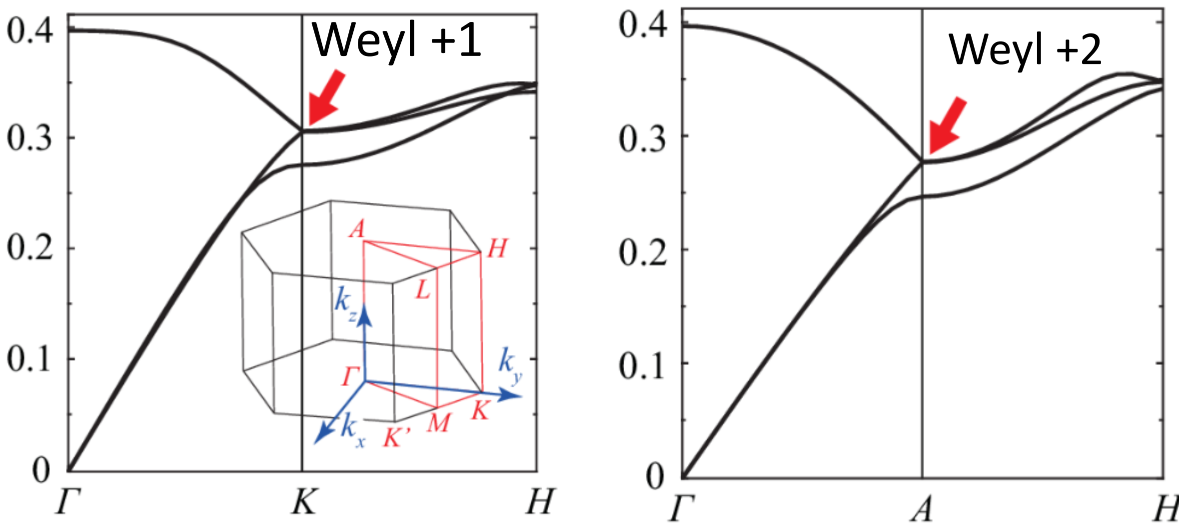
$\Lambda_{\varepsilon_1} = \varepsilon_{//} - 1$
$\Lambda_{\varepsilon_3} = 0$
$\Lambda_{\varepsilon_2} = 0.5\,\Lambda_{\varepsilon_2}$
$\varepsilon_{//} = 29, r = 0.14, h = 1$

(iii) Weyls in Chiral Woodpile ph.c.

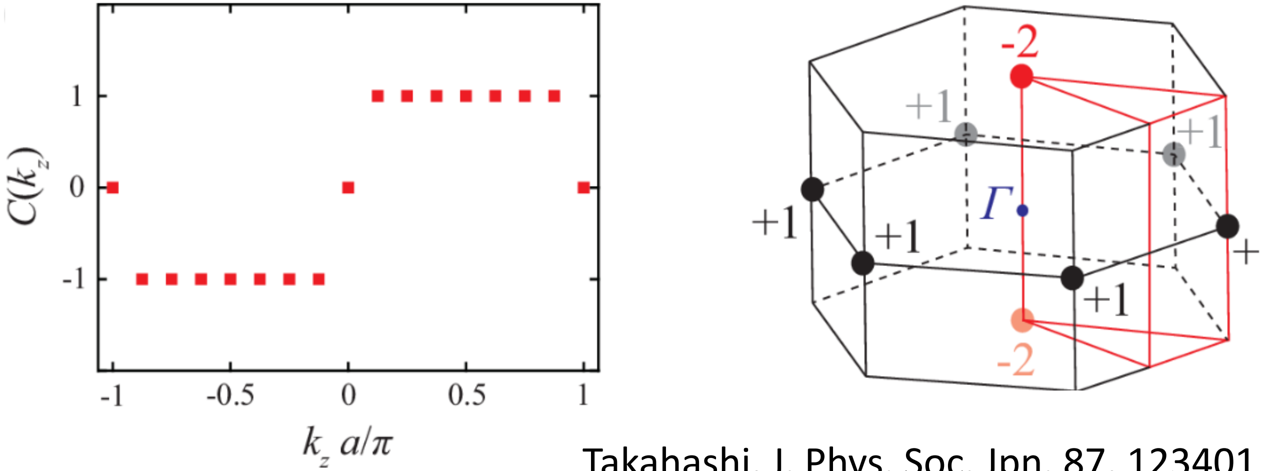
(iii) How to realize 3D Chern $\mathbb{Z}$ photonic phases with no analogue in electronics in chiral woodpiles or chiral liquid crystal blue phases?







Change in section Chern number



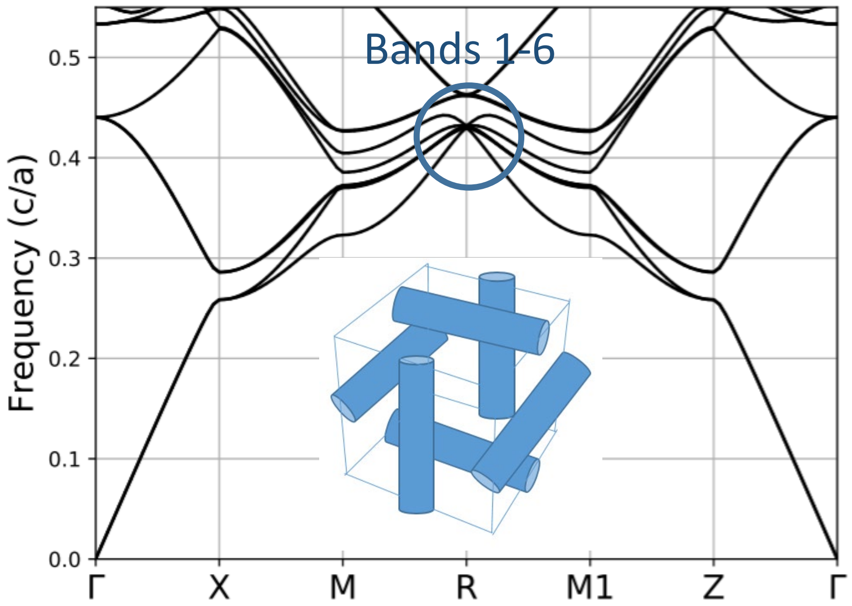
- Weyl gapping? Magnetic field along +z or –z axis: Axial dipendent topology of the gap?

Takahashi, J. Phys. Soc. Jpn. 87, 123401 (2018)

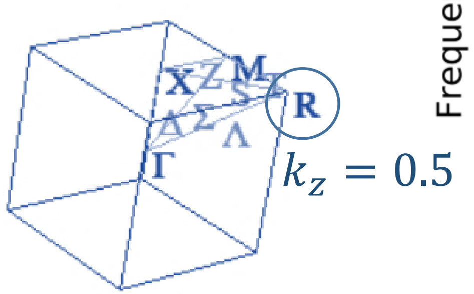
(iv) Six-fold degeneracy in a cubic Oh ph.c.

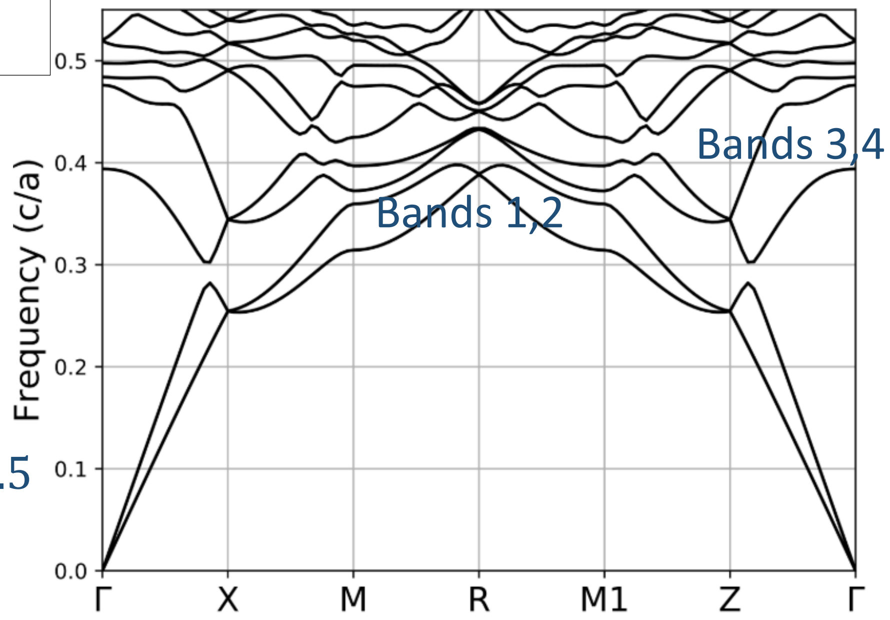
(iv) Which is the possible WL characterization of topological properties of multi-fold degeneracies in ''new-fermions'' beyond Weyl and Dirac?

SG 223 + rods at Wyckoff lines 12g



$B = B_o(1,1,1)$
Broken T





- For the Oh cubic crystal symmetry, there is a 6-dimensional irreducible representation (irrep 'R4') at R=(1/2,1/2,1/2) in SG 223