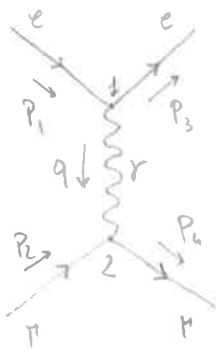
KT II - EXERCISE SHEET 3

Electron-Muon scattering

a)



$e^- \mu^- \to e^- \mu^-$

b) $$-iM = (2\pi)^2 \int d^3q \; \underset{\text{vertex 1}}{\delta^4(p_1 - q - p_3)} \; \underset{\text{vertex 2}}{\delta^4(p_2 + q - p_4)} \times$$

$$\times \; \bar{u}(3)\, i g_e \gamma^\mu u(1) \; \frac{-i g_{\mu\nu}}{q^2} \; \bar{u}(4)\, i g_e \gamma^\nu u(2)$$

$$M = \frac{g_e^2}{(p_1 - p_3)^2} \left[\bar{u}(3)\gamma^\mu u(1)\right]\left[\bar{u}(4)\gamma_\mu u(2)\right]$$

c) $p_1 = (E, 0, 0, p)$

$p_3 = (E, p\sin\theta, 0, p\cos\theta)$

N B: Each helicity combination constitute a separate physical process the helicity states are independent and orthogonal, so there is no interference.

For each combination of initial helicity states, the cross section is given by the sum of the rates for the four possible combination of final helicity states:

$$\sum |M_{RR}|^2 = |M_{RR\to RR}|^2 + |M_{RR\to RL}|^2 + |M_{RR\to LR}|^2 + |M_{RR\to LL}|^2$$

If electrons involved in the collision are unpolarized $\to$ all initial helicity states are possible (4 possible combinations)

$\to$ let's average on the initial helicity states

$$\langle |M_{fi}|^2 \rangle = \frac{1}{4}\left(|M_{RR}|^2 + |M_{RL}|^2 + |M_{LR}|^2 + |M_{LL}|^2\right)$$

$$\downarrow$$

$$\sum_{S_{final}} |M_{RR}|^2$$

The helicity spinors for e particle are

$$u_\uparrow = \sqrt{E+m}\begin{pmatrix} c \\ s e^{i\phi} \\ \frac{p}{E+m} c \\ \frac{p}{E+m} s e^{i\phi} \end{pmatrix} , \quad u_\downarrow = \sqrt{E+m}\begin{pmatrix} -s \\ c e^{i\phi} \\ \frac{p}{E+m} s \\ -\frac{p}{E+m} c e^{i\phi} \end{pmatrix}$$

Given that $p_1 = (E, 0, 0, p)$ and $p_3 = (E, p\sin\theta, 0, p\cos\theta) \Rightarrow E_1 = E_3$

Moreover for initial electron $\theta = 0$ and $\phi = 0$, while for the final electron $\theta \neq 0, \phi = 0$

For $P_1 \rightarrow$ $s = \sin\theta/2 = 0$ and $c = \cos\theta/2 = 1$ because $\theta = 0$

$k = \frac{p}{E+m}$

1) $\bar{u}_\uparrow(3)\gamma^\mu u_\uparrow(1)$

- $\bar{u}_\uparrow(3)\gamma^0 u_\uparrow(1) = (E+m)(c \;\; s \;\; kc \;\; ks)\begin{pmatrix} 1 \\ 0 \\ k \\ 0 \end{pmatrix} = (E+m)(c + k^2 c) =$

$$= (E+m)\, c\, \frac{E^2 + m^2 + 2Em + |\vec{p}|^2}{(E+m)^2} = \frac{2E(E+m)}{E+m} c = 2Ec$$

- $\bar{u}_\uparrow(3)\gamma^1 u_\uparrow(1) = (E+m)(c \;\; s \;\; kc \;\; ks)\begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&-1&0 \\ 0&0&0&-1 \end{pmatrix}\begin{pmatrix} 0&0&0&1 \\ 0&0&1&0 \\ 0&-1&0&0 \\ -1&0&0&0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ k \\ 0 \end{pmatrix} =$

$$= (E+m)(c \;\; s \;\; kc \;\; ks)\begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&-1&0 \\ 0&0&0&-1 \end{pmatrix}\begin{pmatrix} 0 \\ k \\ 0 \\ -1 \end{pmatrix} =$$

$$= (E+m)(c \;\; s \;\; kc \;\; ks)\begin{pmatrix} 0 \\ k \\ 0 \\ 1 \end{pmatrix} = (E+m)\, 2ks = 2p\sin\theta/2 = 2ps$$

$$\bullet\ \bar{u}_\uparrow(3)\,\gamma^2 u_\uparrow(1) = (E+m)\begin{pmatrix} c & s & kc & ks\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\begin{pmatrix}0&0&0&-i\\0&0&i&0\\0&i&0&0\\-i&0&0&0\end{pmatrix}\begin{pmatrix}1\\0\\k\\0\end{pmatrix} =$$

$$= (E+m)\begin{pmatrix} c & s & kc & ks\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\begin{pmatrix}0\\ik\\0\\-i\end{pmatrix} =$$

$$= (E+m)\begin{pmatrix} c & s & kc & ks\end{pmatrix}\begin{pmatrix}0\\ik\\0\\i\end{pmatrix} = (E+m)(2iks) = 2ips$$

$$\bullet\ \bar{u}_\uparrow(3)\,\gamma^3 u_\uparrow(1) = (E+m)\begin{pmatrix} c & s & kc & ks\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\begin{pmatrix}0&0&1&0\\0&0&0&-1\\-1&0&0&0\\0&1&0&0\end{pmatrix}\begin{pmatrix}1\\0\\k\\0\end{pmatrix} =$$

$$= (E+m)\begin{pmatrix} c & s & kc & ks\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\begin{pmatrix}k\\0\\-1\\0\end{pmatrix} =$$

$$= (E+m)\begin{pmatrix} c & s & kc & ks\end{pmatrix}\begin{pmatrix}k\\0\\1\\0\end{pmatrix} = (E+m)\,2ck = 2cp$$

In summary $\bar{u}_\uparrow(3)\gamma^\mu u_\uparrow(1) = 2(Ec, ps, ips, pc)$

Similarly for the other combinations we obtain:

2) $\bar{u}_\downarrow(3)\gamma^\mu u_\downarrow(1) = 2(E, ps, -ips, pc)$

3) $\bar{u}_\uparrow(3)\gamma^\mu u_\downarrow(1) = 2(ms, 0, 0, 0)$

4) $\bar{u}_\downarrow(3)\gamma^\mu u_\uparrow(1) = -2(ms, 0, 0, 0)$

Where

$$u_\downarrow(3) = \sqrt{E+m}\begin{pmatrix}-s\\c\\ks\\-kc\end{pmatrix} \qquad u_\downarrow(1) = \sqrt{E+m}\begin{pmatrix}0\\1\\0\\-k\end{pmatrix}$$

d) the 4-momenta for initial and final muon are

$P_2 = (E, 0, 0, -p)$ where $\theta = \pi$, $\phi = \pi$

$P_4 = (E, -p\,\text{sen}\,\theta', 0, -p\cos\theta')$ where $\theta' = \pi - \theta$, $\phi = \pi$

hence for P_2 $s = \text{sen}\,\theta/_2 = 1$, $c = \cos\theta/_2 = 0$ and $e^{i\phi} = -1$

for P_4 $s' = \text{sen}\,\theta'/_2 = \text{sen}\left(\frac{\pi - \theta}{2}\right) = \cos\frac{\theta}{2} = c$

$c' = \cos\frac{\theta'}{2} = \cos\left(\frac{\pi - \theta}{2}\right) = \text{sen}\frac{\theta}{2} = s$ and $e^{i\phi} = -1$

$$u_\uparrow(2) = \sqrt{E+m}\begin{pmatrix} 0 \\ -1 \\ 0 \\ -k \end{pmatrix}, \quad u_\downarrow(2) = \sqrt{E+m}\begin{pmatrix} -1 \\ 0 \\ k \\ 0 \end{pmatrix}$$

$$u_\uparrow(4) = \sqrt{E+m}\begin{pmatrix} c' \\ -s' \\ kc' \\ -ks' \end{pmatrix}, \quad u_\downarrow(4) = \sqrt{E+m}\begin{pmatrix} -s' \\ -c' \\ ks' \\ kc' \end{pmatrix}$$

let's see the first combination

1) $\bar{u}_\downarrow(4)\gamma^\mu u_\downarrow(2)$

- $\bar{u}_\downarrow(4)\gamma^0 u_\downarrow(2) = (E+m)\begin{pmatrix} -s' & -c' & ks' & kc' \end{pmatrix}\begin{pmatrix} -1 \\ 0 \\ k \\ 0 \end{pmatrix} = (E+m)(s' + k^2 s') = 2E s' = 2Ec$

- $\bar{u}_\downarrow(4)\gamma^1 u_\downarrow(2) = (E+m)\begin{pmatrix} -s' & -c' & ks' & kc' \end{pmatrix}\begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&-1&0 \\ 0&0&0&-1 \end{pmatrix}\begin{pmatrix} 0&0&0&1 \\ 0&0&1&0 \\ 0&-1&0&0 \\ -1&0&0&0 \end{pmatrix}\begin{pmatrix} -1 \\ 0 \\ k \\ 0 \end{pmatrix} =$

$$= (E+m)\begin{pmatrix} -s' & -c' & ks' & kc' \end{pmatrix}\begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&-1&0 \\ 0&0&0&-1 \end{pmatrix}\begin{pmatrix} 0 \\ k \\ 0 \\ 1 \end{pmatrix} =$$

$$= (E+m)\begin{pmatrix} -s' & -c' & ks' & kc' \end{pmatrix}\begin{pmatrix} 0 \\ k \\ 0 \\ -1 \end{pmatrix} = (E+m)(-2c'k) = -2pc' = -2ps$$

• $\bar{u}_\downarrow(4)\gamma^2 u_\downarrow(2) = (E+m)\,(-s' \ \ -c' \ \ ks' \ \ kc')\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix}\begin{pmatrix}0&0&0&-i\\0&0&i&0\\0&i&0&0\\-i&0&0&0\end{pmatrix}\begin{pmatrix}-1\\0\\k\\0\end{pmatrix} =$

$$= (E+m)\,(-s' \ -c' \ \ ks' \ \ kc')\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\begin{pmatrix}0\\ik\\0\\i\end{pmatrix} =$$

$$= (E+m)\,(-s' \ -c' \ \ ks' \ \ kc')\begin{pmatrix}0\\ik\\0\\-i\end{pmatrix} = (E+m)\,(-2ikc') = -2ipS$$

• $\bar{u}_\downarrow(4)\gamma^3 u_\downarrow(2) = (E+m)\,(-s' \ -c' \ \ ks' \ kc')\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}0&0&1&0\\0&0&0&-1\\-1&0&0&0\\0&1&0&0\end{pmatrix}\begin{pmatrix}-1\\0\\k\\0\end{pmatrix} =$

$$= (E+m)\,(-s' \ -c' \ \ ks' \ kc')\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\begin{pmatrix}k\\0\\1\\0\end{pmatrix} =$$

$$= (E+m)\,(-s' \ -c' \ \ ks' \ \ kc')\begin{pmatrix}k\\0\\-1\\0\end{pmatrix} = (E+m)\,(-2s'k) = -2pC$$

the muon currents for the other combinations can be computed similarly

d) $E \gg M$

$$|M_{LL}|^2 = |M_{LL\to LL}|^2 + |M_{LL\to LR}|^2 + |M_{LL\to RL}|^2 + |M_{LL\to RR}|^2$$

$$M = -\frac{e^2}{(P_1-P_3)^2}\, j_e\, j_\mu$$

$$|M_{LL\to LL}| = \frac{e^2}{(P_1-P_3)^2}\left[\bar{u}_\downarrow(3)\gamma^\mu u_\downarrow(1)\right]\left[\bar{u}_\downarrow(4)\gamma^\mu u_\downarrow(2)\right] =$$

$$\frac{e^2}{(P_1-P_3)^2}\left[2(E_1 c, PS, -iPS, PC)\right]\left[2(E_2 c, -PS, -iPS, -PC)\right]$$

$$= \frac{e^2}{(P_1-P_3)^2}\,4\left[E_1E_2\,c^2 + p^2s^2 + p^2s^2 + p^2c^2\right]$$

$$= \frac{e^2}{(P_1-P_3)^2} 4 \left[E^2c^2 + E^2s^2 + E^2s^2 + E^2c^2 \right] = \frac{e^2}{(P_1-P_3)^2} 4 \left[2E^2 \right] = \frac{2e^2 s}{(P_1-P_3)^2}$$

where $4E^2 = s = (P_1+P_2)^2$

this is the unique combination contributing to $|M_{LL}|^2$

So $$|M_{LL}|^2 = \frac{4e^4 s^2}{(P_1-P_3)^4}$$

$$|M_{RR}|^2 = |M_{RR\to RR}| = \frac{4e^4 s^2}{(P_1-P_3)^4}$$

$$|M_{RL}|^2 = |M_{RL\to RL}|^2$$

With $$|M_{RL\to RL}| = \frac{e^2}{(P_1-P_3)^2} \cdot 4 \left[(Ec, ps, ips, pc) \cdot (Ec, -ps, -ips, -pc) \right] =$$

$$= \frac{e^2}{(P_1-P_3)^2} 4 \left[E^2 + E^2(c^2 - s^2) \right] =$$

$$= \frac{e^2}{(P_1-P_3)^2} 4 E^2 (1 + \cos\theta) \quad \text{being } \cos\theta = \cos^2\frac{\theta}{2} - \operatorname{sen}^2\frac{\theta}{2}$$

$$|M_{LR}|^2 = |M_{LR\to LR}|^2 = \left[\frac{e^2}{(P_1-P_3)^2} \cdot 4 \left[(Ec, ps, -ips, pc) \cdot (Ec, -ps, ips, -pc) \right] \right]^2$$

$$= \left[\frac{e^2}{(P_1-P_3)^2} \cdot 4 \left[E^2c^2 + p^2s^2 - p^2s^2 + p^2c^2 \right] \right]^2 =$$

$$= \left[\frac{e^2}{(P_1-P_3)^2} 4 \left[E^2 + E^2\cos\theta \right] \right]^2 = \left[\frac{e^2}{(P_1-P_3)^2} s(1+\cos\theta) \right]^2$$

e) $$\langle |M_{fi}|^2 \rangle = \frac{1}{4} \times \left[|M_{LL}|^2 + |M_{RR}|^2 + |M_{LR}|^2 + |M_{RL}|^2 \right] =$$

$$= \frac{1}{4} \times \left[\frac{8e^4 s^2}{(P_1-P_3)^4} + 2\frac{e^4 s^2}{(P_1-P_3)^4} (1+\cos\theta)^2 \right] =$$

$$= \frac{e^4 s^2}{(P_1-P_3)^4} \left[2 + \frac{1}{2}(1+\cos\theta)^2 \right]$$

Differential cross section for $a+b \to c+d$ process in the centre-of-mass frame:

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 s} \frac{p_f^*}{p_i^*} |M_{fi}|^2$$

In our case $|\vec{p}_f^*| = |\vec{p}_i^*| = E$

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 s} \cdot \frac{e^4 s^2}{(P_1-P_3)^4} \cdot 2 \left[1 + \frac{1}{4}(1+\cos\theta)^2 \right]$$

$$- (P_1-P_3)^4 = \left[(\cancel{E_1-E_3})^2 - (\vec{P}_1-\vec{P}_3)^2 \right]^2 = \left[p^2 \sin^2\theta + p^2(1-\cos\theta)^2 \right]^2$$

with $(\vec{P}_1 - \vec{P}_3) = (-p\sin\theta, 0, p(1-\cos\theta))$

$$= \left[p^2 (\sin^2\theta + 1 + \cos^2\theta - 2\cos\theta) \right]^2 = \left[2p^2(1-\cos\theta) \right]^2 =$$

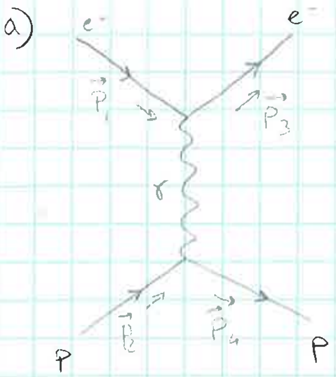
$$= 4p^4 (1-\cos\theta)^2 = s \cdot \frac{s}{4} (1-\cos\theta)^2$$

$$\frac{d\sigma}{d\Omega^*} = \frac{\alpha^2}{4} \frac{s}{s \cdot \frac{s}{4}(1-\cos\theta)^2} 2 \left[1 + \frac{1}{4}(1+\cos\theta)^2 \right] =$$

$$= \frac{2\alpha^2}{s} \frac{1 + \frac{1}{4}(1+\cos\theta)^2}{(1-\cos\theta)^2}$$

ELECTRON - PROTON SCATTERING

a)

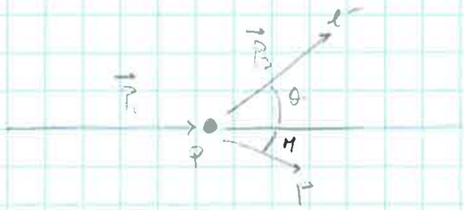


point-like proton approximation

$$u_\uparrow = \sqrt{E+m}\begin{pmatrix} c \\ s e^{i\phi} \\ \frac{p}{E+m} c \\ \frac{p}{E+m} s e^{i\phi} \end{pmatrix} \qquad u_\downarrow = \sqrt{E+m}\begin{pmatrix} -s \\ c e^{i\phi} \\ \frac{p}{E+m} s \\ -\frac{p}{E+m} c e^{i\phi} \end{pmatrix}$$

where $s = \sin\theta/2$ and $c = \cos\frac{\theta}{2}$

b) the energy of the electron does not change in the scattering process $\Rightarrow E_1 = E_3$



$\theta_i = 0 \quad \phi_i = 0$

$\theta_f = \theta \quad \phi_f = 0$

$$u_\uparrow(1) = \sqrt{E+m}\begin{pmatrix} 1 \\ 0 \\ \frac{p}{E+m} \\ 0 \end{pmatrix} \qquad u_\downarrow(1) = \sqrt{E+m}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -\frac{p}{E+m} \end{pmatrix}$$

$$u_\uparrow(3) = \sqrt{E+m}\begin{pmatrix} c \\ s \\ \frac{p}{E+m} c \\ \frac{p}{E+m} s \end{pmatrix} \qquad u_\uparrow(3) = \sqrt{E+m}\begin{pmatrix} -s \\ c \\ \frac{p}{E+m} s \\ -\frac{p}{E+m} c \end{pmatrix}$$

c) $j_{\uparrow\uparrow} = \bar{u}_\uparrow(2)\gamma^\mu u_\uparrow(1)$ with $k = \frac{p}{E+m}$

• $\bar{u}_\uparrow(2)\gamma^0 u_\uparrow(1) = (E+m)\begin{pmatrix} c & s & kc & ks \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ k \\ 0 \end{pmatrix} =$

$$= (E+m)(c + k^2 c) = (E+m)\, c\,(1+k^2)$$

• $\bar{u}_\uparrow(2)\gamma^1 u_\uparrow(1) = (E+m)\begin{pmatrix} c & s & kc & ks \end{pmatrix}\begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&-1&0 \\ 0&0&0&-1 \end{pmatrix}\begin{pmatrix} 0&0&0&1 \\ 0&0&1&0 \\ 0&-1&0&0 \\ -1&0&0&0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ k \\ 0 \end{pmatrix} =$

$$= (E+m)\begin{pmatrix} c & s & kc & ks \end{pmatrix}\begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&-1&0 \\ 0&0&0&-1 \end{pmatrix}\begin{pmatrix} 0 \\ k \\ 0 \\ -1 \end{pmatrix} =$$

$$= (E+m)\begin{pmatrix} c & s & kc & ks \end{pmatrix}\begin{pmatrix} 0 \\ k \\ 0 \\ 1 \end{pmatrix} = 2(E+m)\, sk$$

• $\bar{u}_\uparrow(2)\gamma^2 u_\uparrow(1) = (E+m)\begin{pmatrix} c & s & kc & ks \end{pmatrix}\begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&-1 \end{pmatrix}\begin{pmatrix} 0&0&0&-i \\ 0&0&i&0 \\ 0&i&0&0 \\ -i&0&0&0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ k \\ 0 \end{pmatrix} =$

$$= (E+m)\begin{pmatrix} c & s & kc & ks \end{pmatrix}\begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&-1&0 \\ 0&0&0&-1 \end{pmatrix}\begin{pmatrix} 0 \\ ik \\ 0 \\ -i \end{pmatrix} =$$

$$= (E+m)\begin{pmatrix} c & s & kc & ks \end{pmatrix}\begin{pmatrix} 0 \\ ik \\ 0 \\ i \end{pmatrix} = (E+m)\, 2iks$$

$$\bullet\ \bar{u}_\uparrow(3)\gamma^3 u_\uparrow(1) = (E+m)\,(c\ \ s\ \ kc\ \ ks)\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}0&0&1&0\\0&0&0&-1\\-1&0&0&0\\0&1&0&0\end{pmatrix}\begin{pmatrix}1\\0\\k\\0\end{pmatrix} =$$

$$= (E+m)(c\ \ s\ \ kc\ \ ks)\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}k\\0\\-1\\0\end{pmatrix} = (E+m)(c\ \ s\ \ kc\ \ ks)\begin{pmatrix}k\\0\\+1\\0\end{pmatrix} =$$

$$= (E+m)(c\ \ s\ \ kc\ \ ks)\begin{pmatrix}k\\0\\1\\0\end{pmatrix} = (E+m)2kc$$

In the relativistic limit $E \gg m$ and $k = \frac{p}{E+m} \approx 1$, hence only two of the four combinations give non-zero electron current.

d) Recoiling proton $\theta = \eta$ and $\phi = \pi$

$k = \frac{p}{E+m} = \frac{\beta\gamma}{\gamma+1} \approx 0$ for low velocity protons ($\beta_p \ll 1$) $\Rightarrow$ Moreover $E \approx m_p$

For the initial proton, which is at rest $\Rightarrow E = m_p$

$$u_\uparrow(2) = \sqrt{2m_p}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \qquad u_\downarrow(2) = \sqrt{2m_p}\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

$$u_\uparrow(4) \approx \sqrt{2m_p}\begin{pmatrix}c_\eta\\-s_\eta\\0\\0\end{pmatrix} \qquad u_\downarrow(4) \approx \begin{pmatrix}-s_\eta\\-c_\eta\\0\\0\end{pmatrix}\sqrt{2m_p}$$

where $c_\eta = \cos\frac{\eta}{2}$

$s_\eta = \sin\frac{\eta}{2}$

e) $j_{p\uparrow\uparrow} = \bar{u}_\uparrow(4)\gamma^\mu u(2)$

$$\bullet\ \bar{u}_\uparrow(4)\gamma^0 u_\uparrow(2) = 2m_p\,(c_\eta\ \ -s_\eta\ \ 0\ \ 0)\begin{pmatrix}1\\0\\0\\0\end{pmatrix} = 2m_p\, c_\eta$$

$$\bullet\ \bar{u}_\uparrow(4)\gamma^1 u_\uparrow(2) = 2m_p\,(c_\eta\ \ -s_\eta\ \ 0\ \ 0)\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{pmatrix}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} =$$

$$= 2m_p \begin{pmatrix} c_H & -s_H & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} = 2m_p \begin{pmatrix} c_H & -s_H & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} =$$

$$= 2m_p \cdot 0 = 0$$

- $\bar{u}_1(k)\Gamma^2 u_1(k) = 2m_p \begin{pmatrix} c_H & -s_H & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} =$

$$= 2m_p \begin{pmatrix} c_H & -s_H & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} =$$

$$= 2m_p \begin{pmatrix} c_H & -s_H & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} = 0$$