KTII - Exercise 5

Universität Zürich

Due: 29 April 2020

For particle information, including hadronic quark content, particle masses, particle lifetimes, or other physical constants not given in the problem, please consult the Particle Data Group's Review of Particle Physics. It is available for free on their website: http://pdg.lbl.gov/.

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1. **(2 points)** Use lepton universality and lepton–quark symmetry to estimate the branching ratios for

   (a) the decays $b \to c + e^- + \bar{\nu}_e$, where the b and c quarks are bound in hadrons

   (b) $\tau^- \to e^- + \bar{\nu}_e + \nu_\tau$

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2 **(2 points)**

The couplings of the Z boson to right-handed (R) and left-handed (L) fermions are given by

$$g_R(f) = -q_f \sin^2\theta_\mathrm{W}, \quad g_L(f) = \pm 1/2 - q_f \sin^2\theta_\mathrm{W}$$

,

where q_f is the electric charge of the fermion f in units of e and θ_W is the weak mixing angle. The positive sign in g_L is used for neutrinos and the up-type quarks ($q = u, c, t$); the negative sign is used for charged leptons and the down-type quarks ($q = d, s, b$). If the partial width for $\mathrm{Z} \to f\bar{f}$ is given by

$$\Gamma_f = \frac{G_F M_Z^3 c^6}{3\pi\sqrt{2}(\hbar c)^3}\left[g_R^2(f) + g_L^2(f)\right].$$

(a) Calculate the partial widths for neutrinos, Γ_ν.

(b) Calculate the partials width for $q\bar{q}$ pairs, Γ_q, and explain the relation of Γ_q to the partial width to hadrons Γ_hadron.

(c) The widths to hadrons and to charged leptons are measured to be $\Gamma_\mathrm{hadron} = (1738 \pm 12)$ MeV and $\Gamma_\mathrm{lepton} = (250{\pm}2)$ MeV, and the total width to all final states is measured to be $\Gamma_\mathrm{tot} = (2490 \pm 7)$ MeV. Use these experimental results and your calculated value for the decay width to neutrinos to show that there are only three generations of neutrinos with masses $M_\nu < M_\mathrm{Z}/2$.

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3 **(3 points)** Make back-of-the-envelope estimates for the following:

(a) Estimate the number of solar neutrino interactions occurring in a typical human body over a human lifetime. Consider neutrino-electron elastic scattering only. Assume the human body is made of water and that it has a mass of 80 kg. Assume the solar neutrino flux is $\phi \sim 2 \times 10^6\,\mathrm{cm^{-2}s^{-1}}$.

(b) Estimate the number of $\nu_e - {}^{40}$Ar charged-current neutrino interactions in a 40-kton liquid-argon detector for a core collapse at the centre of the Milky Way. Estimate also the number for a core collapse in M3. As cross section use $\sigma \sim 2 \times 10^{-41}\,\mathrm{cm^2}$. (Do not cheat by reading event rates directly off any plot!)

(c) Assume that on average 7000 neutrinos will be recorded by Super-K (22.5 ktons of water) for a core-collapse supernova at 10 kpc. What is the probability of recording at least one event?

Additional information:

- neutrino-electron elastic scattering cross-section above a few MeV: $\sigma \sim 5 \times 10^{-44}\,\mathrm{cm^2}$
- flux above a few MeV, mostly from ^{8}B neutrinos: $\phi \sim 2 \times 10^6\,\mathrm{cm^{-2}s^{-1}}$
- one year is close to $\pi \times 10^7$ seconds

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4 **(3 points)** The oscillation length between the first and second neutrino mass eigenstates in a vacuum is:

$$\mathrm{L_0/km} = X \frac{(E_\nu/\mathrm{GeV})}{(\delta m^2_{21} c^4/\mathrm{eV}^2)}.$$

(a) What is the value of X?

(b) If you are interested in probing neutrino oscillations for $\delta m^2_{21} c^4 \sim 7.5 \times 10^{-5}\,\mathrm{eV}^2$ using a reactor beam of $\bar{\nu}_e$s, what is the nearest distance at which you would see the maximum effect of oscillations? (Reactor antineutrino spectra are broad, but peak around 3 MeV.)

(c) Answer the same question if $\delta m^2_{21} c^4$ is replaced by $\delta m^2_{32} c^4 \sim 2.3 \times 10^{-3}\,\mathrm{eV}^2$ and a 5 GeV FermiLab ν_μ beam is used. (The FermiLab beam will also be broad, but use this one energy for your calculation.)

(d) Finally, for solar neutrinos of energy ~ 1 MeV, determine the $\delta m^2 c^4$ that corresponds to a vacuum oscillation maximum coincident with the earth-Sun distance (thus determining the minimum $\delta m^2 c^4$ that might be tested with solar neutrinos).