MMP II
Exercise Sheet 1

FS 19
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Exercise 1 [Green's functions (4 points)]

The time-independent Klein-Gordon equation for massive particles ($m > 0$) is given in units where $c = \hbar = 1$ as

$$(\nabla^2 - m^2)\varphi(\vec{x}) = J(\vec{x}),$$

where $J(\vec{x})$ is a given function (source).

a) Following section 8.2 in your lecture notes, express the condition for the fundamental solution of this equation in Fourier space and find the Green's function there.

b) In order to find the solution in c) you will need to prove the identity

$$\mathcal{F}[\int f(\vec{y})g(\vec{x}-\vec{y})\mathrm{d}\vec{y}] = \hat{f}(\vec{p})\hat{g}(\vec{p})$$

where f, g are test functions and $\mathcal{F}[f]$ and $\hat{f}$ represent Fourier transforms of f.

c) Using the Green's function you recovered from part a), express the solution $\hat{\varphi}(\vec{p})$ as a convolution and then apply the above identity.

Exercise 2 [Complex roots (3 points)]

Find all $z \in \mathbb{C}$ which satisfy the equation $z^n = a$. Draw the solution in the complex plane.

a) $a = i$ for $n = 4$

b) $a = -1 - i$ for $n = 2$

Exercise 3 [Calculating with complex numbers (3.5 points)]

a) Write the following numbers in the form $x + iy$:

$$\frac{(1+i)^5}{(1-i)^5}, \quad (1+i)^n + (1-i)^n$$

Hint: Use tha fact that complex conjugation $z \mapsto \bar{z}$ is multiplicative.

b) Write the following complex numbers in polar form:

$$-\sqrt{3}, \quad 4i, \quad 1+i$$

c) Draw the following subsets of the complex plane:

$$1 < |2z-1| < 2, \quad |z-1| < |z|, \quad |Im(z)| < |z|$$

Exercise 4 [Matrix form of $\mathbb{C}$ (3 points)]

Show, that the set K of real 2×2 matrices of the form

$$\begin{pmatrix} x & -y \\ y & x \end{pmatrix} \quad x, y \in \mathbb{R}$$

together with matrix-addition and multiplication form a field, which is isomorphic to $\mathbb{C}$.