MMP II
Exercise Sheet 9

FS 19
Prof. Ph. Jetzer

A. Boitier, M. Haney, A. Primo, S. Tiwari
https://www.physik.uzh.ch/en/teaching/PHY322

Issued: 16.04.2019
Due: 30.04.2019

Exercise 1 [Invariant subgroups (3 points)]
Let G be a group. The commutator of G is define as the set $[G, G] := \{aba^{-1}b^{-1} | a, b \in G\}$. Show that

a) $[G, G]$ is an invariant (also called "normal") subgroup of G.

b) the factor group $G/[G, G]$ is abelian.

Exercise 2 [Group action (2 points)]
Let G be a group with neutral element e and let X be an arbitrary set. The mapping $\mu : G \times X \to X$ is called a (left) group action or group operation on X if $\mu(e, X) = x \; \forall x \in X$ and $\mu(h, \mu(g, x)) = \mu(hg, x) \; \forall g, h \in G$ and $x \in X$. The *stabilizer* of $x \in X$ is defined as $G_x := \{g \in G : \mu(g, x) = x\}$. Show that G_x is a subgroup of G

Exercise 3 [Representations I (6 points)]
Let ρ_1, ρ_2 be the maps

$$\rho_1 : \mathbb{R} \to GL(2, \mathbb{R}) \; : \quad \rho_1(x) = \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix} \tag{1}$$

and

$$\rho_2 : \mathbb{R}_+ \to GL(2, \mathbb{R}) \; : \quad \rho_2(x) = \begin{pmatrix} 1 & \log(x) \\ 0 & 1 \end{pmatrix} \; . \tag{2}$$

a) Of which groups are they representations?

b) Determine a non-trivial invariant subspace for ρ_1.

c) Is ρ_1 completely reducible?

Exercise 4 [Representations II (3 points)]
Are the following maps representations?

a)

$$\rho_1 : (\mathbb{R}, +) \to GL(3, \mathbb{R}) \ : \quad \rho_1(x) = \begin{pmatrix} 1 & x & x \\ 0 & 1 & x \\ 0 & 0 & 1 \end{pmatrix} . \tag{3}$$

b)

$$\rho_2 : (\mathbb{R}, +) \to GL(3, \mathbb{R}) \ : \quad \rho_2(x) = \begin{pmatrix} 1 & x & \frac{x^2}{2} \\ 0 & 1 & x \\ 0 & 0 & 1 \end{pmatrix} . \tag{4}$$

c)

$$\rho_3 : U(1) \to GL(2, \mathbb{C}) \ : \quad \rho_3(e^{i\varphi}) = \begin{pmatrix} \cos(\varphi) & -\sin(\varphi) \\ \sin(\varphi) & \cos(\varphi) \end{pmatrix} . \tag{5}$$