PHY213 - KT II
Exercise Sheet 7

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Exercise 1: CP asymmetry in neutral kaons decays

In the hypothesis of CP violation, the free-particle hamiltonian eigenstates ($|K_{S/L}\rangle$) for the neutral kaons system are written in terms of the CP eigenstates ($|K_{1/2}\rangle$) as

$$|K_S\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}\left(|K_1\rangle + \epsilon|K_2\rangle\right) \quad \text{and} \quad |K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}\left(|K_2\rangle + \epsilon|K_1\rangle\right)$$

a) Determine the time-dependent probability of the weak interaction eigenstates ($|K^0\rangle$,$|\bar{K}^0\rangle$) to decay to the CP even $|K_1\rangle$ state.

b) Show that the CP time dependent asymmetry

$$A_{\pm}(t) = \frac{\Gamma(\bar{K}^0 \to \pi^+\pi^-) - \Gamma(K^0 \to \pi^+\pi^-)}{\Gamma(\bar{K}^0 \to \pi^+\pi^-) + \Gamma(K^0 \to \pi^+\pi^-)}$$

Can be approximated for the neutral kaon system where $|\epsilon|^2 << 1$ by

$$A_{\pm}(t) = 2\mathrm{Re}\{\epsilon\} - \frac{2|\epsilon|e^{(\Gamma_S-\Gamma_L)t/2}\cos(\Delta m t - \phi)}{1+|\epsilon|^2 e^{(\Gamma_S-\Gamma_L)t}}$$

Exercise 2: CP violation in neutral kaons decays

Recall the Hamiltonian $\mathcal{H}$ for a system of oscillating neutral mesons

$$\mathcal{H} = \begin{bmatrix} M & M_{12} \\ M_{12}^* & M \end{bmatrix} + \frac{i}{2}\begin{bmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{bmatrix}$$

a) Using the fact that

$$\frac{1-\epsilon}{1+\epsilon} = \pm\sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

Show that

$$\epsilon = \frac{\mathrm{Im}\{M_{12}\} - i\mathrm{Im}\{\Gamma_{12}/2\}}{\Delta m - i\Delta\Gamma/2}$$

b) For the neutral kaons system, measurements show $\arg(\epsilon) = \phi \sim 45$ and $\Delta m \sim \Delta\Gamma/2$, use this approximation to deduce $\mathrm{Im}\{M_{12}\} >> \mathrm{Im}\{\Gamma_{12}\}$ and therefore:

$$|\epsilon| \sim \frac{1}{\sqrt{2}} \frac{\mathrm{Im}\{M_{12}\}}{\Delta m}$$

Exercise 3: Helicity suppression in pion decays

Show that the ratio of branching ratios

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)}$$

is ~ 5.5 if this interaction were mediated by a pure scalar contribution