Kern- und Teilchenphysik II
Spring Term 2016

Exercise Sheet 3

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1. Electron muon scattering

Consider the electron muon scattering:

a) Draw the allowed QED Feynman diagram/s for this process.

1 pt

b) Use the Feynman rules for QED to calculate the lowest-order matrix element for this process, where the p_1 and p_3 are the four-momenta of the initial and final state electron, and p_2 and p_4 are the four-momenta of the initial and final state muon.

1 pt

c) Working in the centre of mass frame, and writing the four-momenta of the initial and final state electron as $p_1 = (E_1, 0, 0, p)$ and $p_3 = (E_1, p\ \sin\theta, 0, p\ \cos\theta)$ respectively, show that the electron currents for the four possible helicity combinations are:

$$\bar{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1) = 2(\mathrm{E_1c}, \mathrm{ps}, -i\mathrm{ps}, \mathrm{pc}), \quad (1)$$
$$\bar{u}_\uparrow(p_3)\gamma^\mu u_\downarrow(p_1) = 2(\mathrm{ms}, 0, 0, 0), \quad (2)$$
$$\bar{u}_\uparrow(p_3)\gamma^\mu u_\uparrow(p_1) = 2(\mathrm{E_1c}, \mathrm{ps}, ips, \mathrm{pc}), \quad (3)$$
$$\bar{u}_\downarrow(p_3)\gamma^\mu u_\uparrow(p_1) = -2(\mathrm{ms}, 0, 0, 0), \quad (4)$$
$$(5)$$

where m is the mass of the electron, s = sin(θ/2) and c = cos(θ/2).

4 pt

d) Show that the muon currents for the four helicity combinations are:

$$\bar{u}_\downarrow(p_4)\gamma^\mu u_\downarrow(p_2) = 2(\mathrm{E_2c}, -\mathrm{ps}, -i\mathrm{ps}, -\mathrm{pc}), \quad (6)$$
$$\bar{u}_\uparrow(p_4)\gamma^\mu u_\downarrow(p_2) = 2(\mathrm{Ms}, 0, 0, 0), \quad (7)$$
$$\bar{u}_\uparrow(p_4)\gamma^\mu u_\uparrow(p_2) = 2(\mathrm{E_2c}, -\mathrm{ps}, ips, -\mathrm{pc}), \quad (8)$$
$$\bar{u}_\downarrow(p_4)\gamma^\mu u_\uparrow(p_2) = -2(\mathrm{Ms}, 0, 0, 0), \quad (9)$$
$$(10)$$

where M is the mass of the muon.

4 pt

e) Verify that, in the relativistic limit (E>>M), the matrix element squared for the case where the incoming electron and muon are left-handed is:

$$|\mathcal{M}_{LL}|^2 = \frac{4e^4 s^2}{(p_1 - p_3)^4}$$

where s $=(p_1 + p_2)^2$

2 pt

f) Find the corresponding expressions for $|\mathcal{M}_{\mathcal{RL}}|^2$, $|\mathcal{M}_{\mathcal{RR}}|^2$ and $|\mathcal{M}_{\mathcal{LR}}|^2$

2 pt

g) In this limit, show that the differential cross section for unpolarised $e^-\mu^- \to e^-\mu^-$ scattering in the centre-of-mass frame is:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{2\alpha^2}{s} \cdot \frac{1 + \frac{1}{4}(1 + \cos\theta)^2}{(1 - \cos\theta)^2}$$

2 pt

2. Electron proton scattering

Consider the electron proton scattering in the limit of low-energy scattering and in the approximation of point-like proton.The left-handed and right-handed helicity spinors are:

$$u_\uparrow = \sqrt{E+m}\begin{pmatrix} c \\ se^{i\phi} \\ \frac{p}{E+m}c \\ \frac{p}{E+m}se^{i}\phi \end{pmatrix}$$

$$u_\downarrow = \sqrt{E+m}\begin{pmatrix} -s \\ ce^{i\phi} \\ \frac{p}{E+m}s \\ -\frac{p}{E+m}ce^{i}\phi \end{pmatrix}$$

where s = $\sin(\theta/2)$ and c = $\cos(\theta/2)$.

a) Draw the allowed QED Feynman diagram for this process.

1 pt

b) Write the initial- and final-state spinors for the electron, considering that it is scattered by an angle θ and taking the azimuthal angle $\phi = 0$. [Hint: assume that the proton recoil is small, hence its kinetic energy is small and electron energy does not change in the scattering process].

2 pt

c) Write the electron current for the four possible helicity combinations. What does it happen in the relativistic limit?

4 pt

d) In the limit of small proton recoil (low velocity of recoiling proton) and taking $\theta = \eta$ and $\phi = \pi$ as angular direction of the recoiling proton, write the intial- and final-state spinors for the proton. [Hint: $\frac{p}{E+m} \equiv \frac{\beta\gamma}{\gamma+1}$].

1 pt

e) Write the proton four-vector currents for the four possible helicity combinations.

2 pt

3. Feynman diagrams

Draw Feynman diagrams for the following processes, labelling each quark, lepton and boson, indicating clearly the propagator.

a) $e^+ + e^- \to \mu^+ + \mu^-$

b) $\tau^- \to \nu_\tau + \pi^-$

c) $\Delta^{++} \to p + \pi^+$

d) $B^+ \to \bar{D}^0 + \pi^+$

3 pt