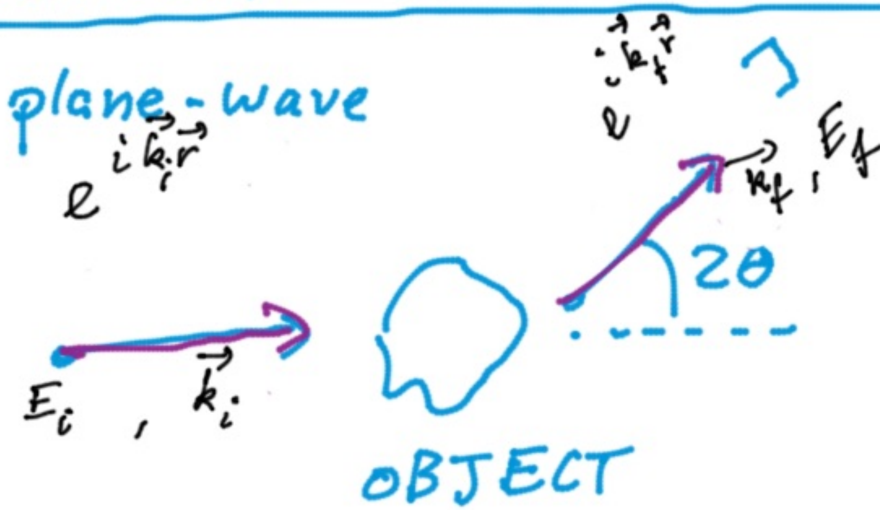
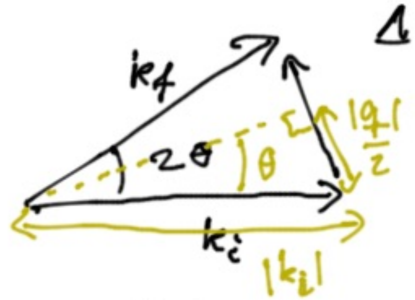
SCATTERING TRIANGLE:



WE CONSIDER ELASTIC SCATTERING: $E_i = E_f$

$\Delta\vec{k} = \vec{q} = \vec{k}_f - \vec{k}_i$ (scattering vector)



$|q| = f(\theta) = 2|k_i| \sin\theta$

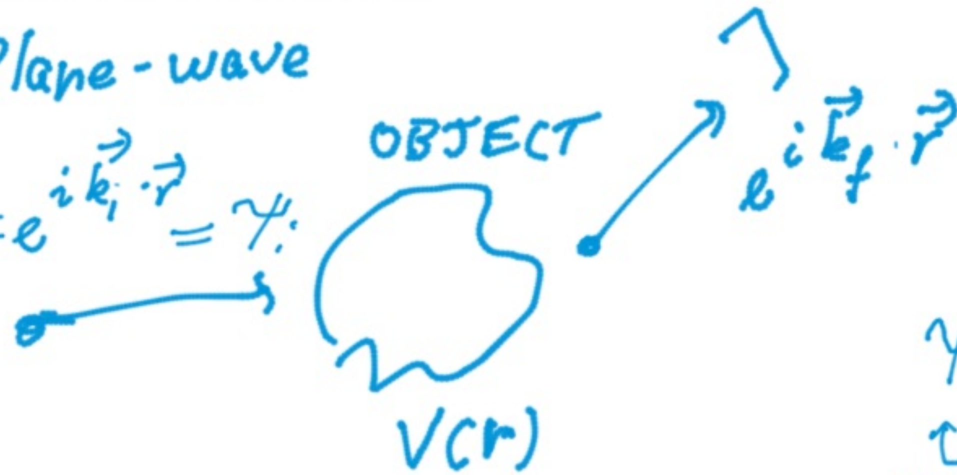
$|k_i| = |k_f|$

Created with Doceri

Scattering Potential:

Plane-wave

$\langle k_i | = e^{i\vec{k}_i \cdot \vec{r}} = \psi_i$



$\psi_i^* = | k_i \rangle$

↑ WAVE FUNCTION

↑ KET NOTATION

Fermi Golden Rule:

$$P = 2\pi \left| \langle k_f | V(r) | k_i \rangle \right|^2 \delta(E_f - E_i)$$

↑ SCATTERING PROBABILITY

Created with Doceri

FORM FACTOR:

$$F(q) = \langle k_f | V(r) | k_i \rangle$$

$$= \int e^{-i\vec{k}_f \cdot \vec{r}} V(r) e^{i\vec{k}_i \cdot \vec{r}} d\vec{r}$$

$$= \int V(r) e^{i(\vec{k}_i - \vec{k}_f)\cdot \vec{r}} d\vec{r}$$

$$= \int V(r) e^{-i\vec{q}\cdot\vec{r}} d\vec{r} = V(q)$$

Created with Doceri

SUMMARY

$$\text{Intensity} \propto P \propto F^2(q)$$

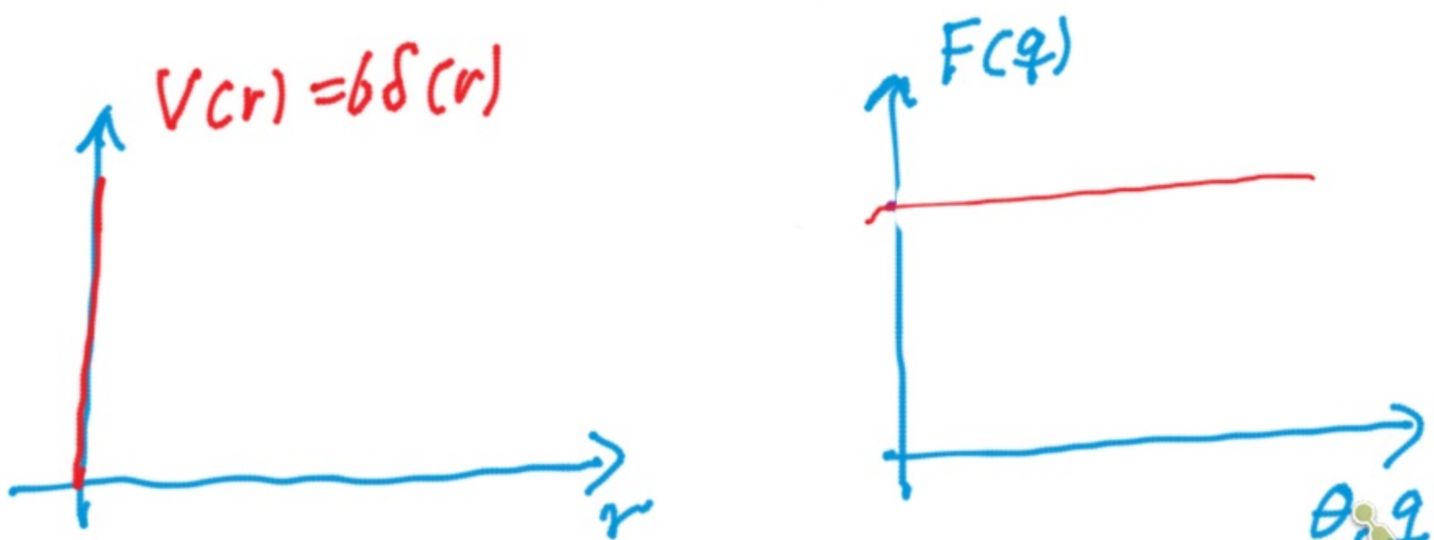
FROM SCATTERING TRIANGLE:

$$|q| = 2|k_i| \sin\theta$$

Created with Doceri

EXAMPLE 1:

NEUTRON SCATTERNG ON AN ATOMIC NEUCLEUS.

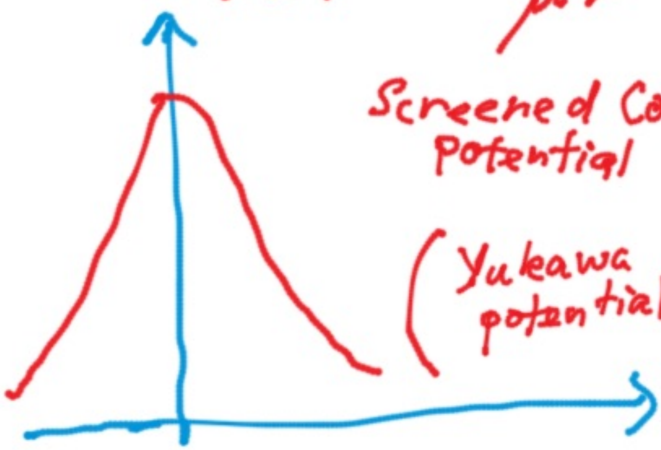


Created with Doceri

EXAMPLE 2:

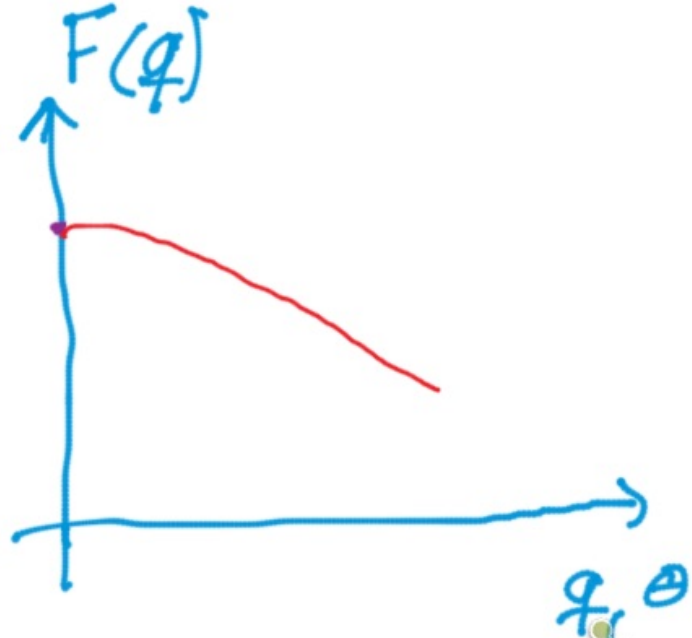
Electron scattering on an atom

$$V(r) = \frac{V_0 e^{-\mu r}}{\mu r}$$



Screened Coulomb potential

(Yukawa potential)



Created with Doceri

Scattering on a CRYSTAL:

SUM OVER ALL Atoms

$$V(r) = \sum_i V_i(r - r_i)$$

$$P = 2\pi \left| \langle k_f | V(r) | k_i \rangle \right|^2 \delta(E_f - E_i)$$

Created with Doceri

Scattering on a CRYSTAL:

$$V(r) = \sum_i V_i(r - r_i)$$

FERMI GOLDEN RULE:

$$P = 2\pi |\langle k_f | V(r) | k_i \rangle|^2 \delta(E_i - E_f)$$

Created with Doceri

SCATTERED INTENSITY:

$$\langle k_f | V(r) | k_i \rangle = \int e^{i\vec{k}_f \cdot \vec{r}} \sum_i V_i(r - r_i)\, e^{-i\vec{k}_i \cdot \vec{r}}\, dr$$

$$= \sum_i \int e^{-i\vec{k}_f \vec{r}'} V(r') e^{ik_i r'} dr'\, e^{-ik_f r_i} e^{ik_i r_i}$$

$$= F(q) \sum_i e^{i\vec{q}\,\vec{r}_i}$$

$$= F(q)\, S(q)$$

change var

$r' \equiv r - r_i$

$r = r' + r_i$

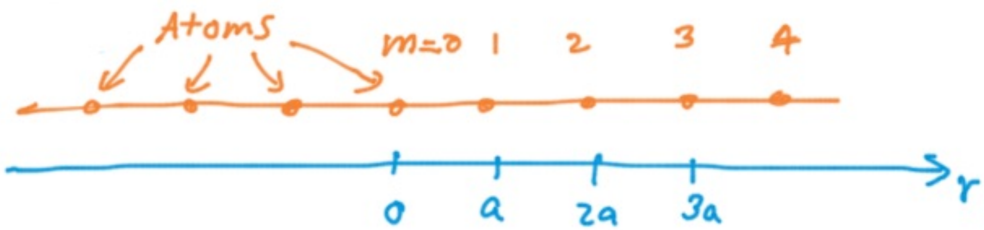
$dr = dr'$

$$S(q) = \sum_i e^{i\vec{q}\,\vec{r}_i} = \text{Structure factor}$$

Created with Doceri

STRUCTURE FACTOR:

LINEAR CHAIN EXAMPLE

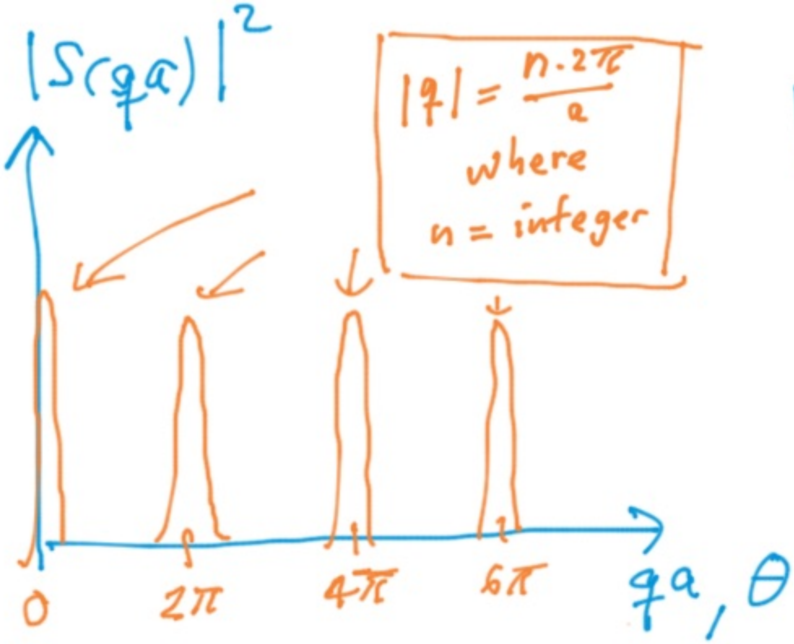


$$S(q) = \sum_i e^{-iqr_i}$$

$$= \sum_m e^{-i\vec{q}\,m\cdot a}$$

EXERCISE!!

$$|S(qa)|^2 = \frac{\sin^2\left(\frac{1}{2}M q a\right)}{\sin^2\left(\frac{1}{2} q a\right)}$$



Created with Doceri

BRAGG'S LAW:

$$|q| = n\frac{2\pi}{a} = 2|k_i| \sin\theta$$

$$\Downarrow$$

$$n \cdot \frac{2\pi}{|k_i|} = 2a \sin\theta$$

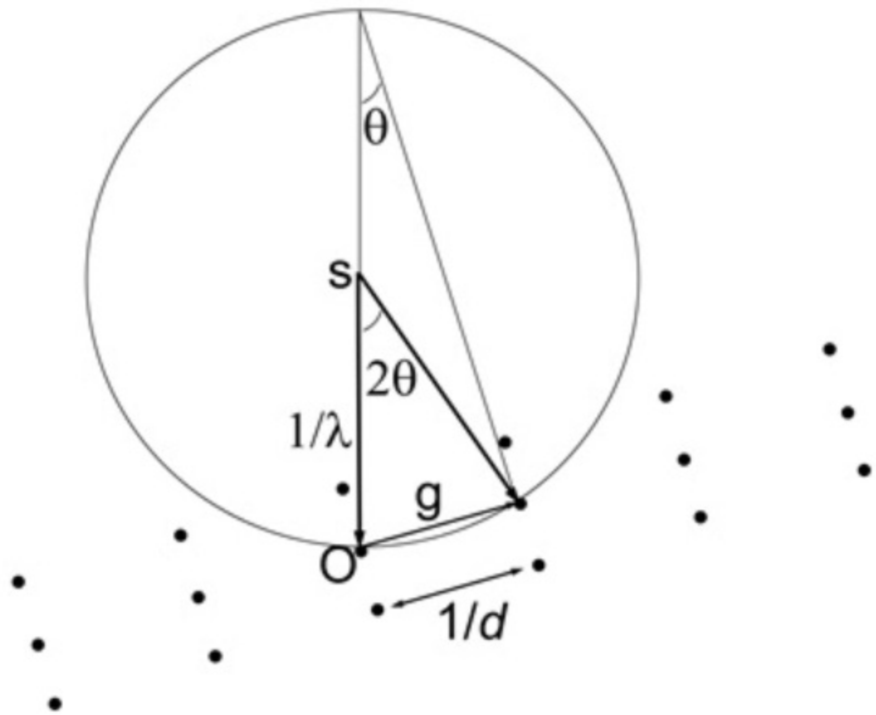
$$\Downarrow$$

$$n \cdot \lambda = 2a \sin\theta$$

$$2a \sin\theta = n \cdot \lambda$$

Created with Doceri

EWALD'S SPHERE:

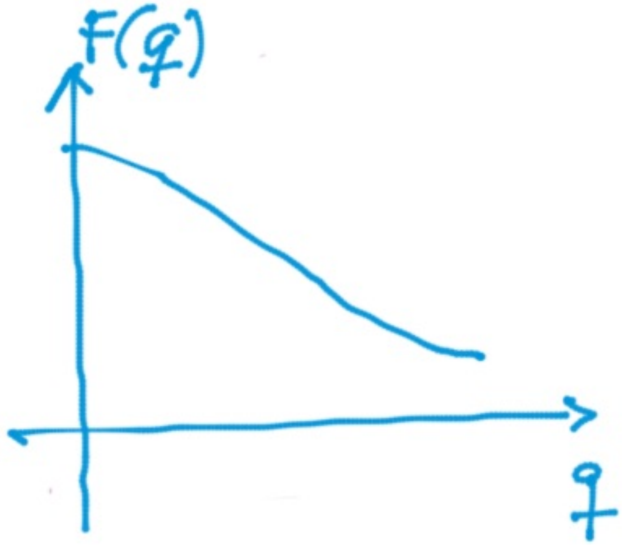


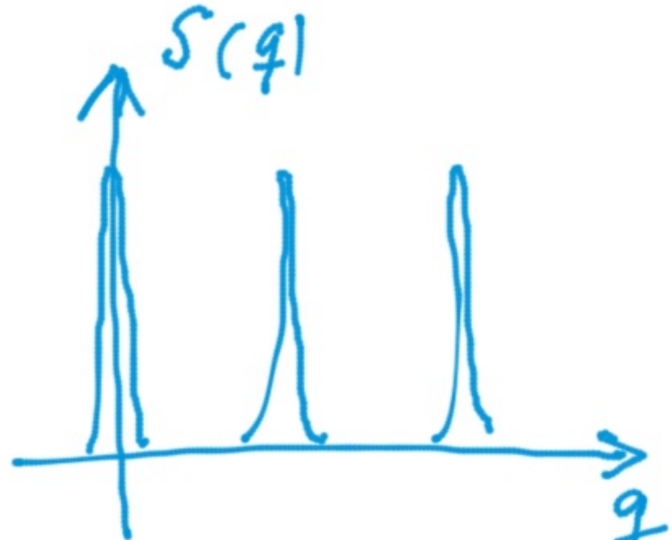
Created with Doceri

OVERVIEW

SCATTERED INTENSITY $\propto$ $F(q)\,S(q)$

_______ $\circ$ _______





Created with Doceri

STRUCTURE FACTOR:

$$S(q) = \sum_i e^{-i\vec{q}\cdot\vec{r}_i}$$

$$= N \sum_{i \in BASIS} e^{-i\vec{q}\cdot\vec{r}_i} \qquad N = \text{Number}$$

$$= N \cdot \sum_{i \in BASIS} e^{-i 2\pi (hx_i + ky_i + lz_i)}$$

$$\vec{q} = hb_1 + kb_2 + lb_3$$

Created with Doceri