**Exercise 1.** ***Parity conservation in QCD***

Consider the QCD Lagrangian with massive fermions,

$$\mathscr{L}^{\text{QCD}} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu,a} - \bar{q}_i\left(\not{D} + m_i\right)q_i + \theta\frac{g^2}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{\mu\nu,a}\,, \tag{1}$$

where all the masses $m_i$ are real (i.e. we have chosen $\theta = \bar{\theta}$).

In a volume $V$, the Euclidean path-integral formula for the ground-state energy is then

$$e^{-VE} = \int \mathscr{D}A\mathscr{D}\bar{q}\mathscr{D}q \exp\left(-\int \mathrm{d}^4x_E\ \mathscr{L}_E^{\text{QCD}}\right)\,, \tag{2}$$

where the subscript $E$ indicates Euclidean quantities.

1. By integrating out the fermions, show that the energy is minimised for $\theta = 0$.
2. Show that this argument holds for any parity-breaking operator, not only the $\theta$-term (and in particular for the axion).

Proceed as follows:

- First, show that the integrand in the Euclidean path-integral is given by
$$\exp\left[-\int \mathrm{d}^4x_E\left(\frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} + \bar{q}_i(\not{D} + m_i)q_i + \theta\frac{g^2}{32\pi^2}i\epsilon^{\mu\mu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}\right)\right]\,.$$
- Next, integrate out the fermions so to obtain the fermion determinant $\det(\not{D} + m)$. Use $\gamma_5\gamma_\mu = -\gamma_\mu\gamma_5$ to show that the non-zero eigenvalues of $i\not{D}$ are paired and use this observation to prove that the fermion determinant is positive.
- Finally, determine the minimum of the energy as a function of $\theta$.

**Exercise 2.** ***Electric dipole moment of the neutron***

The electric dipole moment of the neutron, denoted by $d_E$, is defined as

$$\langle n(p')|J^{em}_\mu|n(p)\rangle\big|_{\text{edm}} = id_E\ \bar{u}(p')\sigma_{\mu\nu}(p'-p)^\nu\gamma^5 u(p)\,. \tag{3}$$

No electric diplole moment is produced at first order in the Standard Model. The main contribution to $d_E$ comes from the $\theta$ parameter that appears in the full Lagrangian of QCD:

$$\mathscr{L}_{\text{QCD}} = \mathscr{L}_{\text{QCD}}^{(\theta=0)} + \theta\frac{g^2}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{\mu\nu,a}\,. \tag{4}$$

Estimate the neutron electric dipole moment $d_E$ as a function of $\bar{\theta} := \theta - \arg\det M$, where $M$ denotes the quark mass matrix.