Elementary Particle Theory – PHY452

Fall Semester 2020

Exercise Sheet 12

Exercise 1: The conjugate Higgs

The conjugate of the Higgs doublet $\Phi = (\phi^+, \phi_0)^T$ is defined as

$$\tilde{\Phi} := i\sigma_2 \Phi^* = \begin{pmatrix} \phi_0^* \\ -\phi^- \end{pmatrix} \tag{1}$$

where we denote $\phi^- := (\phi^+)^*$.

a) Show that $\tilde{\Phi}$ transforms like Φ under $SU(2)$.

b) What is the hypercharge of $\tilde{\Phi}$?

c) What are the electric charges of the field components $\phi_0, \phi^\pm$?

Exercise 2: Symmetry breaking of an $SU(2)$ gauge theory

The Lagrangian for a complex scalar doublet $\Phi^T = (\Phi_1, \Phi_2)$ and $SU(2)$ gauge fields A_μ^a ($a = 1, 2, 3$) is

$$\mathcal{L} \;=\; \mathcal{L}_{\text{Yang-Mills}} + \mathcal{L}_{\text{scalar}} \;=\; -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + (D_\mu \Phi)^\dagger (D^\mu \Phi) + \mu^2 \Phi^\dagger \Phi - \frac{\lambda}{4} (\Phi^\dagger \Phi)\,, \tag{2}$$

with $D_\mu \Phi = (\partial_\mu \mathbf{1} - igT^a A^a_\mu)\Phi$. For $\mu^2 > 0$, the symmetry is spontaneously broken $\langle 0|\Phi^\dagger \Phi|0\rangle = 2\mu^2/\lambda := v^2/2$. A general parameterisation for the scalar doublet is

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^+ \\ v + h + i\chi \end{pmatrix} \tag{3}$$

where ϕ^+, h and χ are scalar fields. We denote $\phi^- := (\phi^+)^*$ for the complex conjugate field.

a) Find all terms originating from $\mathcal{L}_{\text{scalar}}$ in Eq. (2) that are at most quadratic in the fields.

b) Defining a gauge-fixing function of the form

$$C^a \;=\; \partial^\mu A^a_\mu \;+\; c^a_0\, \chi \;+\; c^a_+\, \phi^+ \;+\; c^a_-\, \phi^-\,, \tag{4}$$

fix the coefficients c_0, c_+, c_- in such a way that the gauge-fixed Lagrangian $\mathcal{L} = \mathcal{L} + \mathcal{L}_{\text{fix}}$ with

$$\mathcal{L}_{\text{fix}} \;:=\; -\frac{1}{2\xi}\, C^a C^a, \tag{5}$$

does not contain spurious quadratic terms that mix the gauge and the scalar fields.

c) What are the masses of the fields A^a_μ, h, χ and $\phi^\pm$?

d) Which are the physical fields?

Exercise 3: Higgs and vector-boson interactions.

In the standard electroweak theory the vacuum excitations in unitary gauge are parametrized as

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix} \tag{6}$$

where v is the electro-weak vacuum expectation value $v = 246$ GeV and h is the Higgs field.

a) From the gauge-invariant kinetic term $(D_\mu \Phi)^\dagger D^\mu \Phi$ derive the Feynman rules for the interactions of massive gauge bosons, $W^\pm$ and Z, with the Higgs field h.

b) Compute the total width for the Higgs decay into W boson pairs, $h \to W^+ W^-$.

c) The Higgs field can only couple to photons at the 1-loop level. Draw all possible 1-loop Feynman diagrams for the Higgs decaying to a pair of photons, the so-called Higgs "golden decay channel" $h \to \gamma\gamma$.
Hint: You should find 3 different diagram topologies.