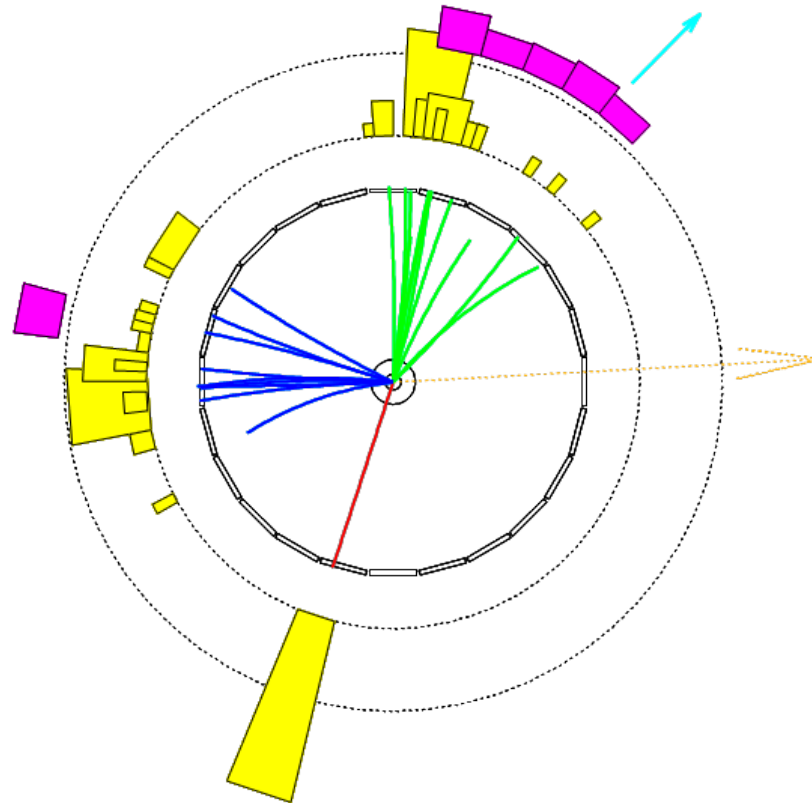# Particle Physics

Handout from Prof. Mark Thomson's lectures
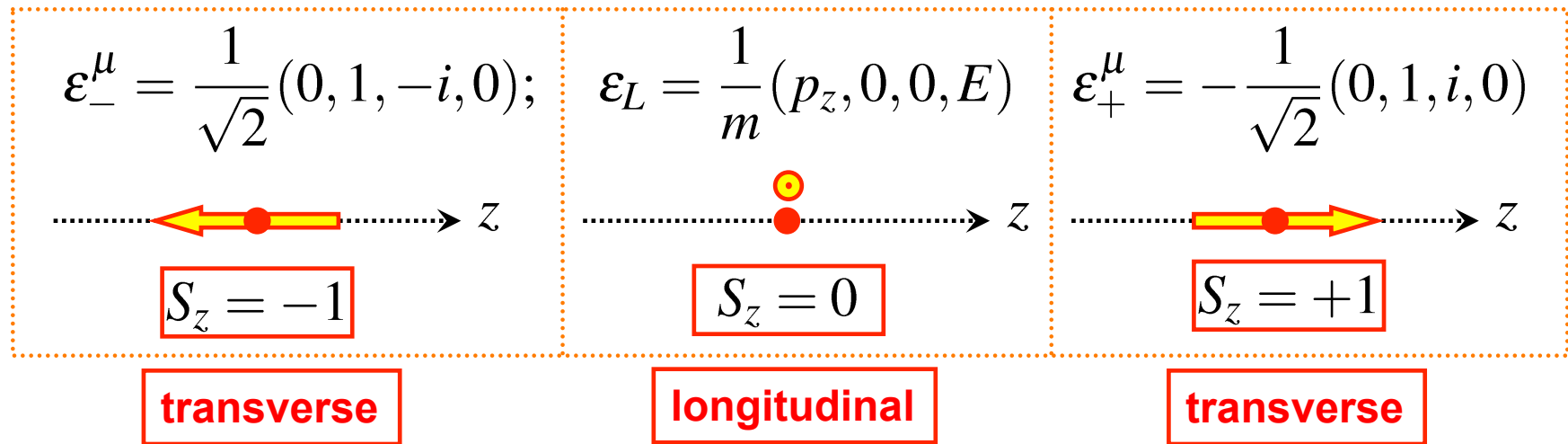**Adapted to UZH by Prof. Canelli and Prof. Serra**

Handout 13 : Electroweak Unification and the W and Z Bosons

Boson Polarization States

★ **In this handout we are going to consider the decays of W and Z bosons, for this we will need to consider the polarization. Here simply quote results although the justification is given in Appendices I and II**

★ **A real (i.e. not virtual) <u>massless</u> spin-1 boson can exist in two transverse polarization states, a massive spin-1 boson also can be longitudinally polarized**

★ **Boson wave-functions are written in terms of the polarization four-vector ε^μ**

$$\boxed{B^\mu = \varepsilon^\mu e^{-ip.x} = \varepsilon^\mu e^{i(\vec{p}.\vec{x} - Et)}}$$

★ **For a spin-1 boson travelling along the z-axis, the polarization four vectors are:**

$$\varepsilon_-^\mu = \frac{1}{\sqrt{2}}(0,1,-i,0); \qquad \varepsilon_L = \frac{1}{m}(p_z,0,0,E) \qquad \varepsilon_+^\mu = -\frac{1}{\sqrt{2}}(0,1,i,0)$$

$$\boxed{S_z = -1} \qquad \qquad \boxed{S_z = 0} \qquad \qquad \boxed{S_z = +1}$$
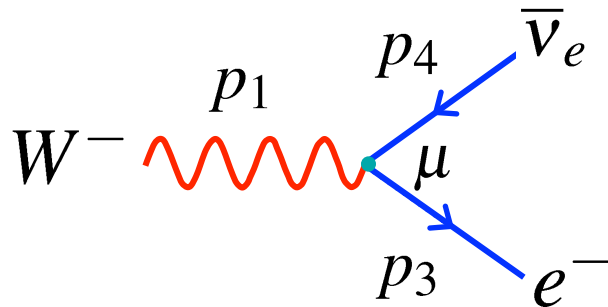
transverse   **longitudinal**   **transverse**

Longitudinal polarization isn't present for on-shell massless particles, the photon can exist in two helicity states $h = \pm 1$ (LH and RH circularly polarized light)

W-Boson Decay

★ **To calculate the W-Boson decay rate first consider** $W^- \to e^- \overline{\nu}_e$

★ **Want matrix element for :**

Incoming W-boson : $\varepsilon_\mu(p_1)$

Out-going electron : $\overline{u}(p_3)$

Out-going $\overline{\nu}_e$: $v(p_4)$

Vertex factor : $-i\dfrac{g_W}{\sqrt{2}}\dfrac{1}{2}\gamma^\mu(1-\gamma^5)$

$$-iM_{fi} = \varepsilon_\mu(p_1).\overline{u}(p_3).-i\frac{g_W}{\sqrt{2}}\gamma^\mu\tfrac{1}{2}(1-\gamma^5).v(p_4)$$

Note, no propagator

$$\Rightarrow \boxed{M_{fi} = \frac{g_W}{\sqrt{2}}\varepsilon_\mu(p_1)\overline{u}(p_3)\gamma^\mu\tfrac{1}{2}(1-\gamma^5)v(p_4)}$$

★ **This can be written in terms of the four-vector scalar product of the W-boson polarization** $\varepsilon_\mu(p_1)$ **and the weak charged current** j^μ

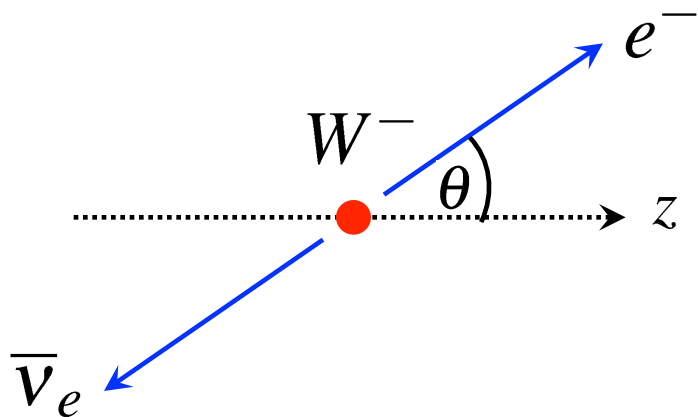$$\boxed{M_{fi} = \frac{g_W}{\sqrt{2}}\varepsilon_\mu(p_1).j^\mu}$$ **with** $$\boxed{j^\mu = \overline{u}(p_3)\gamma^\mu\tfrac{1}{2}(1-\gamma^5)v(p_4)}$$

W-Decay : The Lepton Current

★ **First consider the lepton current** $j^\mu = \overline{u}(p_3)\gamma^\mu \frac{1}{2}(1-\gamma^5)v(p_4)$

★ **Work in Centre-of-Mass frame**

$$p_1 = (m_W, 0, 0, 0);$$
$$p_3 = (E, E\sin\theta, 0, E\cos\theta)$$
$$p_4 = (E, -E\sin\theta, 0, -E\cos\theta)$$

with $\quad E = \dfrac{m_W}{2}$

★ **In the ultra-relativistic limit only <u>LH particles</u> and <u>RH anti-particles</u> participate in the weak interaction so**

$$j^\mu = \overline{u}(p_3)\gamma^\mu \frac{1}{2}(1-\gamma^5)v(p_4) = \overline{u}_\downarrow(p_3)\gamma^\mu v_\uparrow(p_4)$$

Note: $\quad \frac{1}{2}(1-\gamma^5)v(p_4) = v_\uparrow(p_4) \qquad \overline{u}(p_3)\gamma^\mu v_\uparrow(p_4) = \overline{u}_\downarrow(p_3)\gamma^\mu v_\uparrow(p_4)$

Chiral projection operator, e.g. see p.131 or p.294

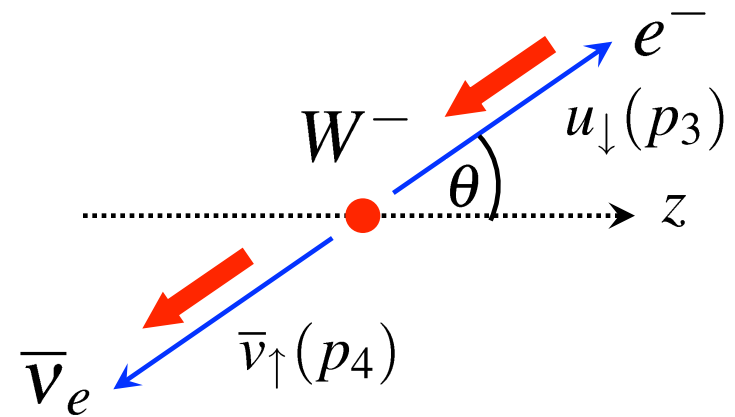**"Helicity conservation", e.g. see p.133 or p.295**

- **We have already calculated the current**

$$j^\mu = \overline{u}_\downarrow(p_3)\gamma^\mu v_\uparrow(p_4)$$

  when considering $e^+e^- \to \mu^+\mu^-$

- **From page 128 we have for** $\mu_L^- \mu_R^+$

$$\boxed{j^\mu_{\uparrow\downarrow} = 2E(0, -\cos\theta, -i, \sin\theta)}$$
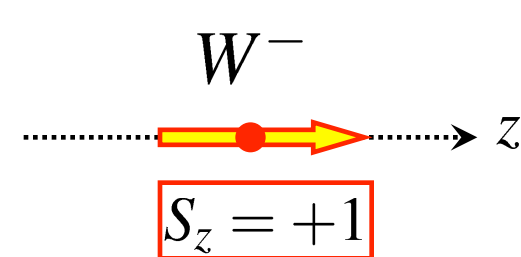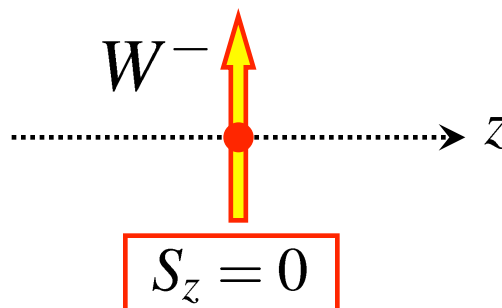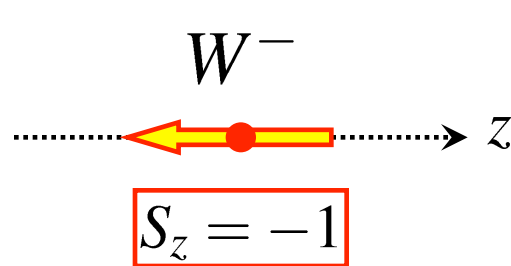
- **For the charged current weak Interaction we only have to consider this single combination of helicities**

$$j^\mu = \overline{u}(p_3)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)v(p_4) = \overline{u}_\downarrow(p_3)\gamma^\mu v_\uparrow(p_4) = 2E(0, -\cos\theta, -i, \sin\theta)$$

  and the three possible W-Boson polarization states:

$$\varepsilon^\mu_- = \frac{1}{\sqrt{2}}(0, 1, -i, 0); \quad \varepsilon_L = \frac{1}{m}(p_z, 0, 0, E) \quad \varepsilon^\mu_+ = -\frac{1}{\sqrt{2}}(0, 1, i, 0)$$

W^- W^- W^-

$\boxed{S_z = -1}$ $\boxed{S_z = 0}$ $\boxed{S_z = +1}$

★ **For a W-boson at rest these become:**

$$\varepsilon_-^\mu = \frac{1}{\sqrt{2}}(0,1,-i,0); \quad \varepsilon_L = (0,0,0,1) \quad \varepsilon_+^\mu = -\frac{1}{\sqrt{2}}(0,1,i,0)$$

★ **Can now calculate the matrix element for the different polarization states**

$$M_{fi} = \frac{g_W}{\sqrt{2}} \varepsilon_\mu(p_1) j^\mu \quad \textbf{with} \quad j^\mu = 2\frac{m_W}{2}(0,-\cos\theta,-i,\sin\theta)$$

Decay at rest : $E_e = E_\nu = m_W/2$

★ **giving**

$\boxed{\varepsilon_-}$ $M_- = \frac{g_W}{\sqrt{2}}\frac{1}{\sqrt{2}}(0,1,-i,0).m_W(0,-\cos\theta,-i,\sin\theta) = \frac{1}{2}g_W m_W(1+\cos\theta)$

$\boxed{\varepsilon_L}$ $M_L = \frac{g_W}{\sqrt{2}}(0,0,0,1).m_W(0,-\cos\theta,-i,\sin\theta) = -\frac{1}{\sqrt{2}}g_W m_W \sin\theta$

$\boxed{\varepsilon_+}$ $M_+ = -\frac{g_W}{\sqrt{2}}\frac{1}{\sqrt{2}}(0,1,i,0).m_W(0,-\cos\theta,-i,\sin\theta) = \frac{1}{2}g_W m_W(1-\cos\theta)$

➡️
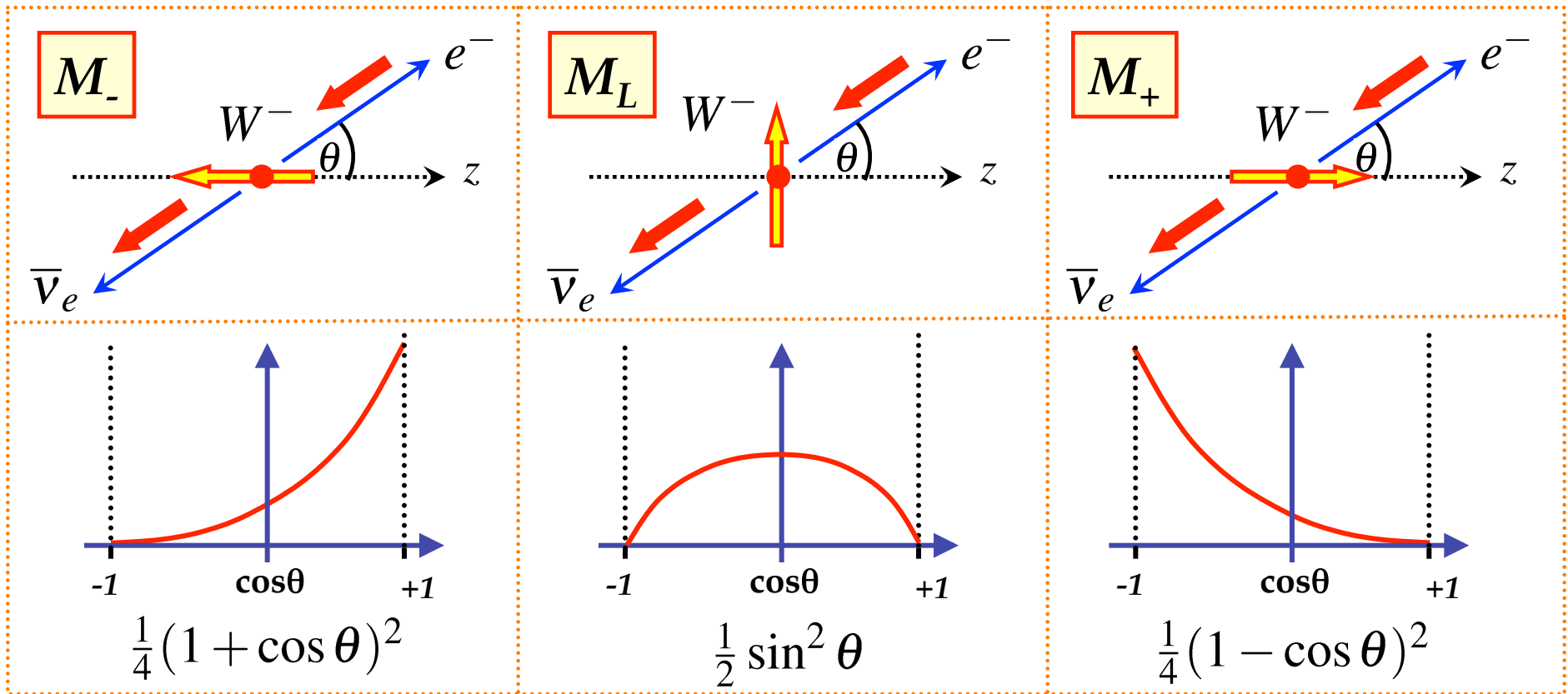$$|M_-|^2 = g_W^2 m_W^2 \tfrac{1}{4}(1+\cos\theta)^2$$
$$|M_L|^2 = g_W^2 m_W^2 \tfrac{1}{2}\sin^2\theta$$
$$|M_+|^2 = g_W^2 m_W^2 \tfrac{1}{4}(1-\cos\theta)^2$$

★ **The angular distributions can be understood in terms of the spin of the particles**

$$\frac{1}{4}(1 + \cos\theta)^2 \qquad \frac{1}{2}\sin^2\theta \qquad \frac{1}{4}(1 - \cos\theta)^2$$

★ **The differential decay rate (see page 26) can be found using:**

$$\frac{d\Gamma}{d\Omega} = \frac{|p^*|}{32\pi^2 m_W^2}|M|^2$$

where p* is the C.o.M momentum of the final state particles, here $p^* = \frac{m_W}{2}$

★ **Hence for the three different polarisations we obtain:**

$$\frac{d\Gamma_-}{d\Omega} = \frac{g_W^2 m_w}{64\pi^2}\frac{1}{4}(1+\cos\theta)^2 \qquad \frac{d\Gamma_L}{d\Omega} = \frac{g_W^2 m_w}{64\pi^2}\frac{1}{2}\sin^2\theta \qquad \frac{d\Gamma_+}{d\Omega} = \frac{g_W^2 m_w}{64\pi^2}\frac{1}{4}(1-\cos\theta)^2$$

★ **Integrating over all angles using**

$$\int \tfrac{1}{4}(1\pm\cos\theta)^2 d\phi\, d\cos\theta = \int \tfrac{1}{2}\sin^2\theta\, d\phi\, d\cos\theta = \frac{4\pi}{3}$$

★ **Gives**

$$\boxed{\Gamma_- = \Gamma_L = \Gamma_+ = \frac{g_W^2 m_W}{48\pi}}$$

★ **The total W-decay rate is independent of polarization; this has to be the case as the decay rate cannot depend on the arbitrary definition of the z-axis**

★ **For a sample of unpolarized W boson each polarization state is equally likely, for the average matrix element sum over all possible matrix elements and average over the three initial polarization states**

$$\begin{aligned}
\langle |M_{fi}|^2\rangle &= \tfrac{1}{3}(|M_-|^2 + |M_L|^2 + |M_+|^2)\\
&= \tfrac{1}{3}g_W^2 m_W^2 \left[\tfrac{1}{4}(1+\cos\theta)^2 + \tfrac{1}{2}\sin^2\theta + \tfrac{1}{4}(1-\cos\theta)^2\right]\\
&= \tfrac{1}{3}g_W^2 m_W^2
\end{aligned}$$

★ **For a sample of unpolarized W-bosons, the decay is isotropic (as expected)**

★ **For this isotropic decay**

$$\frac{d\Gamma}{d\Omega} = \frac{|p^*|}{32\pi^2 m_W^2}\langle|M|^2\rangle \implies \Gamma = \frac{4\pi|p^*|}{32\pi^2 m_W^2}\langle|M|^2\rangle$$

$$\implies \boxed{\Gamma(W^- \to e^- \overline{\nu}) = \frac{g_W^2 m_W}{48\pi}}$$

★ **The calculation for the other decay modes (neglecting final state particle masses) is same. For quarks need to account for colour and CKM matrix. No decays to top – the top mass (175 GeV) is greater than the W-boson mass (80 GeV)**

$$W^- \to e^- \overline{\nu}_e \qquad W^- \to d\overline{u} \boxed{\times 3|V_{ud}|^2} \qquad W^- \to d\overline{c} \boxed{\times 3|V_{cd}|^2}$$
$$W^- \to \mu^- \overline{\nu}_\mu \qquad W^- \to s\overline{u} \boxed{\times 3|V_{us}|^2} \qquad W^- \to s\overline{c} \boxed{\times 3|V_{cs}|^2}$$
$$W^- \to \tau^- \overline{\nu}_\tau \qquad W^- \to b\overline{u} \boxed{\times 3|V_{ub}|^2} \qquad W^- \to b\overline{c} \boxed{\times 3|V_{cb}|^2}$$

★ **Unitarity of CKM matrix gives, e.g.** $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$
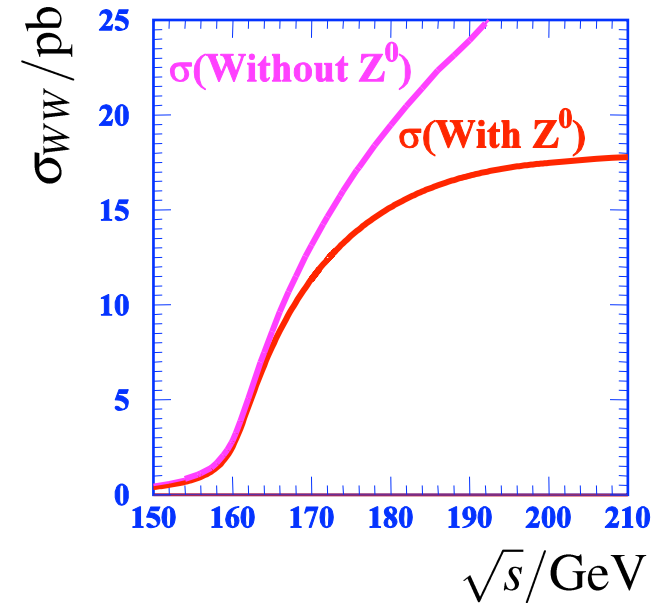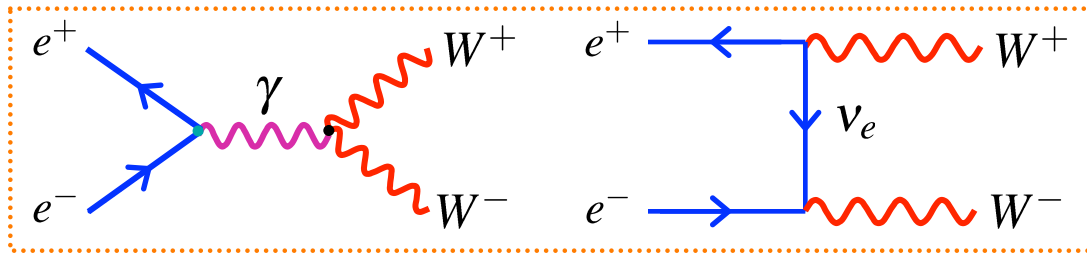
★ **Hence** $BR(W \to qq') = 6BR(W \to e\nu)$

and thus the total decay rate :

$$\boxed{\Gamma_W = 9\Gamma_{W \to e\nu} = \frac{3g_W^2 m_W}{16\pi} = 2.07\,\text{GeV}}$$

Experiment: 2.14±0.04 GeV
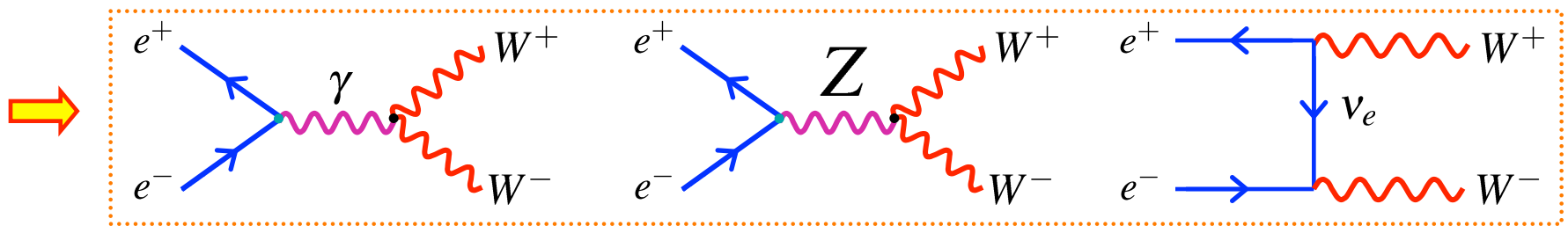(our calculation neglected a 3% QCD correction to decays to quarks)

From W to Z

★ **The W± bosons carry the EM charge - suggestive Weak are EM forces are related.**

★ **W bosons can be produced in e⁺e⁻ annihilation**

★ **With just these two diagrams there is a problem: the cross section increases with C.o.M energy and at some point violates QM unitarity**

> **UNITARITY VIOLATION: when QM calculation gives larger flux of W bosons than incoming flux of electrons/positrons**

★ **Problem can be "fixed" by introducing a new boson, the Z. The new diagram interferes negatively with the above two diagrams fixing the unitarity problem**

$$|M_{\gamma WW} + M_{ZWW} + M_{\nu WW}|^2 < |M_{\gamma WW} + M_{\nu WW}|^2$$

★ **Only works if Z, γ, W couplings are related: need ELECTROWEAK UNIFICATION**

SU(2)$_L$: The Weak Interaction

★ **The Weak Interaction arises from SU(2) local phase transformations**

$$\psi \rightarrow \psi' = \psi e^{i\vec{\alpha}(x).\frac{\vec{\sigma}}{2}}$$

where the $\vec{\sigma}$ are the generators of the SU(2) symmetry, i.e the **three** Pauli spin matrices

⟹ **3 Gauge Bosons** $\quad W_1^\mu, W_2^\mu, W_3^\mu$

★ **The wave-functions have two components which, in analogy with isospin, are represented by "weak isospin"**

★ **The fermions are placed in isospin doublets and the local phase transformation corresponds to**

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \rightarrow \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}' = e^{i\vec{\alpha}(x).\frac{\vec{\sigma}}{2}} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$$

★ **Weak Interaction only couples to LH particles/RH anti-particles, hence only place LH particles/RH anti-particles in weak isospin doublets:** $I_W = \frac{1}{2}$
RH particles/LH anti-particles placed in weak isospin singlets: $I_W = 0$

Weak Isospin

$$I_W = \frac{1}{2} \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, \begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} t \\ b' \end{pmatrix}_L \quad \begin{array}{l} \leftarrow I_W^3 = +\frac{1}{2} \\ \leftarrow I_W^3 = -\frac{1}{2} \end{array}$$

$$I_W = 0 \quad (\nu_e)_R, (e^-)_R, ...(u)_R, (d)_R, ... \qquad \text{Note: RH/LH refer to chiral states}$$

★ **For simplicity only consider** $\quad \chi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$

• **The gauge symmetry specifies the form of the interaction: one term for each of the 3 generators of SU(2) – [note: here include interaction strength in current]**

$$j_\mu^1 = g_W \overline{\chi}_L \gamma^\mu \tfrac{1}{2} \sigma_1 \chi_L \qquad j_\mu^2 = g_W \overline{\chi}_L \gamma^\mu \tfrac{1}{2} \sigma_2 \chi_L \qquad j_\mu^3 = g_W \overline{\chi}_L \gamma^\mu \tfrac{1}{2} \sigma_3 \chi_L$$

★ **The charged current W⁺/W⁻ interaction enters as a linear combinations of W₁, W₂**

W₁

$$W^{\pm\mu} = \tfrac{1}{\sqrt{2}}(W_1^\mu \pm W_2^\mu)$$

★ **The W± interaction terms**

$$j_\pm^\mu = \tfrac{g_W}{\sqrt{2}}(j_1^\mu \pm i j_2^\mu) = \tfrac{g_W}{\sqrt{2}} \overline{\chi}_L \gamma^\mu \tfrac{1}{2}(\sigma_1 \pm i\sigma_2)\chi_L$$

★ **Express in terms of the weak isospin ladder operators** $\quad \sigma_\pm = \tfrac{1}{2}(\sigma_1 \pm i\sigma_2)$

$$\boxed{j_\pm^\mu = \tfrac{g_W}{\sqrt{2}} \overline{\chi}_L \gamma^\mu \sigma_\pm \chi_L}$$

$\Big\}$ **Origin of** $\tfrac{1}{\sqrt{2}}$ **in Weak CC**

W⁺ $\qquad \nu_e \xrightarrow{\quad g_W \quad} e^- \quad W^+$ **corresponds to** $\quad \boxed{j_+^\mu = \tfrac{g_W}{\sqrt{2}} \overline{\chi}_L \gamma^\mu \sigma_+ \chi_L}$

Bars indicates adjoint spinors

which can be understood in terms of the weak isospin doublet

$$j_+^\mu = \tfrac{g_W}{\sqrt{2}} \overline{\chi}_L \gamma^\mu \sigma_+ \chi_L = \tfrac{g_W}{\sqrt{2}} (\overline{\nu}_L, \overline{e}_L) \gamma^\mu \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_L = \tfrac{g_W}{\sqrt{2}} \overline{\nu}_L \gamma^\mu e_L = \boxed{\tfrac{g_W}{\sqrt{2}} \overline{\nu} \gamma^\mu \tfrac{1}{2}(1-\gamma^5) e}$$

★ **Similarly**

$$W^-$$ $$e^- \xrightarrow{g_W} \nu_e \quad W^-$$ **corresponds to** $$j^\mu_- = \frac{g_W}{\sqrt{2}} \overline{\chi}_L \gamma^\mu \sigma_- \chi_L$$
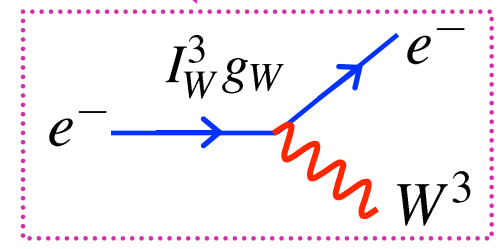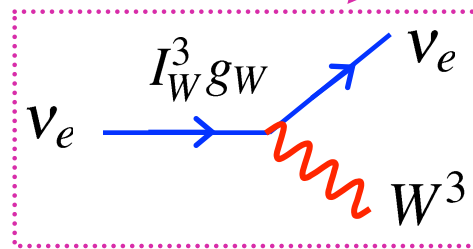
$$j^\mu_- = \frac{g_W}{\sqrt{2}} \overline{\chi}_L \gamma^\mu \sigma_- \chi_L = \frac{g_W}{\sqrt{2}} (\overline{\nu}_L, \overline{e}_L) \gamma^\mu \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_L = \frac{g_W}{\sqrt{2}} \overline{e}_L \gamma^\mu \nu_L = \frac{g_W}{\sqrt{2}} \overline{e} \gamma^\mu \tfrac{1}{2}(1 - \gamma^5) \nu$$

★**However have an additional interaction due to W³**

$$j^\mu_3 = g_W \overline{\chi}_L \gamma^\mu \tfrac{1}{2} \sigma_3 \chi_L$$

expanding this:

$$I^3_W = \pm \tfrac{1}{2}$$

$$j^\mu_3 = g_W \tfrac{1}{2} (\overline{\nu}_L, \overline{e}_L) \gamma^\mu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}_L = g_W \tfrac{1}{2} \overline{\nu}_L \gamma^\mu \nu_L - g_W \tfrac{1}{2} \overline{e}_L \gamma^\mu e_L$$

$$\nu_e \xrightarrow{I^3_W g_W} \nu_e \quad W^3$$

$$e^- \xrightarrow{I^3_W g_W} e^- \quad W^3$$

⟹ **NEUTRAL CURRENT INTERACTIONS !**

Electroweak Unification

★ **Tempting to identify the W^3 as the Z**

★ **However this is not the case, have two physical neutral spin-1 gauge bosons, γ, Z and the W^3 is a mixture of the two,**

★ **Equivalently write the photon and Z in terms of the W^3 and a new neutral spin-1 boson the B**

★ **The physical bosons (the Z and photon field, A) are:**

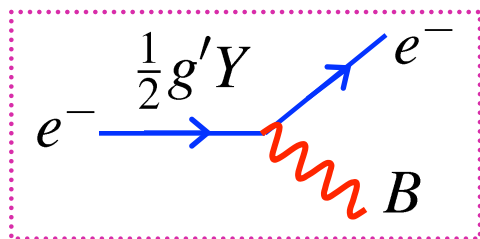$$A_\mu = B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W$$
$$Z_\mu = -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W$$

θ_W **is the weak mixing angle**

★ **The new boson is associated with a new gauge symmetry similar to that of electromagnetism : U(1)$_Y$**

★ **The charge of this symmetry is called WEAK HYPERCHARGE Y**

$$\boxed{Y = 2Q - 2I_W^3}$$

Q is the EM charge of a particle
I_W^3 is the third comp. of weak isospin

• **By convention the coupling to the B$_\mu$ is $\frac{1}{2}g'Y$**

$$e_L : Y = 2(-1) - 2(-\tfrac{1}{2}) = -1 \qquad \nu_L : Y = +1$$
$$e_R : Y = 2(-1) - 2(0) = -2 \qquad \nu_R : Y = 0$$

(this identification of hypercharge in terms of Q and I$_3$ makes all of the following work out)

★ **For this to work the coupling constants of the W³, B, and photon must be related**
  e.g. consider contributions involving the neutral interactions of electrons:

$\boxed{\gamma}$ $\quad j_\mu^{em} = e\overline{\psi}Q_e\gamma_\mu\psi = e\bar{e}_L Q_e \gamma_\mu e_L + e\bar{e}_R Q_e \gamma_\mu e_R$

$\boxed{W^3}$ $\quad j_\mu^{W^3} = -\frac{g_W}{2}\bar{e}_L\gamma_\mu e_L$

$\boxed{B}$ $\quad j_\mu^Y = \frac{g'}{2}\overline{\psi}Y_e\gamma_\mu\psi = \frac{g'}{2}\bar{e}_L Y_{e_L}\gamma_\mu e_L + \frac{g'}{2}\bar{e}_R Y_{e_R}\gamma_\mu e_R$

★ **The relation** $A_\mu = B_\mu\cos\theta_W + W_\mu^3\sin\theta_W$ **is equivalent to requiring**

$$\boxed{j_\mu^{em} = j_\mu^Y\cos\theta_W + j_\mu^{W^3}\sin\theta_W}$$

•**Writing this in full:**

$e\bar{e}_L Q_e \gamma_\mu e_L + e\bar{e}_R Q_e \gamma_\mu e_R = \frac{1}{2}g'\cos\theta_W[\bar{e}_L Y_{e_L}\gamma_\mu e_L + \bar{e}_R Y_{e_R}\gamma_\mu e_R] - \frac{1}{2}g_W\sin\theta_W[\bar{e}_L\gamma_\mu e_L]$

$-e\bar{e}_L\gamma_\mu e_L - e\bar{e}_R\gamma_\mu e_R = \frac{1}{2}g'\cos\theta_W[-\bar{e}_L\gamma_\mu e_L - 2\bar{e}_R\gamma_\mu e_R] - \frac{1}{2}g_W\sin\theta_W[\bar{e}_L\gamma_\mu e_L]$

which works if: $\boxed{e = g_W\sin\theta_W = g'\cos\theta_W}$ **(i.e. equate coefficients of L and R terms)**

★ **Couplings of electromagnetism, the weak interaction and the interaction of the**
  U(1)$_Y$ symmetry are therefore related.

The Z Boson

★**In this model we can now derive the couplings of the Z Boson**

$$Z_\mu = -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W \qquad \boxed{I_W^3} \qquad \text{for the electron } I_W^3 = \tfrac{1}{2}$$

$$j_\mu^Z = -\tfrac{1}{2} g' \sin\theta_W [\bar{e}_L Y_{e_L} \gamma_\mu e_L + \bar{e}_R Y_{e_R} \gamma_\mu e_R] - \tfrac{1}{2} g_W \cos\theta_W [e_L \gamma_\mu e_L]$$

•**Writing this in terms of weak isospin and charge:**

$$j_\mu^Z = -\tfrac{1}{2} g' \sin\theta_W [\bar{e}_L (2Q - 2I_W^3) \gamma_\mu e_L + \bar{e}_R (2Q) \gamma_\mu e_R] + I_W^3 g_W \cos\theta_W [e_L \gamma_\mu e_L]$$

For RH chiral states I$_3$=0

•**Gathering up the terms for LH and RH chiral states:**

$$j_\mu^Z = \left[g' I_W^3 \sin\theta_W - g' Q \sin\theta_W + g_W I_W^3 \cos\theta_W \right] \bar{e}_L \gamma_\mu e_L - \left[g' Q \sin\theta_W \right] e_R \gamma_\mu e_R$$

•**Using:** $\quad e = g_W \sin\theta_W = g' \cos\theta_W \quad$ **gives**

$$j_\mu^Z = \left[g' \frac{(I_W^3 - Q\sin^2\theta_W)}{\sin\theta_W} \right] \bar{e}_L \gamma_\mu e_L - \left[g' \frac{Q\sin^2\theta_W}{\sin\theta_W} \right] e_R \gamma_\mu e_R$$

$$\boxed{j_\mu^Z = g_Z(I_W^3 - Q\sin^2\theta_W)[\bar{e}_L \gamma_\mu e_L] - g_Z Q\sin^2\theta_W [e_R \gamma_\mu e_R]}$$
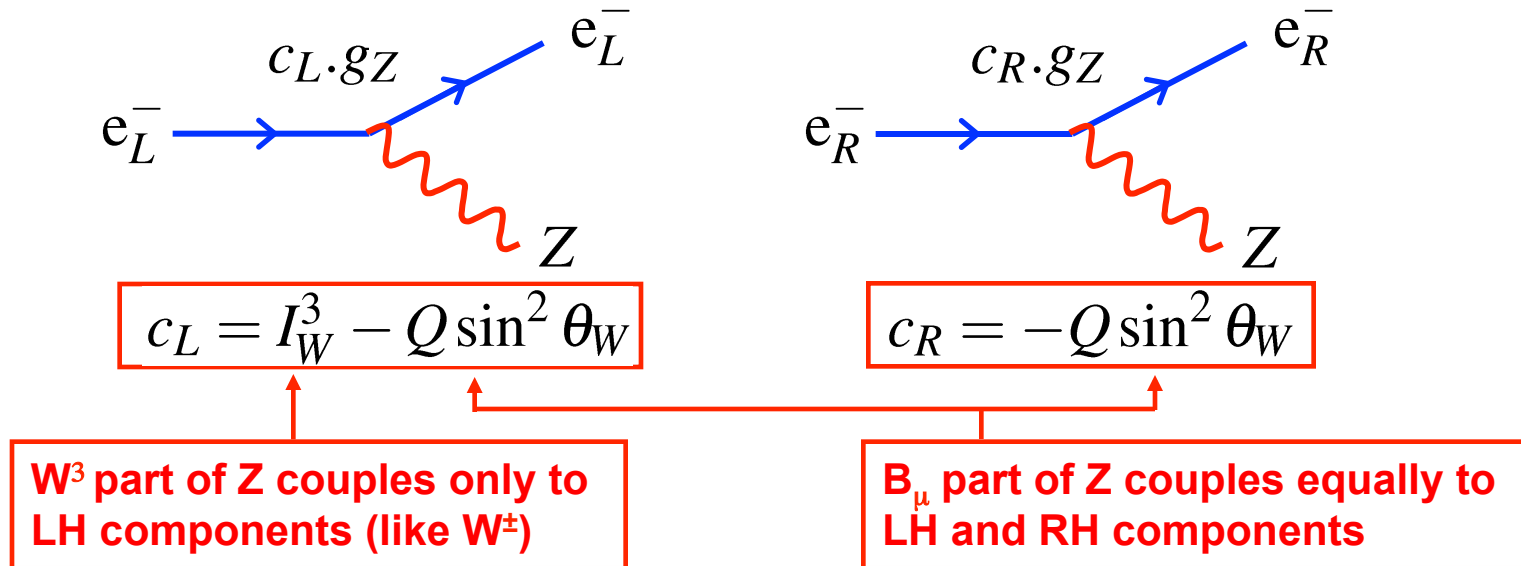
with $\quad \boxed{e = g_Z \cos\theta_W \sin\theta_W} \quad$ **i.e.** $\quad \boxed{g_Z = \dfrac{g_W}{\cos\theta_W}}$

★ **Unlike for the Charged Current Weak interaction (W) the Z Boson couples to both LH and RH chiral components, but not equally…**

$$j_\mu^Z = g_Z(I_W^3 - Q\sin^2\theta_W)[\bar{e}_L\gamma_\mu e_L] - g_Z Q\sin^2\theta_W[e_R\gamma_\mu e_R]$$

$$= g_Z c_L[\bar{e}_L\gamma_\mu e_L] + g_Z c_R[e_R\gamma_\mu e_R]$$

$$\boxed{c_L = I_W^3 - Q\sin^2\theta_W} \qquad \boxed{c_R = -Q\sin^2\theta_W}$$

W^3 part of Z couples only to LH components (like $W^\pm$)

B_μ part of Z couples equally to LH and RH components

★ **Use projection operators to obtain vector and axial vector couplings**

$$\bar{u}_L\gamma_\mu u_L = \bar{u}\gamma_\mu\tfrac{1}{2}(1-\gamma_5)u \qquad \bar{u}_R\gamma_\mu u_R = \bar{u}\gamma_\mu\tfrac{1}{2}(1+\gamma_5)u$$

$$j_\mu^Z = g_Z\bar{u}\gamma_\mu\left[c_L\tfrac{1}{2}(1-\gamma_5) + c_R\tfrac{1}{2}(1+\gamma_5))\right]u$$

$$j_\mu^Z = \frac{g_Z}{2}\overline{u}\gamma_\mu\left[(c_L + c_R) + (c_R - c_L)\gamma_5)\right]u$$

★ **Which in terms of V and A components gives:**
$$j_\mu^Z = \frac{g_Z}{2}\overline{u}\gamma_\mu\left[c_V - c_A\gamma_5\right]u$$
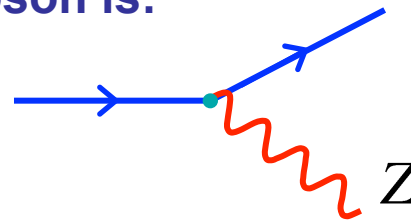
with $\boxed{c_V = c_L + c_R = I_W^3 - 2Q\sin^2\theta_W}$ $\boxed{c_A = c_L - c_R = I_W^3}$

★ **Hence the vertex factor for the Z boson is:**

$$\boxed{-ig_Z\tfrac{1}{2}\gamma_\mu\left[c_V - c_A\gamma_5\right]}$$

★ **Using the experimentally determined value of the weak mixing angle:**

$\boxed{\sin^2\theta_W \approx 0.23}$

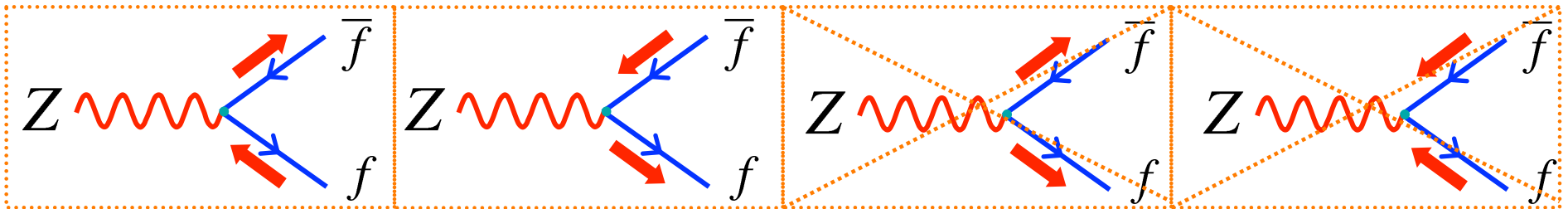| Fermion	Q	I_W^3	c_L	c_R	c_V	c_A
ν_e, ν_μ, ν_τ	0	$+\frac{1}{2}$	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$
e^-, μ^-, τ^-	-1	$-\frac{1}{2}$	-0.27	0.23	-0.04	$-\frac{1}{2}$
u, c, t	$+\frac{2}{3}$	$+\frac{1}{2}$	0.35	-0.15	$+0.19$	$+\frac{1}{2}$
d, s, b	$-\frac{1}{3}$	$-\frac{1}{2}$	-0.42	0.08	-0.35	$-\frac{1}{2}$

Z Boson Decay : Γ_Z

★ **In W-boson decay only had to consider one helicity combination of** (assuming we can neglect final state masses: helicity states = chiral states)

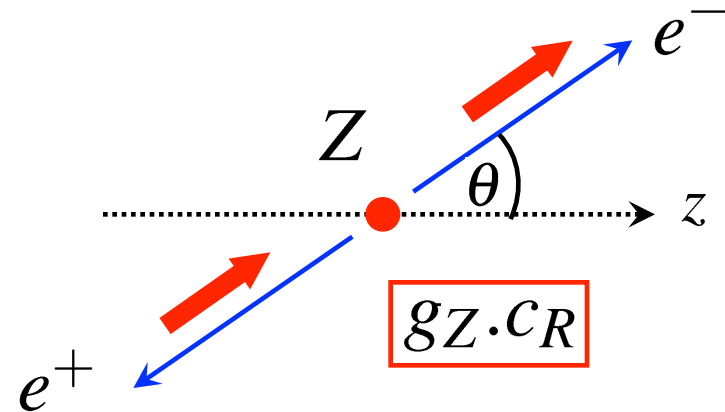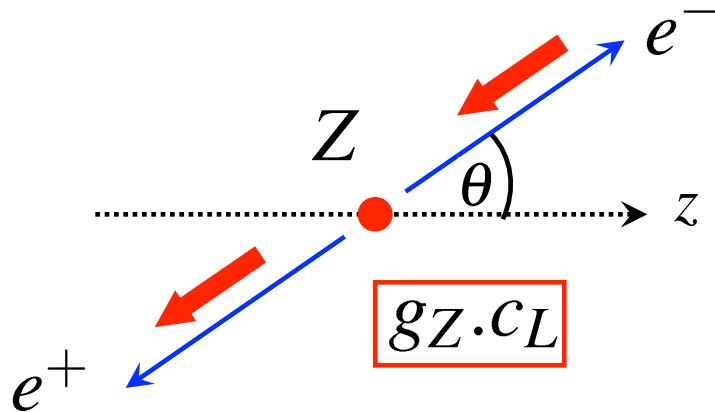**W-boson couples:**
to LH particles
and RH anti-particles

★ **But Z-boson couples to LH and RH particles (with different strengths)**
★ **Need to consider only two helicity (or more correctly chiral) combinations:**

This can be seen by considering either of the combinations which give zero

e.g.

$$\bar{u}_R \gamma^\mu (c_V + c_A \gamma_5) v_R = u^\dagger \tfrac{1}{2}(1 + \gamma^5) \gamma^0 \gamma^\mu (c_V + c_A \gamma^5) \tfrac{1}{2}(1 - \gamma^5) v$$

$$= \tfrac{1}{4} u^\dagger \gamma^0 (1 - \gamma^5) \gamma^\mu (1 - \gamma^5)(c_V + c_A \gamma^5) v$$

$$= \tfrac{1}{4} \bar{u} \gamma^\mu (1 + \gamma^5)(1 - \gamma^5)(c_V + c_A \gamma_5) v = 0$$

★ **In terms of left and right-handed combinations need to calculate:**

★ **For unpolarized Z bosons:**

$$\langle |M_{fi}|^2 \rangle = \tfrac{1}{3}[2c_L^2 g_Z^2 m_Z^2 + 2c_R^2 g_Z^2 m_Z^2] = \tfrac{2}{3} g_Z^2 m_Z^2 (c_L^2 + c_R^2)$$

average over polarization

★ **Using** $\quad c_V^2 + c_A^2 = 2(c_L^2 + c_R^2) \quad$ **and** $\quad \dfrac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \dfrac{|p^*|}{32\pi^2 m_W^2}|M|^2$

$$\Rightarrow \quad \boxed{\Gamma(Z \to e^+ e^-) = \dfrac{g_Z^2 m_Z}{48\pi}(c_V^2 + c_A^2)}$$

Z Branching Ratios

★ **(Neglecting fermion masses) obtain the same expression for the other decays**

$$\Gamma(Z \to f\overline{f}) = \frac{g_Z^2 m_Z}{48\pi}(c_V^2 + c_A^2)$$

- **Using values for c_V and c_A on page 471 obtain:**

$$Br(Z \to e^+e^-) = Br(Z \to \mu^+\mu^-) = Br(Z \to \tau^+\tau^-) \boxed{\approx 3.5\%}$$

$$Br(Z \to \nu_1\overline{\nu}_1) = Br(Z \to \nu_2\overline{\nu}_2) = Br(Z \to \nu_3\overline{\nu}_3) \boxed{\approx 6.9\%}$$

$$Br(Z \to d\overline{d}) = Br(Z \to s\overline{s}) = Br(Z \to b\overline{b}) \boxed{\approx 15\%}$$

$$Br(Z \to u\overline{u}) = Br(Z \to c\overline{c}) \boxed{\approx 12\%}$$

- **The Z Boson therefore predominantly decays to hadrons**

$$Br(Z \to \text{hadrons}) \approx 69\%$$

Mainly due to factor 3 from colour

- **Also predict total decay rate (total width)**

$$\Gamma_Z = \sum_i \Gamma_i = 2.5\,\text{GeV}$$

Experiment: $\boxed{\Gamma_Z = 2.4952 \pm 0.0023\,\text{GeV}}$

Summary

★ **The Standard Model interactions are mediated by spin-1 gauge bosons**
★ **The form of the interactions are completely specified by the assuming an underlying local phase transformation** ➡ **GAUGE INVARIANCE**

$\boxed{\textbf{U(1)}_{\textbf{em}}}$	➡	**QED**
$\boxed{\textbf{SU(2)}_{\textbf{L}}}$	➡	**Charged Current Weak Interaction + W³**
$\boxed{\textbf{SU(3)}_{\textbf{col}}}$	➡	**QCD**

★ **In order to "unify" the electromagnetic and weak interactions, introduced a new symmetry gauge symmetry : U(1) hypercharge**

$\boxed{\textbf{U(1)}_{\textbf{Y}}}$ ➡ $\boxed{\textbf{B}_{\mu}}$

★ **The physical Z boson and the photon are mixtures of the neutral W boson and B determined by the Weak Mixing angle**

$$\boxed{\sin\theta_W \approx 0.23}$$

★ **Have we really unified the EM and Weak interactions ? Well not really…**
- **Started with two independent theories with coupling constants** g_W, e
- **Ended up with coupling constants which are related but at the cost of introducing a new parameter in the Standard Model** θ_W
- **Interactions not unified from any higher theoretical principle… but it works!**

Appendix I : Photon Polarization

• **For a free photon (i.e.** $j^\mu = 0$ **) equation (A7) becomes**

$$\boxed{\Box^2 A^\mu = 0}$$ **(B1)**

(note have chosen a gauge where the Lorentz condition is satisfied)

★ **Equation (A8) has solutions (i.e. the wave-function for a free photon)**

$$A^\mu = \varepsilon^\mu(q)e^{-iq.x}$$

where ε^μ **is the four-component polarization vector and** q **is the photon four-momentum**

$$0 = \Box^2 A^\mu = -q^2 \varepsilon^\mu e^{-iq.x}$$
$$\implies q^2 = 0$$

★ **Hence equation (B1) describes a massless particle.**
★ **But the solution has four components – might ask how it can describe a spin-1 particle which has three polarization states?**

★ **But for (A8) to hold we must satisfy the Lorentz condition:**

$$0 = \partial_\mu A^\mu = \partial_\mu(\varepsilon^\mu e^{-iq.x}) = \varepsilon^\mu \partial_\nu(e^{-iq.x}) = -i\varepsilon^\mu q_\mu e^{-iq.x}$$

Hence the Lorentz condition gives $\boxed{q_\mu \varepsilon^\mu = 0}$ **(B2)**

i.e. only 3 independent components.

★ **However, in addition to the Lorentz condition still have the addional gauge freedom of** $A_\mu \to A'_\mu = A_\mu + \partial_\mu \Lambda$ **with (A8)** $\square^2 \Lambda = 0$

•**Choosing** $\Lambda = iae^{-iq.x}$ **which has** $\square^2 \Lambda = q^2 \Lambda = 0$

$$
\begin{aligned}
A_\mu \to A'_\mu = A_\mu + \partial_\mu \Lambda &= \varepsilon_\mu e^{-iq.x} + ia\partial_\mu e^{-iq.x} \\
&= \varepsilon_\mu e^{-iq.x} + ia(-iq_\mu)e^{-iq.x} \\
&= (\varepsilon_\mu + aq_\mu)e^{-iq.x}
\end{aligned}
$$

★ **Hence the electromagnetic field is left unchanged by**

$$
\varepsilon_\mu \to \varepsilon'_\mu = \varepsilon_\mu + aq_\mu
$$

★ **Hence the two polarization vectors which differ by a mulitple of the photon four-momentum describe the same photon. Choose** a **such that the time-like component of** ε_μ **is zero, i.e.** $\varepsilon_0 \equiv 0$

★ **With this choice of gauge, which is known as the COULOMB GAUGE, the Lorentz condition (B2) gives**

$$
\boxed{\vec{\varepsilon} \cdot \vec{q} = 0}
$$

(B3)

i.e. only 2 independent components, both transverse to the photons momentum

★ **A massless photon has two transverse polarisation states. For a photon travelling in the z direction these can be expressed as the transversly polarized states:**

$$\varepsilon_1^\mu = (0,1,0,0); \qquad \varepsilon_2^\mu = (0,0,1,0)$$

★ **Alternatively take linear combinations to get the circularly polarized states**

$$\varepsilon_-^\mu = \frac{1}{\sqrt{2}}(0,1,-i,0); \qquad \varepsilon_+^\mu = -\frac{1}{\sqrt{2}}(0,1,i,0)$$

★ **It can be shown that the ε_+ state corresponds the state in which the photon spin is directed in the +z direction, i.e. $S_z = +1$**

Appendix II : Massive Spin-1 particles

- **For a massless photon we had (before imposing the Lorentz condition) we had from equation (A5)**

$$\Box^2 A^\mu - \partial^\mu(\partial_v A^v) = j^\mu$$

★ **The Klein-Gordon equation for a spin-0 particle of mass m is**

$$(\Box^2 + m^2)\phi = 0$$

    suggestive that the appropriate equations for a massive spin-1 particle can be obtained by replacing $\Box^2 \rightarrow \Box^2 + m^2$

★ **This is indeed the case, and from QFT it can be shown that for a massive spin 1 particle equation (A5) becomes**

$$(\Box^2 + m^2)B^\mu - \partial^\mu(\partial_v B^v) = j^\mu$$

★ **Therefore a free particle must satisfy**

$$(\Box^2 + m^2)B^\mu - \partial^\mu(\partial_v B^v) = 0 \qquad \textbf{(B4)}$$

- **Acting on equation (B4) with** ∂_ν **gives**

$$(\Box^2 + m^2)\partial_\mu B^\mu - \partial_\mu \partial^\mu (\partial_\nu B^\nu) = 0$$

$$(\Box^2 + m^2)\partial_\mu B^\mu - \Box^2(\partial_\nu B^\nu) = 0$$

$$m^2 \partial_\mu B^\mu = 0 \qquad \text{(B5)}$$

★ **Hence, for a massive spin-1 particle, unavoidably have** $\partial_\mu B^\mu = 0$ **; note this is not a relation that reflects to choice of gauge.**

- **Equation (B4) becomes**

$$\boxed{(\Box^2 + m^2)B^\mu = 0} \qquad \text{(B6)}$$

★ **For a free spin-1 particle with 4-momentum,** p^μ **, equation (B6) admits solutions**

$$B_\mu = \varepsilon_\mu e^{-ip.x}$$

★ **Substituting into equation (B5) gives**

$$\boxed{p_\mu \varepsilon^\mu = 0}$$

★**The four degrees of freedom in** ε^μ **are reduced to three, but for a massive particle, equation (B6) does <u>not</u> allow a choice of gauge and we can not reduce the number of degrees of freedom any further.**

★ **Hence we need to find three orthogonal polarisation states satisfying**

$$\boxed{p_\mu \varepsilon^\mu = 0} \tag{B7}$$

★ **For a particle travelling in the z direction, can still admit the circularly polarized states.**

$$\varepsilon_-^\mu = \frac{1}{\sqrt{2}}(0, 1, -i, 0); \qquad \varepsilon_+^\mu = -\frac{1}{\sqrt{2}}(0, 1, i, 0)$$

★ **Writing the third state as**

$$\varepsilon_L^\mu = \frac{1}{\sqrt{\alpha^2 + \beta^2}}(\alpha, 0, 0, \beta)$$

equation (B7) gives $\quad \alpha E - \beta p_z = 0$

$$\implies \qquad \varepsilon_L^\mu = \frac{1}{m}(p_z, 0, 0, E)$$

★ **This <u>longitudinal</u> polarisation state is only present for massive spin-1 particles, i.e. there is no analogous state for a free photon (although off-mass shell virtual photons can be longitudinally polarized – a fact that was alluded to on page 114).**