PHY213 - KT II
Exercise Sheet 2

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Prof. N. Serra

D. Lancierini
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Exercise 1: Symmetries of the Dirac Lagrangian

Consider the Dirac Lagrangian

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \tag{1}$$

a) Derive the Noether current associated to the variation of the fields

$$\psi \to \psi' = exp(i\alpha\gamma^5)\psi \tag{2}$$

b) Show that the Dirac Lagrangian is invariant under such transformation only if it describes massless leptons

Exercise 2: The decay of the Z boson

The matrix element for the decay of the Z boson in two fermions is given by:

$$\mathcal{M}^\mu = \frac{-ig_Z}{2}\bar{u}(p)\gamma^\mu(c_V - c_A\gamma^5)v(p') \tag{3}$$

Where p, p' are the four momenta of the outgoing leptons, such that $p^2 = p'^2 = m_f^2$, and $q = p + p'$ such that $q^2 = m_Z^2$ is the four momentum of the decaying Z boson.

a) Write $\mathcal{M}^{*\nu}$, the complex conjugate of the amplitude

b) Using the Casimir's trick, it is found that

$$\sum_{spins} \mathcal{M}^\mu \mathcal{M}^{*\nu} = \frac{g_Z^2}{4}\mathrm{Tr}\left[(\gamma^\alpha p_\alpha + m)\gamma^\nu(c_V - c_A\gamma^5)(\gamma^\alpha p'_\alpha - m)(c_V + c_A)\gamma^\mu\right] \tag{4}$$

Using the massless fermions approximation, and the following identities derive the expression for the square modulus of the amplitude $|\mathcal{M}|^2$
Useful relations:

$$\gamma^\mu\gamma_\mu = -2I$$
$$\mathrm{Tr}[\gamma^\alpha\gamma^\beta\gamma^5] = 0$$

$$\mathrm{Tr}[\gamma^\alpha p_\alpha \gamma^\beta p'_\beta] = 4p \cdot p'$$

c) The differential decay width is defined as:

$$d\Gamma = \frac{1}{2m_Z}|\mathcal{M}|^2 d\phi_2$$

Using the result from the 2 body phase space calculation

$$d\phi_2 = \frac{1}{32\pi^2} d(cos\theta)d\phi$$

Derive the partial decay width of the Z boson to the massless fermion f