

# Today's program

(1) Tasks & Course overview

(2) Exam structure

(3) Quantum oscillations (continued)

(4) Magneto-resistance

#	Dates	Title	Exercise	(1=easy, 10=hard)	<b>Tasks</b>
1	22.2	Introduction	VESTA	<b>2-3</b>	Read Chap. 1
2	01.3	Crystal structures	Daniel - info	<b>4</b>	Read Chap. 2, Ex. 1
3	08.03	Reciprocal space	Discuss Ex. 1	<b>6</b>	Read Chap. 2, Ex. 2
4	15.03	Scattering Theory	Discuss Ex. 2	<b>8-9</b>	Read Chap. 3, Ex. 3
5	22.03	Crystal bindings	Discuss Ex. 3	<b>5</b>	Read Chap. 4, Ex. 4
6	29.03	Phonons	Discuss Ex. 4	<b>5-6</b>	Read Chap. 5, Ex. 5
7	05.04	Thermal properties	Discuss Ex. 5	<b>5-6</b>	Read Chap. 6, Ex. 6
8	12.04	Electron gasses, $C_{el}$	Discuss Ex. 6	<b>5-6</b>	Read Chap. 7, Ex. 7
--	19.04	EASTER HOLIDAY	-----	<b>0</b>	<b>RECAP</b>
9	26.04	Electronic band struc.	Discuss Ex. 7	<b>5-6</b>	Read Chap. 8, Ex. 8
10	03.05	Semi-conductors	Discuss Ex. 8	<b>6</b>	Read Chap. 9, Ex. 9
11	10.05	Fermi surfaces & Metals - I	Discuss Ex. 9	<b>8</b>	Read Chap. 9, Ex. 10
12	17.05	Fermi surfaces & Metals - II	Discuss Ex. 10	<b>8</b>	Read Chap. 9, Ex. 11
13	24.05	Guest lecture	Discuss Ex. 11	--	
14	31.05	Repetition		<b>4</b>	

#	Dates	Title	Tasks
10	03.05	Semi-conductors	<p>Read Chap. 6: Motion in magnetic fields p. 163-167</p> <p>Read Chap. 9: Introduction to Fermi surfaces p. 235-244</p> <p>Read Chap. 9: Experimental methods in FS studies p. 255-265</p>
11	10.05	Fermi surfaces & Metals - I	<p>Read Chap. 9: Experimental methods in FS studies p. 255-265</p>
12	17.05	Fermi surfaces & Metals - II	<p>Read Chap. 9 : Calculation of energy bands 244 -255</p>
13	24.05	Guest lecture	<p><b>Electronic band structure: Tight-binding model</b></p>
14	31.05	Repetition	

# Course overview

## Crystal lattice phenomena's

- Crystal structures ( Real and reciprocal space)
- Scattering theory ( Bragg's law, Form Factor, Structure factor)
- Crystal bindings ( Equilibrium lattice constants, binding energies)
- Lattice vibrations ( Phonon dispersions, density of state, heat capacity)

## Electronic phenomena's

- Free electron gas ( Fermi Dirac distribution, density of states)
- Band structure ( electronic masses, Fermi surfaces)
- Electronic measurements ( Heat capacity, resistivity, Hall effect, quantum oscillations)
- Electronic phases ( metals, semi-metals, semi-conductors, band insulators)

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(1) Tasks & Course overview

**(2) Exam structure**

(3) Quantum oscillations (continued)

(4) Magneto-resistance

# Exam – time line

31th of May (13h00-17h00) – Last lecture: Repetition of course content

6<sup>th</sup> of June (14.00-17.00) in Y36-J-33 – Questions and Answers session with Stefan

**8-9th of June (9h00 – 17h30) in Y-36-H-48 – Oral exam**

9th of June (17h30): Beer in StudiBar

## **MY AVAILABILITY BEFORE EXAM:**

1<sup>st</sup> of June in office.

After 1<sup>st</sup> of June no around but can be reached on:

[johan.chang@physik.uzh.ch](mailto:johan.chang@physik.uzh.ch)

skypename: johan.chang7

# Exam Structure

## **~7 min – Presentation:**

- Topics:
- (1) Crystal structures,
  - (2) Crystal Bindings,
  - (3) Reciprocal lattice+ scattering theory,
  - (4) Crystal vibrations (Phonons),
  - (5) Heat capacity
  - (6) Band structure
  - (7) Semiconductors
  - (8) Resistivity & Hall effect

## **~7 min – Discussion 1:**

Questions to the lecture material (Example next slide)

## **~7 min – Discussion 2:**

Questions to the exercises (Example next slide)

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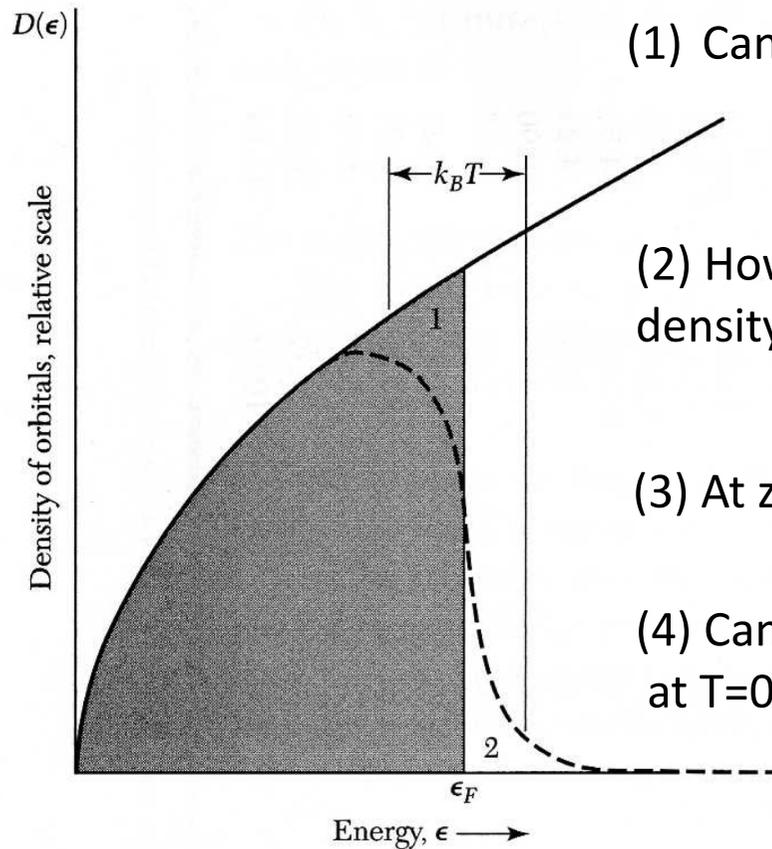
End Exam

**5 min** - evaluation

**2 min** – Results: Passed / failed, grade will be known at a later point.

# Question example:

## Electronic density of state & Fermi Dirac



(1) Can you draw or state the dispersion of a free electron?

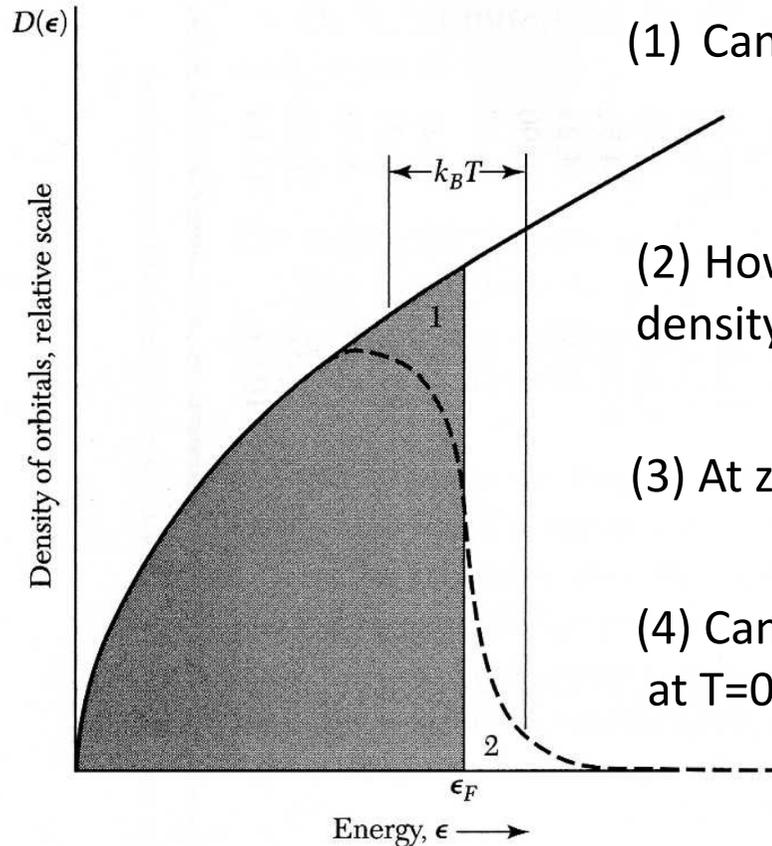
(2) How would you derive (rough arguments) the density of states for a free electron gas?

(3) At zero – temperature how is the Fermi energy defined?

(4) Can you draw schematically the Fermi Dirac distribution at  $T=0$  and  $T=300\text{ K}$ ?

# Discussion 1:

## Electronic density of state & Fermi Dirac



(1) Can you draw or state the dispersion of a free electron?

(2) How would you derive (rough arguments) the density of states for a free electron gas?

(3) At zero – temperature how is the Fermi energy defined?

(4) Can you draw schematically the Fermi Dirac distribution at  $T=0$  and  $T=300\text{ K}$ ?

# Discussion 2:

## Exercise 5 *Sphere packings*

Calculate the ratio  $c/a$  of an ideal hexagonal dense sphere packing (hcp) and its packing density.

Compare the packing density to that of an fcc lattice and explain your findings.

# Questionnaire

The university is asking us / you to evaluate this course:  
Some of you have already given feedback using the link below:

Festkörperphysik (PHY210.1, VVZ-Nr. 1872)

– Link: <https://www.sae.uzh.ch/lvb/UU6MC>

It would be great to have the opinions of as many of you as possible  
before 21th of May where the evaluation is closing.

# Today's program

(1) Tasks & Course overview

(2) Exam structure

**(3) Quantum oscillations (continued)**

(4) Magneto-resistance

# Summary of previous lecture

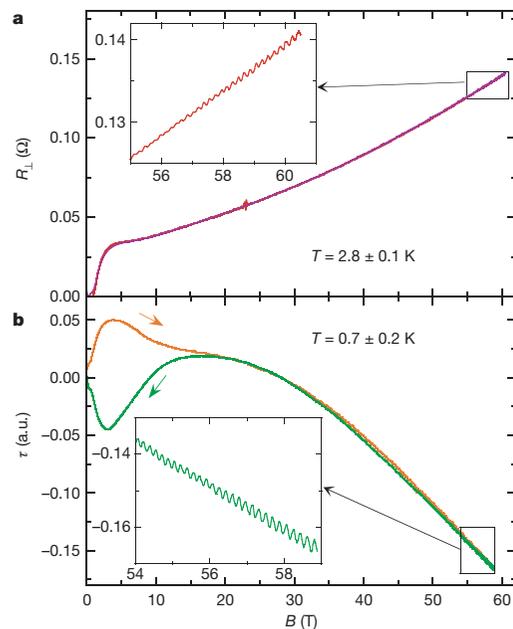
## (1) Quantum Oscillation experiments:

### **Exercise 2** *Quantum oscillations on quasi two-dimensional systems*

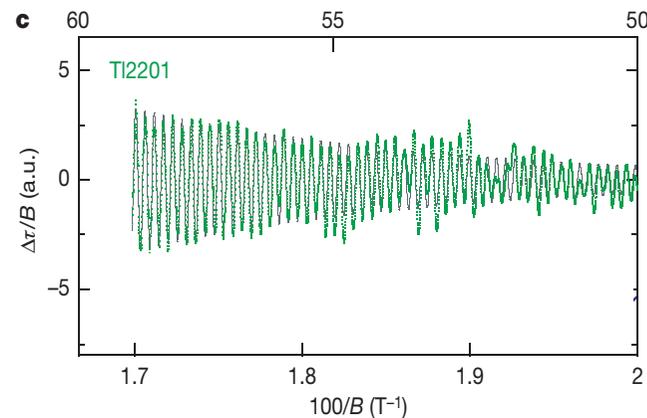
In  $\text{Ti}_2\text{Ba}_2\text{CuO}_{6+\delta}$ , quantum oscillations with a frequency of  $F = 18.1 \text{ kT}$  are observed (B. Vignolle et al., Nature **455**, 952-955 (2008)).

- Use the Onsager relation ( $S = 2\pi \frac{eF}{\hbar}$ ) to calculate the Fermi surface area.
- If we assume a circular Fermi surface shape what is the Fermi momentum?

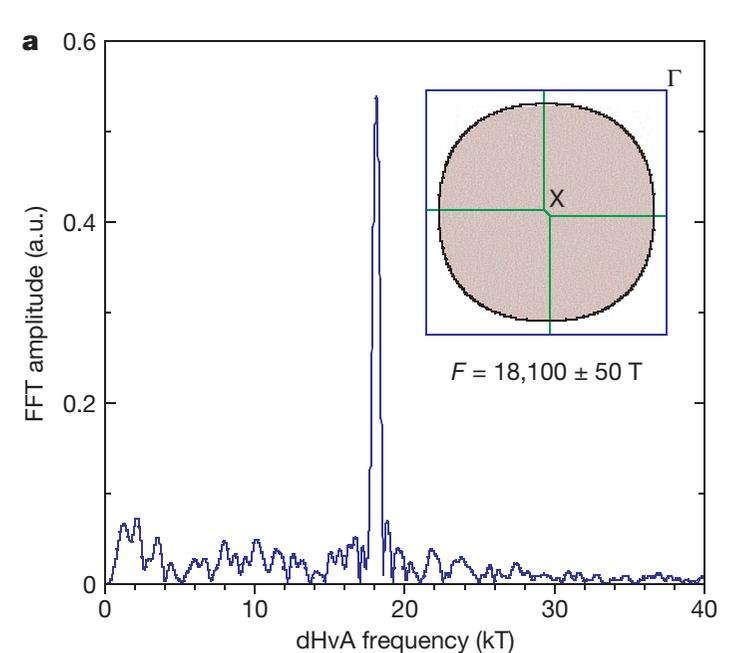
**(a) RAW DATA**



**(b) OSCILATIONS VERSUS  $1/B$**



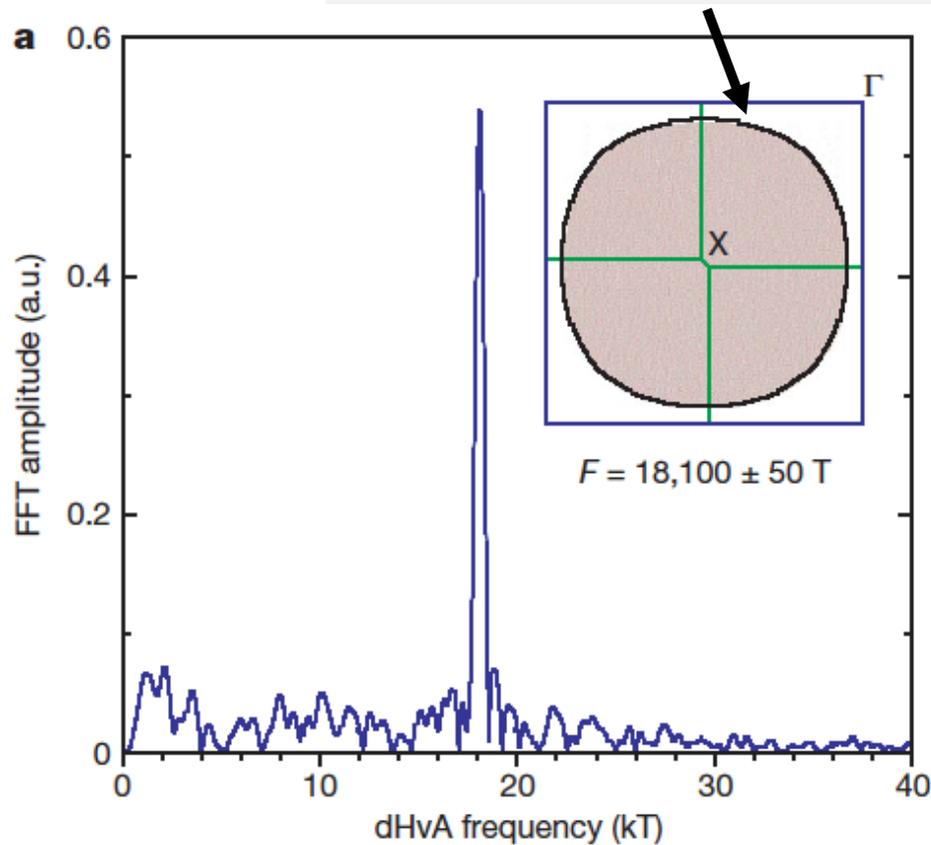
**(c) Fourier Transform**



# Fermi surface: $Tl_2Ba_2CuO_{6+y}$ (Tl2201)

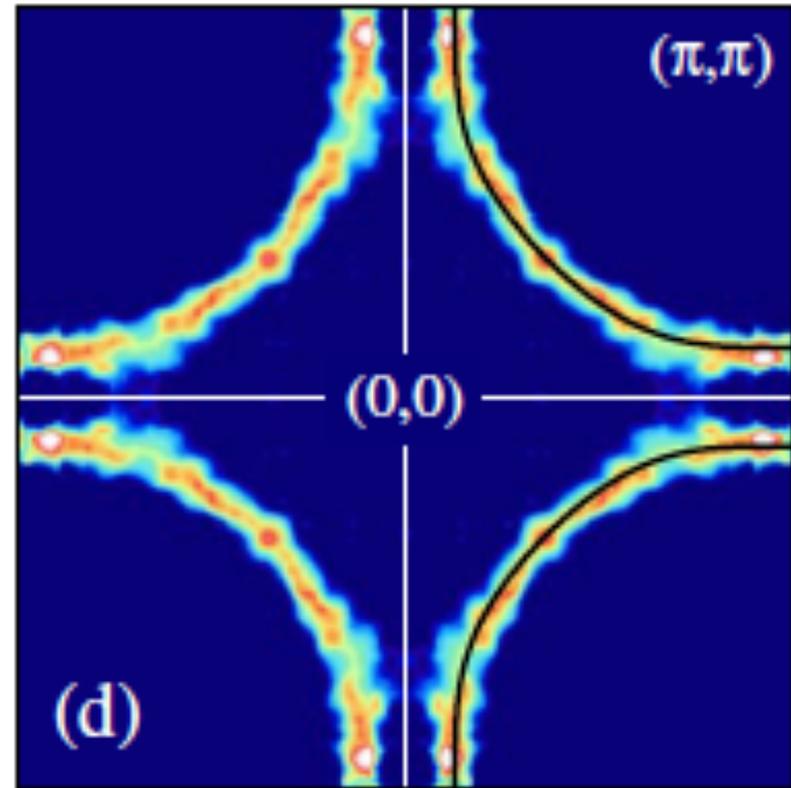
*ARPES vs Quantum Oscillations*

## Band Structure Calculations



Nature 455, 952 (2008)

Quantum Oscillation (QO) experiments

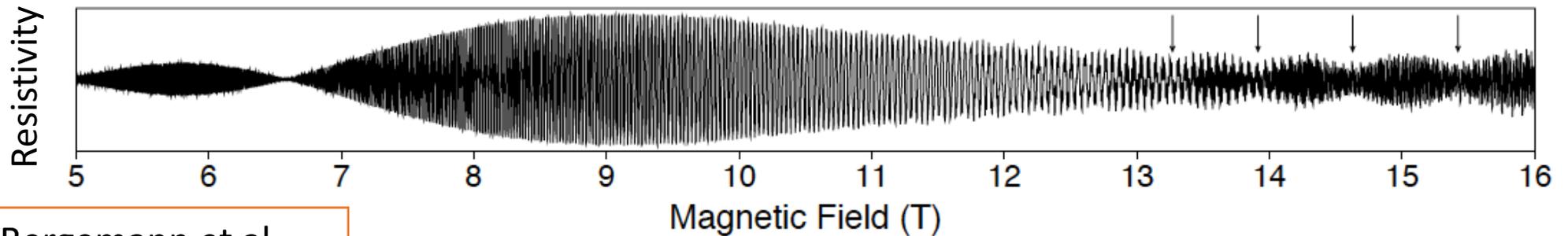


PRL 95, 077001 (2005)

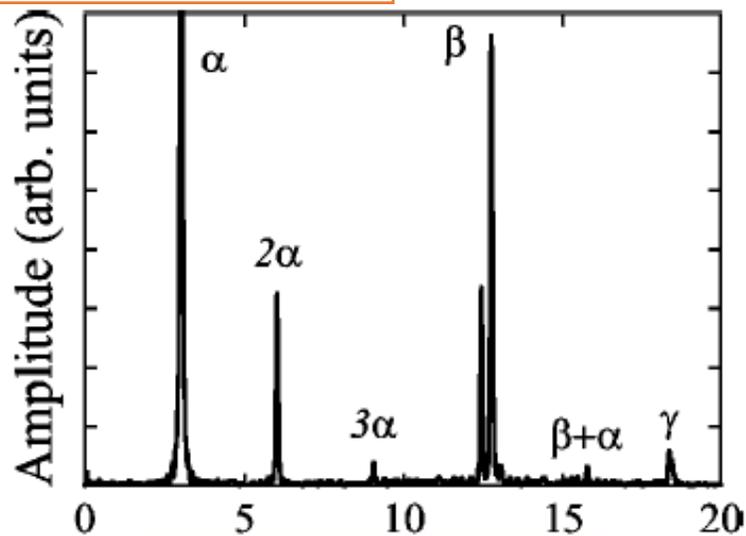
Data taken @ Swiss Light Source

Angle-resolved Photo-Emission Spectroscopy (ARPES)

# Multi – band metals: $\text{Sr}_2\text{RuO}_4$

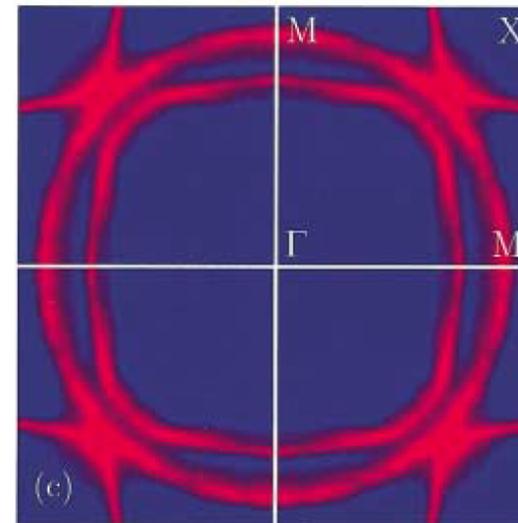


Bergemann et al,  
PRL 84, 2662 (2000)



A.P. Mackenzie et al,  
JPSJ 67, 385 (2003)

Quantum Oscillation (QO) experiments



A. Damascelli et al,  
PRL 85, 5194 (2000)

Angle-resolved Photo-Emission Spectroscopy  
(ARPES)

# QUANTUM OSCILLATIONS:

*Temperature dependence*

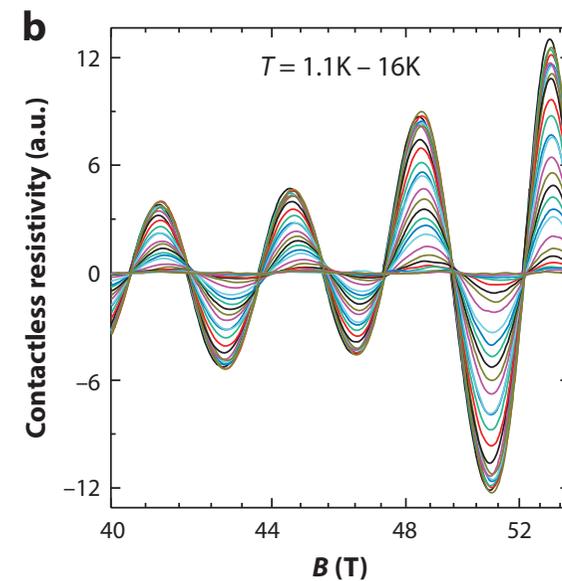
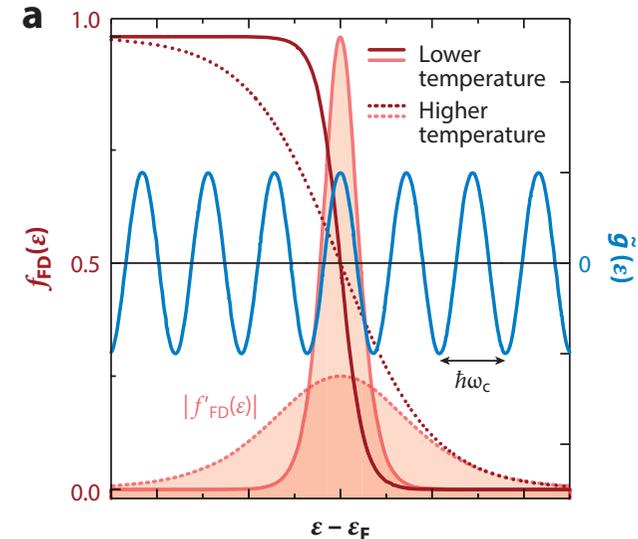
Thermal Condition:

$$\hbar\omega_c > k_B T$$

Landau level splitting > thermal energy

$$\omega_c = \frac{eB}{m^*}$$

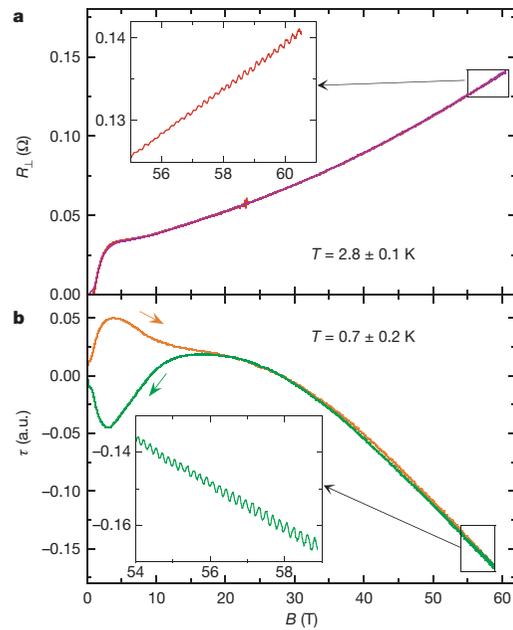
Temperature dependence of the oscillatory amplitude yield information about the electronic mass.



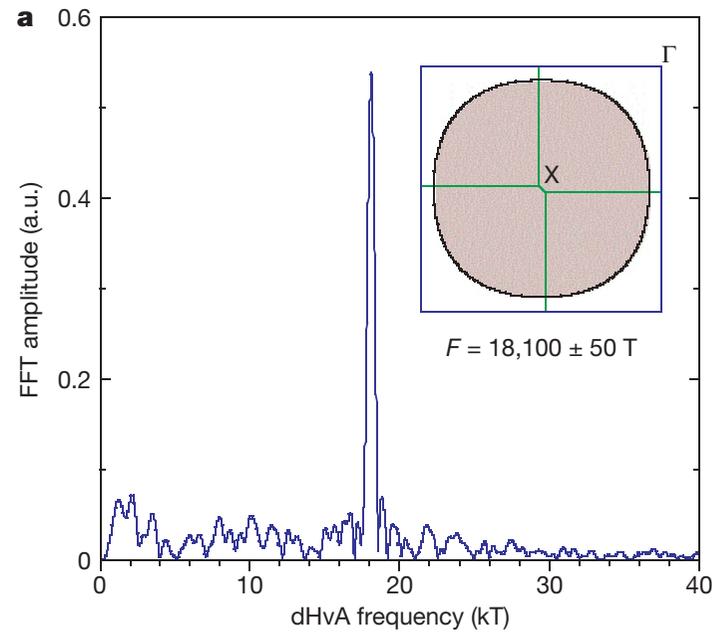
# Electronic Mass

## (1) Quantum Oscillation experiments:

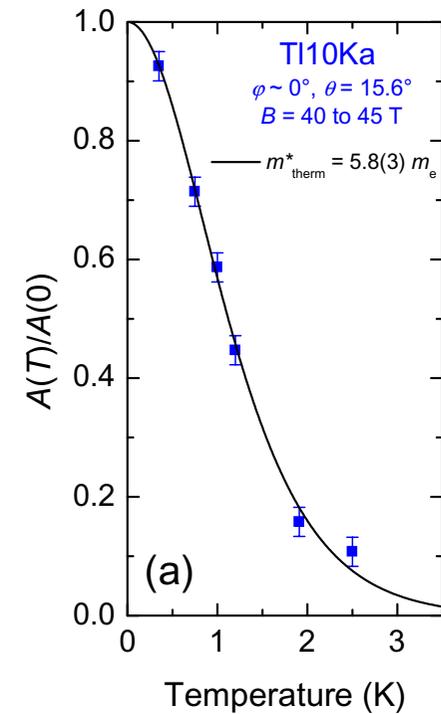
(a) RAW DATA



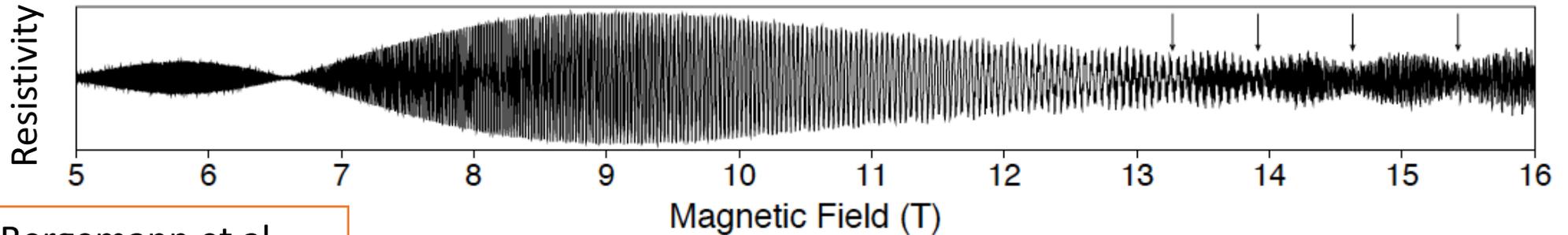
(b) Fourier Transform



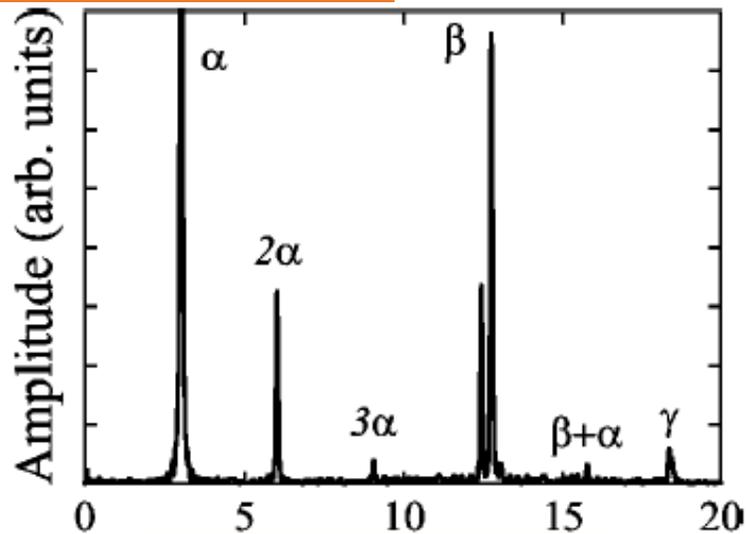
(c) T-dependence



# Multi – band metals: $\text{Sr}_2\text{RuO}_4$



Bergemann et al,  
PRL 84, 2662 (2000)



A.P. Mackenzie et al,  
JPSJ 67, 385 (2003)

TABLE II. Summary of quasiparticle parameters of  $\text{Sr}_2\text{RuO}_4$ .

Fermi-surface sheet	$\alpha$	$\beta$	$\gamma$
Character	Holelike	Electronlike	Electronlike
$k_F$ ( $\text{\AA}^{-1}$ ) <sup>a</sup>	0.304	0.622	0.753
$m^*$ ( $m_e$ ) <sup>b</sup>	3.3	7.0	16.0

A.P. Mackenzie et al,  
RMP 75, 657 (2003)

Quantum Oscillation (QO) experiments

# Fermi surface – Gold and Copper

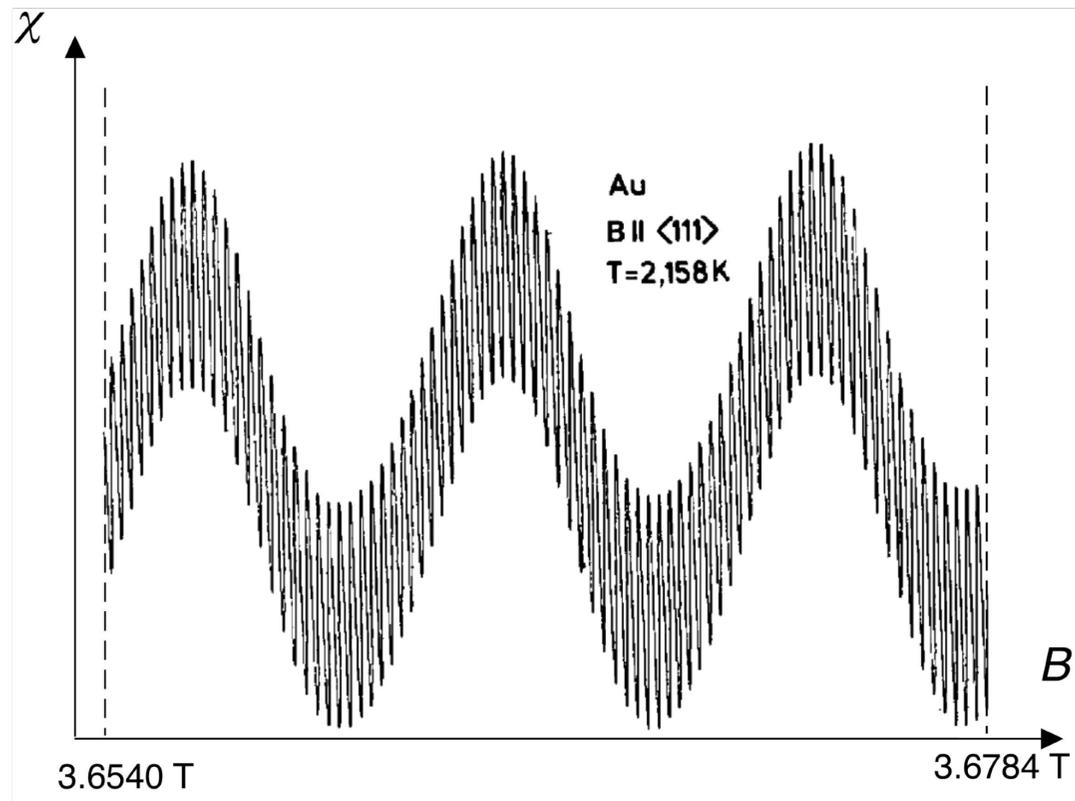
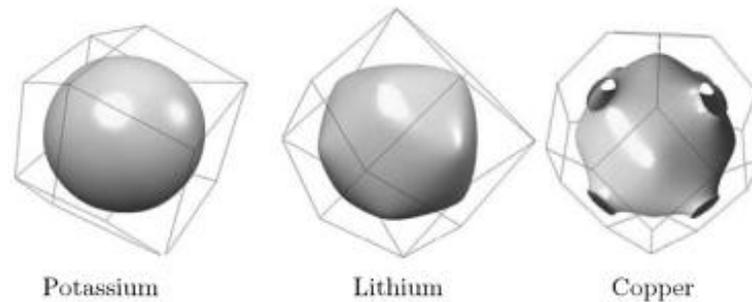
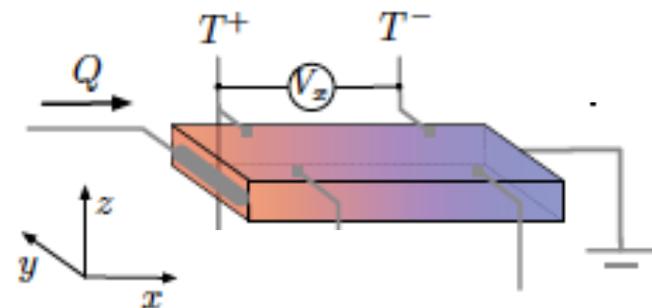
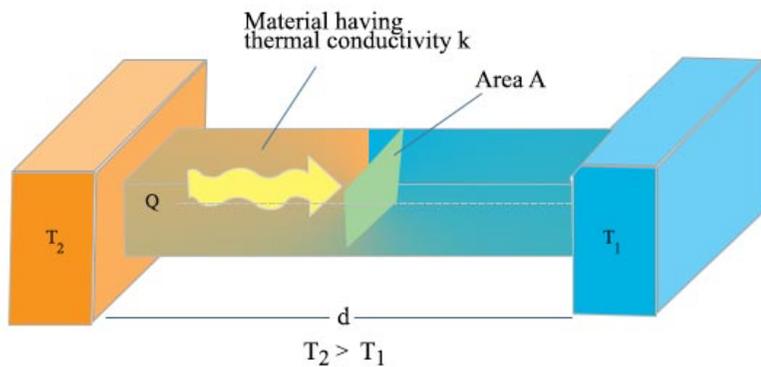


Figure 1: The spin susceptibility of gold in a magnetic field.



Material Property	Coefficient	In classed
Resistivity		
Hall coefficient		
Heat capacity		
Thermal conductivity		
Thermopower / Seebeck effect		



Material Property	Coefficient	In classed
Resistivity	$\rho = \frac{m^*}{ne^2\tau}$ or $\sigma = \frac{ne^2\tau}{m^*}$	Discussed / Proven
Hall coefficient	$R_H = \frac{-1}{en}$	Discussed / Proven
Heat capacity	$\frac{C}{T} = \gamma = \frac{1}{3}\pi^2 k_B^2 \text{DOS}(\epsilon_F) \propto m^*$	Discussed / Proven
Thermal conductivity	$\frac{\kappa}{T} = \frac{nk_B^2\pi^2\tau}{3m^*}$	Not proven
Thermopower / Seebeck effect	$\frac{S}{T} = \frac{\pm\pi^2 k_B}{2} \frac{1}{e T_F} \propto \frac{m^*}{n}$	Not proven

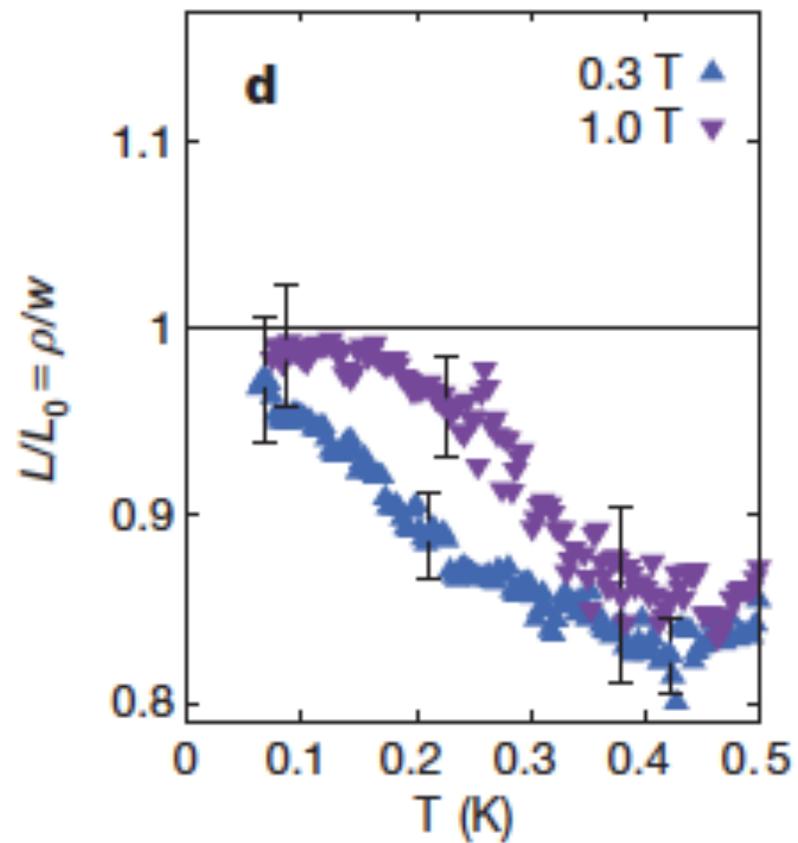
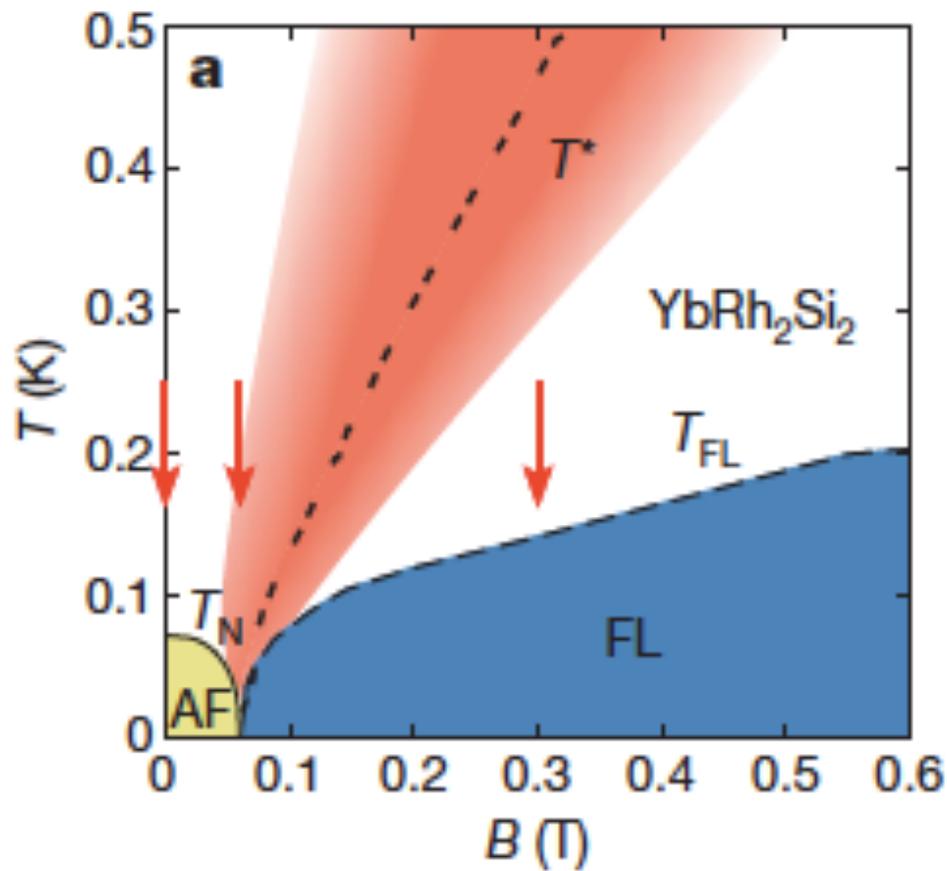
## Wiedemann Franz Law

$$\frac{\kappa}{\sigma T} = \boxed{\phantom{\frac{\pi^2 k_B^2}{3e^2}}}$$

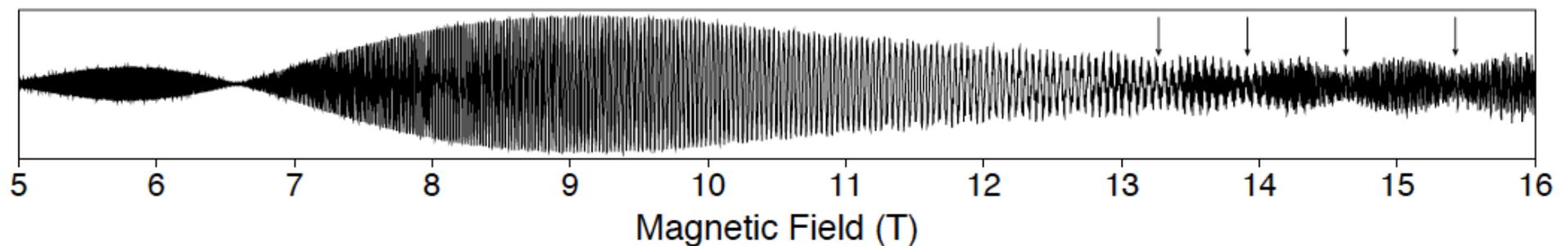
## Lorenz Number

$$L_0 = \frac{\pi^2 k_B^2}{3 e^2}$$

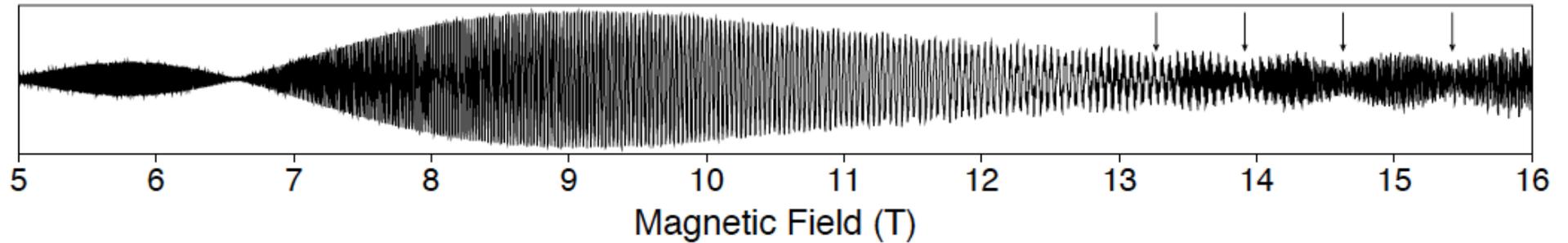
# Wiedemann Franz Law



Material Property	Coefficient	In classed
Resistivity	$\rho = \frac{m^*}{ne^2\tau}$ or $\sigma = \frac{ne^2\tau}{m^*}$	Discussed / Proven
Hall coefficient	$R_H = \frac{-1}{en}$	Discussed / Proven
Heat capacity	$\frac{C}{T} = \gamma = \frac{1}{3}\pi^2 k_B^2 DOS(\epsilon_F) \propto m^*$	Discussed / Proven
Thermal conductivity	$\frac{\kappa}{T} = \frac{nk_B^2\pi^2\tau}{3m^*}$	Not proven
Thermopower / Seebeck effect	$\frac{S}{T} = \frac{\pm\pi^2 k_B}{2} \frac{1}{e T_F} \propto \frac{m^*}{n}$	Not proven



# Multi – band metals: $\text{Sr}_2\text{RuO}_4$



Heat Capacity

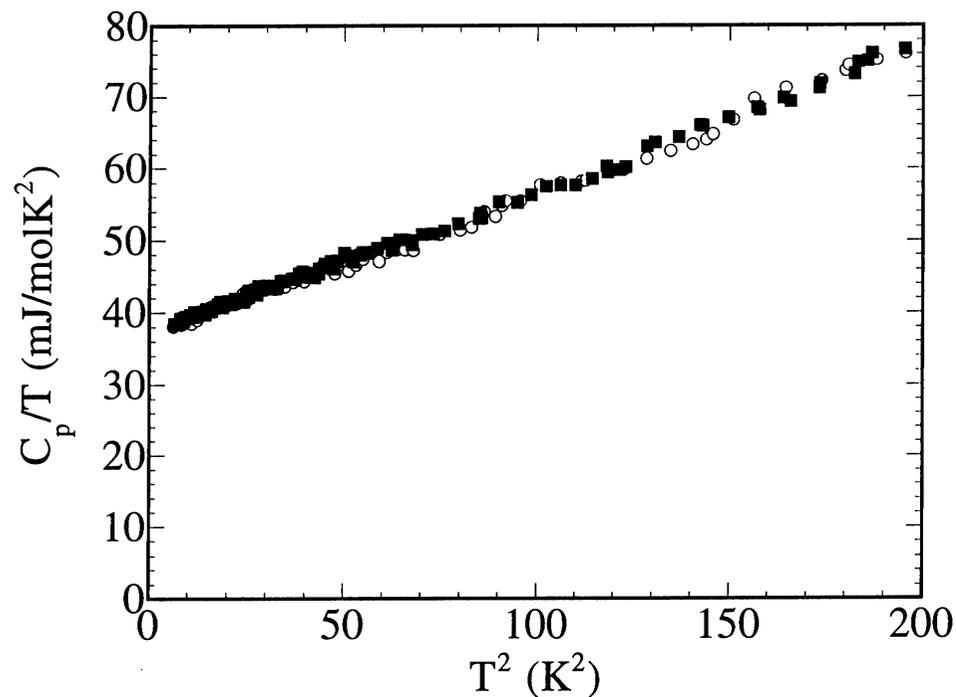


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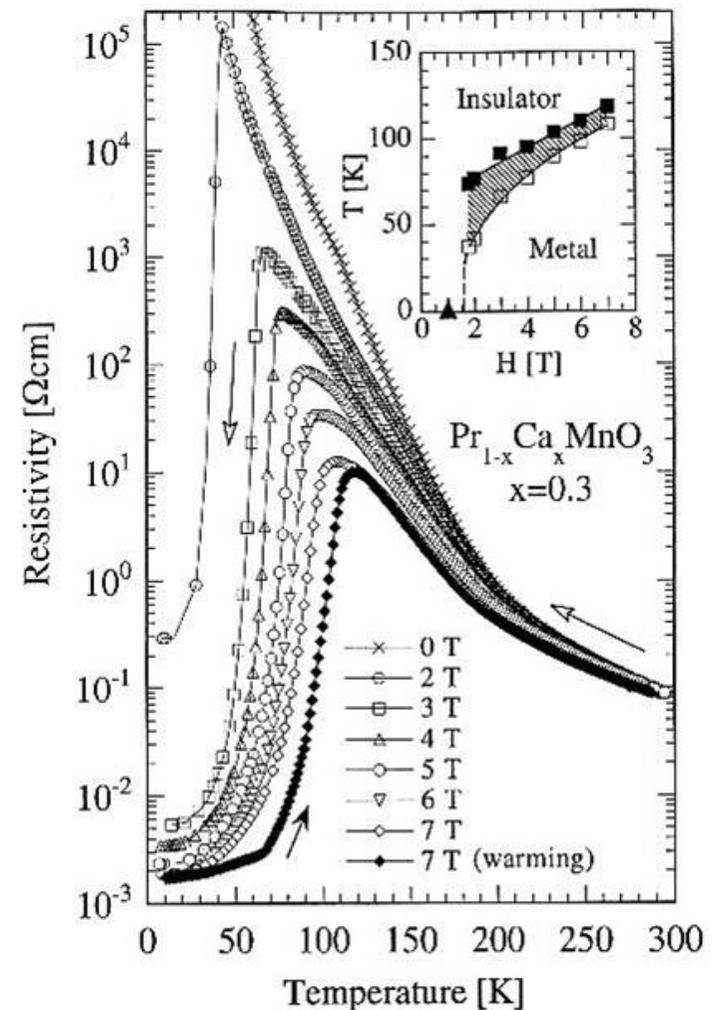
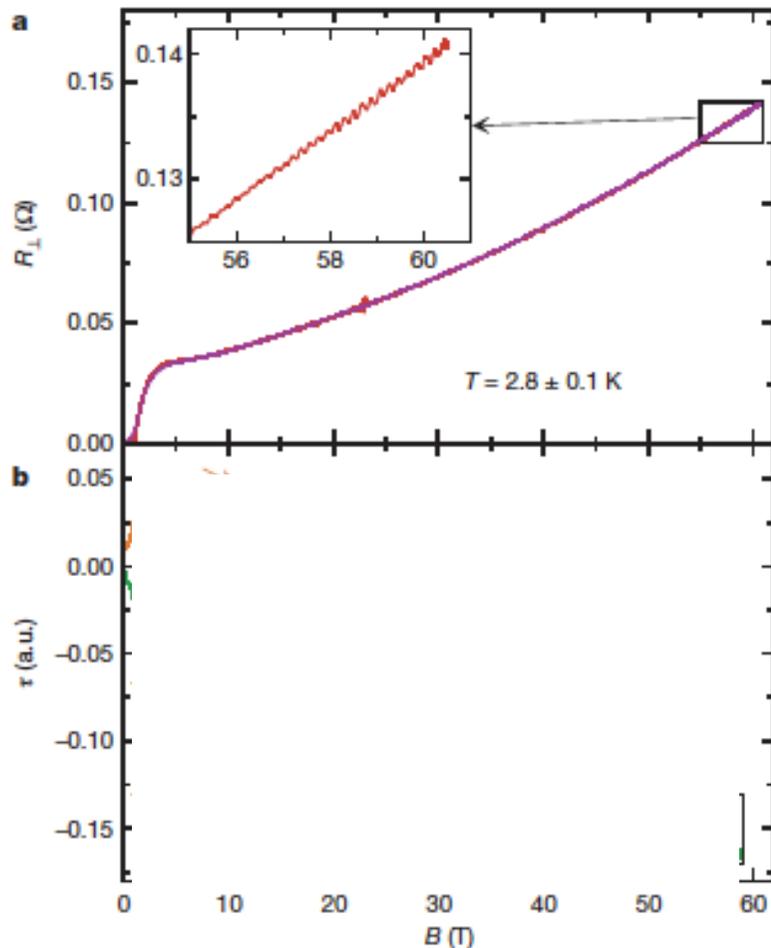
(2) Exam structure

(3) Quantum oscillations (continued)

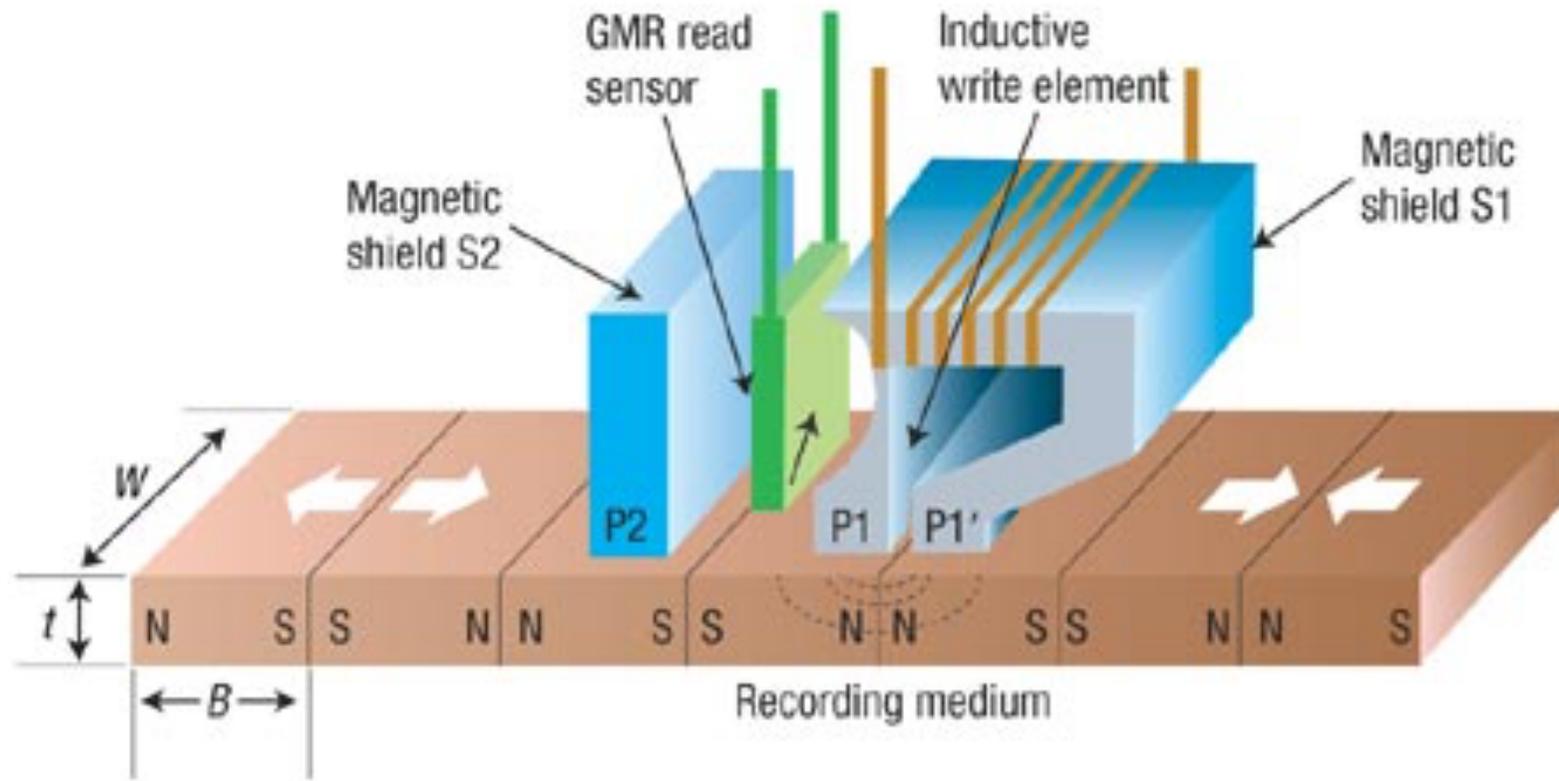
**(4) Magneto-resistance**

# Magneto-resistance

**Magneto-resistance** is the tendency of a material to change the value of its [electrical resistance](#) in an externally-applied [magnetic field](#).



# Application of Magneto-resistance



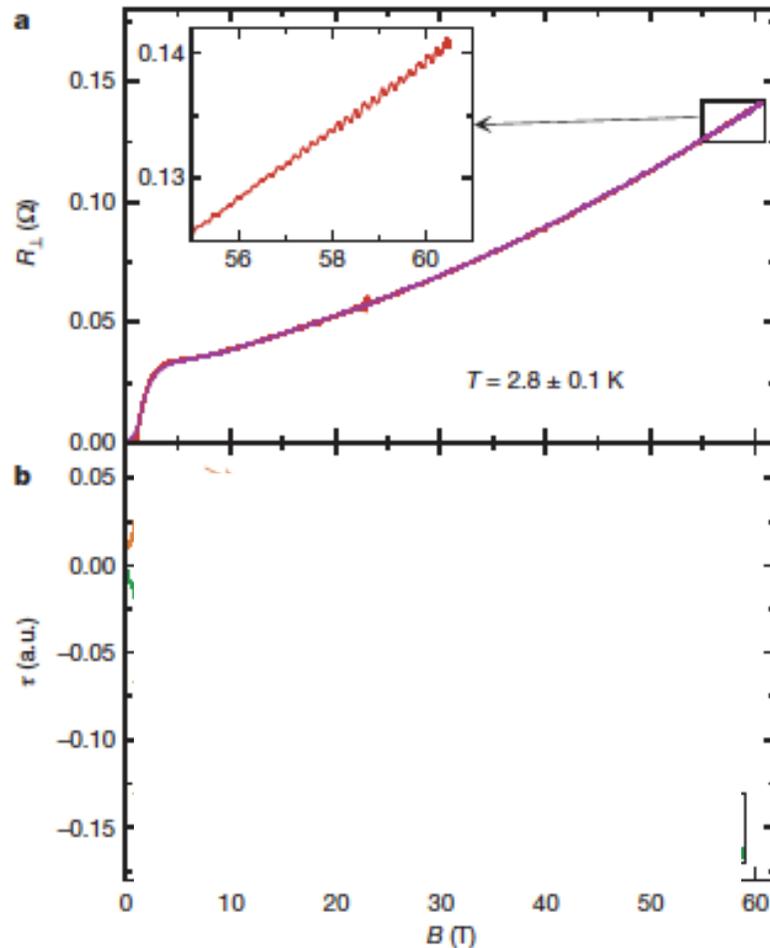
# Nobel Prize 2007: Magneto-resistance

The Nobel Prize in Physics 2007 was awarded jointly to Albert Fert and Peter Grünberg *"for the discovery of Giant Magnetoresistance"*



# Magneto-resistance

**Magneto-resistance** is the tendency of a material to change the value of its [electrical resistance](#) in an externally-applied [magnetic field](#).



# One route to Magneto-resistance

## Exercise 2 *Hall effect: Multiband scenario*

In the lecture we derived single-band expressions for the resistivity  $\rho = m/ne^2\tau$  and the Hall coefficient  $R_H = -1/ne$ . It is convenient to write the relation between the current density  $\mathbf{j}$  and the electric field  $\mathbf{E}$  as  $\mathbf{E} = \boldsymbol{\rho}\mathbf{j}$  where:

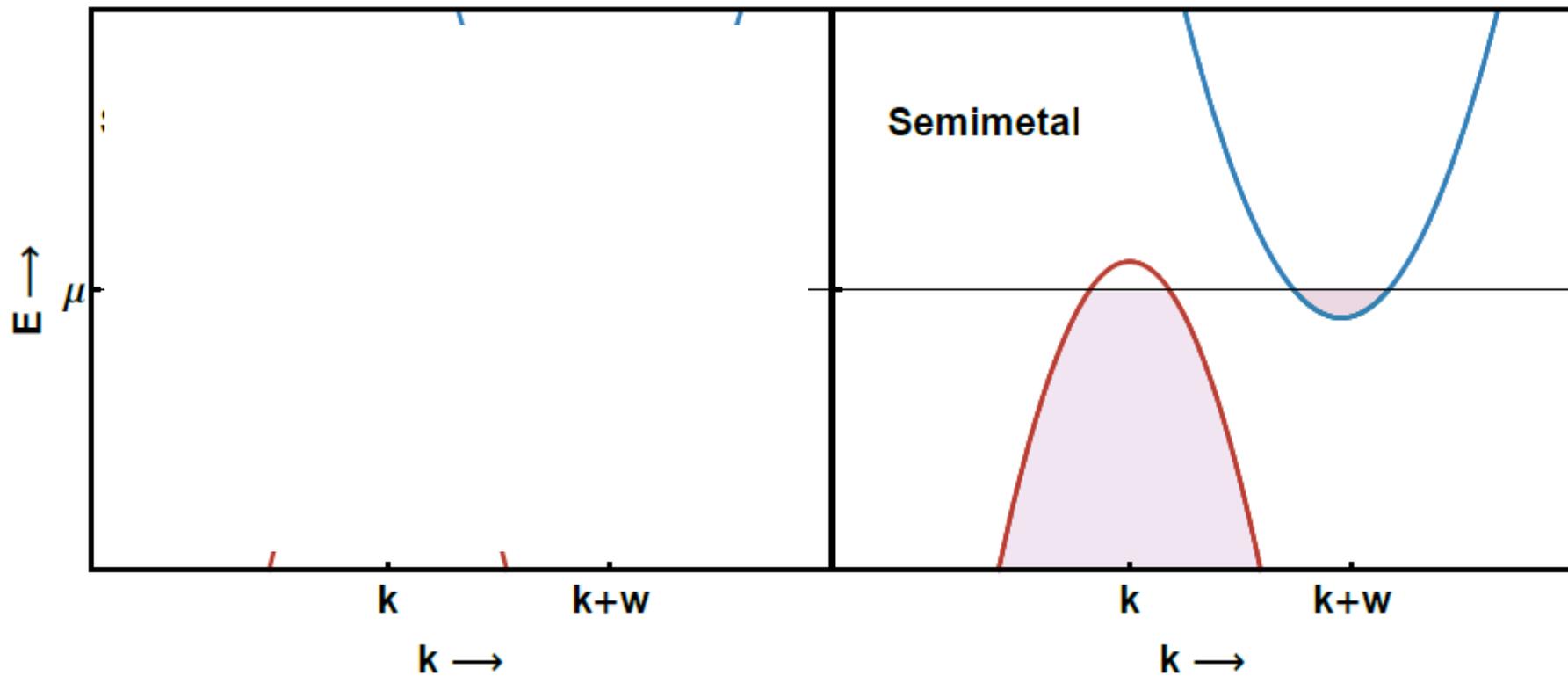
$$\boldsymbol{\rho} = \begin{pmatrix} \rho & -R_H B \\ R_H B & \rho \end{pmatrix} \quad (1)$$

(a) Let us consider a metal where more than one band crosses the Fermi level. When applying an electric field  $\mathbf{E}$ , the current  $\mathbf{j}_n$  on the  $n^{\text{th}}$  band is:  $\mathbf{j}_n = \boldsymbol{\rho}_n^{-1}\mathbf{E}$  where

$$\boldsymbol{\rho}_n = \begin{pmatrix} \rho_n & -R_{H,n} B \\ R_{H,n} B & \rho_n \end{pmatrix}. \quad (2)$$

Show that the total induced current  $\mathbf{j}$  is given by  $\mathbf{E} = \boldsymbol{\rho}\mathbf{j}$  where  $\boldsymbol{\rho} = (\sum \boldsymbol{\rho}_n^{-1})^{-1}$ .

# Consider two bands crossing the Fermi level



# One route to Magneto-resistance

(b) If only two bands are crossing the Fermi level, show that:

$$R_H = \frac{R_{H,1}\rho_2^2 + R_{H,2}\rho_1^2 + R_{H,1}R_{H,2}(R_{H,1} + R_{H,2})B^2}{(\rho_1 + \rho_2)^2 + (R_{H,1} + R_{H,2})^2 B^2} \quad (3)$$

$$\rho = \frac{\rho_1\rho_2(\rho_1 + \rho_2) + (\rho_1 R_{H,2}^2 + \rho_2 R_{H,1}^2)B^2}{(\rho_1 + \rho_2)^2 + (R_{H,1} + R_{H,2})^2 B^2} \quad (4)$$

Hint: It is allowed to use Mathematica. If you do so, print out the code and the output.

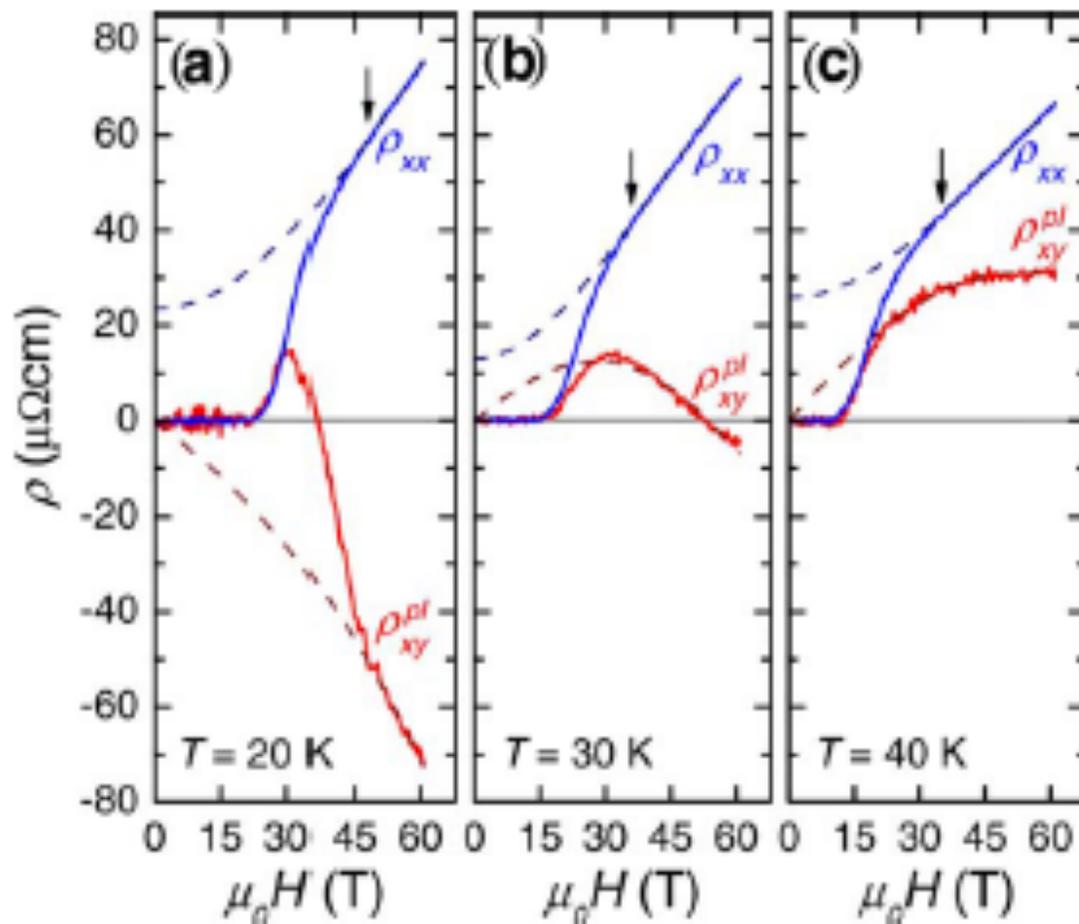
(c) Magnetic field dependence of resistivity is called magneto-resistance. If the two-band system has both electron-like and hole-like carries so that  $|R_{H,1}| \approx |R_{H,2}|$ , what is the field dependence of  $\rho$ .

(d) The mobility can be expressed as  $\mu = e\tau/m$ . Use the low-field limits of equations 3 and 4 to derive the following two expressions:

$$\sigma = ne\mu_e + pe\mu_h \quad (5)$$

$$R_H = \frac{p\mu_h^2 - n\mu_e^2}{e(p\mu_h + n\mu_e)^2}. \quad (6)$$

# Magneto-resistance



PHYSICAL REVIEW B 82, 020514(R) (2010)