## Course overview

## Crystal lattice phenomena's

-- Crystal structures (Real and reciprocal space)
-- Scattering theory (Bragg's law, Form Factor, Structure factor)
-- Crystal bindings ( Equilibrium lattice constants, binding energies)
-- Lattice vibrations ( Phonon dispersions, density of state, heat capacity)

## Electronic phenomena's

-- Free electron gas ( Fermi Dirac distribution, density of states)
-- Band structure ( electronic masses, Fermi surfaces)
-- Electronic measurements ( Heat capacity, resistivity, Hall effect, quantum oscillations)
-- Electronic phases ( metals, semi-metals, semi-condductors, band insulators)

## Exam Structure

## 10 min - Presentation:

Topics: (1) Crystal structures, (2) Crystal Bindings, (3) Reciprocal lattice+ scattering theory,
(4) Crystal vibrations (Phonons), (5) Heat capacity (6) Band structure
(7) Semiconductors

20 min - Discussion:
(a) Questions to the lecture material
(b) Questions to the exercises

End Exam

5 min - evaluation
5 min - Results: Passed / failed, grade will be known at a later point.

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MY AVAILABILITY BEFORE EXAM:
30th-31th May
1st and 6 6
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## Exercise 5 Sphere packings

Calculate the ratio $c / a$ of an ideal hexagonal dense sphere packing (hcp) and its packing density. Compare the packing density to that of an fcc lattice and explain your findings.

## http://quiz.thefullwiki.org/Solid-state_physics

Question 1: Here, the electrons are modelled as a Fermi gas, a gas
of particles which obey the quantum mechanical
(1) Dirac delta function
(2) Fermi-Dirac statistics
(3) Boltzmann distribution
(4) Maxwell-Boltzmann distribution

Question 2: This structure can be investigated using a range of crystallographic techniques, including , neutron diffraction and electron diffraction.
(1) X-ray crystallography
(2) Atom
(3) Protein
(4) Protein structure

Question 5: Phonons are also necessary for understanding the lattice heat capacity of a solid, as in the Einstein model and the later
(1) Copper
(2) Debye model
(3) Zinc
(4) Carbon
https://www.crcpress.com/downloads/IP662/IP662_Questions.pdf
2.5 The cohesive energy of a solid is $9 \mathrm{eV} / \mathrm{atom}$. What does this tell you about the solid?
2.6 What types of bonding are present in a graphite crystal?
2.10 Calculate the packing fraction of the face-centered cubic structure.
4.10(a) What are alloys of copper and zinc commonly known as?
5.1(a) State the Bragg law.
5.4(a) What types of radiation other than X-rays are commonly used to obtain diffraction patterns?
6.6(a) Does the resistivity of a metal increase or decrease with temperature?
6.8(a) Which principle do electrons in a Fermi gas obey?

## Today's program

## Physics 481: Condensed Matter Physics - Midsemester test

Friday, March 4, 2011
Problem 1: Structure determination (70 points)
X-ray diffraction is used to study a powder specimen of a monoatomic substance that is known to crystallize in a cubic lattice structure with primitive vectors $\vec{a}_{1}=$ $(a, 0,0), \vec{a}_{2}=(0, a, 0)$ and $\vec{a}_{3}=(0,0, a)$. The wavelength of the X-rays is $1.4 \AA$.
a) Find the primitive vectors of the reciprocal lattice. (15 points)

## Problem 2: One-dimensional Morse solid (80 points points)

Consider $N$ identical atoms of mass $M$ whose motion is restricted to the $x$-axis. Nearest neighbor atoms are coupled by the so-called Morse potential

$$
V_{M}(r)=D\left(1-e^{-\alpha\left(r-r_{0}\right)}\right)^{2}-D
$$

where $r$ is the distance between them and $D, \alpha$, and $r_{0}$ are positive constants.
a) Calculate $V_{M}(0), V_{M}(\infty)$ and qualitatively sketch the Morse potential. (10 points)

Problem 3: Phonons of a square lattice (50 points)
Consider a two-dimensional solid of identical atoms of mass $M$ on a square lattice of lattice constant $a$. In this problem, we investigate vibrations perpendicular to the lattice plane. The equations of motion for the displacements $u_{j, l}$ read

$$
M \ddot{u}_{j, l}=K\left(u_{j+1, l}-u_{j, l}\right)+K\left(u_{j-1, l}-u_{j, l}\right)+K\left(u_{j, l+1}-u_{j, l}\right)+K\left(u_{j, l-1}-u_{j, l}\right)
$$

Here, $j$ and $l$ index the atom position in the $x$ and $y$ directions, respectively.
a) Determine the dispersion relation ( $\omega$ as a function of $\vec{q}$ ) of the phonons for a wave with a wave vector $\vec{q}=\left(q_{x}, q_{y}\right)$. (30 points)
b) Calculate the speed of sound in terms of $K$ and $M$. Does it depend on the direction of $\vec{q}$ ? (20 points).

## Dispersions (Dispersion relations)






## Phonon Density of States




## Van Hove Singularity

## Exercise 3 Singularity in density of states

(a) From the dispersion relation derived in the lecture for a monoatomic linear lattice of N atoms with nearest neighbour interactions, show that the density of modes is

$$
\begin{equation*}
D(\omega)=\frac{2 N}{\pi} \cdot \frac{1}{\sqrt{\omega_{\mathrm{m}}^{2}-\omega^{2}}} \tag{4}
\end{equation*}
$$

where $\omega_{\mathrm{m}}$ is the maximum frequency.
(b) Suppose that an optical phonon branch has the form $\omega(K)=\omega_{0}-A K^{2}$, near $K=0$ in three dimensions. Show that $D(\omega)=\left(\frac{L}{2 \pi}\right)^{3}\left(\frac{2 \pi}{A^{3 / 2}}\right)\left(\omega_{0}-\omega\right)^{\frac{1}{2}}$ for $\omega<\omega_{0}$ and $D(\omega)=0$ for $\omega>\omega_{0}$. Here the density of modes is discontinuous.

## Electronic Density of States



## Physics 481: Condensed Matter Physics - Test prep homework

Problem 1: Tightly bound electrons in 1D (10 points)
Consider a one-dimensional electron system with lattice constant $a$ in tight binding approximation. The energy-momentum relation reads

$$
\epsilon(k)=-2 t \cos (k a) .
$$

a) Calculate the electronic density of states $D(\epsilon)$.
b) Does it have van-Hove singularities? If so, discuss their character!
c) Calculate the Fermi energy for 0.5 , 1, and 2 electrons per unit cell.
d) For one electron per unit cell, calculate the low-temperature specific heat (per cell)!

