# Kern- und Teilchenphysik II <br> Spring Term 2016 <br> <br> Exercise Sheet 5 

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## 1. Tau decay

Consider the decay $\tau^{-} \rightarrow \pi^{-} \nu_{\tau}$.
a) Draw the Feynman diagram and show that the corresponding matrix element is

$$
\mathcal{M} \approx \sqrt{2} G_{F} f_{\pi} \bar{u}\left(p_{\nu}\right) \gamma^{\mu} \frac{1}{2}\left(1-\gamma^{5}\right) u\left(p_{\tau}\right) g_{\mu \nu} p_{\pi}^{\nu}
$$

b) Taking the $\tau^{-}$spin to be in the $z$-direction and the four-momentum of the neutrino to be

$$
p_{\nu}=p *(1, \sin \theta, 0 \cos \theta)
$$

show that the leptonic current is

$$
j^{\mu}=\sqrt{2 m_{\tau} p^{*}}(-s,-c,-i c, s)
$$

where $s=\sin \left(\frac{\theta}{2}\right)$ and $c=\cos \left(\frac{\theta}{2}\right)$. Note that, for this configuration, the spinor for the $\tau^{-}$can be taken to be $u_{1}$ for a particle at rest.

$$
2 \mathrm{pt}
$$

c) Write down the four-momentum of the $\pi^{-}$and show that

$$
|\mathcal{M}|^{2}=4 G_{F}^{2} f_{\pi}^{2} m_{\tau}^{3} p^{*} \sin ^{2}\left(\frac{\theta}{2}\right)
$$

d) Hence show that

$$
\Gamma\left(\tau^{-} \rightarrow \pi^{-} \nu_{\tau}\right)=\frac{G_{F}^{2} f_{\pi}^{2}}{16 \pi} m_{\tau}^{3}\left(\frac{m_{\tau}^{2}-m_{\pi}^{2}}{m_{\tau}^{2}}\right)^{2}
$$

## 2. Kaon decay

Consider the kaon ( $\mathrm{K}^{-}$) decay into leptons in its rest frame.
a) Determine its decay rate and the ratio

$$
\frac{\Gamma\left(K^{-} \rightarrow e^{-} \bar{\nu}_{e}\right)}{\Gamma\left(K^{-} \rightarrow \mu^{-} \bar{\nu}_{\mu}\right)}
$$

## 3. Weak Interactions

Draw the Feynman diagrams of the following processes and write the related matrix element (ignore spectator quarks in matrix element computation).
a) $\pi^{-} \rightarrow \pi^{0} e^{-} \bar{\nu}_{e}$
b) $K^{+} \rightarrow \mu^{+} \nu_{\mu}$
c) $\tau^{-} \rightarrow e^{-} \bar{\nu}_{e} \nu_{\tau}$

