



Spectral properties of hybrid bilayer graphene

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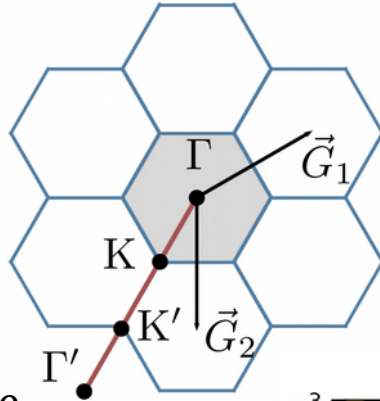
³Aalto University, Finland

Motivation and strategy

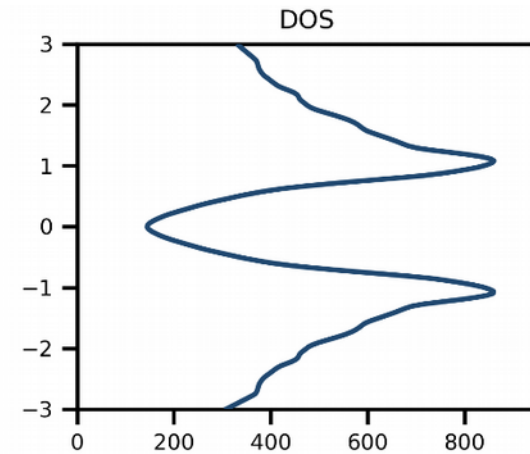
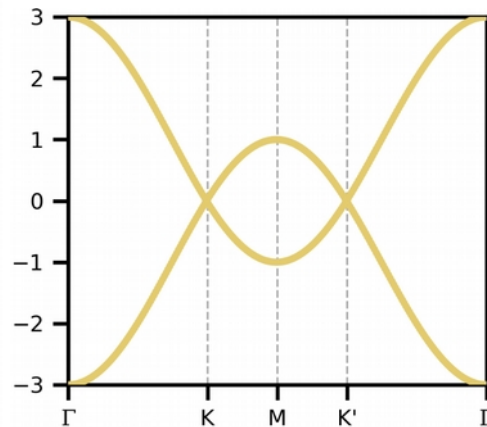
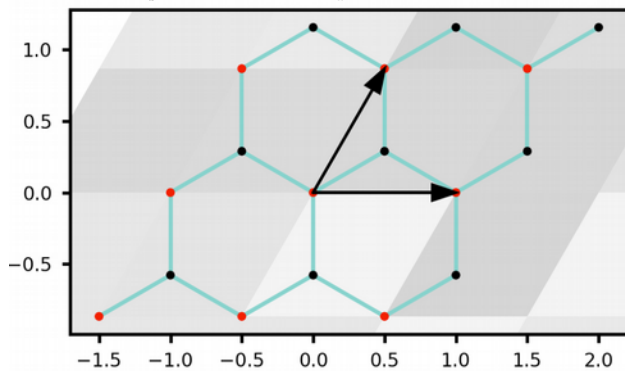
- Flat band superconductivity in experimentally tunable systems, using superstructures of weakly correlated 2D materials
- Experimental tools
 - twist angle (Bistritzer & MacDonald, PNAS2011)
 - atomic defects (Lopez-Bezanilla PRM2019, Ramires PRB2019)
- Compare physics of superlattices: defects vs. moiré
- Enhance DOS in order to suppress the kinetic term
 - possibly high correlations → unconventional SC

Graphene

$$H = \sum_{\langle i,j \rangle} t c_i^\dagger c_j$$

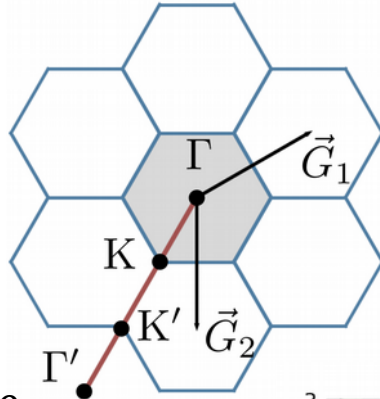


Honeycomb, bipartite



Graphene

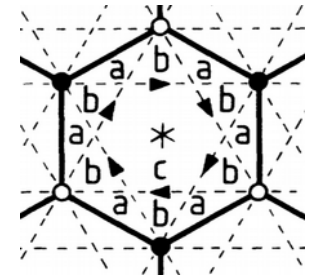
$$H = \sum_{\langle i,j \rangle} t c_i^\dagger c_j$$



Valley flavour $\eta_{ij} = \pm$

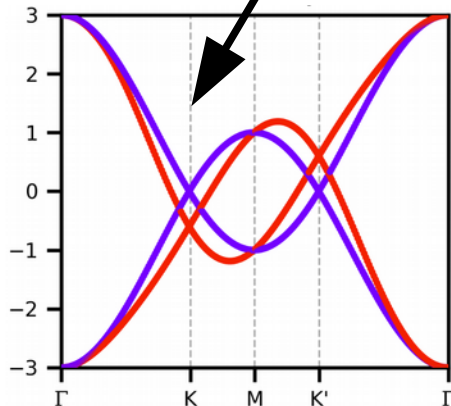
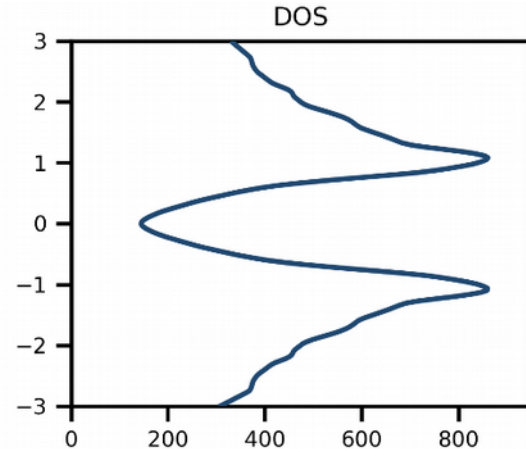
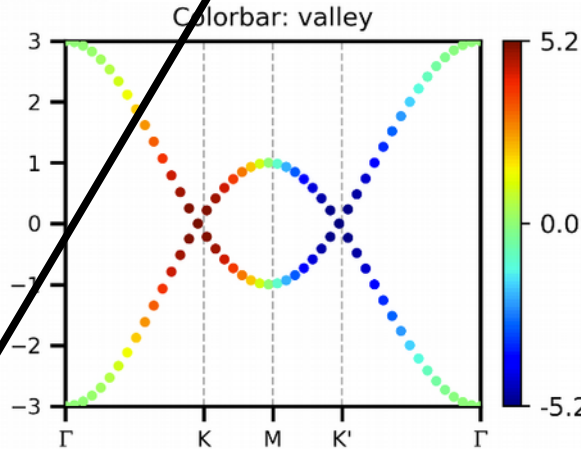
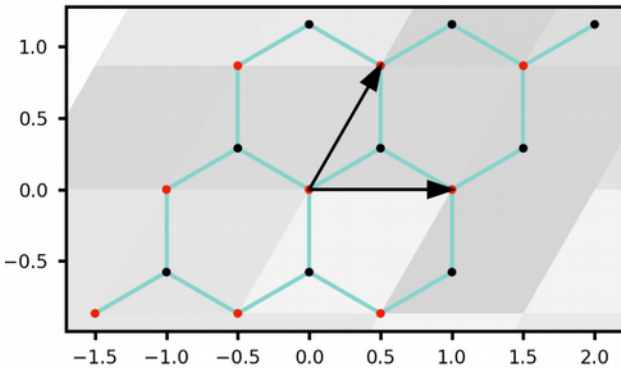
$$H_{aH} = \frac{i}{3\sqrt{3}} \sum_{\langle\langle i,j \rangle\rangle} \eta_{ij} \sigma_z^{ij} c_i^\dagger c_j$$

Ramires PRB2019



Haldane PRL1998

Honeycomb, bipartite



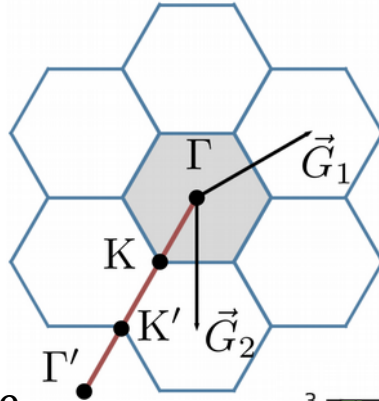
The “Anti-Haldane“ hopping term shifts the energy of the Dirac points at K and K' in opposite directions

- purple: H
- red: $H + H_{aH}$

Use it as projection operator: $\text{color} = \langle \psi | H_{aH} | \psi \rangle$

Graphene

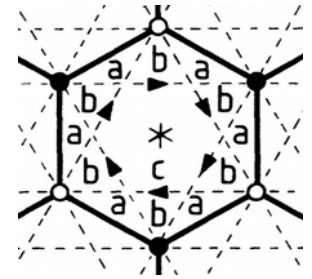
$$H = \sum_{\langle i,j \rangle} t c_i^\dagger c_j$$



Valley flavour $\eta_{ij} = \pm$

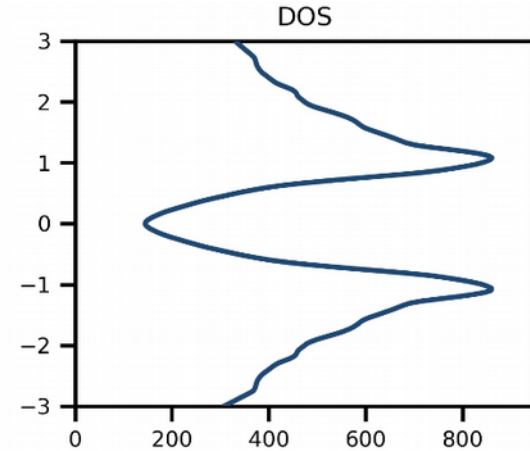
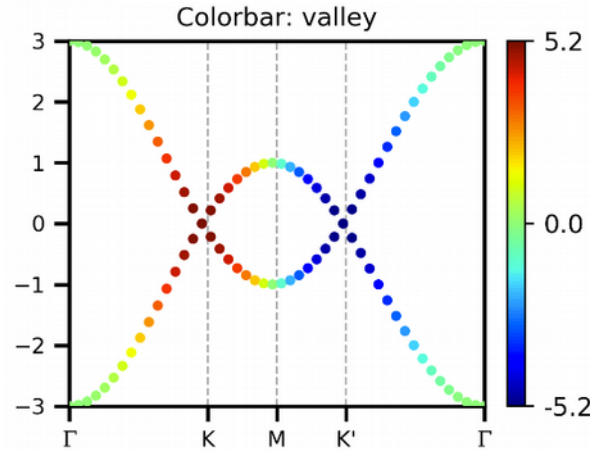
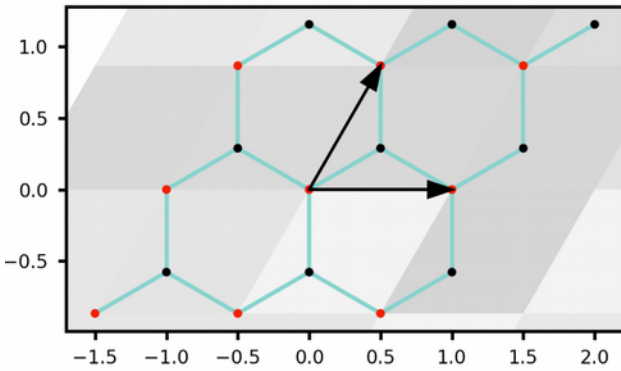
$$H_{\text{aH}} = \frac{i}{3\sqrt{3}} \sum_{\langle\langle i,j \rangle\rangle} \eta_{ij} \sigma_z^{ij} c_i^\dagger c_j$$

Ramires PRB2019

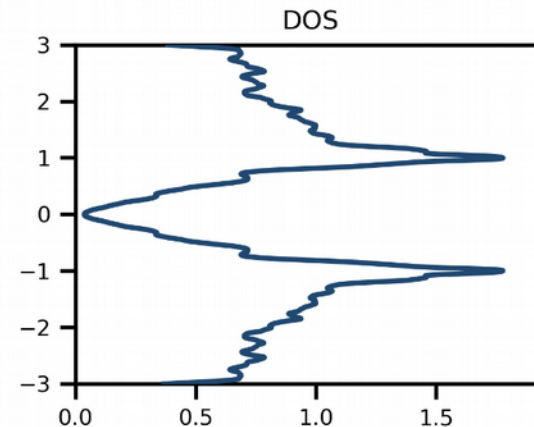
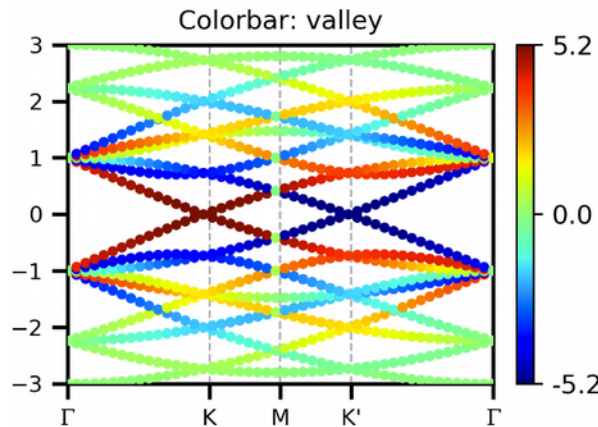
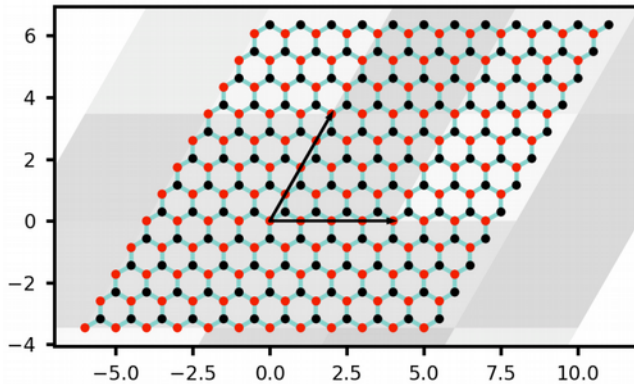


Haldane PRL1998

Honeycomb, bipartite



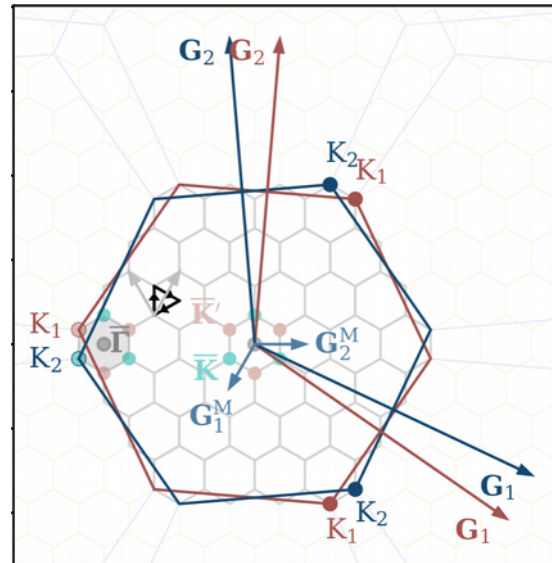
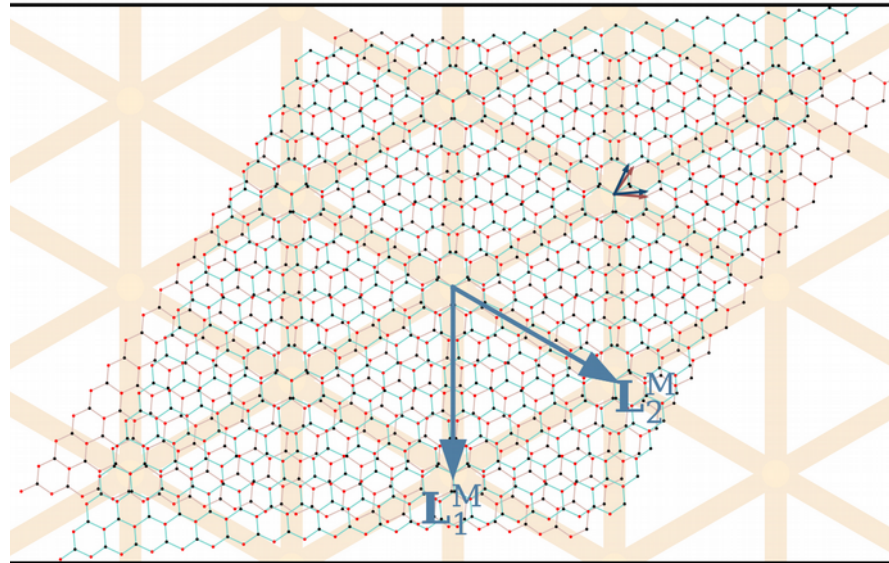
4x4, flatter bands



Moiré pattern and van Hove singularities

small relative twist \rightarrow moiré superlattice

Certain angles: periodic



$r=1$

$$\cos\theta(m,r) = \frac{3m^2 + 3mr + r^2/2}{3m^2 + 3mr + r^2}$$

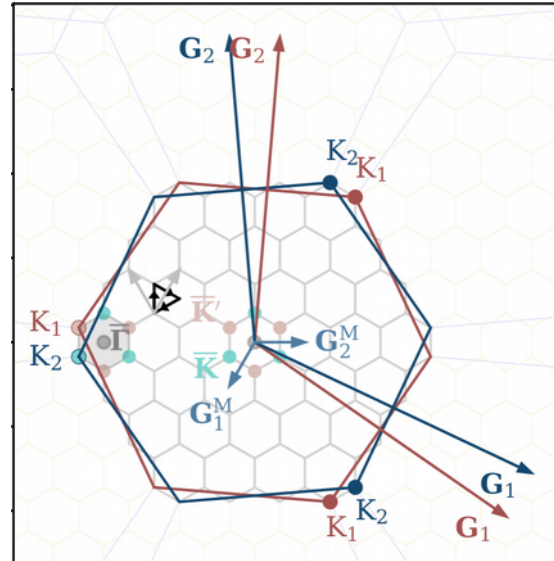
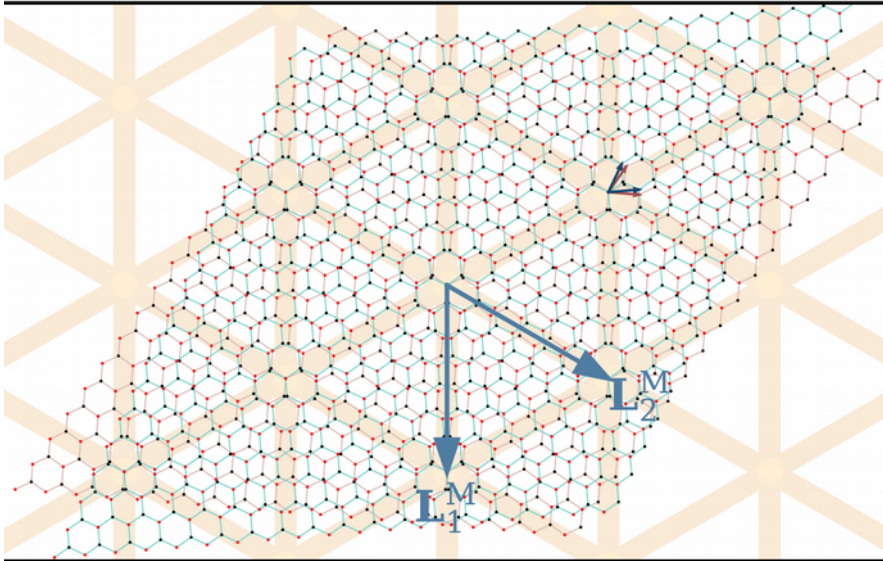
$$\begin{bmatrix} \mathbf{t}_1 \\ \mathbf{t}_2 \end{bmatrix} = \begin{bmatrix} m & m+r \\ -(m+r) & 2m+r \end{bmatrix} \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{bmatrix}$$

Neto PRB2012

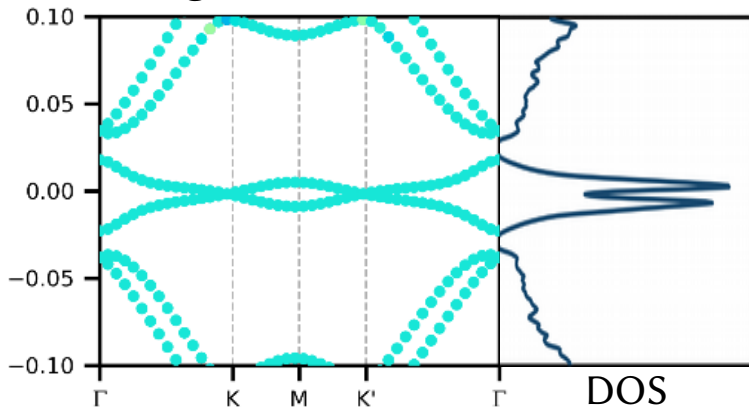
Moiré pattern and van Hove singularities

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BZ folding flattens the bands at low energy



$$\cos\theta(m,r) = \frac{3m^2 + 3mr + r^2/2}{3m^2 + 3mr + r^2}$$

$r=1 \rightarrow$

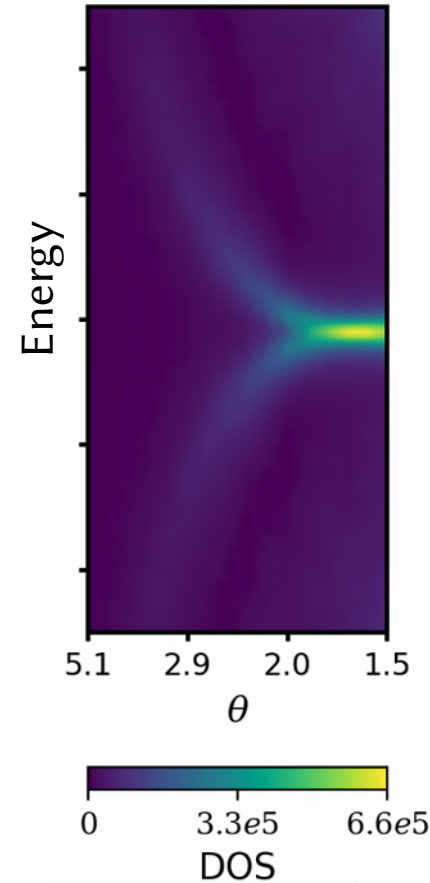
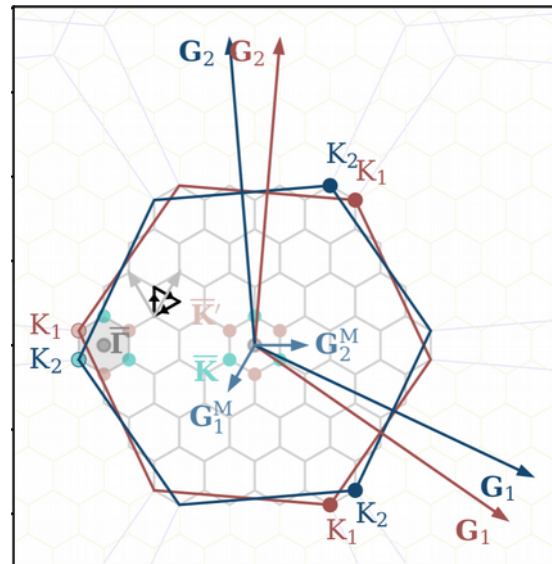
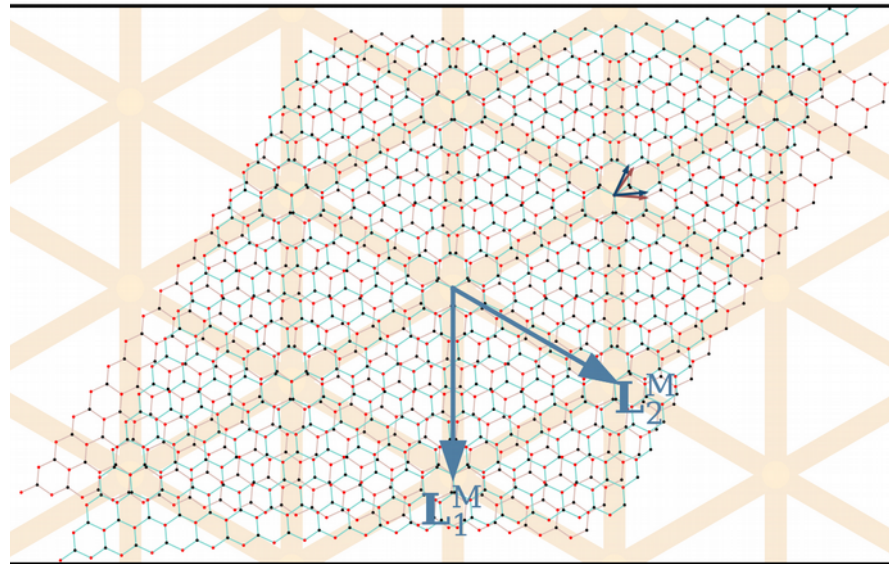
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Neto PRB2012

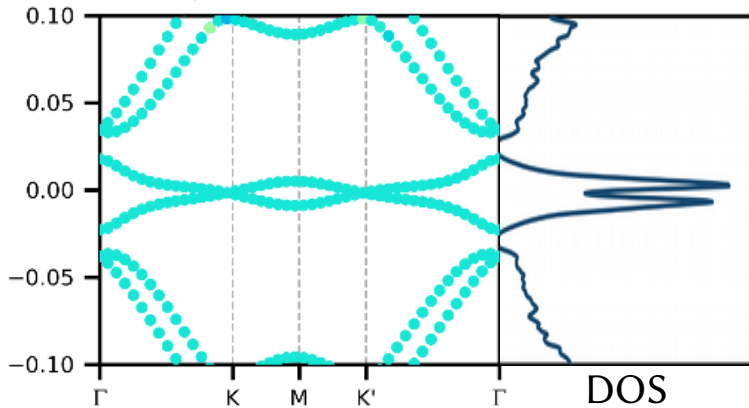
Moiré pattern and van Hove singularities

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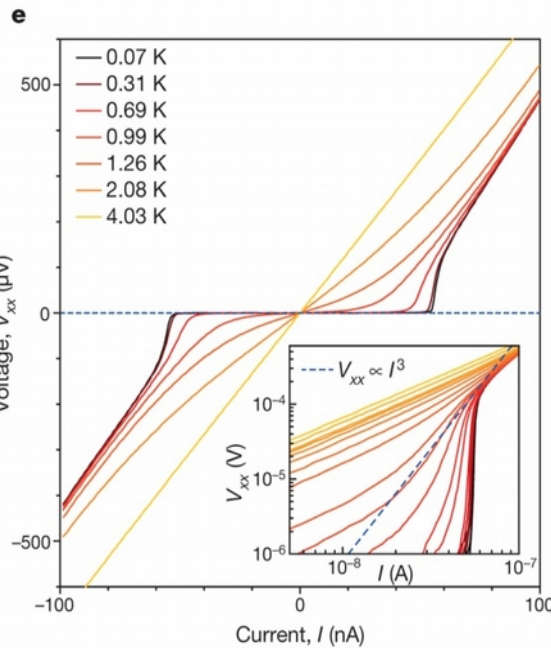
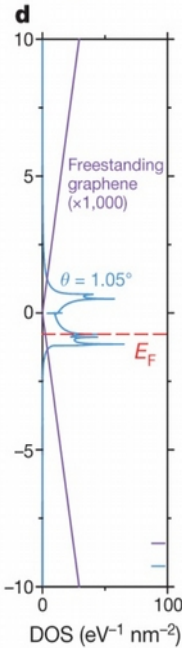
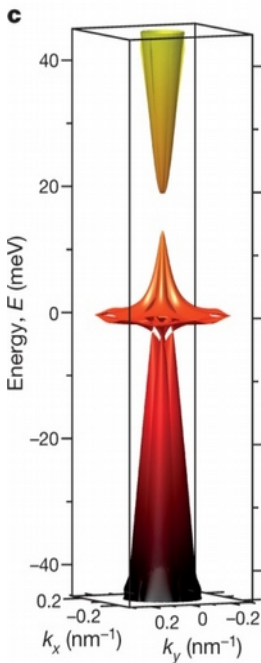
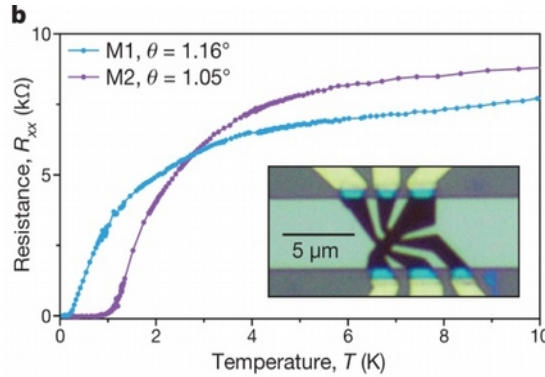
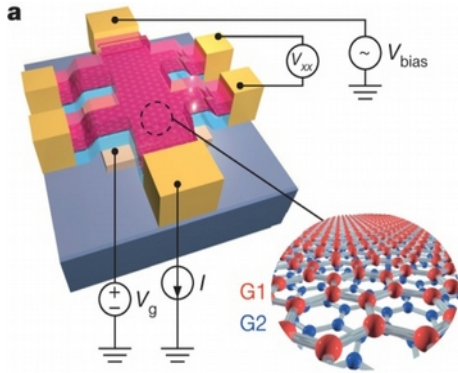


$$\cos\theta(m,r) = \frac{3m^2 + 3mr + r^2/2}{3m^2 + 3mr + r^2}$$

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Neto PRB2012

Superconductivity in TBG



Superfluidity in system with fermion condensate

V. A. Khodel' and V. R. Shaginyan

I. V. Kurchatov Institute of Atomic Energy, Moscow; B. P. Konstantinov Institute of Nuclear Physics, Academy of Sciences of the USSR

(Submitted 4 April 1990)

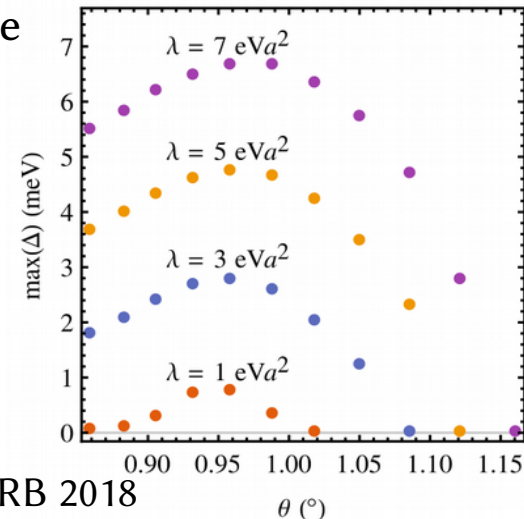
Pis'ma Zh. Eksp. Teor. Fiz. **51**, No. 9, 488–490 (10 May 1990)

The properties of Fermi systems beyond the phase transition point, at which the group velocity of the quasiparticles changes sign on the Fermi surface, are analyzed. A Fermi condensate arises in the new phase: The energies $\epsilon(\mathbf{p})$ of quasiparticles with momenta $p_{1c} < p < p_{2c}$ ($p_{1c} < p_F, p_{2c} > p_F$) turn out to be identical and equal to the chemical potential μ . If a Cooper pairing can occur in this phase **the gap Δ is a linear function of the pairing constant λ .**

$$\text{BCS: } \Delta \propto e^{-1/\mathcal{D}(0)\lambda}$$

Inside van Hove singularity:

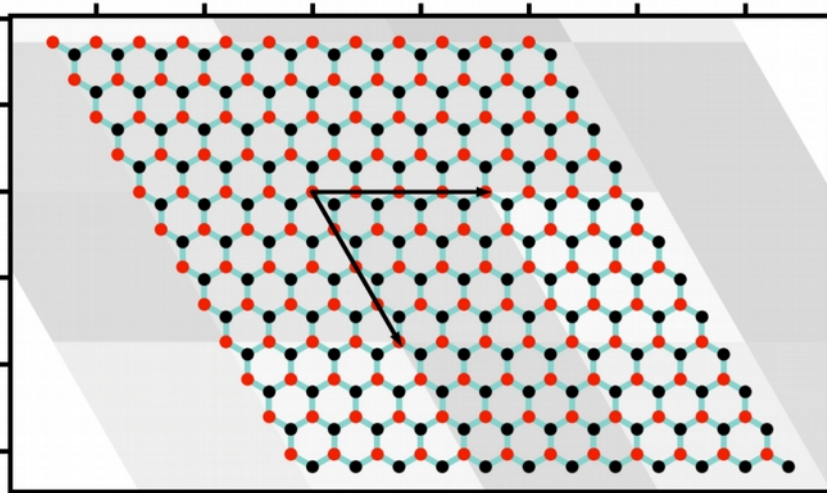
$$\Delta \propto \lambda$$



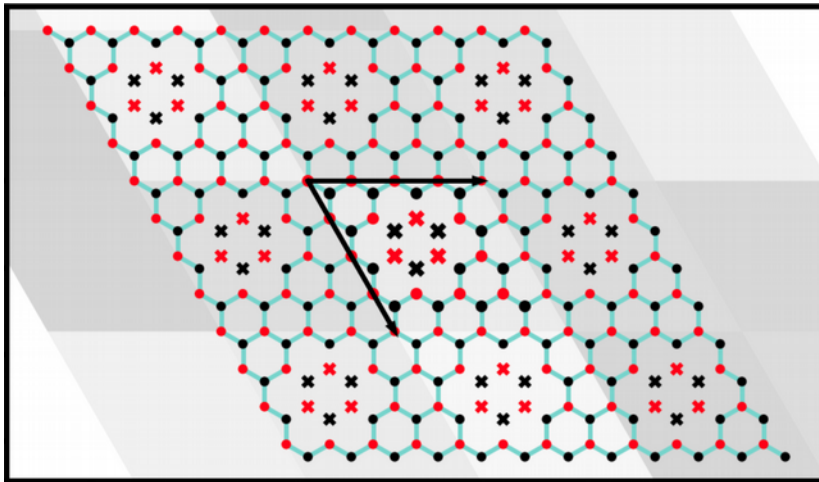
Peltonen PRB 2018

θ (°)

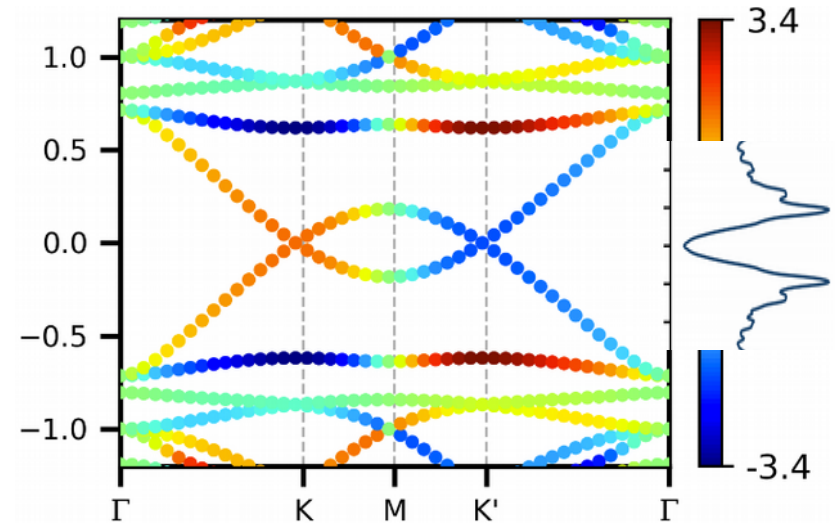
Other mechanism which flattens the low-energy bands



Other mechanism which flattens the low-energy bands

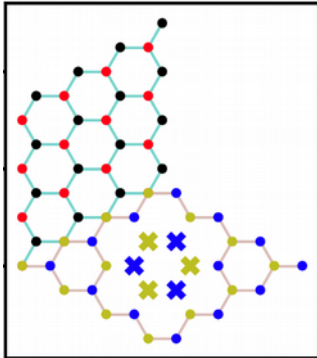


- The UC has $4 \times 4 \times 2 - 6 = 28$ atoms
- The position of the removed hexagon in the UC is irrelevant

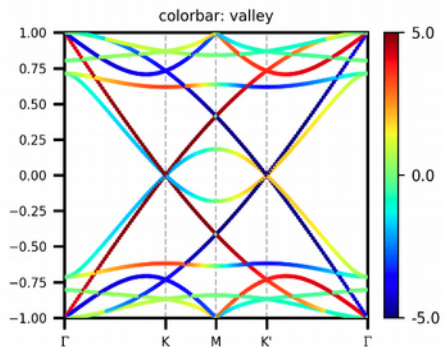
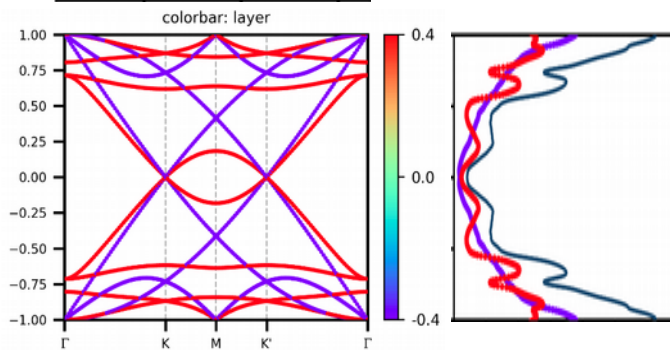


- Low-energy \sim graphene spectrum
 - Renormalized t
 - Weaker valley polarization
- Combined with a pristine layer ?

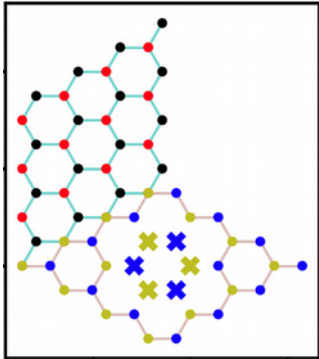
Hybrid TBG



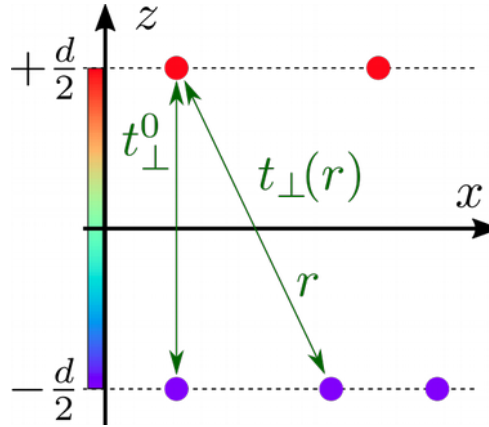
4x4, 60deg
No coupling



Hybrid TBG



4x4, 60deg
No coupling



$$H_{\text{inter}} = \sum_{i\uparrow, j\downarrow} t_{\perp}(\mathbf{r}_i - \mathbf{r}_j) c_i^{\dagger} c_j + \text{H.c.}$$

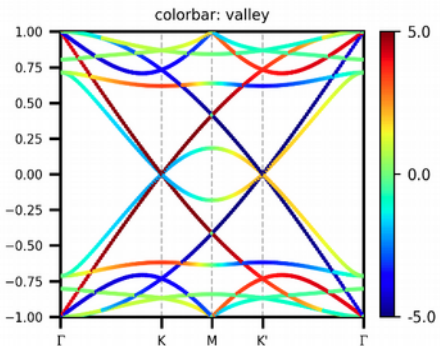
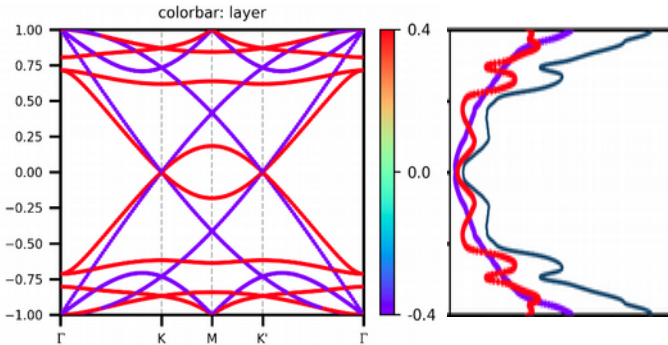
$$t_{\perp}(\mathbf{r}) = t_{\perp}^0 \frac{z^2}{r^2} e^{-(r-d)/l}$$

Sboychakov PRB2015
≈ only σ bonds

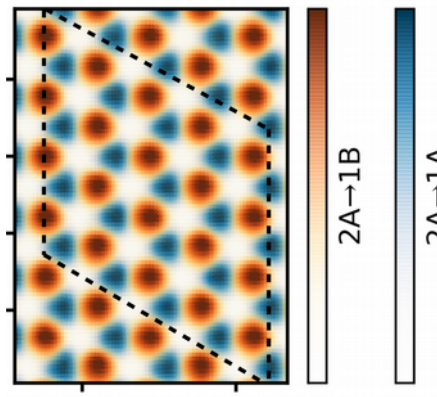
$$t_{\perp}^0 = 0.25t \quad d = 1.4a \quad l = 0.1a$$

(2x larger, angle 2x smaller)

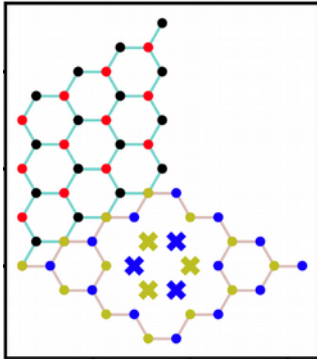
Gonzalez-Arraga, PRL2017



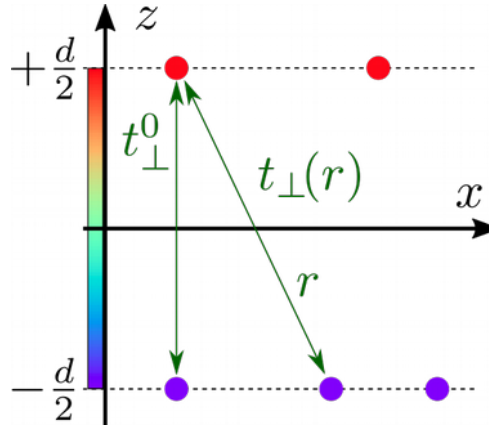
4x4, 7.3deg



Hybrid TBG



4x4, 60deg
No coupling



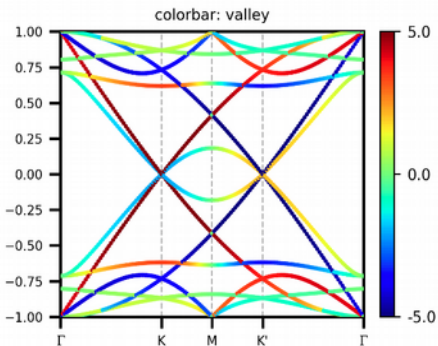
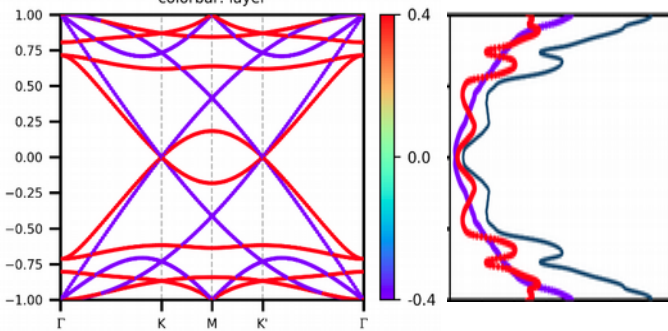
$$H_{\text{inter}} = \sum_{i\uparrow, j\downarrow} t_{\perp}(\mathbf{r}_i - \mathbf{r}_j) c_i^{\dagger} c_j + \text{H.c.}$$

$$t_{\perp}(\mathbf{r}) = t_{\perp}^0 \frac{z^2}{r^2} e^{-(r-d)/l}$$

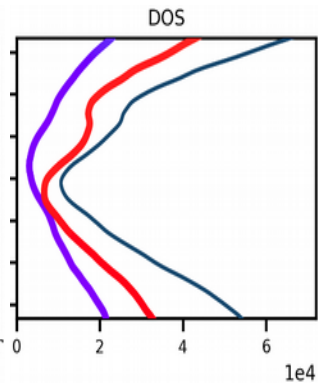
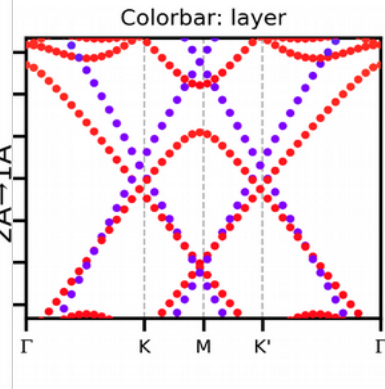
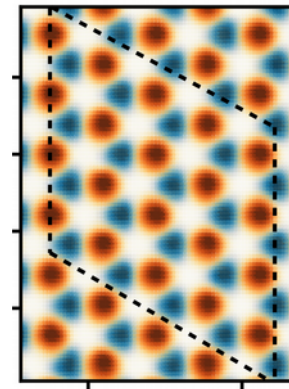
Sboychakov PRB2015
≈ only σ bonds

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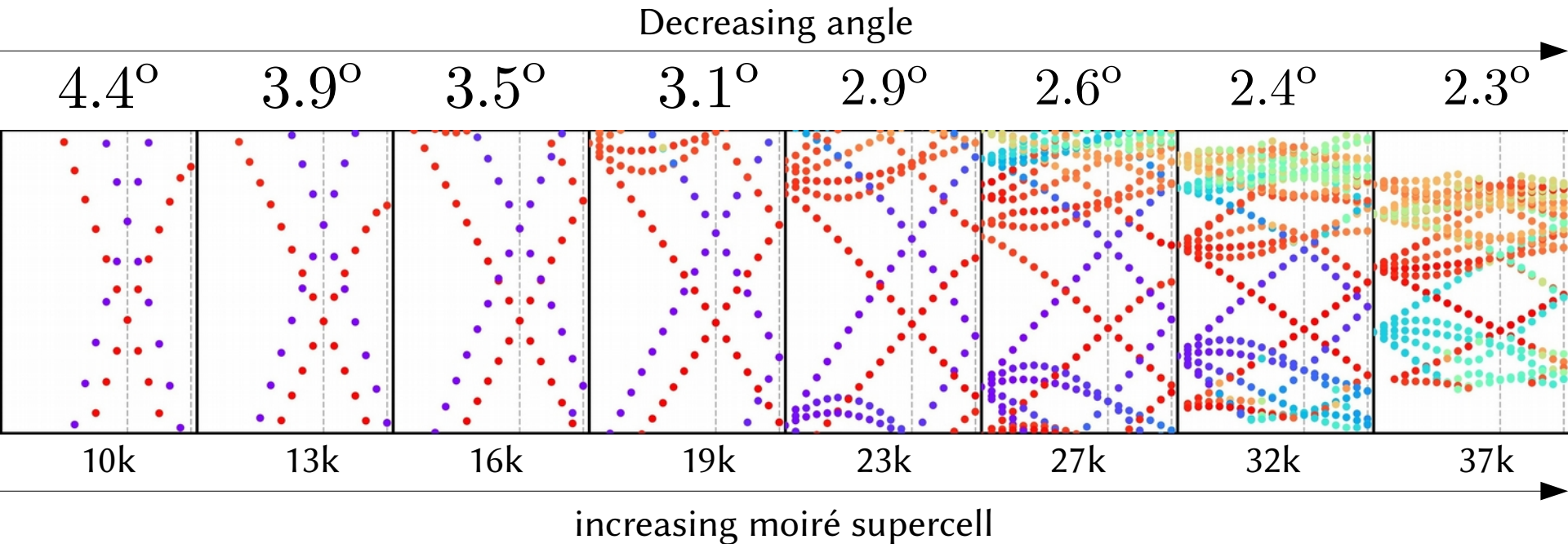


4x4, 7.3deg



- The E_n of the states in the upper layer is shifted down
- Their Fermi velocity is reduced
- Intervalley scattering only in the upper layer

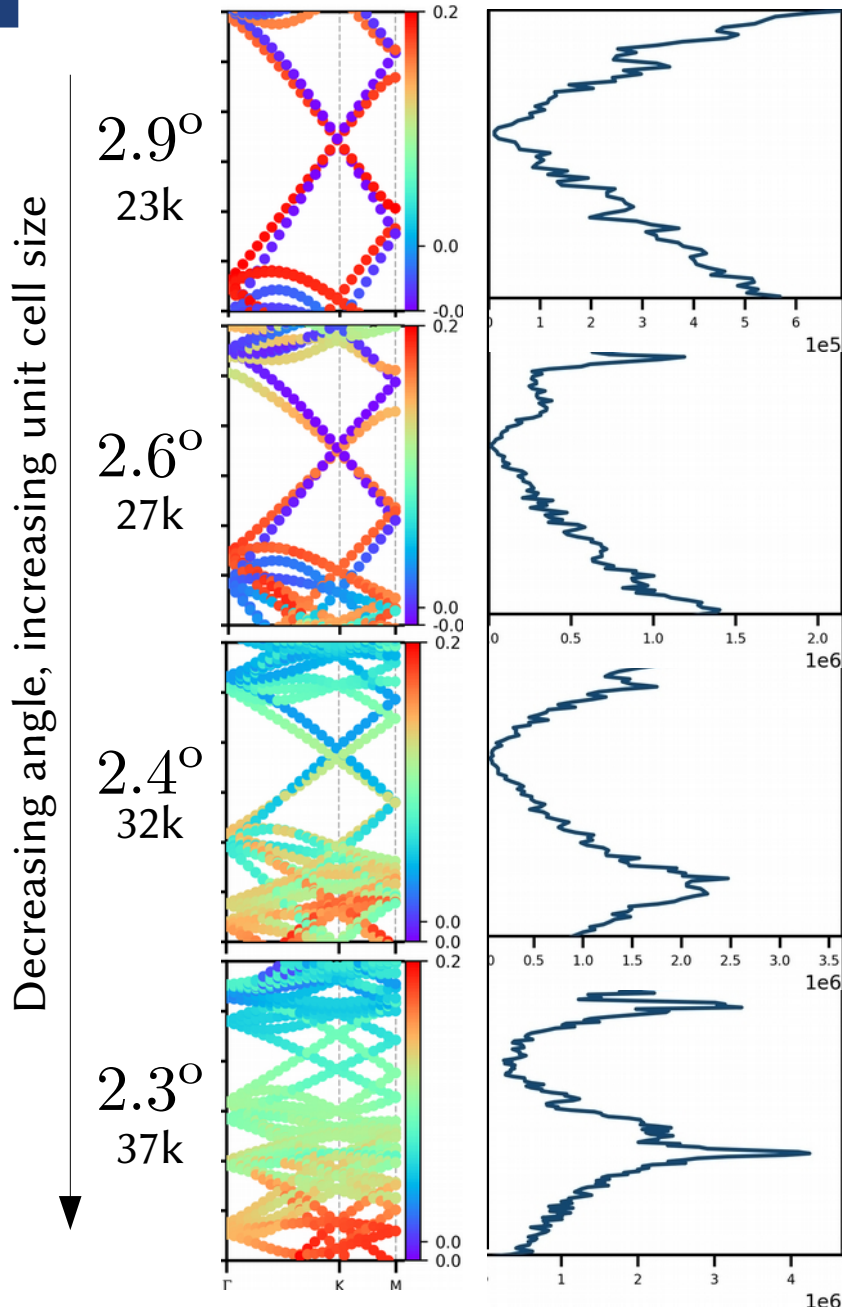
Small-angle hybrid TBG



The Energy difference between the cones is almost constant

→ related to the coupling strength and the vacancy concentration

→ the degeneracy of the cones can be restored using an electric field

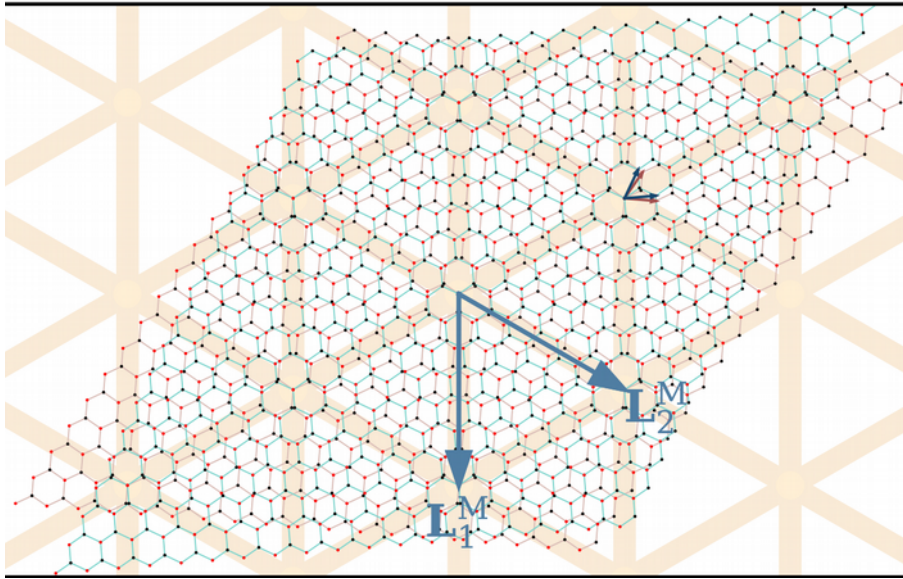


Degenerate cones?

$$\begin{aligned}
 H = & t_{\uparrow} \sum_{\langle i,j \rangle \in \uparrow} c_i^{\dagger} c_j + t_{\downarrow} \sum_{\langle i,j \rangle \in \downarrow} c_i^{\dagger} c_j \\
 & + \sum_{i \in \uparrow, j \in \downarrow} t_{\perp} (\mathbf{r}_i - \mathbf{r}_j) c_i^{\dagger} c_j \\
 & + E \sum_{i \in \{\uparrow, \downarrow\}} z_i c_i^{\dagger} c_i
 \end{aligned}$$

- Apply electric field E to restore the degeneracy of the Dirac points
- The states tend to localize in the upper layer as the interlayer coupling becomes stronger
- At small angles the Electric field is ineffective because of the weak layer polarization.

Toy model



2 honeycomb layers, no vacancies

- Displacement of the two pairs of Dirac cones using an electric field
- Reduced Fermi velocity using a smaller t in the upper layer

$$H = \overbrace{t_{\uparrow} \sum_{\langle i,j \rangle \in \uparrow} c_i^{\dagger} c_j + t_{\downarrow} \sum_{\langle i,j \rangle \in \downarrow} c_i^{\dagger} c_j}^{\text{Intra-layer hoppings}} + \overbrace{\sum_{i \in \uparrow, j \in \downarrow} t_{\perp} (\mathbf{r}_i - \mathbf{r}_j) c_i^{\dagger} c_j}_{\text{Inter-layer hopping}} + \overbrace{E \sum_{i \in \{\uparrow, \downarrow\}} z_i c_i^{\dagger} c_i}_{\text{Electric field}}$$

$t_{\uparrow} < t$ $t_{\uparrow} = t$

Bottom layer:
pristine graphene

Toy model: spectra

$$H = t_{\uparrow} \sum_{\langle i,j \rangle \in \uparrow} c_i^{\dagger} c_j + t_{\downarrow} \sum_{\langle i,j \rangle \in \downarrow} c_i^{\dagger} c_j + \sum_{i \in \uparrow, j \in \downarrow} t_{\perp} (\mathbf{r}_i - \mathbf{r}_j) c_i^{\dagger} c_j + E \sum_{i \in \{\uparrow, \downarrow\}} z_i c_i^{\dagger} c_i$$

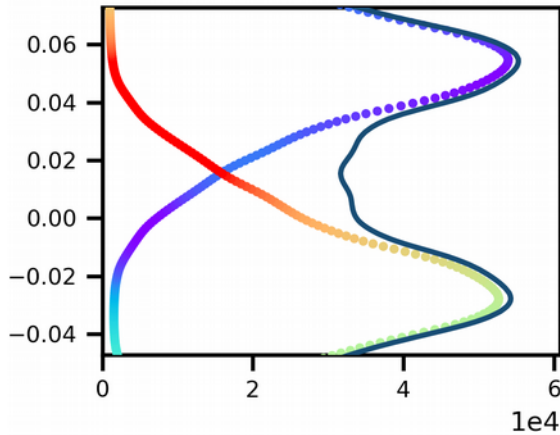
$$t_{\uparrow} = 0.65t$$

$$t_{\downarrow} = t$$

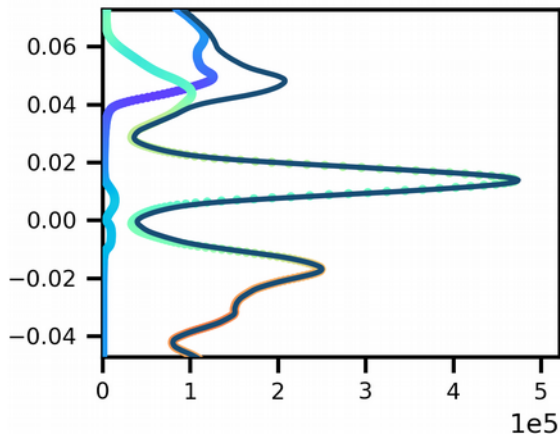
$$E = 0.19t$$

DOS per layer & total

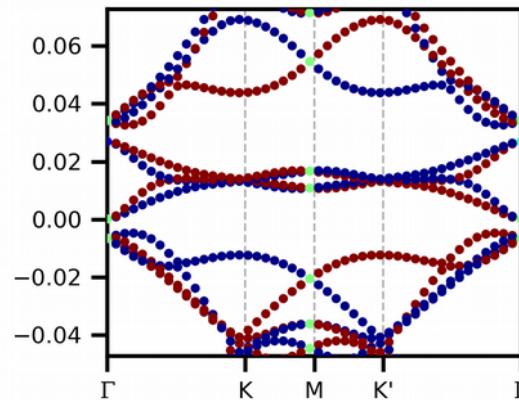
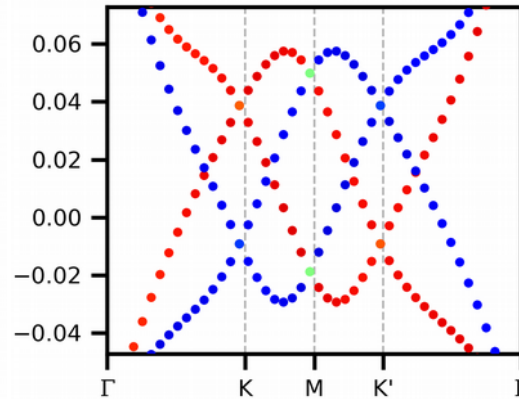
4.4 deg



2.0 deg



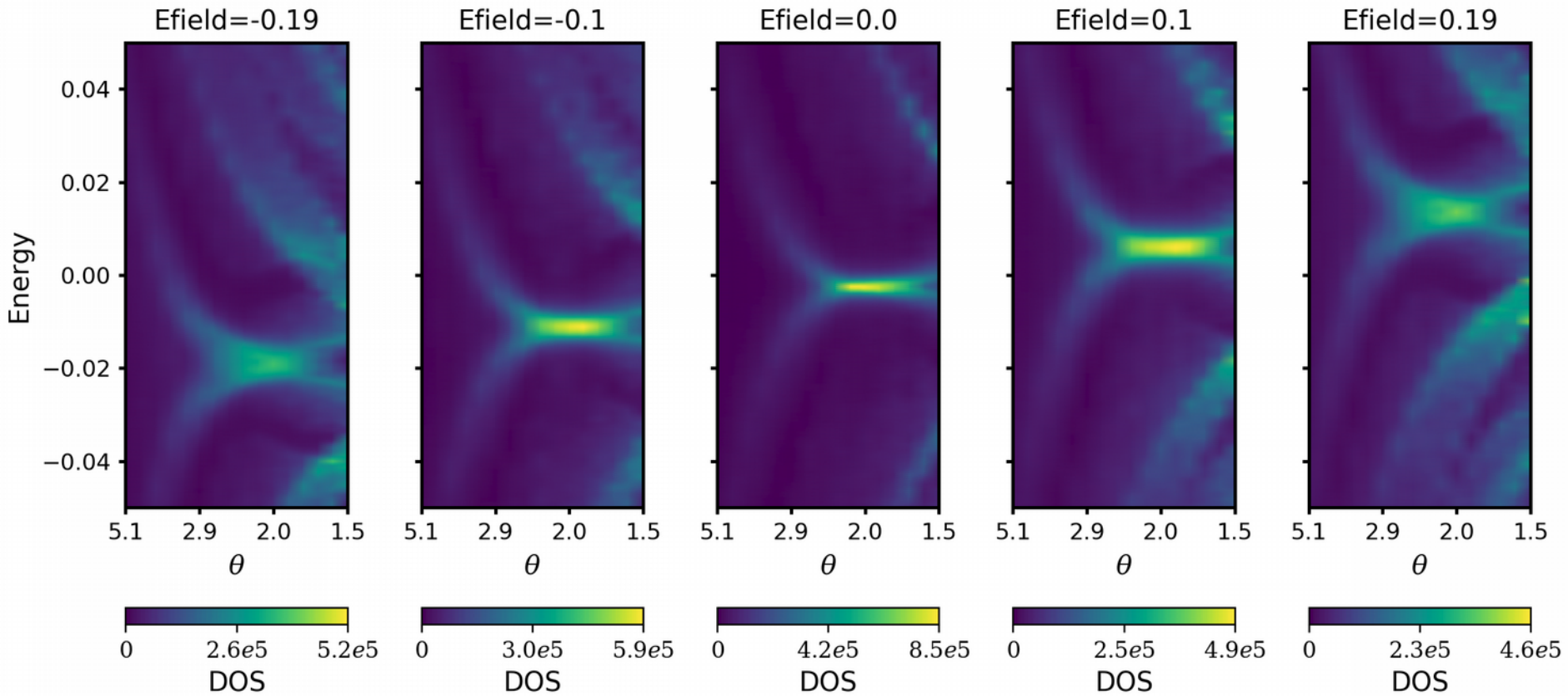
Bands, valley flavour



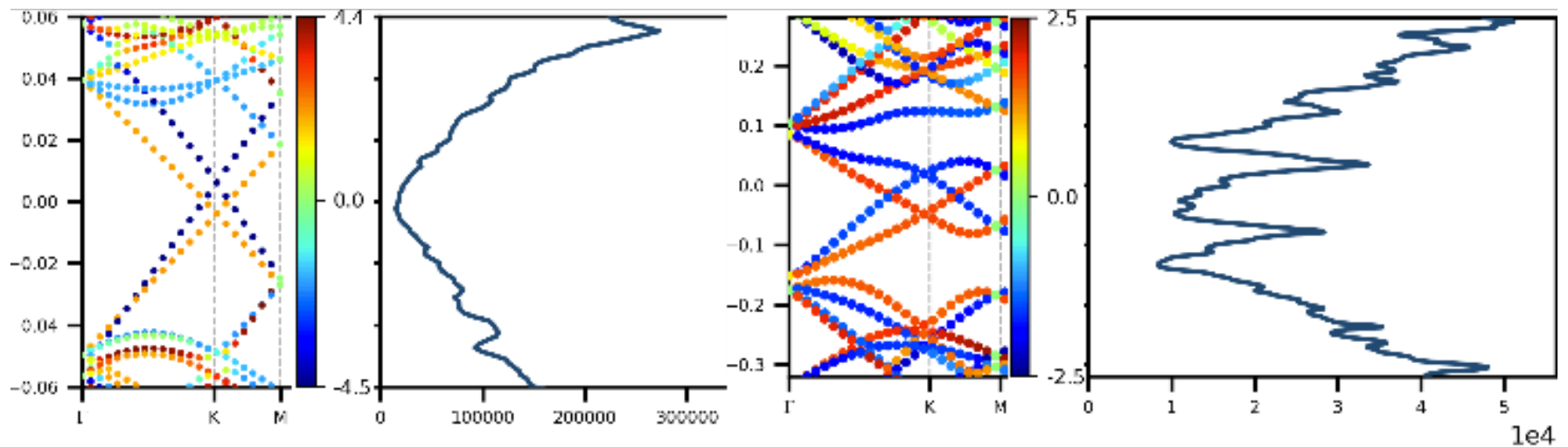
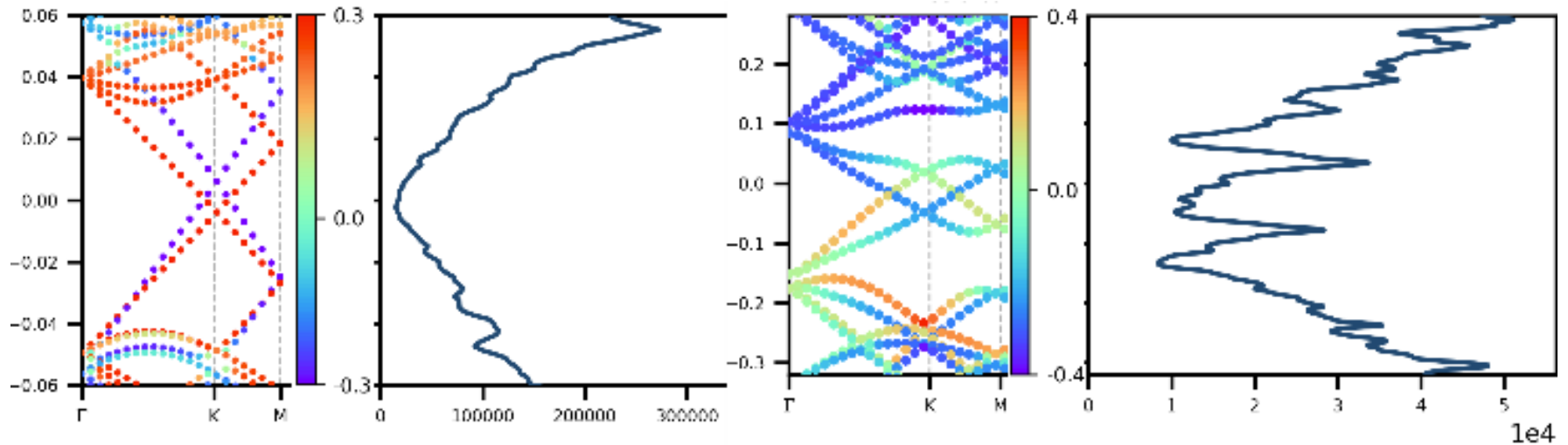
E acts like a valley-dependent anti-Haldane term.

Larger magic angle(s)

Toy model: DOS vs angle



Yet to understand: hybrid bilayer vs. toy model



Thank you for
your attention!

